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TIME AND AMPLITUDE DEPENDENT RESPONSE OF STRUCTURES

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SUMMARY

An understanding of the precise nature of the non-linear response of structures is essential for future improvement of earthquake resistant design procedures. This paper presents a summary of observations of dynamic behaviour which were made on two typical modern buildings during a period of about ten years. These structures underwent numerous tests and experienced three strong earthquake ground motions. The data presented should prove useful for calibration of parameters in theoretical non-linear models.

For buildings having an apparent soft-spring-type non-linearity, a partial or complete recovery of the structural stiffness appears to occur following the large strains created by strong ground shaking. The rate and extent of this recovery appear to depend strongly on the strain levels throughout the excitation.

INTRODUCTION

The main purpose of most experiments on full-scale structures is to find their dynamic properties which, in the majority of applications, can be expressed through the natural frequencies, modes of vibration and the corresponding equivalent viscous dampings. Because these properties determine the initial linear response of structures during strong earthquake ground shaking, strong winds and artificial blasts, much effort has been devoted in the past several years to the development of new techniques and the refinement of existing experimental methods for full-scale structural testing.[1-12]

A majority of the experimental procedures and methods of analysis in the full-scale testing of structures are developed around the assumption that the systems studied are time-invariant and linear, or that an equivalent time-invariant and linear system can be found. While it has been repeatedly demonstrated that these assumptions are quite adequate for many applications which involve small amplitude excitations, large excitations bring out the non-linear features of the system, and erroneous results may appear if the above assumptions are still applied. The tendency to use linear models in mathematical formulations of vibration problems has also been motivated by the fact that the tools of the theory of linear systems are readily available for use in the analysis.

A rather helpful aspect of the time-invariant linear system approach to structural vibration problems is the relative ease with which the system can be characterized (through its transfer function) by experimentally measuring the input to the system and its subsequent response. Though this approach may not be helpful in the actual design process, it might substantially aid in calculating the response of existing structures to ground shaking, if indeed the linear time-invariant criterion is satisfied. This is because the non-parametric approach (transfer function method) bypasses the lengthy and involved procedures in the system definition utilizing parameters that would otherwise have to be estimated from structural details, i.e. the formulation of a theoretical parametric model. Because of this important application it becomes essential to determine the amplitudes of vibration to which the linear time-invariant system approach can be carried out without introducing serious errors into the analysis.

If an experiment on a non-linear time-variant structural system is analysed by means of a 'transfer function' as though the system were a linear time-invariant one, several apparent effects will be introduced into the results. The 'natural frequency' peaks will become broader and less well defined. In some cases, especially for the higher modes, the peaks may branch into two or three. Consequently, damping estimates, derived from the shape of the frequency peak, will become too large and ill defined.[13]

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The purpose of this paper is to examine the quantitative aspects of the above-mentioned time and amplitude variations of structural characteristics by presenting the experimental results for two modern buildings. These structures were tested repeatedly over a span of several years by almost all experimental procedures currently in use. During the same period of time, three earthquakes were recorded in one building and two in the other, thus allowing comparisons to be made of the responses to small, intermediate and large excitations and the apparent system changes as a function of time.

One of the structures studied is a nine-storey reinforced concrete building (the Robert Millikan Library on the Campus of the California Institute of Technology in Pasadena, California). A forced vibration experiment was carried out on this structure by Kuroiwa[14] and was briefly summarized by Jennings and Kuroiwa.[15] The first ambient vibration test was performed in March 1967 and was reported by Blanford *et al.*[4] In April 1968 the Borrego Mountain earthquake occurred but was recorded only in the basement of the library.[16] Subsequent ambient vibration tests were carried out in April 1968[13] and July 1969.[13] The Lytle Creek earthquake[17] was recorded in September 1970 and the San Fernando earthquake[18] was recorded in February 1971. Shortly after the San Fernando earthquake the last complete ambient vibration test was carried out.[19]

The second structure studied in this paper is a ten-storey steel-frame building (Building 180 at the Jet Propulsion Laboratory of the California Institute of Technology, Pasadena). Forced vibration tests in this building were carried out from January to November 1963.[20] Three earthquakes (the Borrego Mountain in April 1968,[16] the Lytle Creek in September 1970[17] and the San Fernando in February 1971[18]) were recorded in the basement and on the roof. The only complete, two-dimensional ambient test of this structure was carried out in July 1971.[21]

Only a small number of modern buildings have so far been tested using the ambient and forced vibration methods. Prior to the San Fernando earthquake of February 1971 there were only a few measurements of large amplitude building response to strong earthquake ground motion. Since our understanding of the dynamics of full-scale structures can expand and improve only on the basis of many sound and complete measurements covering a wide amplitude and frequency range, it is clear that present vibration measuring techniques should be extended and that more detailed and complete instrumentation of typical structures should be developed.

EXPERIMENTAL PROCEDURES

Several methods of testing full-scale structures are currently being used. According to the desired level of excitation, the available instrumentation and the methods of analysis, the details of the selected procedures may vary considerably. In the following we briefly summarize some of the main characteristics of the experimental procedures which have been employed in testing the two structures studied in this paper. More detailed descriptions of these and other experimental methods can be found in the earthquake engineering literature.[22]

Microtremors and wind represent examples of naturally occurring excitations which are often used for full-scale structural testing. Microtremors and earthquakes excite the structural motions in the same way. The random trains of transient waves excited by various noise sources close to the surface of the earth are transmitted into the building through its foundation. Wind excitations, on the other hand, act on the whole building directly, thus leading to a time- and space-dependent forcing function. While wind excitations can lead to vibration amplitudes which are significantly larger than those excited by microtremors, both excitations are acting simultaneously and usually no effort is made to separate their effects. In analysing building vibrations caused by weak winds and microtremors, because the strains are small it is usually assumed that the materials respond linearly, and consequently that the principle of superposition applies. These assumptions then allow the system to be expressed as a linear combination of its modes of vibration. The experimental measurements are then analysed using the Fourier transform approach if the recorded vibrations are of a transient nature and the power spectrum method if they are assumed to be steady-state members of a stationary random process.

Forced vibration tests are carried out by means of one or more vibration generators which effectively exert a sinusoidal point force on the structure. By suitable arrangements of several vibration generators, it is possible to excite translational, torsional or two-dimensional modes of vibration.[20,21] The main characteristic of these tests is that the structure oscillates at the excitation frequency imposed by the vibration generator. This facilitates the data analysis and reduction since no Fourier transform or power spectrum techniques are called for as in the case of microtremors and wind excitations. However, forced vibration tests take significantly more time in the field since the structural characteristics have to be studied point by point in the selected frequency band.

Man-excited structural vibration testing is a simple and expedient method of finding information about the fundamental and only rarely the higher natural frequencies of a structure. For this type of testing, the operator moves his body back and forth in synchronism with the natural period of the system. This can be accomplished by his continuous observation of the visual display of the induced structural motions.[9]

Numerous other procedures have been developed and used for the testing of full-scale structures.[22,23] These methods involve different forms of excitation and a variety of data analysis procedures, but are all characterized by relatively small levels of the induced structural vibrations. It has been demonstrated that even at these small levels the structures may exhibit non-linear force deflection relationships.[12,14] However, the observed changes of natural frequency have usually been of the order of 10 per cent at most, there being no detectable irreversible change in the structural properties.

Strong earthquake ground motions represent one of the most powerful excitation sources for buildings. Though these strong motions represent a serious threat to human life, thus motivating us towards acquiring a better understanding of them by comprehensive instrumentation programs, records of structural vibrations generated by them also represent invaluable and unique sources of information for the detailed analysis of non-linear response.

LINEAR AND NON-LINEAR MODELS: METHODS OF ANALYSIS

In most ambient and forced vibration tests of buildings it is assumed that the structure can be idealized as a damped, linear, discrete or continuous system whose properties vary with reference to a line or a plane.[13] When the measurements indicate that the floor structures are sufficiently stiff, a one-dimensional line model may be acceptable.

A linear, time-invariant, structural system can be thought of as a device or a process which operates on a time history and changes it in some way. The most basic description of such a process may be given through the unit impulse function $h(\tau)$ and the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\,\mathrm{d}\tau \tag{1}$$

where we have assumed zero initial conditions; $x(t)$ is the input time history and $y(t)$ is the result of the action of the linear system on $x(t)$. Taking the Fourier transform of both sides of equation (1) and designating the Fourier transforms of $x(t), y(t)$ and $h(t)$ by $X(f)$, $Y(f)$ and $H(f)$ respectively, it is easy to show that

$$Y(f) = H(f)\, X(f) \tag{2}$$

Equation (2) is of great importance in the analysis of linear structures because it states that the Fourier transform of the output is equal to the product of the Fourier transform of the input and the Fourier transform of the unit impulse response $h(\tau)$. The latter transform is also often referred to as the transfer function of the system. The significance of this formulation lies in the fact that the structure can be represented by a 'black box' whose unit response function and therefore the output $y(t)$, for any given $x(t)$, can be determined without a detailed knowledge of the structural system.

For a system with non-linear characteristics superposition is no longer applicable, hence the concept of the 'transfer function' breaks down, and the simple relationship depicted in equation (1) becomes invalid. However, a 'moving window' Fourier analysis may be used to determine the 'apparent transfer function' of the 'equivalent linear system' that may characterize the structure in a short interval of time when non-linearities are small and the excitation amplitudes and frequency of excitation change slowly.

Moving window Fourier analysis is a commonly used tool in time series analysis. It is used to describe the changes in the frequency content of a signal, which are slow relative to the smallest frequency resolved by the Fourier transform, i.e. $2\pi/T$, where T is the length of the time window. Under these circumstances equations (1) and (2) may be used to determine the 'apparent transfer function' of the system which would correspond to the equivalent linear model that describes approximately the system response in a short interval of time T. This approach, of course, can serve only as an approximate tool for determining the equivalent linear system with slowly changing parameters, since the effects of the input before the time interval T, which are introduced into the observation window through the memory characteristics of the non-linear system, have been neglected. For the purpose of this approximate analysis we will neglect the effects caused by the memory characteristics of the system and will assume that the apparent frequencies of the system, for the time interval T, can be determined from the ratio of the output and input Fourier transforms as indicated by equation (2) for the same time interval T. We will use this approximate method to study the apparent frequency changes of building vibrations during strong earthquake ground motion.

TWO CASE STUDIES

Nine-storey reinforced concrete building

General description. The Robert Millikan Library on the Campus of the California Institute of Technology is a nine-storey reinforced concrete building with basement. Figure 1 shows a NS section, the typical floor plan and the overall dimensions. The structural system is characterized by the two shear walls which are designed to carry lateral loads in the NS direction and by the central core which houses the elevator shaft and provides resistance to EW loads. Other structural details, phases of construction and the properties of the underlying soil have been described in detail by Kuroiwa.[14]

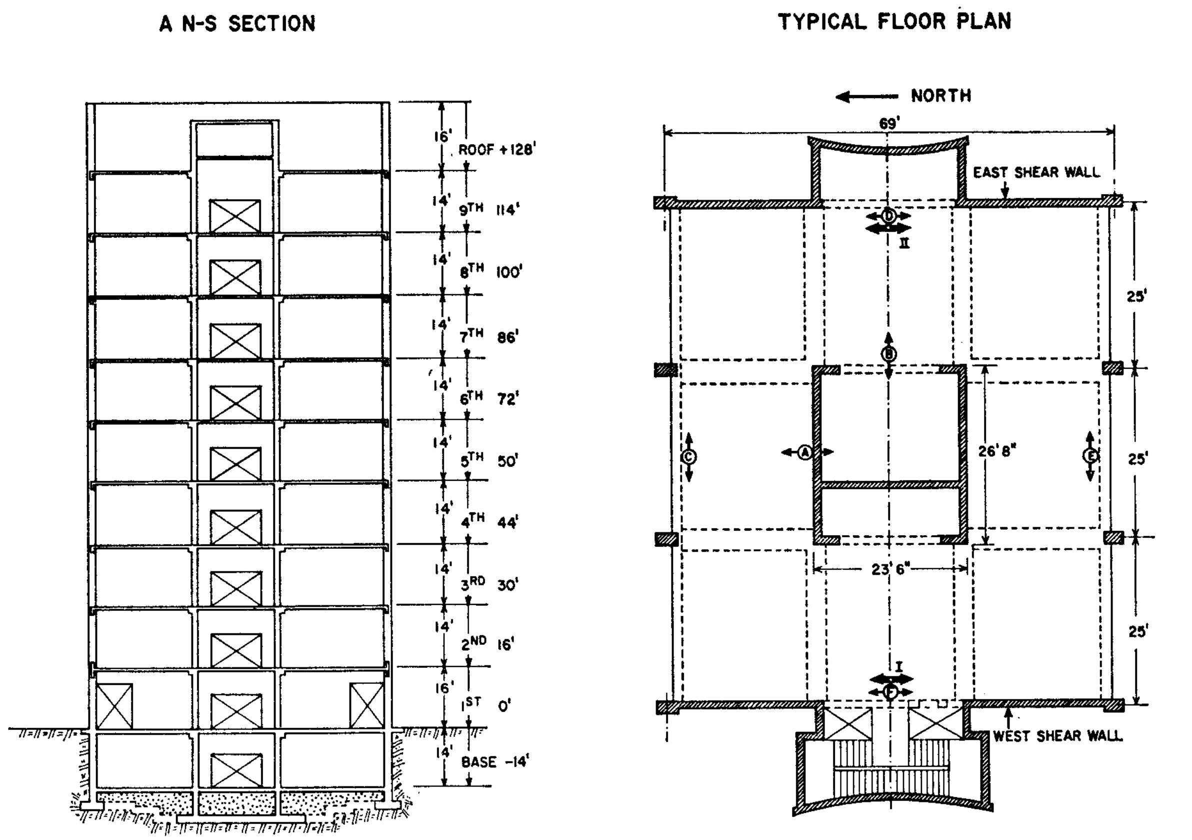

Figure 1. Millikan library building

Summary of measuring equipment, procedures and the different tests. Forced vibration tests of the Millikan Library were carried out during 1966–67 by Kuroiwa.[14] During these tests two shakers were mounted on the 9th floor at locations I and II which are shown in Figure 1. For the NS and EW translational experiments the shakers were synchronized to move in phase, while for the torsional excitation out-of-phase motion was used. Acceleration transducers were located in position A for NS and in position B for EW vibration tests (Figure 1). Positions C, D, E and F were used during the torsional experiments. Typical acceleration amplitudes during the forced vibration tests were of the order of 10^{-2} g. To test the linearity of response, Kuroiwa[14] used different force levels and found that in the response amplitude range between 3×10^{-2} cm and 15×10^{-2} cm the E–W fundamental soil-structure frequency changed by about 3 per cent. These changes could be interpreted by a softening-spring type of non-linearity.

Ambient vibration experiments on the Millikan Library were first carried out during March 1967.[4] The measuring system consisted of Ranger seismometers, a signal conditioner and an analogue magnetic tape recorder. The transducer locations coincided with those used by Kuroiwa[14] in the forced vibration experiment (Figure 1). Seven measuring stations for translational modes were located at the roof, 8th, 6th, 4th, 2nd and 1st floors and in the basement. One torsional experiment was conducted on the roof.

The other three ambient vibration tests were carried out during 1968,[13] 1969[13] and 1971.[19] The measuring equipment, the methods of measurement and the data analysis were essentially the same as those during the first experiment in 1967. Changes in the apparent frequency of building vibration prior to, during and after the San Fernando earthquake of 9 February 1971 and the possible incomplete recovery of the pre-earthquake stiffness of the soil-structure system motivated an additional man-excited test which was carried out during 1972.[13]

Comparison of ambient, forced and earthquake-excited vibrations. Figures 2 and 3 show the NS and EW transfer functions of the Millikan Library building for the excitations during the Lytle Creek earthquake of 1970[17] and the San Fernando earthquake.[18] These transfer functions have been computed from the Fourier transforms of the complete earthquake records obtained at the roof and basement of this building. For the San Fernando earthquake the figures show marked reductions of the apparent natural frequencies relative to values obtained from the ambient and forced vibration tests (shown by arrows in the figures). The same phenomenon is present to a lesser degree in the response to the Lytle Creek earthquake. The broad peaks of the transfer functions seem to represent the integrated effect of gradual changes in the structural natural frequencies for different levels of earthquake excitation. No significant change in the structural system following the Borrego Mountain earthquake of 1968[16] could be identified from the ambient vibration tests in 1969[13] (Figures 2 and 3). Vibrations during this earthquake could most probably be characterized by a transfer function similar to that for the Lytle Creek earthquake. However, following the San Fernando earthquake, ambient vibration tests indicated appreciable reductions of natural frequencies. This reduction for the first mode in the NS direction was 4·8 per cent, while for the softer EW direction the percentage reductions in the 1st and 2nd modes were 14 per cent and 9 per cent, respectively. Ambient vibration tests[19] after the San Fernando earthquake indicated a fundamental EW period of 0·80 sec (Figure 3). A smaller aftershock during March 1971[13] indicated this period to be about 0·75 sec, while a subsequent ambient test gave a period of 0·77 sec.[13] The man-excited ambient tests conducted during December 1971,[13] indicated a period of 0·73 sec.

Moving window Fourier analysis of earthquake-excited vibrations. Figures 4, 5, 6 and 7 present the moving window Fourier analysis for EW and NS vibrations of the Millikan Library during the Lytle Creek and San Fernando earthquakes. The transfer functions have been calculated for 8 sec window lengths of both EW and NS records starting from the beginning and displacing the window each time by 2 sec. The apparent natural frequencies were picked by hand and plotted versus the time corresponding to the middle of each window (Figure 4). The horizontal lines (Figure 4) indicate the natural frequencies as determined from various tests done on the structure at various times. The error bars show the maximum possible error in picking a frequency peak from the computed transfer functions. It is clear that this choice of the window duration permits detection of the changes in the system that occur over time intervals comparable to the window length T. Thus, for example, the short bursts of larger roof response amplitudes at 3, 7 and 15 sec in Figure 5 are too short for possible changes in the apparent frequency of vibration to be detected.

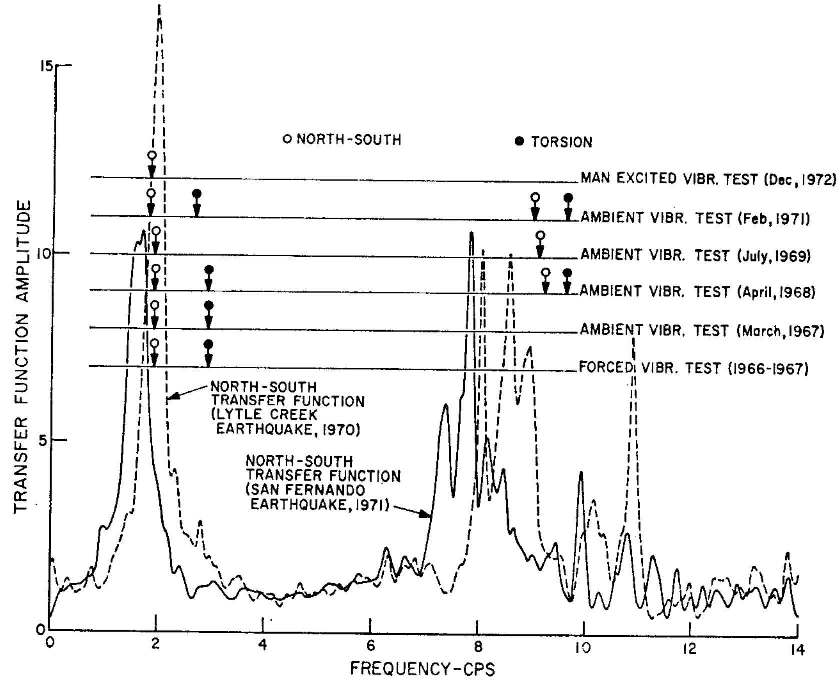

Figure 2. Earthquake, forced vibration and ambient vibration test data, Millikan library.

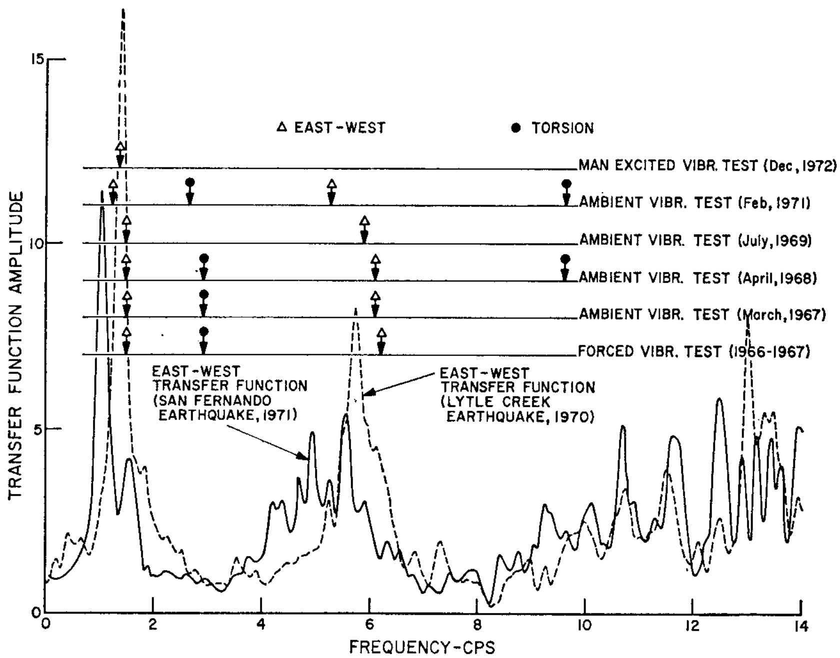

Figure 3. Earthquake, forced vibration and ambient vibration test data, Millikan library

The EW response (Figure 4) of the structure indicates systematic changes in the apparent frequency. A gradual increase in the fundamental frequency is observed up to about 12 sec, after which the original pre-earthquake frequency is recovered. The reason for this behaviour is attributable to the fact that only the tail end of the strong motion was recorded. The initial reduced value of the apparent frequency (as compared to the pre-earthquake values) during the interval of time when the roof accelerations were about 0·03 g, and its recovery when the roof accelerations decreased to less than 0·01 g, are clearly indicated in Figure 4.

The signal-to-noise ratio for the moving window Fourier analysis of the second NS mode of vibration (Figure 5) for the Lytle Creek data was low and the transfer function often indicated several peaks. Rather than delete the data, we selected up to three peaks and shaded the area between the peaks in Figure 5 to

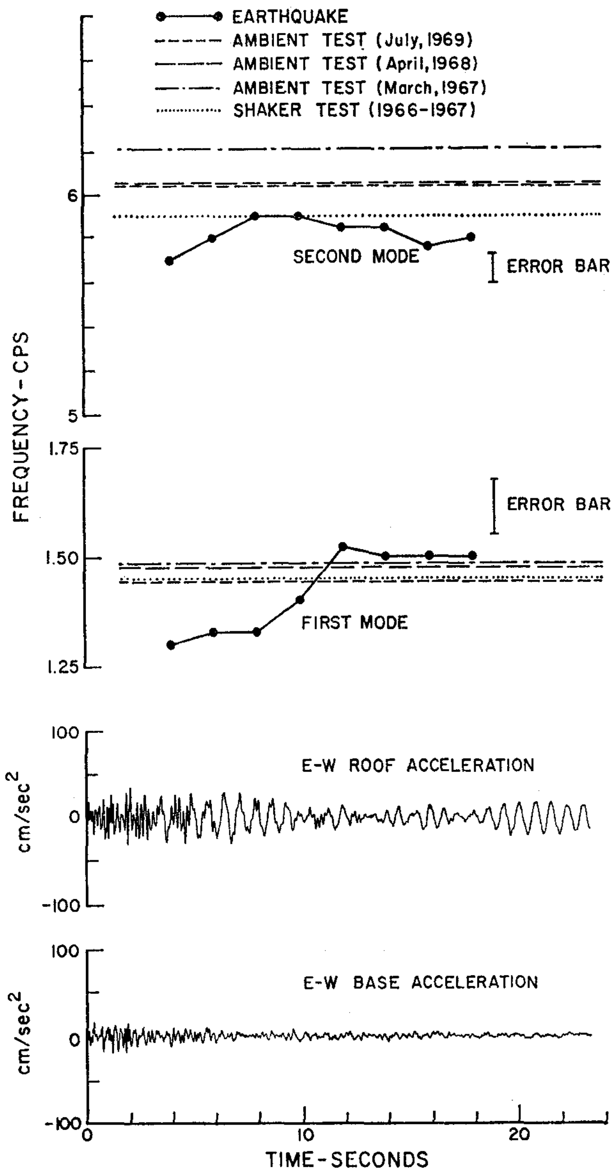

Figure 4. Frequency variation of first and second translational modes, Lytle Creek earthquake, 1970, data on Millikan library east–west response

indicate the approximate changes of this frequency. The purpose of doing this was to illustrate the complications and problems that can arise in using this rather simple heuristic approach to determining time-dependent frequency characteristics, especially in the presence of noise. The time changes of the fundamental NS frequency and its departures from the frequencies measured in ambient and forced vibration tests (Figure 5) are smaller than the possible errors in selecting the frequency of a peak from the computed transfer function and thus may not be significant.

The first 8 sec of the EW and NS motions during the San Fernando earthquake (Figures 6 and 7) show fundamental frequencies of 1·3 cps and 1·9 cps. These are essentially the same as those determined by the small amplitude ambient and shaker tests prior to the earthquake. However, with the arrival of large motions that resulted in a roof acceleration in excess of 0·2 g, the apparent fundamental frequency quickly reduced to about 1·0 cps for the EW (Figure 6) and 1·5 cps for the NS (Figure 7) motions. At about 16 sec the EW

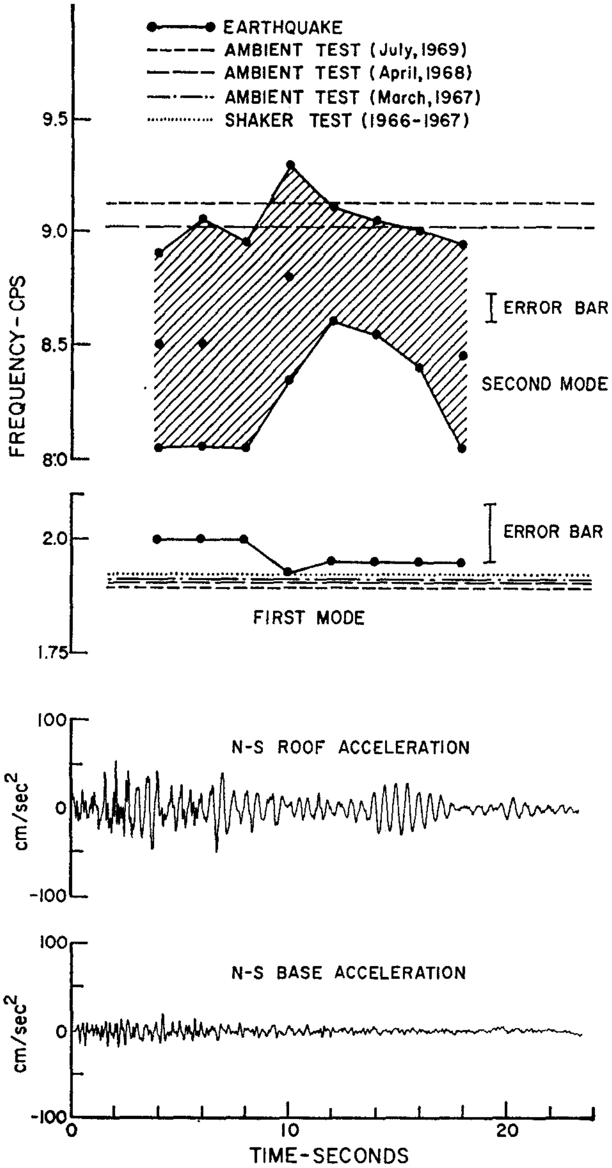

Figure 5. Frequency variation of first and second translational modes Lytle Creek earthquake, 1970, data on Millikan library north–south response

(Figure 6) building accelerations show much lower amplitudes and the transfer function at this time indicates a double peak, the higher frequency peak being more predominant. As the window shifts out of this low response zone, the apparent fundamental frequency drops again to 1 cps and remains there until the quiescent segment of response between 44 and 54 sec (Figure 6), when the frequency jumps back to 1·33 cps. Subsequent motions cause the apparent fundamental frequency to drop back to 1·0 cps after which a recovery to about 1·23 cps for the window centred at 80 sec occurs. A similar behaviour for the NS component of motion is shown for the first two modes in Figure 7. The bifurcation in the frequency peaks for the first mode is indicated by the break in the solid dotted line at around 18 sec in Figure 6 and at about 40 sec in Figure 7.

A comparison of the apparent fundamental frequencies determined during the large earthquake excitations (approximately 0·2 g), with the frequencies measured during the ambient and forced vibration tests, shows not only a frequency reduction of the order of 35 per cent, but also a partial or complete recovery following these large excitations. During the Lytle Creek earthquake (Figure 4) the 15 per cent reduction of the apparent EW fundamental frequency was completely recovered following the earthquake. During the San Fernando earthquake (Figure 6) the EW motion was characterized by the apparent fundamental frequency

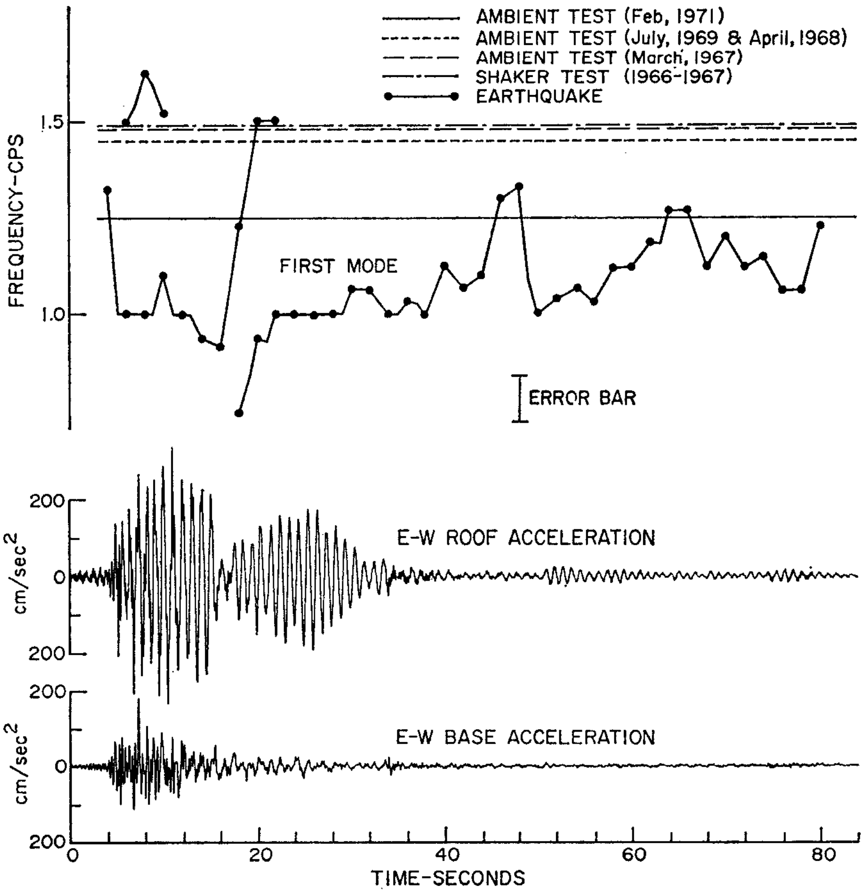

Figure 6. Frequency variation of lowest translational mode San Fernando earthquake, 1971, data on Millikan library east–west response

reduction of 50 per cent. A few weeks after the earthquake the frequency had partly recovered to a value which was about 18 per cent lower than the pre-earthquake frequency.[19] Twenty-two months after the San Fernando earthquake the man-excited test indicated that an additional apparent recovery had taken place to a value of about 12 per cent lower than the pre-earthquake frequency.[13]

Ten-storey steel frame building

General description. The Engineering Building 180, located at the site of the Jet Propulsion Laboratory of the California Institute of Technology in Pasadena, California, is a ten-storey symmetric steel-frame structure. The overall dimensions of the building and some construction details are shown in Figure 8 together with the typical floor plan and the NS, EW sections. The structural system is characterized by a steel frame and relatively rigid girders in the long EW direction (Figure 8). Other structural characteristics, construction details and soil properties have been discussed in detail by Nielsen.[20]

Summary of measuring equipment and procedures for the different tests. Forced- and man-excited vibration tests were carried out between January and November 1963[20] during the various stages of construction, and after the building had been completed. For the preliminary torsional and translational tests only one shaker was used on the 6th floor (Figure 8). It was located off the building centre to enable an adequate excitation of the torsional modes. The results obtained from the preliminary tests were used to find the optimal locations for two shakers (B and C in Figure 8) to be used in the final tests.[20] The detailed description of the measuring equipment, the results of the numerous tests and their interpretation have been given by Nielsen.[20] A linear computer model of this structure was studied by Wood,[24] while its non-linear hysteritic behaviour was deduced by Jennings.[25]

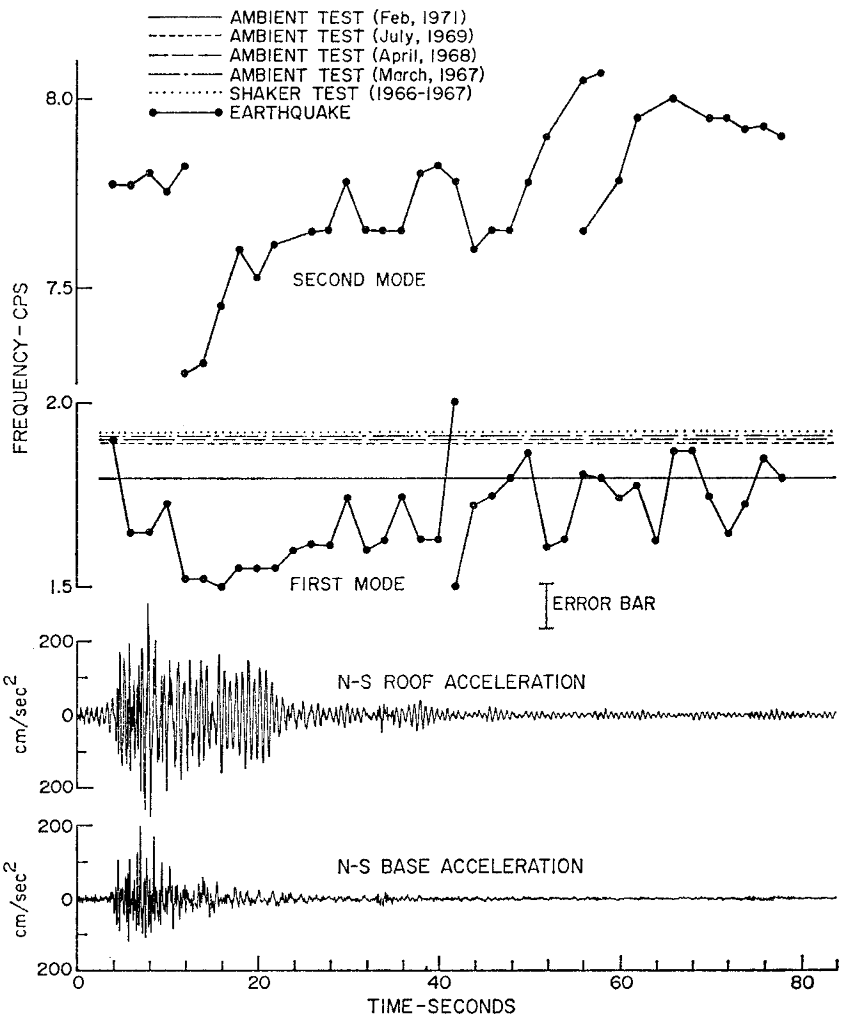

Figure 7. Frequency variation of first and second translational modes San Fernando earthquake, 1971, data on Millikan library north–south response

Ambient vibration tests on Building 180 were carried out in July 1971 following the San Fernando earthquake.[21] These tests revealed the detailed two-dimensional vibrational characteristics for the NS vibrations and clarified and augmented the information previously gathered during the forced vibration tests. The measurements were carried out by simultaneously using six Ranger seismometers, one signal conditioner and an analogue magnetic tape recorder. The vertical reference axis of the vibration mode shapes was a vertical line close to column 13 (Figure 8). The horizontal variations of modal amplitudes were measured at six levels (basement, 2nd, 4th, 6th and 8th floors and roof) near columns 2, 4, 6, 9, 11 and 13[21] (Figure 8).

Comparison of ambient, forced and earthquake-excited vibrations. The NS and EW transfer functions computed from the Fourier transforms of the complete available length of the strong-motion records are

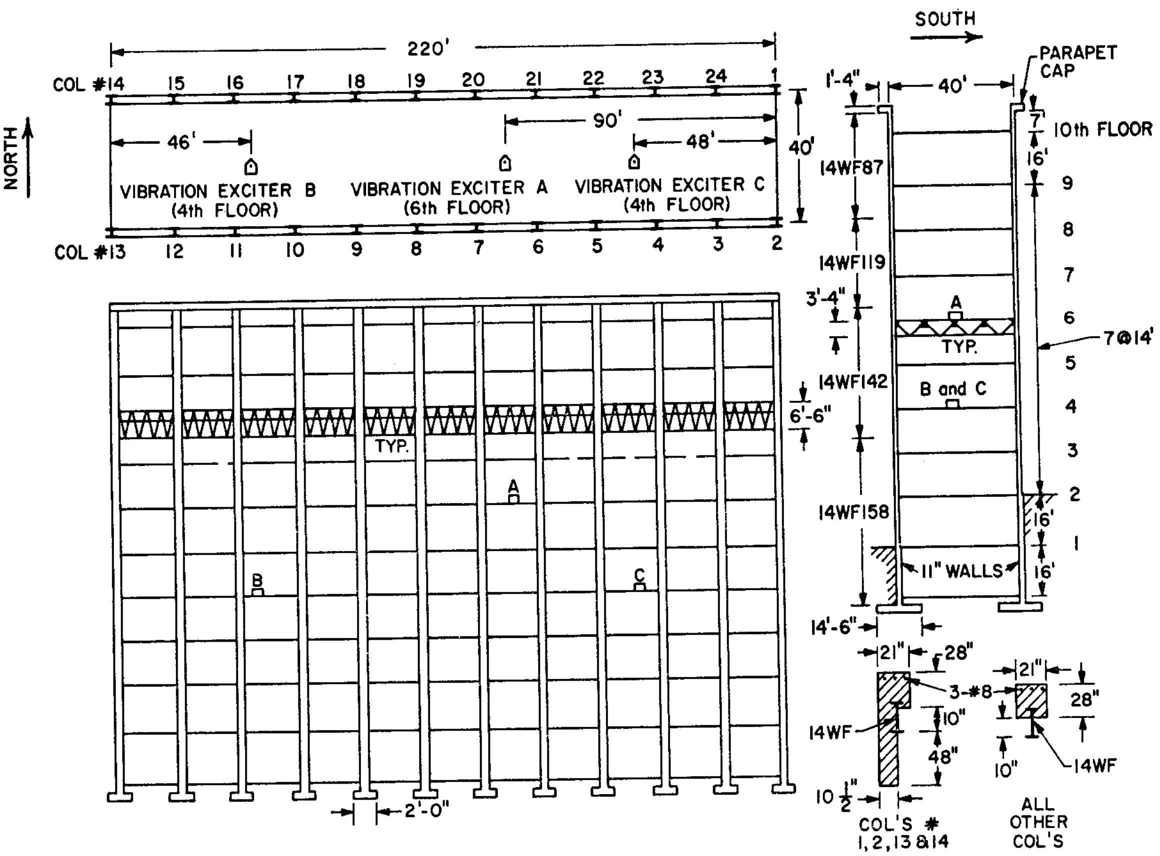

Figure 8. J.P.L. building

shown in Figures 9 and 10. These transfer functions are presented for the Borrego Mountain, the Lytle Creek and the San Fernando earthquakes. For the distant Borrego Mountain earthquake, recorded motions consisted predominantly of long period surface waves. Thus, the highly attenuated high frequency waves did not provide an adequate signal-to-noise ratio causing the transfer function for this earthquake (Figures 9 and 10) to show large unreliable amplitudes for frequencies higher than about 5 cps. As is shown in Figures 11–14, vibration amplitudes during the Borrego Mountain and Lytle Creek earthquakes were nearly the same. Consequently the apparent natural frequencies in the transfer functions in Figures 9 and 10 are essentially the same. The San Fernando earthquake generated about ten times larger vibration amplitudes resulting in the reduction of both NS and EW apparent natural frequencies (Figures 9 and 10).

Moving window Fourier analysis. Figures 11–16 show the results of the moving window Fourier analysis for the data from the three earthquakes. Again, the transfer functions have been calculated for 8 sec window lengths starting from the beginning and moving the window in steps of 2 sec each in the direction of increasing time. As for the data in Figures 6 and 7, the apparent peaks of the moving window transfer functions were picked by hand and plotted versus time corresponding to the middle of each window.

Figures 11–14 show the changes of apparent natural frequencies for the motions recorded during the Borrego Mountain and Lytle Creek earthquakes. Although these changes are only slightly larger than the errors involved in identifying the frequency peaks, they seem to follow the usual trend of decreasing apparent frequency with increasing excitation amplitudes. After these two earthquakes, the apparent frequencies essentially returned to those determined by the forced vibration tests.[20] During the San Fernando earthquake (Figures 15 and 16) the apparent fundamental frequency reduction in the EW and NS directions was about 15 per cent and 20 per cent respectively. Six months later the fundamental frequencies were only about 6 per cent lower than those determined by the forced vibration test in 1963. Similar changes were also indicated for the apparent frequencies of the second modes of vibration.

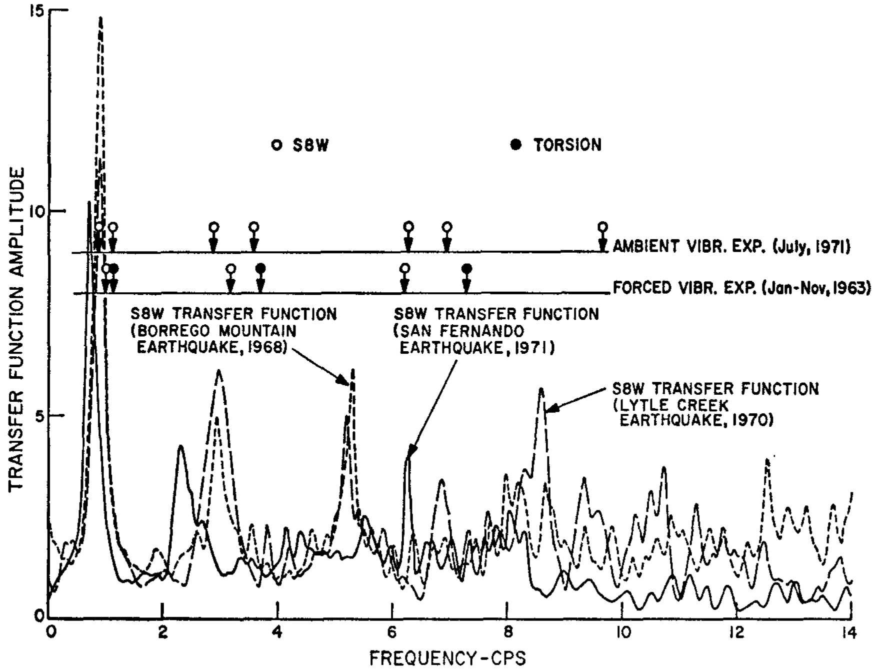

Figure 9. Earthquake, forced vibration and ambient vibration test data, J.P.L. building

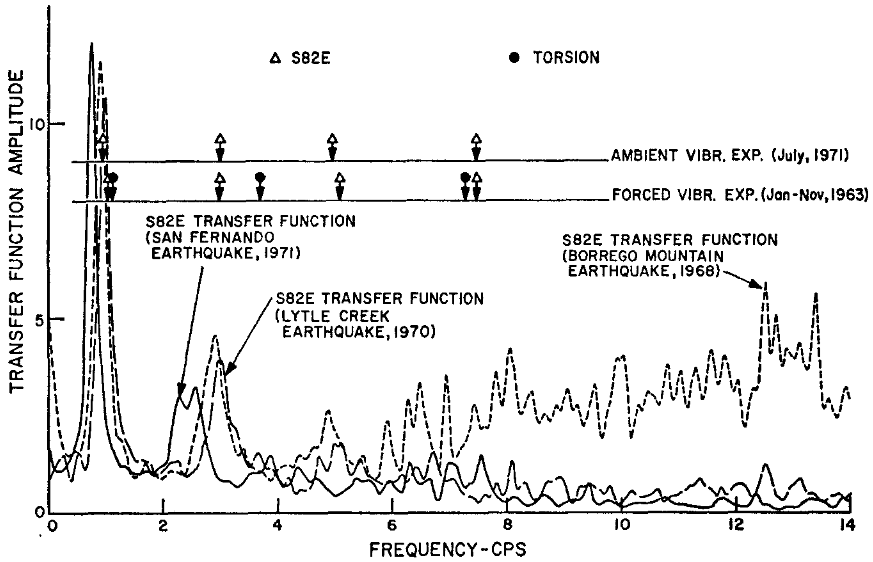

Figure 10. Earthquake, forced vibration and ambient vibration test data, J.P.L. building

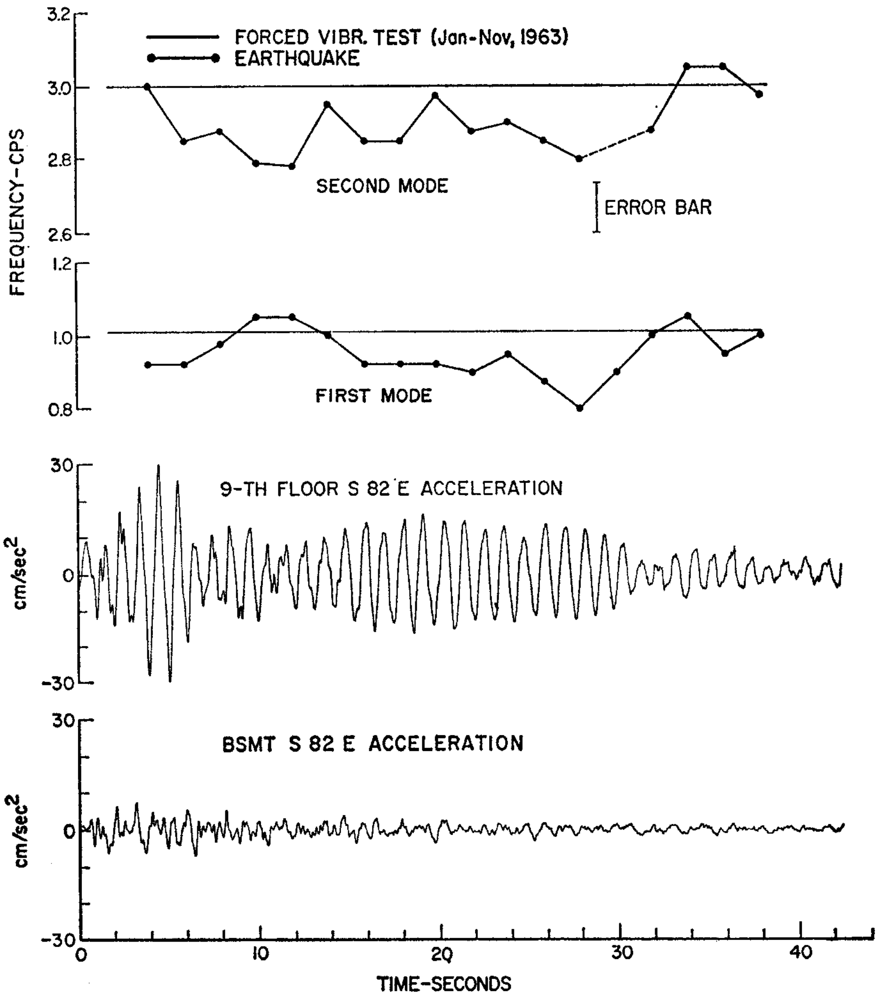

Figure 11. Frequency variation of first and second translational modes Borrego Mountain earthquake, 1968, data on J.P.L. building S 82 E response

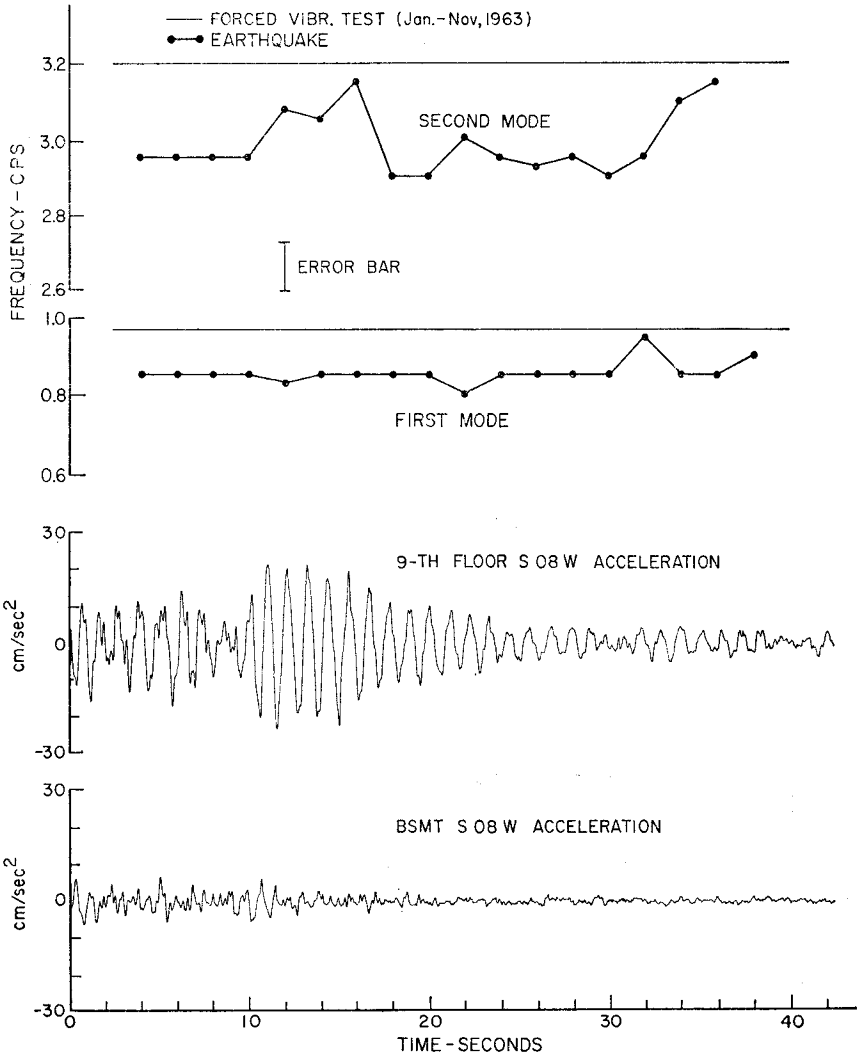

Figure 12. Frequency variation of first and second translational modes Borrego Mountain earthquake, 1968, data on J.P.L. building S 08 W response

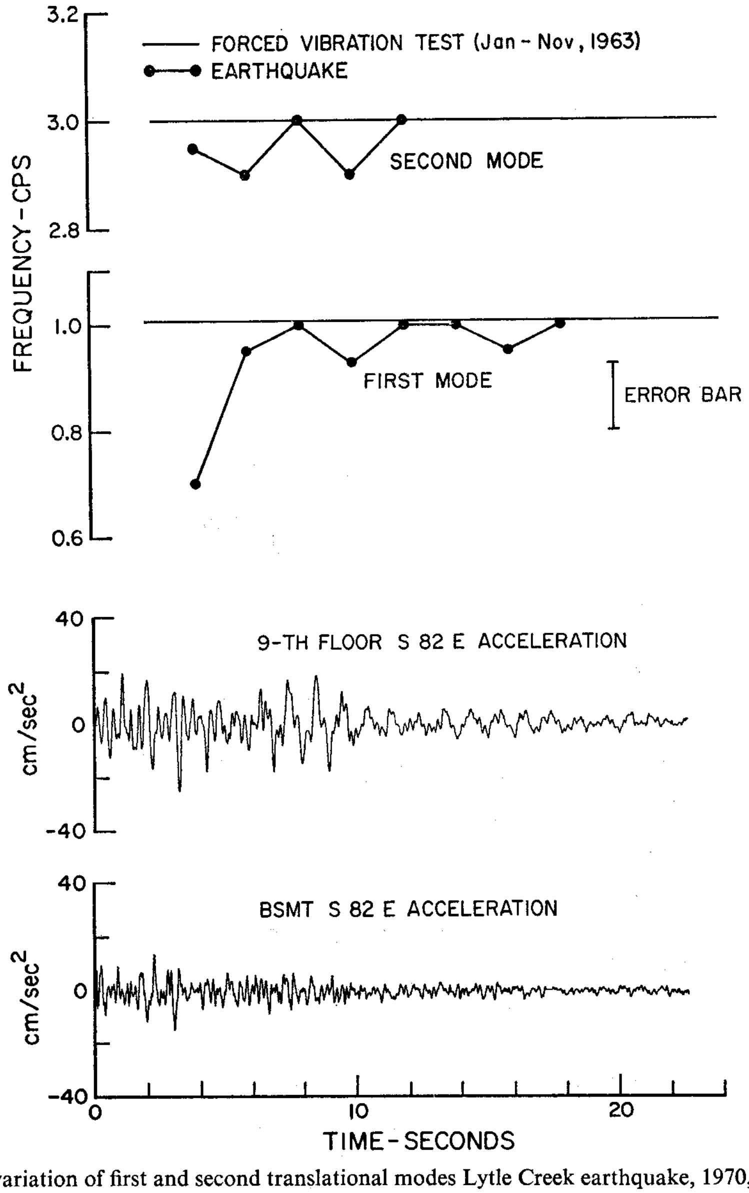

Figure 13. Frequency variation of first and second translational modes Lytle Creek earthquake, 1970, data on J.P.L. building S 82 E response

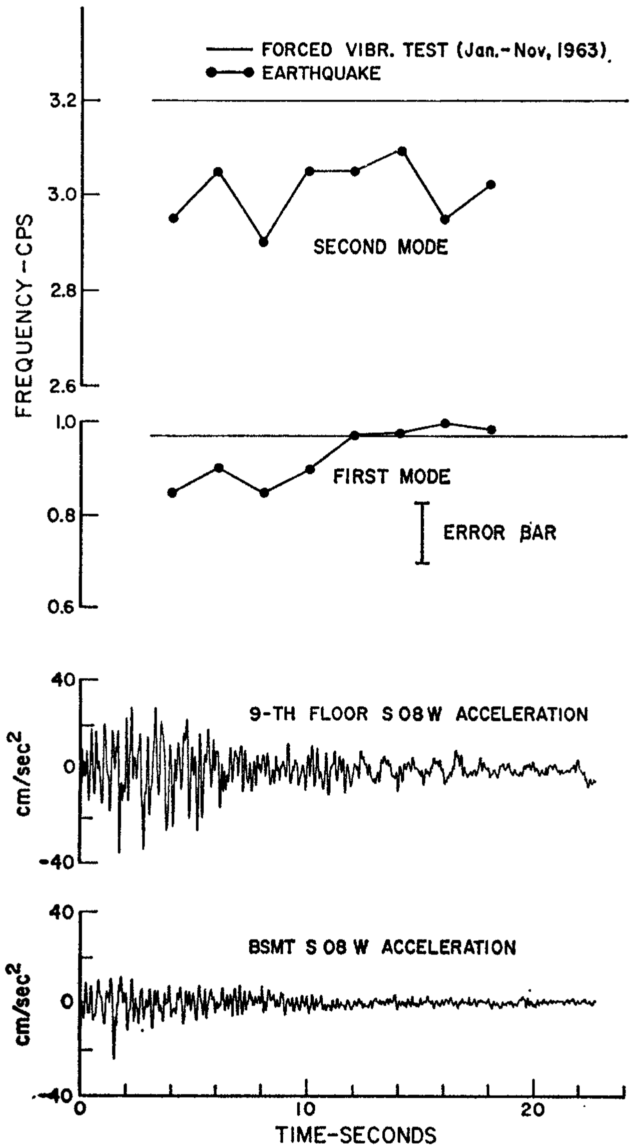

Figure 14. Frequency variation of first and second translational modes Lytle Creek earthquake, 1970, data on J.P.L. building S 08 W response

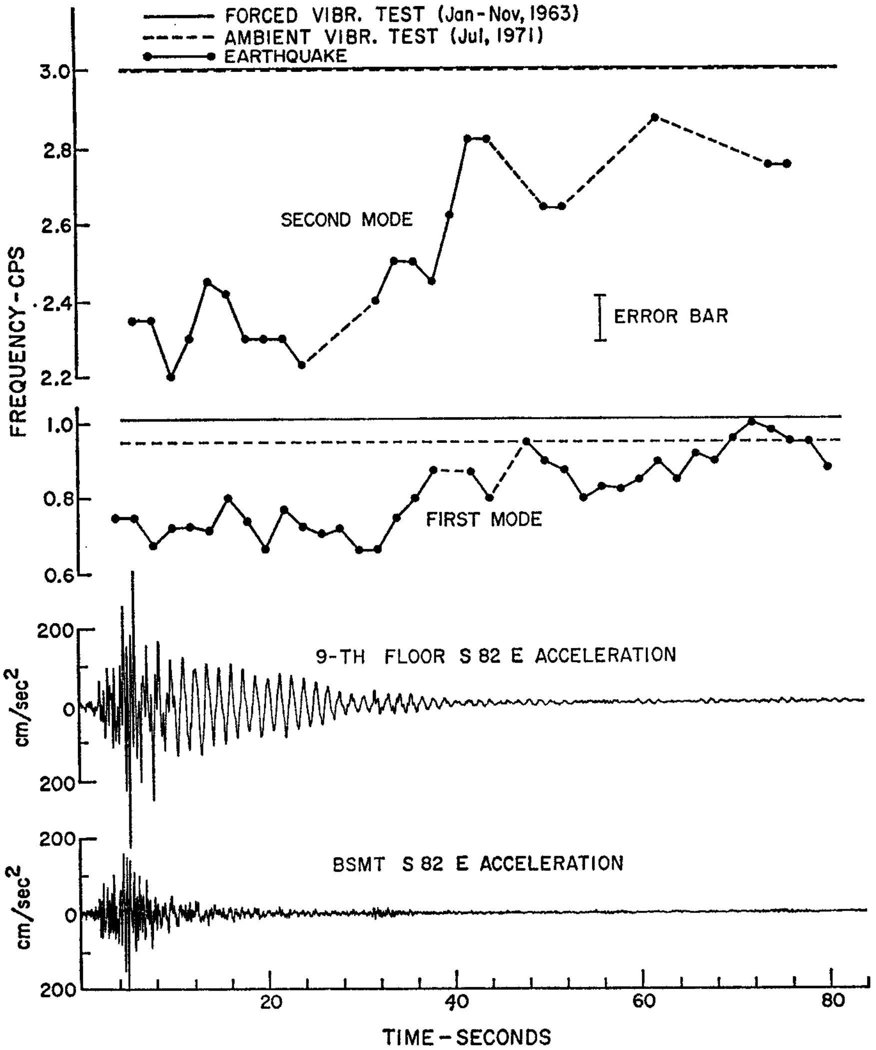

Figure 15. Frequency variation of first and second translational modes San Fernando earthquake, 1971, data on J.P.L. building S 82 E response

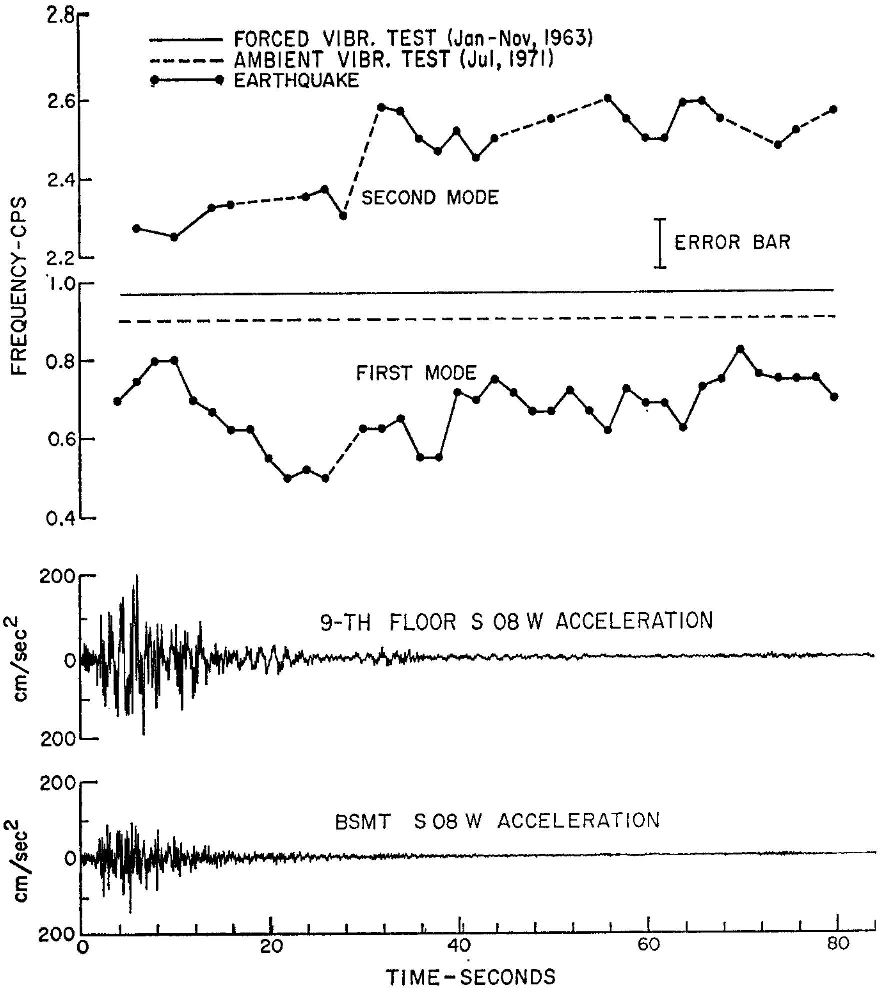

Figure 16. Frequency variation of first and second translational modes San Fernando earthquake, 1971, data on J.P.L. building S 08 W response

SIGNIFICANCE OF THE OBSERVED NON-LINEARITIES AND CONCLUSIONS

The foregoing summary of different tests on two typical modern buildings shows trends that may prove to be of value in assessing the potential damage of other similar buildings subjected to earthquakes, blasts or high wind loads. It has been known for some time that the natural frequencies determined from ambient vibration tests after an earthquake could be lower by as much as 10–20 per cent when compared with the measurements from the same or similar pre-earthquake tests.[26] This detailed study of two typical buildings shows, however, that these changes may be significantly larger than the reductions inferred from post-earthquake ambient vibration tests and that these changes are strongly time-dependent.

Two case studies, of course, are not sufficient to permit drawing any general conclusions at this time and many other experiments will have to be carried out before we begin to understand the full significance of the apparent frequency changes that accompany the large amplitude vibrations of structures. Nevertheless, the picture that emerges from the data presented in this paper is that reductions in the apparent fundamental frequency of vibration during moderate earthquake excitations may amount to as much as 50 per cent without being accompanied by observable damage. Following the earthquake there is an apparent recovery of structural stiffness. Whether this recovery is complete or only partial appears to depend on the amplitude levels of the response throughout the complete excitation sequence. This recovery appears to be almost instantaneous for small earthquake excitations which are characterized in this paper by the Borrego Mountain and Lytle

Creek earthquakes. However, following the San Fernando earthquake the apparent frequencies of the two buildings studied here may still be in the process of recovery. Although some structures undergo repairs after large earthquakes, thus causing the frequency to recover, in the two case studies illustrated here this does not seem to be a governing factor.

The type of frequency reduction described in the above two case studies may be due to the effects of soil-structure interaction, non-linear response of soils and/or the non-linear response of structural elements. The failure of non-structural elements during strong ground shaking (like the cracking of plaster, facade walls, etc.) could also be responsible for the reduction of the fundamental periods observed during strong ground shaking. Irrespective of the fact that we still do not know the precise causes for the observed reductions of the apparent frequencies of structural response to intermediate and large excitations, it is already possible to see the significance that these observations may have for accurate characterization of the energy absorption potential of structures. The apparent frequency changes may be thought of as being caused by a time-dependent non-linear hysteretic behaviour.

At the present time we have no detailed knowledge of the mechanism of 'frequency recovery' as observed in these two structures. Though several causes for such a behaviour can be proposed, a considerable amount of research needs to be done to establish the nature of the factors that primarily affect the extent of such a recovery.

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