Time-Consistent Majority Rules and Heterogenous Preferences in Group Decision-Making

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Abstract

This paper studies a collective decision problem in which a group of individuals with interdependent preferences vote whether or not to implement a public project of unknown value. A utilitarian social planner aggregates these votes according to a majority rule; but, unlike what is commonly assumed in the literature, the planner is unable to commit to the rule before votes are cast. Characterizing the time-consistent majority rules, we find that the ex ante optimal majority rule is time-consistent; but for groups whose members have sufficiently homogenous preferences, there is an ex ante suboptimal rule that is also time-consistent. Thus, in the absence of an ex ante commitment, the social planner prefers a relatively heterogeneous group in which strategic voting incentives are weak. This finding is in sharp contrast with the observation that under an exogenously given majority rule, the social planner prefers the most homogenous group. Applications to trial jury and advisory committee formations as well as academic hiring decisions are discussed.

JEL Classifications: C7, D7

Keywords: time-consistency; majority rule; heterogeneity, group decision-making

1 Introduction

Many collective decisions are made by a group of individuals with conflicting or heterogenous preferences: even if all the information about an alternative were publicly available, individuals would not unanimously agree whether or not it should replace the status quo.

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In this paper, we argue that it may nonetheless be in a group's best interest to have members with heterogenous preferences because such members vote less strategically, enabling a utilitarian social planner to credibly commit to the (ex ante) optimal voting rule. Thus, at the heart of our argument lies a potential time-consistency problem associated with the decision rule to aggregate votes.

Examples abound in which the issue of time-consistency of voting rule could be paramount – either because one is not announced to voting individuals, or because the announced rule is not binding. For instance, in an academic hiring or promotion case, the faculty members in the relevant department often submit confidential yes/no votes about the case, without exactly knowing the administration's voting rule. Similarly, in the publication process, a set of independent referees express their opinions as to the publication, without being told of the editor's aggregation process. Finally, as with many advisory committees, when the members of a Food and Drug Administration (FDA) advisory committee evaluate a new drug application, they commonly take a simultaneous approve/disapprove vote, without fully knowing the FDA's acceptance standard. The voting procedures in these examples contrast with those in trial juries and congressional committees, where individuals are informed of the majority rule in advance of casting their votes.[1]

Nevertheless, it is conceivable that even in the absence of a publicly announced voting rule, rational agents will form rational expectations about the rule and tailor their voting strategies accordingly. Our main objective in this paper is to determine equilibrium or *time-consistent* majority rules that the social planner can credibly adopt.[2] Our formal model is a modified Condorcet Jury setup. A group of n agents vote whether or not to implement a public project of unknown value. Before casting his binary vote, each agent costlessly receives an independent private signal about the project,[3] but his valuation also depends on others' signals. Thus, agents' valuations are interdependent,[4] ranging from the

[1] We elaborate on some of these examples in Section 5.

[2] Our objective is, however, not to propose a theory as to why the social planner may choose not to commit to a voting rule, because such a theory would require a more context-dependent modeling than ours. For instance, a university administration may not announce a voting rule for one department because it may then be required to apply the same rule to other departments.

[3] While the assumption of costless information is standard in many Jury decision problems, in many others, it may seem extreme. We maintain this assumption since, as explained below, it distinguishes our argument for heterogeneity from the ones that rely on costly information, and since it is quite possible that voters are presented with the information source such as candidate dossiers, drugs' clinical trials, etc.

[4] Interdependency of valuations is a convenient form of introducing correlation among agents' preferences and has been widely exploited in the mechanism design literature (e.g., Jehiel and Moldovanu (2001), Maskin (2000), and Myerson (1981)).

most heterogenous group with pure private values to the most homogenous group with pure common values. Based on their private signals (and, on the event of being pivotal) agents simultaneously submit their votes to a utilitarian social planner, who, by construction, does not have a direct preference for group heterogeneity. The planner accepts the project if and only if it receives k or more affirmative votes. As suggested above, she may, however, be unable to commit to the rule before votes are cast.

For an exogenous majority rule, k, we show that there is a unique symmetric voter equilibrium such that each agent approves the project if and only if his signal exceeds a cutoff. Using this equilibrium, we find that the social planner prefers an increasingly homogenous group. This is surprising due to the fact that agents with more homogenous preferences are also the ones who vote more strategically as they place a greater weight on others' signals and thus on the information gleaned from being pivotal. Nonetheless, homogenous agents' overall incentives turn out to be better aligned with those of the social planner's.

When the social planner could choose the majority rule, we show that she would commit to the one that leads to "sincere" voting in the sense that each individual votes based only on his signal (and not on being pivotal). This is intuitive since the social planner in our model cares simply about the information held by agents, and voting is most informative when it is sincere. What is less intuitive is the observation that the optimal voting rule is time-consistent. That is, in the absence of an ex ante commitment, there is an equilibrium in which the social planner sets the optimal voting rule and agents vote sincerely. In fact, under a mild hazard rate condition on signal distribution, we find that this "optimal" equilibrium is unique provided that agents' preferences are sufficiently heterogenous so that their strategic voting incentives are sufficiently weak. Thus, in sharp contrast with our finding for an exogenously given majority rule, when the rule is endogenously chosen, it is in the best interest of the group to be the most heterogenous.

When preferences are sufficiently homogenous, however, strategic voting incentives are so strong that there also exists a (unique) suboptimal equilibrium in which the social planner deviates from the optimal rule and agents vote strategically. The direction of the planner's deviation is such that as the group becomes more homogeneous, she lowers her majority requirement, and in anticipation, agents adopt a higher standard of approval. Thus, all else equal, we predict that with the same number of affirmative votes, projects are more likely to be accepted if these votes come from a more homogenous group; though individuals in such a group are less likely to vote affirmatively.

Our characterization of time-consistent majority rules also reveals that the unanimity rule is unlikely to be optimal. Moreover, as the group size grows, any time-consistent *percentage* rule approaches the rule for the most heterogenous group. This makes sense: in a larger group, each agent has a better prediction of the average of others' signals, which weakens the preference interdependence and increases heterogeneity.

Our key results on social planner's preference for group heterogeneity appear consistent with trial jury and the FDA's advisory committee selection procedures, which we discuss in Section 5.

Related Literature. Building on Condorcet's (1785) pioneering work, there is an extensive early literature on voting as a means of information aggregation in committees, which is ably summarized by Grofman and Feld (1988), and Li and Suen (2009). Austen-Smith and Banks (1996) first pointed out that sincere voting assumed in this literature is unlikely to occur in equilibrium "even when individuals have [such] a common preference."[5] Our analysis reveals that individuals with a common preference may actually vote the most strategically; but, from an ex ante viewpoint, it may still be in the group's best interest to have members with a common interest. In a series of papers, Feddersen and Pesendorfer (1996, 1997, 1998) have investigated the consequences of strategic voting on information aggregation in large common value elections for a given majority rule; and in particular, they have demonstrated that the unanimity rule performs poorly in this regard. Ignoring the integer problem, we also show that the unanimity rule is never socially optimal under more general preferences with interdependence.

More closely related to our analysis is the recent strand of the strategic voting literature concerned with committee design in which a social planner forms a committee and commits to the decision rule to achieve the dual goal of motivating agents to collect costly information and aggregating this information efficiently. Allowing for more general decision rules than simple threshold rules, Gerardi and Yariv (2008), Gersbach (1995), Gershkov and Szentes (2009), and Li (2001) present various common-values settings and show that the ex ante optimal decision rule is, in general, ex post inefficient. That is, the social planner is willing to commit to wasting some information ex post to incentivize agents to gather information ex ante. Persico (2004) examines a similar committee design problem but with threshold voting rules, as in our model. The optimal majority rule in his common-values setup turns

[5] Ali et al. (2008) offer some experimental evidence in favor of strategic voting in a Condorcet-type model.

out to be ex post optimal. Unlike this set of papers, we consider committees composed of members with heterogenous preferences, and how heterogeneity affects the social planner's choice of the majority rule if she cannot commit to one. Moreover, we do not have costly information acquisition; so any divergence between the ex ante and ex post optimal voting rules must be due to strategic voting incentives.

Perhaps, most pertinent to our work are the papers by Dewatripont and Tirole (1999), Cai (2009), and Che and Kartik (2009), among others, who construct cheap-talk type models and find that some degree of preference heterogeneity in the group may be desirable because such heterogeneity encourages agents to invest in information, while partially compromising with information transmission. Hence, in these models, if information were costless, then it would be socially optimal to have a homogenous group. In ours, on the other hand, a heterogenous group may still be socially desirable to allow a credibly commitment to the optimal voting rule. Finally, Gruner and Kiel (2004) consider a collective decision situation with interdependent valuations much like ours but where private signal is one's continuous policy preference. They compare the performances of mean and median aggregation rules; so they do not consider optimal or time-consistent rules, which are at the crux of our paper.

The remainder of the paper is organized as follows. In the next section, we lay out the model. In Section 3, we investigate voter equilibrium and group welfare under a fixed majority rule. In Section 4, we allow the social planner to choose the majority rule with and without commitment. Finally, we offer some applications of our key results in Section 5. Proofs that do not appear in the text are relegated to an appendix.

2 The Model

A group such as an academic department or FDA advisory committee contains $n \geq 3$ risk-neutral agents who need to make a collective decision whether or not to implement a public project such as a faculty hiring or a new drug. At the time of the decision, the exact value of the project is unknown, but agent i costlessly receives an independent private signal, θ_i, say through the candidate's scholarly work or drug's clinical tests, about the value from a differentiable c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$ over $[\underline{\theta}, \overline{\theta}]$ where $\underline{\theta} < 0 < \overline{\theta}$ and $E[\theta] = 0$. We assume that i's valuation of the project takes the following form:

$$v_i = (1-\alpha)\theta_i + \alpha \sum_j \theta_j / n, \tag{1}$$

where $\alpha \in [0,1]$. In words, agent i's valuation is a convex combination of signals of all group members, placing more weight on his own.[6] The parameter α in (1) traces the degree of (preference) heterogeneity, or conflict, in the group in the sense that as α increases, agents' realized payoffs get closer.[7] In particular, while $\alpha = 0$ refers to the most heterogenous group with pure private values, i.e., $v_i = \theta_i$, $\alpha = 1$ refers to the most homogenous group with pure common values, i.e., $v_i = \sum_j \theta_j/n$. We normalize agents' reservation payoffs from *not* implementing the project to 0. Since $E[v_i] = 0$, this normalization implies that no ex ante bias for or against the project exists, allowing us to highlight the informational issues.[8] For analytical convenience, we will restrict attention in the analysis to non-extreme values of α, i.e., $\alpha \in (0,1)$, though one can always take the limits.

Upon obtaining their private signals, the agents simultaneously submit their binary approve(+)/disapprove(−) votes for the project to a risk-neutral social planner, e.g., the FDA or the university administration. The planner aggregates these votes and renders a decision according to a majority rule, $k \in \{1, ..., n\}$ such that the project is implemented if and only if it receives k or more affirmative votes. The planner is a utilitarian agent acting on behalf of the group whose payoff from the project is

$$w \equiv \sum_i v_i/n = \sum_i \theta_i/n, \tag{2}$$

where the equality follows from (1). Note that w is independent of α, which means that the group heterogeneity does not have a direct effect on the group's welfare, though it will have an indirect effect through equilibrium voting. Note also that w is the average welfare per group member, which eliminates the scale effect of group size.

It is worth emphasizing that the social planner in our model does not have her own information and she cares only about the group's welfare. This is without loss of generality if, as in the mechanism design literature, the social planner's problem is a metaphor for group members' joint decision as to how to aggregate information. However, if the social planner is considered a real decision-maker such as the FDA or an editor, then the assumption of being utilitarian is important, as it is conceivable that these decision-makers may have their

[6] As alluded to in Footnote 4, such (linear) forms of interdependent valuations are commonly used in the mechanism design literature. Maskin (2000) provides a Bayesian inference interpretation as to why agent i's valuation depends directly on others' signals (see his Example 2.2).

[7] Formally, $|v_i - v_j| = (1-\alpha)|\theta_i - \theta_j|$.

[8] We have also extended the model to include a status quo bias, i.e., $E[\theta] \neq 0$, but, given that such extension had no qualitative effects on our results, we have chosen to present the simpler case with no such bias.

own agenda or biases weighing against the group's welfare. Modeling these biases seems to be context-dependent, in which case our investigation here without them can be viewed as a first step in this direction.[9]

We begin the analysis with characterizing voters' equilibrium behavior for an exogenously given majority rule, and then proceed to endogenizing the rule.

3 Exogenous Majority Rules

Let the majority rule, k, be fixed and publicly known by all group members before they cast their votes. Aside from serving as a building block for the next section, the analysis of an exogenous decision rule is of independent interest because in many collective decision-making situations such as jury trials and congressional committees, majority rules are set in the "constitution," much in advance of the arrival of any issue or project, and thus any particular group to consider it.

Note that since signals about the project are independently drawn and v_i strictly increases in θ_i, it is readily verified that agent i follows a cutoff voting strategy such that for some signal $\widehat{\theta}_i$, he approves the project if $\theta_i > \widehat{\theta}_i$ and disapproves it if $\theta_i < \widehat{\theta}_i$.[10] Given ex ante symmetry among agents, we consider symmetric voter equilibrium throughout.

Suppose that all agents but i adopt a cutoff, $\widehat{\theta}$. In determining his cutoff, agent i needs to evaluate only the event in which his vote is pivotal; namely the event in which there are exactly $k-1$ approve $(+)$ and $n-k$ disapprove $(-)$ votes except for his. Using (1), agent i's expected payoff conditional on being pivotal and privately observing θ_i is

$$V(\theta_i;\widehat{\theta},k,n,\alpha) \equiv (1-\alpha+\frac{\alpha}{n})\theta_i + \frac{\alpha}{n}[(k-1)E^+(\widehat{\theta}) + (n-k)E^-(\widehat{\theta})],$$

where $E^+(\widehat{\theta}) \equiv E[\theta|\theta > \widehat{\theta}]$ and $E^-(\widehat{\theta}) \equiv E[\theta|\theta < \widehat{\theta}]$. The cutoff $\widehat{\theta}$ is part of a symmetric voter equilibrium if and only if the signal $\theta_i = \widehat{\theta}$ also leaves agent i indifferent between approving and disapproving the project, or equivalently $\widehat{\theta}$ solves

$$V(\widehat{\theta};\widehat{\theta},k,n,\alpha) = 0. \tag{3}$$

Lemma 1. *For any feasible $k, n,$ and α, there exists a unique symmetric voter equilibrium.*

[9] Even in these examples, the anecdotal evidence suggests that the decision-makers rarely overrule the group's recommendation – an indication of how heavily they care about the group's welfare.

[10] His decision when indifferent is, of course, immaterial to our analysis as $\theta_i = \widehat{\theta}_i$ is a zero probability event.

The existence follows from the fact that at a symmetric cutoff with the lowest signal, agents always reject the project and at the cutoff with the highest signal, they always accept it. The uniqueness follows because the expected valuation in (3) strictly increases in the cutoff. In the next two lemmas, we further characterize the voter equilibrium.[11]

Lemma 2. *In the voter equilibrium,* $\underline{\theta} < \widehat{\theta}(k,n,\alpha) < \overline{\theta}$; $\widehat{\theta}(k+1,n,\alpha) < \widehat{\theta}(k,n,\alpha)$; *and* $\widehat{\theta}(k,n,\alpha) < \widehat{\theta}(k,n+1,\alpha)$.

Lemma 2 says that the voter equilibrium is strictly interior, and that it satisfies some well-known properties articulated in the strategic voting literature (e.g., Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997, 1998)): Given a fixed group size, the majority rule, k, and individuals' equilibrium approval standards are inversely related. As the decision rule requires more affirmative votes for acceptance, individuals relax their equilibrium standards to vote affirmatively because they have a more positive view of the project in the event of being pivotal. By the same logic, fixing k, an increase in group size means more disapproval votes in the pivotal event, leading each agent to raise his standard.

To investigate how $\widehat{\theta}(k,n,\alpha)$ changes with α, let us first introduce the notion of "sincere" or nonstrategic voting in our model. Agent i is said to vote sincerely if he conditions his vote only on his private information (see, e.g., Austen-Smith and Banks (1996)). Given that the reservation payoff is 0 and $E[\theta_j] = 0$ (so no ex ante bias toward the project is present), sincere voting is equivalent to adopting a cutoff of 0 in our model. That is, an agent who votes sincerely and receives a positive (resp. negative) signal about the project approves (resp. disapproves). In general, as implied by Lemma 2, strategic voting does not lead to sincere voting owing to the fact that rational agents try to infer others' information in equilibrium; and since the amount of this information depends on the majority rule, they tend to correct the "bias" caused by the majority rule in their strategies. The extent of this correction depends on the weight an agent attaches to others' signals, or equivalently on the degree of group homogeneity. Nonetheless, for one specific majority rule, sincere voting may result. Let $k = k^s(n)$ be this rule. Setting $\widehat{\theta} = 0$ in (3), and solving and simplifying for k, it follows,[12]

$$k = k^s(n) \equiv F(0) + (1 - F(0)) \times n. \tag{4}$$

[11] The reader may wish to review Examples 1 and 2 below along with the results.

[12] To be more precise, $k^s(n) = \frac{E^+(0) - nE^-(0)}{E^+(0) - E^-(0)}$; but, since, by conditional expectations, $F(0) \times E^-[0] + (1 - F(0)) \times E^+[0] = E[\theta]$, and by assumption, $E[\theta] = 0$, the expression in (4) is obtained.

For instance, for a symmetric signal distribution, we have $F(0) = \frac{1}{2}$; so sincere voting equilibrium is obtained whenever $k^s(n) = \frac{n+1}{2}$ for an odd n, coinciding with the Austen-Smith and Banks (1996) finding for symmetric binary signals. For expositional convenience, we assume $k^s(n)$ is an integer in the remainder.[13]

Lemma 3. *In the voter equilibrium,* $\widehat{\theta}_\alpha(k, n, \alpha) =^{sign} \widehat{\theta}(k, n, \alpha) =^{sign} k^s(n) - k$. *Moreover,* $E^-(\widehat{\theta}(.)) < 0 < E^+(\widehat{\theta}(.))$.

To understand Lemma 3, suppose $k < k^s(n)$. Since the majority rule $k^s(n)$ leads to sincere voting, a less stringent rule induces agents to adopt a more demanding approval standard (or a higher cutoff): knowing that the acceptance of the project requires few affirmative votes, each agent has a negative expectation of others' signals in the event of being pivotal and compensates this by raising his standard of approval. More importantly, in a more homogenous group, i.e., a greater α, this negative expectation of others' signals is reinforced, making the agent in question raise his standard further. When the majority requirement is more stringent than $k^s(n)$, a similar line of reasoning shows that each agent possesses a positive expectation of others' signals and thus reduces his approval standard in a more homogenous group. Together, we can say that agents vote more strategically in a more homogenous group in the sense of moving their cutoff away from the sincere voting cutoff of 0. Although strategic voting can induce an individual to approve the project when his signal is negative or disapprove it when his signal is positive, the last part of Lemma 3 reveals that an agent who approves (resp. disapproves) the project must have a strictly positive (resp. negative) *expected* signal in equilibrium.

Armed with voters' equilibrium strategies, we ask the following two questions: given the majority rule, is the group better off being more or less homogenous? And, is a more homogenous group more or less likely to accept the project? To answer these questions, note that for an arbitrary voting cutoff, x, the probability that there are exactly m approval and $n - m$ disapproval votes is $b(x; m, n) = \binom{n}{m}[1 - F(x)]^m F(x)^{n-m}$, and with this vote profile, the ex post group welfare (before payoffs are realized) is

$$w(x; m, n) \equiv \frac{mE^+(x) + (n - m)E^-(x)}{n}. \tag{5}$$

[13] If it were not an integer, sincere voting equilibrium would simply not exist for any k; but none of our results depends on such existence. What matters for our results, say for Lemma 3, is that the equilibrium cutoff changes sign for some k, which is always true.

Hence, the ex ante group welfare (before private signals are obtained) and the acceptance probability are, respectively,

$$\overline{w}(x;k,n) = \sum_{m=k}^{n} b(x;m,n)w(x;m,n), \tag{6}$$

and

$$P(x;k,n) = \sum_{m=k}^{n} b(x;m,n). \tag{7}$$

For $k = k^s(n)$, neither the welfare nor the acceptance probability is affected by the group homogeneity, because sincere voting is obtained independent of α. The following result shows that this observation changes dramatically for $k \neq k^s(n)$.

Proposition 1. *Fix the majority rule at $k \neq k^s(n)$. Then, the ex ante welfare in equilibrium strictly increases as the group becomes more homogenous. In addition, a more homogenous group is strictly less (resp. more) likely to accept the project if $k < k^s(n)$ (resp. $k > k^s(n)$).*

Proof. Suppose $k \neq k^s(n)$. Then, $\widehat{\theta}(k,n,\alpha) \neq 0$. To prove the first part, note from Lemma A1 in the Appendix that

$$\overline{w}_x(x;k,n) = -b(x;k-1,n-1) \times f(x) \times \left[(k-1)E^+(x) + (n-k)E^-(x) + x\right].$$

Thus,

$$\begin{aligned}\frac{\partial}{\partial\alpha}\overline{w}(\widehat{\theta}(k,n,\alpha),k,n) &= \overline{w}_x(\widehat{\theta}(k,n,\alpha),k,n) \times \widehat{\theta}_\alpha(.)\\ &=^{sign} -\left[(k-1)E^+(\widehat{\theta}(.)) + (n-k)E^-(\widehat{\theta}(.)) + \widehat{\theta}(.)\right] \times \widehat{\theta}_\alpha(.).\end{aligned}$$

Now, note that the equilibrium condition in (3) implies that $(k-1)E^+(\widehat{\theta}(.)) + (n-k)E^-(\widehat{\theta}(.)) + \widehat{\theta}(.) = -n\frac{1-\alpha}{\alpha}\widehat{\theta}(.)$. Inserting this fact along with $\widehat{\theta}_\alpha(k,n,\alpha) =^{sign} \widehat{\theta}(.)$ from Lemma 3, we obtain

$$\frac{\partial}{\partial\alpha}\overline{w}(\widehat{\theta}(k,n,\alpha),k,n) =^{sign} n\frac{1-\alpha}{\alpha}(\widehat{\theta}(.))^2 > 0.$$

To prove the second part, observe that by using simple algebra $P_x(x;k,n) = -n \times b(x;k-1,n-1) \times f(x) < 0$. Then,

$$\begin{aligned}\frac{\partial}{\partial\alpha}P(\widehat{\theta}(k,n,\alpha);k,n) &= P_x(\widehat{\theta}(k,n,\alpha);k,n) \times \widehat{\theta}_\alpha(k,n,\alpha)\\ &=^{sign} -\widehat{\theta}_\alpha(.) =^{sign} k - k^s(n),\end{aligned}$$

where the last equality is due to Lemma 3. □

Proposition 1 is a key finding of this paper. It reveals that when the voting rule is fixed at a level that induces strategic or "insincere" voting in equilibrium, the social planner strictly prefers the group to be the *most* homogenous. This finding is surprising for two reasons: first, recall from (2) that the social planner does not have a direct preference for group composition in our model; so any such preference must come from voters' equilibrium behavior; second, in light of Lemma 3, members of the most homogenous group are also the ones who engage in the most strategic voting, and thus mostly likely to ignore their own private information [see Lemma 3]. To see the intuition behind the planner's preference, notice that for a given majority rule, the ex ante welfare is single-peaked in voters' cutoff: Conditional on accepting the project, the planner wants the cutoff to be high so both an approve and a disapprove vote would mean a relatively positive signal (see eq. (5)); but a high cutoff makes approve votes unlikely. Thus, for a fixed majority rule, there is a unique socially optimal voting cutoff, which can only be reached in the voter equilibrium by agents who are social-minded, or in our context, by agents who have no preference conflict, i.e., $\alpha = 1$. When $\alpha < 1$, the social optimum cannot be reached, and how close it can be approached in equilibrium depends on how well agents correct the bias introduced by the majority rule in their voting strategies. Consider, for instance, $k < k^s(n)$. As indicated above, when the majority requirement for acceptance is low, agents fear that the project may be accepted too easily and therefore adopt a strictly positive voting cutoff. However, since they place strictly more weight on their own information about the project, they tend to choose too low a cutoff in equilibrium. As implied by Lemma 3, a positive equilibrium cutoff increases as α increases, or group becomes more homogenous, bringing the ex ante welfare closer to the optimal one. A similar logic applies to the case of $k > k^s(n)$: agents choose a strictly negative cutoff, which is too high; and again, as α increases, the cutoff decreases, improving the ex ante welfare. Overall, it follows that for $k \neq k^s(n)$, the social planner strictly prefers the group to be composed of more homogenous members.

Proposition 1 also reveals that how the equilibrium probability of accepting the project changes with the degree of group homogeneity depends critically on the majority rule, as implied by Lemma 3. For $k < k^s(n)$, agents choose a high (positive) approval standard in equilibrium, which increases in the degree of group homogeneity, and reduces the probability of acceptance. The opposite conclusion holds for $k > k^s(n)$: agents choose a low (negative) approval standard, which decreases in the degree of group homogeneity, and in turn increases the probability of acceptance.

Proposition 1 helps put some basic observations regarding strategic voting in Condorcet-type models in perspective. As Austen-Smith and Banks (1996) first demonstrated, for a fixed majority rule, sincere voting is unlikely to occur in equilibrium (except for a specific majority rule; $k^s(n)$ here) "even when individuals have [such] a common preference". Our investigation uncovers that voters with a common preference may actually behave the most strategically as they place the highest weight on the information inferred from being pivotal. While this seems bad for information transmission and thus for the group welfare, Proposition 1 says that from an ex ante point of view, the group may, nonetheless, benefit from having members with a common preference due to its effect on probability of acceptance. The following example illustrates most of our findings thus far.

Example 1. *Consider a group of* 5 *agents, who each independently draw a signal from a uniform distribution on* $[-1, 1]$. *Trivial algebra shows that agents' equilibrium cutoff is given by* $\widehat{\theta} = \frac{\alpha}{5-2\alpha}(3-k)$, *and* $k = k^s = 3$ *induces sincere voting. Clearly,* $\widehat{\theta} > 0$ *and strictly increases in* α *for* $k < 3$ *whereas for* $k > 3$, $\widehat{\theta} < 0$ *and strictly decreases in* α. *Additional algebra shows that the ex ante welfare is*

$$\overline{w} = \begin{cases} \frac{3125(5-4\alpha)}{64(5-2\alpha)^6} & for \quad k = 1, 5 \\ \frac{3}{32} & for \quad k = 3 \\ \frac{(5-3\alpha)^2(5-\alpha)^4}{16(5-2\alpha)^6} & for \quad k = 2, 4. \end{cases}$$

Three remarks about Example 1 are in order. First, the ex ante welfare is equal across $k = 1, 5$ and across $k = 2, 4$. This is a consequence of strategic voting: agents adjust their voting strategies to the voting rule, and for a symmetric signal distribution, like the uniform used here, this adjustment is complete. For instance, $\widehat{\theta} = \frac{2\alpha}{5-2\alpha}$ and $\widehat{\theta} = -\frac{2\alpha}{5-2\alpha}$ for $k = 1$ and $k = 5$, respectively. Second, for each $k \neq 3$, the ex ante welfare strictly increases in α, as indicated by Proposition 1. And third, the ex ante welfare is hump-shaped in k, attaining a maximum at $k = k^s = 3$– an observation we will prove holds in general.

4 Time-Consistent Majority Rules

Up to now, we have examined environments in which the majority rule to aggregate votes is exogenous. While, as mentioned above, there are many such environments including jury trials and congressional committees, in many others, the majority rule is tailored to the specific group or committee in question. In fact, the recent literature on committee design deals with identifying ex ante optimal majority rules that the social planner commits to before votes are cast (see, Li and Suen (2009) for a survey). Our focus here is on complementary settings where the social planner is unable to commit to a decision rule when asking for votes. For instance, in academic hiring cases, the faculty members in the relevant department often submit confidential yes/no votes, which are then relayed to the university administration. Similarly, when the members of an FDA advisory committee evaluate a new drug application, they frequently convey their recommendation to the FDA by taking a simultaneous approve/ disapprove vote. Finally, in the publication process, an editor solicits independent opinions of a group of experts, and renders the final decision by aggregating these opinions. In all these examples, the individuals who vote are rarely told – if at all – exactly how many positive votes are needed for a positive decision on the project. In such environments without an ex ante commitment, the social planner cannot act as a Stackelberg leader when choosing the majority rule; rather she can choose the majority rule that best responds to agents' voting strategies, and in anticipation, agents best respond to the planner's majority rule when submitting votes, effectively playing a simultaneous-move game. Suppressing parameters n and α for now, let (k^*, θ^*) be an equilibrium pair of majority rule and voting cutoff in this game, which, by definition, lies at the intersection of the players' best responses:

$$k^* = \arg\max_k \sum_{m=k}^{n} b(\theta^*; m, n) w(\theta^*; m, n) \tag{8}$$

and

$$\theta^* = \widehat{\theta}(k^*, n, \alpha). \tag{9}$$

Since the ex post welfare, $w(\theta^*; m, n)$, strictly increases in the number of approve votes, m, and $b(\theta^*; m, n) > 0$ given that $\theta^* \neq \underline{\theta}, \overline{\theta}$ by Lemma 2, (8) can be simplified as $k^* = \arg\min_k \; w(\theta^*; k, n)$ subject to $w(\theta^*; k, n) \geq 0$. That is, the social planner's equilibrium choice of majority rule must be ex post optimal. This makes sense. Lacking the ex ante commitment to a majority rule, it is best for the social planner to choose one after observing the votes.[14] Note that k^* is the majority rule that can be credibly adopted by the social planner, or said differently, it is the rule that is *time-consistent*. Note also that the planner

[14]Notice, though, we do not require the rule to be ex post optimal; rather it is a consequence of the social planner's equilibrium choice.

need not publicly announce k^* as it can be inferred by group members in equilibrium. To establish a benchmark and understand the value of ex ante commitment, we now find the ex ante optimal majority rule, $k^o(n)$, and determine if it is time-consistent.

Proposition 2. *The ex ante optimal majority rule is the one that induces sincere voting, i.e.,* $k^o(n) = k^s(n) = F(0) + (1 - F(0)) \times n$; *and it is time-consistent.*

Proof. The social planner's ex ante problem can be stated as

$$\max_{k,x} \overline{w}(x;k,n) \text{ s.t. } x = \widehat{\theta}(k,n,\alpha).$$

By Lemma A1, $\overline{w}_x(x;k,n) = -b(x;k-1,n-1) \times f(x) \times [(k-1)E^+(x) + (n-k)E^-(x) + x]$. Since the expression, $(k-1)E^+(x) + (n-k)E^-(x) + x$, is strictly increasing in x; strictly negative at $x = \underline{\theta}$; and strictly positive at $x = \overline{\theta}$, it follows that $\overline{w}(x;k,n)$ is strictly quasi-concave in x, with an interior maximum. Given the (equilibrium) constraint, $x = \widehat{\theta}(k,n,\alpha)$, this maximum must occur when $\overline{w}_x(\widehat{\theta}(k,n,\alpha),k,n) = 0$, or equivalently when $\left[(k-1)E^+(\widehat{\theta}(k,.)) + (n-k)E^-(\widehat{\theta}(k,.))\right] + \widehat{\theta}(k,.) = 0$. In addition, $V(\widehat{\theta}(k,.);\widehat{\theta}(k,.),k,n,\alpha) = 0$ by (3). Thus, the optimal cut-off must be $x^o = \widehat{\theta}(k,.) = 0$. This means that k^o must satisfy: $(k^o - 1)E^+(0) + (n - k^o)E^-(0) + 0 = 0$, whose unique solution is $k^o = k^s(n)$, as given in (4).

To prove that k^o is time-consistent, we need to prove that the social planner does not have an ex post incentive to change the majority rule from k^o upon observing the votes and conjecturing a cutoff, $x^o = 0$. Given that $(k^o - 1)E^+(0) + (n - k^o)E^-(0) = 0$, the ex post welfare with $m \geq k^o$ approve votes is positive because $w(0,m,n-m) \geq w(0;k^o,n-k^o) = \frac{E^+(0)}{n} > 0$, whereas the ex post welfare with $m \leq k^o - 1$ approve votes is strictly negative because $w(0,m,n-m) \leq w(0;k^o-1,n-k^o+1) = -\frac{E^-(0)}{n} < 0$. Hence, k^o is ex post optimal given $x^o = 0$. Since, given $k^o = k^s(n)$, we have $x^o = 0$ as a best response, k^o is time-consistent. $\square$

Proposition 2 has three implications. First, the ex ante optimal majority rule results in sincere voting. This is intuitive because, being a utilitarian agent, the social planner's objective given in (2) is independent of the group heterogeneity. Namely, the planner cares only about individuals' signals, which are most informative when votes are sincere. Second, (ignoring the integer problem) the ex ante optimal rule is *always* less than unanimity.[15]

[15] For instance, for a symmetric signal distribution, we have $F(0) = \frac{1}{2}$ and thus the optimal rule is $k^o = \frac{n+1}{2}$.

In particular, as negative signals become more likely, the optimal rule moves away from unanimity, and vice versa. This may appear counter-intuitive, because the social planner should require a larger consensus in order to avoid a negative value project; but given our normalization that $E[\theta] = 0$, a greater probability of negative signals, $F(0)$, also means a higher positive value attached to an affirmative vote, $E^+(0)$, to keep the mean at zero, requiring fewer positive votes to accept the project. This observation may lend additional support to Feddersen and Pesendorfer (1998) who point out the weaknesses of the unanimity rule in a Condorcet Jury problem in the presence of strategic voting. Finally, Proposition 2 indicates that upon inducing individuals to vote sincerely by committing to an ex ante optimal majority rule, the social planner has no ex post incentive to deviate from it. To see why, consider the marginal event in which the planner receives $k^o - 1$ approve and $n - k^o + 1$ disapprove votes. The ex ante rule, k^o dictates that the project be rejected in this event. And this is exactly what the planner does ex post, because sincere voting requires that the expected sum of $n - 1$ signals with $k^o - 1$ approve and $n - k^o$ disapprove votes be 0, which, in turn, requires that with one additional disapprove vote, the ex post welfare be strictly negative.

Given that the majority rule, k^o generates the highest ex ante welfare and it is time-consistent, we call the pair $(k^*, \theta^*) = (k^o, 0)$ the *optimal* equilibrium. The existence of the optimal equilibrium suggests that despite the social planner's lack of commitment to an ex ante decision rule, group members may still vote sincerely by holding an equilibrium belief that the ex ante optimal rule will be used. Such a belief, however, may not be unique. In particular, when the group is sufficiently homogenous, we will show that there is often a *suboptimal* equilibrium in which group members believe that the social planner will deviate from the optimal rule and vote strategically as a result. The characterization of suboptimal equilibrium is important since it not only points to a welfare loss due to the commitment problem, but also points to what other majority rules can be time-consistent depending on the group composition. In what follows, we impose a mild distributional assumption to provide a full characterization.

Condition HR. (Monotone Hazard Rate) $\frac{d}{d\theta}\left[\frac{1-F(\theta)}{f(\theta)}\right] \leq 0.$

Condition HR is a familiar one from the mechanism design literature and satisfied by most well-known distributions, including the uniform and normal (see, Bagnoli and

Bergstrom (2005) for an extensive list.).[16] An implication of this condition is that the difference $E^+(x) - x$ decreases in x (see Lemma A2), leading us to

Proposition 3. *Suppose that Condition HR holds. Then,*

(i) *there is a lower bound of group homogeneity, $\underline{\alpha}(n) \in (0,1)$ such that a unique suboptimal equilibrium exists if and only if $\alpha \geq \underline{\alpha}(n)$.*

(ii) *When it exists, the suboptimal equilibrium, (k^{so}, θ^{so}) is characterized by*[17]

$$k^{so}(\alpha, n) = \lceil (1 - F(\theta_h(\alpha, n))) \times n \rceil, \tag{10}$$

and $\theta^{so}(\alpha, n) = \widehat{\theta}(k^{so}(\alpha, n), n, \alpha)$, where $\theta_h(\alpha, n)$ is the unique solution to $h(\widetilde{\theta}; \alpha, n) \equiv \frac{E^+(\widetilde{\theta}) - \widetilde{\theta}}{n} - \frac{1-\alpha}{\alpha}\widetilde{\theta} = 0$.

Proposition 3 is another key finding of this paper (along with Proposition 1). Part (i) indicates that for a sufficiently heterogenous group, i.e. $\alpha < \underline{\alpha}(n)$, only the optimal equilibrium exists. To see this, consider the most heterogenous group, i.e. $\alpha = 0$, where each member cares only about his own signal. In this case, each member has a dominant strategy of voting sincerely independent of the majority rule. This means that the social planner can implement the ex ante optimal rule without publicly committing to it. By continuity of voting strategies, commitment is still of no value to the social planner for a group that is not too homogenous because strategic voting incentives in such a group is still relatively weak. When the group is sufficiently homogenous, however, agents' strategies deviate from sincere voting so much that the social planner may respond by deviating from the optimal rule, engendering a suboptimal equilibrium.

A major implication of part (i) is that in the absence of ex ante commitment, the social planner would not prefer the group to be the most homogenous to *avoid* the suboptimal equilibrium. This is in sharp contrast with Proposition 1, which shows that under an exogenously set majority rule, the planner would prefer the group to be the most homogenous.

[16] Many well-known distributions that are differentiable, and that satisfy $E[\theta] = 0$ and Condition HR appear to be symmetric, but it is easy to construct asymmetric distributions with the same properties such as this one: $f(\theta) = \begin{cases} a + b\theta^3 & if \quad \theta \in [\sqrt[3]{-a/b}, 0] \\ a - \theta^2 & if \quad \theta \in [0, \sqrt{a}] \end{cases}$, where $a \approx .82$ and $b \approx 1.45$. Besides, as indicated in Footnote 8, our model could easily be extended to signal distributions with nonzero means.

[17] $\lceil . \rceil$ denotes the usual ceiling function.

And, it is also in contrast with Proposition 2, which shows that under an optimally set majority rule, the planner is neutral to the group composition because she is able to engender sincere voting for any degree of group homogeneity.[18]

Part (ii) of Proposition 3 provides an explicit description of the majority rule in a suboptimal equilibrium, which allows us to determine all time-consistent rules in our model. Given that the social planner deviates from the optimal majority rule in a suboptimal equilibrium, it is important to discern the direction of this deviation. That is, does the planner adopt too stringent or too lenient decision rule in a suboptimal equilibrium? And, how does this rule change with the degree of group homogeneity and group size? The following result answers these questions.

Proposition 4. *Suppose $\alpha \geq \underline{\alpha}(n)$. Then, $k^{so}(\alpha, n)$ decreases in α, and $k^{so}(\alpha, n) < k^{o}(n)$. Moreover, there exists $\overline{\alpha}(n) \in (\underline{\alpha}(n), 1)$ such that $k^{so}(\alpha, n) = 1$ for $\alpha \geq \overline{\alpha}(n)$.*

According to Proposition 4, in a suboptimal equilibrium, the social planner requires fewer affirmative votes to accept the project than it is optimal. She further relaxes her majority requirement as the group becomes more homogenous; because members of such a group adopt a higher standard of approval in equilibrium. In fact, for a sufficiently homogenous group, only *one* affirmative vote may be enough for acceptance. To gain some intuition why the suboptimal decision rule is less stringent than the optimal one (as opposed being more stringent), recall that the social planner picks the majority rule that is ex post optimal, or formally $k^{so}E^{+}[\theta^{so}] + (n - k^{so})E^{-}(\theta^{so}) = 0$ (ignoring the integer problem). This implies that, in the event of being pivotal, each agent holds a strictly negative expectation of others' signals because $(k^{so} - 1)E^{+}(\theta^{so}) + (n - k^{so})E^{-}(\theta^{so}) = -E^{+}(\theta^{so}) < 0$,[19] and in turn, chooses a strictly positive cutoff in equilibrium. From Lemma 3, we know that this positive cutoff rises, or equivalently strategic voting incentives intensify, as the group homogeneity, or α, increases. In particular, in a more homogenous group, agents' equilibrium strategies diverge from sincere voting, and in response, the social planner relaxes her equilibrium majority requirement.

One implication of Proposition 4 is that all else equal, with the same number of positive votes, a project has a greater chance of being implemented if these votes come from a homogenous group; or said differently, a greater consensus is required for projects submitted

[18] Recall that $k^{s}(n)$ is independent of α.

[19] because, by Lemma 3, $E^{+}[\theta^{so}] > 0$ in equilibrium.

by a heterogenous group.[20] This does not mean, however, projects that are evaluated by a homogenous group are more likely to be accepted because members of such groups are less likely to vote positively. We now illustrate our findings in this section.

Example 2. *Consider the setting in Example 1. Consistent with Proposition 2, the ex ante optimal majority rule is* $k^o = 3$. *To find the suboptimal equilibrium, we solve for* (k^*, θ^*) *from the best-responses:* $w(\theta^*; k^*, n) = 0$ *and* $\theta^* = \widehat{\theta}(k^*, n, \alpha)$*; and find* $k^* = \frac{25(1-\alpha)}{10-9\alpha}$ *and* $\theta^* = \frac{\alpha}{10-9\alpha}$. *But this solution is an equilibrium if and only if* k^* *is an integer, or else we take its ceiling as indicated in Proposition 3. From here, it follows that for* $\alpha \in [0, \frac{5}{7})$*, the unique time-consistent majority rule is* $k^* = k^o = 3$. *For* $\alpha \in [\frac{5}{7}, \frac{15}{16})$*, there is a unique suboptimal equilibrium with* $k^{so} = 2$*; hence time-consistent rules are* $k^* = 2$ *and 3. Finally, for* $\alpha \in [\frac{15}{16}, 1]$*, time-consistent rules are* $k^* = 1$ *and 3. In terms of ex ante welfare along equilibrium path,* $\overline{w}^o = .094$ *for all* α *at the optimal equilibrium since* $k^o = 3$ *is independent of* α. *Using Example 1, the ex ante welfare at the suboptimal equilibrium is,*

$$\overline{w}^{so}(\alpha) = \begin{cases} \frac{(5-3\alpha)^2(5-\alpha)^4}{16(5-2\alpha)^6} & \text{for} \quad \frac{5}{7} \leq \alpha < \frac{15}{16} \\ \frac{3125(5-4\alpha)}{64(5-2\alpha)^6} & \text{for} \quad \frac{15}{16} \leq \alpha \leq 1 \end{cases}$$

Note that $\overline{w}^{so}(\alpha)$ *is non-monotonic in* $\alpha \in [\frac{5}{7}, 1]$*: it strictly increases within each subinterval because* k^{so} *remains fixed and Proposition 1 applies; but at the neighborhood of* $\alpha = \frac{15}{16}$*,* $\overline{w}^{so}(\alpha)$ *jumps down from* .087 *to* .065 *as* k^{so} *switches from* 2 *to* 1*, and diverges further from the optimal majority rule,* $k^o = 3$.

Our investigation up to now can also inform us how equilibrium majority rules change with group size, n. Using (4) and (10), it follows that both the optimal and suboptimal majority rules, $k^{so}(\alpha, n)$ and $k^o(n)$, increase in n. This is not surprising, however, given the scale effect associated with the group size. To distill this effect, we look at the percentage rules, $\frac{k^{so}(\alpha,n)}{n}$ and $\frac{k^o(n)}{n}$.

Proposition 5. *Take a sequence of* n *such that integer problems do not arise. Then, along this sequence, the percentage majority rule,* $\frac{k^{so}(\alpha,n)}{n}$*, increases whereas* $\frac{k^o(n)}{n}$ *decreases*

[20] This observation yields the following testable prediction: all else equal, acceptance of papers in a general interest journal is likely to require a greater consensus among reviewers than those in a field journal because the reviewers of the former are more likely to possess heterogenous preferences owing to their potentially different fields of research.

in n. As $n \to \infty$, both percentage rules converge to $1 - F(0)$, which would be obtained for any n if $\alpha = 0$.

To grasp intuition behind Proposition 5, note that the correlation between agents' valuations gets weaker in a larger group because, by the logic of the law of large numbers, each agent has a sharper prediction of the average signal of others.[21] Thus, much like in a more heterogenous group, agents vote less strategically in a larger group, alleviating the social planner's commitment problem and allowing her to raise the percentage rule in a suboptimal equilibrium. As group size grows without bound, the strategic voting incentive vanishes completely, and the percentage rule converges to the one that would be obtained in a group with pure private values, i.e., $\alpha = 0$. While the same limit applies, the percentage rule in the optimal equilibrium decreases in group size.

5 Applications and Concluding Remarks

Our analysis yields two main results. When the voting rule is fixed, a utilitarian social planner wants the group to have members with the most *homogenous* preferences despite the fact that they tend to vote the most strategically. However, when the planner chooses the voting rule, but cannot commit to it before votes are cast, she wants the group to have members with the most *heterogenous* preferences because they tend to vote the least strategically, relaxing the planner's commitment problem.

There seems to be supporting evidence for these results. For instance, in jury trials where the voting rule is fixed by the constitution,[22] our theory suggests that jurors should be selected to have as homogenous preferences as possible; and the strict jury selection process, called *voir dire*, in the U.S. and other common law countries appears to do just that. By allowing both sides' attorneys to examine potential jurors, the process aims to eliminate strongly prejudiced or unqualified jurors from the pool to ensure a fair trial. In contrast to jury formation, the FDA encourages its advisory committees to be composed of members with diverse preferences in that the committees should contain not only the technical experts but also consumers and industry advocates as voting members.[23] Similar heterogeneity among voting members seems to be in place in faculty hiring cases too, because

[21] It is easy to verify that $Cov(v_i, v_j) = \frac{\alpha(2-\alpha)}{n}$.

[22] See Starr and McCormick (2001) for various unanimity and nonunanimity verdict requirements, and the details of jury selection process in general.

[23] See http://www.fda.gov/Drugs/ResourcesForYou/Consumers/ucm143538.htm

often all faculty in a department are eligible to vote on a candidate regardless of their fields of research. These two examples are also consistent with our theory, since, unlike the jury trials, a voting rule is usually not announced to the FDA committee or to the faculty before votes are submitted. Thus, according to Proposition 3, the social planner should indeed favor a heterogenous group.

In closing, we should note several issues that were not addressed here. As mentioned, our model does not explain why the social planner such as a university administration or the FDA may not want to commit to a voting rule. In our opinion, a more context-dependent model as to the role of the social planner above and beyond aggregating information from the committee is needed to achieve this objective. For instance, the planner may have other economic and political concerns weighing against the group's welfare. Another issue we have not addressed is pre-voting communication of private information. Although many committee voting models assume away such communication, some recent papers have pointed out its potential importance on voting outcomes (Coughlan (2000), Gerardi and Yariv (2007)). While some communication between voters does occur in many real examples, like Persico (2004), we believe that there are probably certain institutional and physical barriers to this communication, and the assumption of no communication may not be totally unrealistic. Nonetheless, it would interesting to enrich the present model with this dimension and see how it interacts with the time-consistency problem and the preference for group heterogeneity. Finally, one may introduce "asymmetric" heterogeneity within the group in that each member may attach a different weight on others' signals, which may or may not be privately known.

6 Appendix

Proof of Lemma 1. To save on notation, let $V(x;k,n,\alpha) \equiv V(x;x,k,n,\alpha)$. Since $E^+(x) > E^-(x)$ for any $x \in [\underline{\theta}, \overline{\theta}]$, $V(x;k,n,\alpha)$ strictly increases in k. Together with the assumption that $E[\theta]=0$, it follows $V(\underline{\theta};k,n,\alpha) \leq V(\underline{\theta};n,n,\alpha) = [1-(n-1)\frac{\alpha}{n}]\underline{\theta} < 0$, and $V(\overline{\theta};k,n,\alpha) \geq V(\overline{\theta};1,n,\alpha) = [1-(n-1)\frac{\alpha}{n}]\overline{\theta} > 0$. In addition, since (with appropriate limit arguments for $x = \underline{\theta}$ and $\overline{\theta}$)

$$E^{+\prime}(x) = \frac{f(x)}{1-F(x)}[E^+(x)-x] > 0 \text{ and } E^{-\prime}(x) = \frac{f(x)}{F(x)}\left[x - E^-(x)\right] > 0,$$

it also follows $V_x(x;k,n,\alpha) > 0$. From these three facts, we conclude that there exists a unique solution, $\widehat{\theta}(k,n,\alpha)$, to $V(\widehat{\theta};k,n,\alpha)=0$. □

Proof of Lemma 2. The fact that $\underline{\theta} < \widehat{\theta}(k,n,\alpha) < \overline{\theta}$ is directly obtained from the proof of Lemma 1. Next, suppose that, in the voter equilibrium, $\widehat{\theta}(k+1,n,\alpha) \geq \widehat{\theta}(k,n,\alpha)$ for some k. Then, since, by the proof of Lemma 1, $V(x;k,n,\alpha)$ strictly increases in x and k, we have

$$V(\widehat{\theta}(k,.);k,n,\alpha) \leq V(\widehat{\theta}(k+1,.);k,n,\alpha) < V(\widehat{\theta}(k+1,.);k+1,n,\alpha),$$

which implies $V(\widehat{\theta}(k,.);k,n,\alpha) \neq V(\widehat{\theta}(k+1,.);k+1,n,\alpha)$. But, in equilibrium,

$$V(\widehat{\theta}(k,.);k,n,\alpha) = V(\widehat{\theta}(k+1,.);k+1,n,\alpha)(=0),$$

yielding a contradiction. Hence, $\widehat{\theta}(k+1,n,\alpha) < \widehat{\theta}(k,n,\alpha)$. Using a similar line of argument and noting that $V(x;k,n,\alpha)$ strictly decreases in n, it follows $\widehat{\theta}(k,n,\alpha) < \widehat{\theta}(k,n+1,\alpha)$. □

Proof of Lemma 3. Differentiating both sides of the equilibrium condition in (3) with respect to α, we find: $\widehat{\theta}_\alpha(.) = -\frac{V_\alpha(\widehat{\theta}(.),.)}{V_x(.)}$. Next, observe that $V_\alpha(\widehat{\theta}(.),.) = -\frac{\widehat{\theta}(.)}{\alpha}$. Since, in addition, $V_x(.) > 0$ and $\alpha > 0$, it follows that $\widehat{\theta}_\alpha(k,n,\alpha) =^{sign} \widehat{\theta}(k,n,\alpha)$. To prove the second sign, fix any $\alpha \in (0,1)$. By Lemma 2, $\widehat{\theta}(.) \in (\underline{\theta}, \overline{\theta})$. Using (3), simple algebra shows that $\widehat{\theta}(1,.) = \frac{n-1}{n}\alpha[\widehat{\theta}(1,.) - E^-(\widehat{\theta}(1,.))] > 0$, and $\widehat{\theta}(n,.) = \frac{n-1}{n}\alpha[\widehat{\theta}(n,.) - E^+(\widehat{\theta}(n,.)] < 0$. Since, by Lemma 2, $\widehat{\theta}(k,n,\alpha)$ strictly decreases in k, there must be a unique $k^0 \in \{2,...,n-1\}$ such that $\widehat{\theta}(k,n,\alpha) > 0$ for $k < k^0$, and $\widehat{\theta}(k,n,\alpha) \leq 0$ for $k \geq k^0$, with equality only if k^0 is an integer. But, by definition, $k^0 = k^s$, as given in (4). To prove the last part, suppose, to the contrary, that $E^+(\widehat{\theta}(.)) \leq 0$. Since $E^-(x) < E^+(x)$, this implies that $E^-(\widehat{\theta}(.)) < 0$. Moreover, since $V(\widehat{\theta}(.);k,n,\alpha) = 0$ by (3), we must have: $\widehat{\theta}(k,n,\alpha) > 0$. But, this means $E^+(\widehat{\theta}(.)) > 0$, yielding a contradiction. Hence, $E^+(\widehat{\theta}(.)) > 0$. A similar argument shows $E^-(\widehat{\theta}(.)) < 0$. □

Lemma A1. *The ex ante welfare stated in (6) satisfies*

$$\overline{w}_x(x;k,n) = -b(x;k-1,n-1) \times f(x) \times \left[(k-1)E^+(x) + (n-k)E^-(x) + x\right],$$

where $b(x;m,n) = \binom{n}{m}[1-F(x)]^m F(x)^{n-m}$ *as defined in text.*

Proof. In this proof, we do not impose the assumption $E[\theta] = 0$. To save space, let $p \equiv 1 - F(x)$ in this proof. Then,

$$\begin{aligned}\overline{w}(x;k,n) &\equiv \phi(p,x,k,n) \equiv \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i} \left[\frac{iE^+(x) + (n-i)E^-(x)}{n}\right] \\ &= \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i} [\frac{i}{n} \frac{\int_x^{\overline{\theta}} \theta dF(\theta)}{p} + \frac{n-i}{n} \frac{\int_{\underline{\theta}}^{x} \theta dF(\theta)}{1-p}].\end{aligned}$$

Since

$$\frac{i}{n}\binom{n}{i} = \binom{n-1}{i-1}, \frac{n-i}{n}\binom{n}{i} = \binom{n-1}{i}$$

and

$$\sum_{i=k}^{n} \binom{n-1}{i-1} p^{i-1}(1-p)^{n-i} = \sum_{i=k-1}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i},$$

we have

$$\begin{aligned}\phi(p,x,k,n) &= \int_x^{\overline{\theta}} \theta dF(\theta) \left[\sum_{i=k-1}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i}\right] + \int_{\underline{\theta}}^{x} \theta dF(\theta) \left[\sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i}\right] \\ &= [\int_{\underline{\theta}}^{x} \theta dF(\theta) + \int_x^{\overline{\theta}} \theta dF(\theta)][\sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} + \int_x^{\overline{\theta}} \theta dF(\theta) \left[\binom{n-1}{k-1} p^{k-1}(1-p)^{n-k}\right] \\ &= E[\theta] \sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} + \int_x^{\overline{\theta}} \theta dF(\theta) [\binom{n-1}{k-1} p^{k-1}(1-p)^{n-k}].\end{aligned}$$

Next, observe that

$$\frac{\partial}{\partial p} \sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} = (n-1)\binom{n-2}{k-1} p^{k-1}(1-p)^{n-1-k}.$$

Thus, we have

$$\begin{aligned}\overline{w}_x(x;k,n) &= \phi_p(.) \times \frac{\partial p}{\partial x} + \phi_x(.) = -f(x) \left[E[\theta](n-1)\binom{n-2}{k-1} p^{k-1}(1-p)^{n-1-k}\right. \\ &\quad + \left.\binom{n-1}{k-1} \left[(k-1)p^{k-2}(1-p)^{n-k} - (n-k)p^{k-1}(1-p)^{n-1-k}\right] \int_x^{\overline{\theta}} \theta dF(\theta)\right] \\ &\quad + \binom{n-1}{k-1} p^{k-1}(1-p)^{n-k}(-xf(x)).\end{aligned}$$

Since $\int_x^{\overline{\theta}} \theta dF(\theta) = pE^+(x)$, and $pE^+(x) + (1-p)E^-(x) = E[\theta]$, we further have

$$\begin{aligned}\phi_p(.) \times \frac{\partial p}{\partial x} + \phi_x(.) &= -f(x)\left\{E[\theta](n-1)\binom{n-2}{k-1}p^{k-1}(1-p)^{n-1-k}\right.\\ &\quad + \binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}\left[(k-1)E^+(x) + (n-k)E^-(x)\right]\\ &\quad - \binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}(n-k)\frac{E[\theta]}{1-p}\\ &\quad \left. + \binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}x\right\}.\end{aligned}$$

Observe that the first and the third terms inside the curly brackets on the r.h.s. cancel out, leaving

$$\begin{aligned}\overline{w}_x(x;k,n) &= -f(x)\left\{\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}\left[(k-1)E^+(x) + (n-k)E^-(x)\right]\right.\\ &\quad \left. + \binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}x\right\}\\ &= -f(x)\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}\left\{(k-1)E^+(x) + (n-k)E^-(x) + x\right\}.\end{aligned}$$

Substituting back for $p \equiv 1 - F(x)$, the desired result for $\overline{w}_x(x;k,n)$ is then obtained. $\square$

Lemma A2. *Under Condition HR,* $\frac{d}{dx}[E^+(x) - x] \leq 0$ *for any* $x \in [\underline{\theta}, \overline{\theta}]$.

Proof. Let $\Delta(x) \equiv E^+(x) - x$, and $\lambda(x) \equiv \frac{f(x)}{1-F(x)}$. By Condition HR, $\lambda'(x) \geq 0$. Simple differentiation shows that $E^{+\prime}(x) = \lambda(x)[E^+(x) - x]$,

$$\Delta'(x) = \lambda(x)\Delta(x) - 1 \text{ and } \Delta''(x) = \lambda'(x)\Delta(x) + \lambda(x)\Delta'(x).$$

Let $\delta \equiv \Delta'$. Note that since $F(x)$ is differentiable, $\delta(x)$ is continuous. Moreover, a recursive limit argument implies that $\delta(x) \to -\frac{1}{2}$ as $x \to \overline{\theta}^-$. Next, suppose $\delta(x_0) > 0$ for some $x_0 \in [\underline{\theta}, \overline{\theta})$. We will argue that this should imply $\delta(x) > 0$ for all $x \in (x_0, \overline{\theta})$, and yield a contradiction to $\delta(\overline{\theta}^-) = -\frac{1}{2} < 0$.

Suppose $\delta(x_1) \leq 0$ for some $x_1 \in (x_0, \overline{\theta})$. Then, there exists some $\widehat{x} \in (x_0, x_1)$ such that $\delta(\widehat{x}) > 0$ and $\delta'(\widehat{x}) \leq 0$. On the other hand, given $\lambda'(\widehat{x}) \geq 0$, $\delta(\widehat{x}) > 0$ implies $\delta'(\widehat{x}) > 0$ – a contradiction. Thus, $\delta(x) > 0$ for all $x \in (x_0, \overline{\theta})$. But, since $\delta(\overline{\theta}^-) < 0$, $\delta(\overline{\theta} - \varepsilon) < 0$ for a small $\varepsilon > 0$ – a contradiction. Hence, $\delta(x) = \Delta'(x) \leq 0$ for all $x \in [\underline{\theta}, \overline{\theta}]$. $\square$

Proof of Proposition 3. It is more convenient to first prove part (ii). Suppose that Condition HR holds. Then, Lemma A2 above implies that $E^+(\widetilde{\theta}) - \widetilde{\theta}$ is decreasing in $\widetilde{\theta}$, which, in turn, implies that $h(\widetilde{\theta}; \alpha, n)$ is strictly decreasing in $\widetilde{\theta}$. Moreover, since

$E^+[\underline{\theta}] = E[\theta] = 0$ and $E^+[\overline{\theta}] = \overline{\theta}$, we have $h(\underline{\theta}; \alpha, n) > 0$ and $h(\overline{\theta}; \alpha, n) < 0$. Together, there must be a unique solution $\theta_h(\alpha, n) \in (\underline{\theta}, \overline{\theta})$ to $h(\widetilde{\theta}; \alpha, n) = 0$. Next, observe that $w(\theta^*; k, n) = h(\theta^*; \alpha, n)$ because $V(\theta^*; k, n, \alpha) = 0$ by (3) and $w(\theta^*; k, n) = \frac{kE^+(\theta^*)+(n-k)E^-(\theta^*)}{n}$ by (5). This means that the equilibrium conditions (8) and (9) can be replaced by $h(\theta^*; \alpha, n) \geq 0$ and $\theta^* = \widehat{\theta}(k^*, n, \alpha)$, where k^* is the smallest integer that satisfies the inequality. Suppose $\theta^* = \theta_h(\alpha, n)$ is an equilibrium cutoff. Then, $w(\theta_h(\alpha, n); k, n) = 0$, which, solving for k, yields $\widetilde{k} = \frac{-E^-(\theta_h(\alpha,n))}{-E^-(\theta_h(\alpha,n))+E^+(\theta_h(\alpha,n))} n$. Using conditional expectations, note that in general, $E[\theta] = \Pr\{\theta \geq \theta_h(.)\} E[\theta | \theta \geq \theta_h(.)] + \Pr\{\theta \leq \theta_h(.)\} E[\theta | \theta \leq \theta_h(.)]$, which, given $E[\theta] = 0$, reveals that

$$1 - F(\theta_h(.)) = \frac{-E^-(\theta_h(\alpha, n))}{-E^-(\theta_h(\alpha, n)) + E^+(\theta_h(\alpha, n))}.$$

Hence, $\widetilde{k} = [1 - F(\theta_h(\alpha, n))] \times n$. Clearly, $\widetilde{k} \in (0, n)$. But, $\theta^* = \theta_h(\alpha, n)$ is part of an equilibrium only if $\widetilde{k}$ is an integer, in which case $k^* = \widetilde{k}$, as given in Lemma 4. If $\widetilde{k}$ is not an integer, then $k^* = \left\lfloor \widetilde{k} \right\rfloor$ or $\left\lceil \widetilde{k} \right\rceil$. Suppose $k^* = \left\lfloor \widetilde{k} \right\rfloor$. Then, $\theta^* = \widehat{\theta}(\left\lfloor \widetilde{k} \right\rfloor, n, \alpha) > \theta_h(\alpha, n)$ by Lemma 2, implying $h(\theta^*; \alpha, n) < 0$, which contradicts the equilibrium requirement. Hence, $k^* = \left\lceil \widetilde{k} \right\rceil$, and in turn, $\theta^* = \widehat{\theta}(\left\lceil \widetilde{k} \right\rceil, n, \alpha)$ as stated in part (ii). Given the uniqueness of $\theta_h(\alpha, n)$, there can be at most one equilibrium pair such that $(k^*, \theta^*) \neq (k^o, 0)$, completing the proof of part (ii).

To prove part (i) of Proposition 3, note first that $h(\theta; \alpha, n)$ strictly increases in α and strictly decreases in n; and, as a result, $\theta_h(\alpha, n)$ strictly increases in α and strictly decreases in n. In addition, $\theta_h(\alpha, n)$ has the following limit properties: $\lim_{\alpha \to 0} \theta_h(\alpha, n) = 0$; $\lim_{\alpha \to 1} \theta_h(\alpha, n) = \overline{\theta}$; and $\lim_{n \to \infty} \theta_h(\alpha, n) = 0$. Together, these imply that $k^{so}(\alpha, n)$ decreases in α and increases in n. Moreover, $\lim_{\alpha \to 0} k^{so}(\alpha, n) = \lceil (1 - F(0)) \times n \rceil$ and $\lim_{\alpha \to 1} k^{so}(\alpha, n) = 1$. Note that $(1 - F(0)) \times n = k^s(n) - \frac{E^+(0)}{-E^-(0)+E^+(0)}$ by (4). Since $k^s(n)$ is an integer by assumption and $\frac{E^+(0)}{-E^-(0)+E^+(0)} \in (0, 1)$, we have $\lim_{\alpha \to 0} k^{so}(\alpha, n) = k^s(n)$. Given that $k^{so}(\alpha, n)$ decreases in α; $\lim_{\alpha \to 0} k^{so}(\alpha, n) = k^s(n)$; and $\lim_{\alpha \to 1} k^{so}(\alpha, n) = 1$, there is some $\underline{\alpha}(n) \in (0, 1)$ such that $k^{so}(\alpha, n) = k^s(n)$ for all $\alpha < \underline{\alpha}(n)$, and $k^{so}(\alpha, n) < k^s(n)$ for $\alpha \geq \underline{\alpha}(n)$. Since, by definition, $k^{so}(\alpha, n)$ is part of a suboptimal equilibrium whenever $k^{so}(\alpha, n) \neq k^o(n)$ and $k^o(n) = k^s(n)$ by Proposition 2, the desired conclusion in part (i) is obtained. □

Proof of Proposition 4. The finding that $k^{so}(\alpha, n)$ decreases in α, and $k^{so}(\alpha, n) < k^o(n)$ directly follows from the proof of Proposition 3. Again, from the same proof, we know $\lim_{\alpha \to 1} k^{so}(\alpha, n) = 1$. Thus, there exists $\overline{\alpha}(n) \in (\underline{\alpha}(n), 1)$ such that $k^{so}(\alpha, n) = 1$ for

$\alpha \geq \overline{\alpha}(n)$. $\square$

Proof of Proposition 5. Take a sequence of n such that integer problems do not arise. Along this sequence, since $k^o(n) = k^s(n)$ by Proposition 2, $\frac{k^o(n)}{n} = \frac{F(0)}{n} + 1 - F(0)$, which strictly decreases in n, and converges to $1 - F(0)$ as $n \to \infty$. Next, using Proposition 3, note that $\frac{k^{so}(\alpha,n)}{n} = 1 - F(\theta_h(\alpha, n))$ when integer problem is ignored. Then, since $\theta_h(\alpha, n)$ strictly decreases in n and converges to 0, it follows that $\frac{k^{so}(\alpha,n)}{n}$ increases in n and converges to $1 - F(0)$ as $n \to \infty$. As mentioned in the text, for $\alpha = 0$, sincere voting is obtained for any n, and thus $\frac{k^o(n)}{n} = \frac{k^{so}(\alpha,n)}{n} = 1 - F(0)$. $\square$

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