TIME COMPLEXITY AND LEARNING

Elda Guala and Paolo Boero
Department of Mathematics, Genoa University
Via Dodecaneso 35 - 16146, GENOA, ITALY
guala@dima.unige.it, tel. +39-010-3536958, fax +39-10-3536752

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1. Introduction

The debate about the physical existence of time ([5], [6], [9]) suggests the possibility that time could also be considered as an intellectual construction in order to "treat" (that is, to describe/ order/ analyse) the flux of external events; in addition, it raises the problem of intellectual constructions suitable for "treating" the flux of internal events. On this point, we can speak about "mind times", metaphors which may help in "treating" mental processes, especially those intervening in complex problem solving. Bearing in mind our competencies (cognitive and epistemological aspects of teaching and learning mathematics), we will consider in a phenomenological manner the variety of "times" that the mind must manage in mathematical problem solving. We will also consider the intertwining amongst them, mentioning some examples ([3], [4]) in which success or failure seems to depend on the capacity to manage such time complexity. Finally, we will consider the hypothesis that the analysis of "mind times" may be useful (in an "embodied cognition" perspective) for singling out some mental processes on which basic mathematical ideas and skills are founded.

2. Phenomenology of some mind times occurring in the mental dynamics of problem solving

Our aim is to focus on some "time components" of "mental dynamics" which are relevant to mathematical problem solving activities. By "mental dynamics" we mean the mental processes of creation, exploration, transformation of space-time environments (in a proper or metaphoric sense). As to the focused "time components", they seem to be appropriate for describing some important mental activities and/or interpreting their misfunctioning.

Let us consider the following examples of times (the list is not exhaustive):

a) "time of past experience" traced by means of memory tracks and supported by the ordering activity the mind performs on those tracks. How tracing works (quickly or slowly, in detail or not) can depend on the involuntary perception of the quality of an event (linked to the subject's experience and its emotional intensity) and/or on the voluntary reconstruction of past experience (performed by skipping episodes irrelevant to investigation and zooming in on relevant aspects). However we may remark that interesting hints may emerge from episodes considered irrelevant. In our opinion, both polarities (involuntary and voluntary) are important: time perception probably develops out of their dialectic relationship;

b) "contemporaneity times": we can distinguish how the observer records the subject's observable behaviours on the time line; and how the subject experiences contemporaneity while is involved in a situation. This time can be evaluated by the subject (with estimations slowed down by wishes and accelerated by fears), who may graft on it virtual deplacements towards the future or past. We may note that these two contemporaneity times interact with each other (perhaps, it would be better to consider only one time of the observer-observer couple), and in particular intertwine during auto-observation;

c) "exploration times" in open-ended tasks requiring the subject to find and concatenate suitable arithmetic operations, plan a geometric construction, build up a proof, etc.; time projections can be realised from the past onward ("How will he have gone about solving ...") or in the future and then towards the past ("I think up a solution and explore it in order to find the operations to perform, depending on available resources");

d) "synchronous connection time" as a perception of coordinated functioning of the components of the real or virtual system under scrutiny, or as a discovery of the links among the variables of the problem; this can graft onto c) as the solving moment. We may note that synchronous connection is nowadays a fascinating subject of investigation from different perspectives - including that of neuronal biology ([10]).

For a person, the borders between one time and another are not clear, and intertwining and grafting of one time onto another are possible (see above; see also Sections 3 and 4).

One interesting research problem concerns the relationships between these "inner times" and the "external" chronological time, whose existence is subject to much debate ([6] and [8]).

3. Mind times and difficulties in problem solving

This section refers to episodes and situations taken from students' work in classes that are involved in the Genoa Group Projects for primary and lower-secondary school. These projects include diagnostic and remedial activities for logic-linguistic and mathematical skills.

As concerns diagnosis and remedial work on some pupils' difficulties in the logic-linguistics and mathematics areas, "mind times" offer interesting interpretative keys ([3], [4]). For instance, some pupils still fail at grade IV level in subtraction problems where no easy, concrete analogic model (e.g. thermometer, ruler, etc.) is available: particularly, in the mental calculation of "How much is needed to make...?" they do not succeed in coordinating (d) the onward counting process while underway and (b) control of its ending. Another example concerns the task of "drawing a small 23 cm plant with a 20 cm ruler" set for grade II pupils who are able to perform measurement within the length of the ruler. Some pupils are unable to project themselves into the time of the solved problem (see the drawn plant: c) in order to discover the operations to perform with the ruler. The teacher's intervention on the exploration time (c) ("with your ruler, extend the 20 cm segment already drawn") together with the pupil's intuition (d) in his contemporaneity time (b) ("I see 20 and 23 written close together... I must add 3!") may allow the pupil to solve the problem.

We studied many cases like those reported above; frequently by taking into account collected information about the pupils' personalities and comparing their performances in different tasks, we arrived at the hypothesis that the pupils' difficulties depended less on cognitive resources than on serious problems related to the affective and emotional sphere (lack of hope for the future, lack of trust in themselves and/or in other people). In some cases positive changes were activated through the teacher's coordinated, repeated interventions in pupils' past experience times (a: reconstruction of experienced situations - also including emotional involvement - and discussion about the difficulties experienced in the process) and in exploration times (c) grafted on contemporaneity time (b) (immersion in present reality, acquiring a sense of security in moving backwards and forwards during the exploration of virtualities). In these cases a possibility of self-regulation of learning processes seemed to emerge gradually in students' behaviours ([9]).

4. Mind times and embodied cognition

From an embodied cognition perspective [7], particular body experiences are "grounding metaphors" for important basic mathematics concepts.

Let us consider a gear system of coplanar wheels and analyse primary school pupils' behaviour, partially described in [1]; we will try to find embodied mathematical contents and skills. The pupil's impulse starts the movement of one wheel and propagates through the gear, provoking clockwise and anticlockwise rotations (d). We can split the pupil's perceptions into touch and view senses, and formalise the chain of impulses, which the pupil perceives through touch as a causal chain, by means of differential relations Fdt=Rdv, where F indicates the physical effort and R the impression of inertia of the system; dt stands for the duration of the impulse, and dv the corresponding increase in rotation speed. When the pupil pays visual attention to rotation directions, he/she discovers alternance and may represent it by two different colors or letters; we may represent the direction of rotation of the K-th wheel in an algebraic manner, by the formula: (-1)K-1 (initial wheel: K=1).

When started, the movement can be observed in the contemporaneity time (b). The system is harmonious, we might say Ptolemaic. Pupils discover differences in speeds and tempos, then count the number N of turns and grasp (d) the relationship with wheel diameters D. We know that by multiplying ratios we get the angular speed VK of the K-th wheel: VK/V1=NK/N1=D1/DK. Proportion theory is the theoretical reference for the pupils' intuitions derived from their time experiences.

As another example, let us consider the production of a conjecture about the possibility that two non-parallel sticks produce parallel shadows [3]. Many VIII-grade students imagine possible movements of the sun or perform/imagine possible movements of the sticks until stating "when sunrays...", a sentence which then becomes, in the statement expressing the conjecture, "if sunrays...". The conditionality of the statement is generated as a time section (d) in a process of dynamic exploration (c) in which the movement of the sun is accelerated, or virtual (or real) stick movements are performed by stopping the sun.

We believe that the analysis outlined above may lead to interesting developments for:

I) helping teachers find reference situations for concept construction; here we refer to Vergnaud's definition of a concept ([11]) as "reference situations", "operational invariants" and "linguistic representation tools" related to that concept;

II) interpreting mathematical behaviour when (in difficult problem situations) we feel the need for heuristics related to "concrete" referents;

III) suggesting further investigations about "embodiment" of important mathematical skills. For instance, the example concerning the genesis of conditionality of statements indicates only one possible root for conditionality. In [2] another root is indicated for another statement. What are the roots of conditionality? Is it possible to characterize them in terms of mind times?

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