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Time dependence of advection-dominated accretion flow with a toroidal magnetic field

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ABSTRACT

The present study examines the self-similar evolution of advection-dominated accretion flow (ADAF) in the presence of a toroidal magnetic field. In this research, it was assumed that angular momentum transport is due to viscous turbulence and the α-prescription was used for the kinematic coefficient of viscosity. The flow does not have a good cooling efficiency and so a fraction of energy accretes along with matter on to the central object. The effect of a toroidal magnetic field on such a system with regard to the dynamical behaviour was investigated. In order to solve the integrated equations that govern the dynamical behaviour of the accretion flow, a self-similar solution was used. The solution provides some insights into the dynamics of quasi-spherical accretion flow, and avoids many of the strictures of steady self-similar solutions. The solutions show that the behaviour of the physical quantities in a dynamical ADAF is different from that for a steady accretion flow or a disc using a polytropic approach. The effect of the toroidal magnetic field is considered using additional variable $\beta\,(= p_{\rm mag}/p_{\rm gas}$, where $p_{\rm mag}$ and $p_{\rm gas}$ are the magnetic and gas pressure, respectively). Also, to consider the effect of advection in such systems, the advection parameter f, which stands for the fraction of energy that accretes by matter on to the central object, was introduced. The solution indicates a transonic point in the accretion flow for all selected values of f and β. Also, by increasing the strength of the magnetic field and the degree of advection, the radial thickness of the disc decreases and the disc compresses. The model implies that the flow has differential rotation and is sub-Keplerian at small radii and super-Keplerian at large radii, and that different results were obtained using a polytropic accretion flow. The β parameter obtained was a function of position, and increases with increasing radii. Also, the behaviour of ADAF in a large toroidal magnetic field implies that different results are obtained using steady self-similar models in large magnetic fields.

Key words: accretion, accretion discs – MHD.

1 INTRODUCTION

During recent years, one type of accretion disc has been studied in which it is assumed that the energy released through viscous processes in the disc may be trapped within the accreting gas. This kind of flow is known as advection-dominated accretion flow (ADAF). The basic ideas of ADAF models have been developed by a number of researchers (e.g. Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1994, 1995; Abramowicz et al. 1995; Ogilvie 1998; Akizuki & Fukue 2006, hereafter AF06).

It is thought that accretion discs, whether in star-forming regions, X-ray binaries, cataclysmic variables or the centres of active galactic nuclei, are likely to be threaded by magnetic fields. Consequently, the role of magnetic fields on ADAF has been analysed in detail by a number of investigators (Bisnovatyi-Kogan & Lovelace 2001; Shadmehri 2004; AF06; Ghanbari, Salehi & Abbassi 2007; Abbassi, Ghanbari & Najjar 2008). The existence of toroidal magnetic fields has been proven in the outer regions of young stellar object (YSO) discs (Greaves, Holland & Ward-Thompson 1997; Aitken et al. 1993; Wright et al. 1993) and in the Galactic Centre (Novak et al. 2003; Chuss et al. 2003). Thus, accretion discs with a toroidal magnetic field have been studied by several authors (AF06; Begelman & Pringle 2007; Abbassi et al. 2008; Khesali & Faghei 2008 and references within, hereafter KF08). KF08 considered the dynamical behaviour of a polytropic accretion flow in the presence of a toroidal magnetic field. In a dynamic approach they showed that the radial behaviour of the physical quantities was different from the results achieved by those who considered the accretion flow using steady self-similar methods (Shadmehri 2004; AF06;

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Ghanbari et al. 2007; Abbassi et al. 2008). For example, KF08 proposed that the ratio of the magnetic pressure to the gas pressure is not constant and varies with radius. The results of KF08 were based on the polytropic equation, which implies that the accreting gas has a good cooling efficiency, while the results of some authors have shown that the behaviour of physical quantities is very sensitive to the fraction of the energy that is trapped within the accreting gas (AF06). In the present study it is therefore intended to investigate the dynamical behaviour of an ADAF in the presence of a toroidal magnetic field.

The paper is organized as follows. In Section 2, the general problem of constructing a model for a quasi-spherical magnetized advection-dominated accretion flow will be defined. In Section 3, a self-similar method for solving the integrated equations that govern the dynamic behaviour of the accreting gas is utilized. A summary of the model appears in Section 4.

2 BASIC EQUATIONS

In this section, we derive the basic equations that describe the physics of accretion flow with a toroidal magnetic field. We use the spherical coordinates (r, θ, ϕ) centred on the accreting object and make the following standard assumptions.

(i) The accreting gas is a highly ionized gas with infinite conductivity.

(ii) The magnetic field has only an azimuthal component.

(iii) The gravitational force on a fluid element is characterized by the Newtonian potential of a point mass, $\Psi = -GM_*/r$, with G representing the gravitational constant and M_* standing for the mass of the central star.

(iv) The equations written in spherical coordinates are considered in the equatorial plane $\theta = \pi/2$ and terms with any θ and φ dependence are neglected, hence all quantities will be expressed in terms of spherical radius r and time t.

(v) For simplicity, self-gravity and general relativistic effects have been neglected.

Under the assumptions, the approximation of quasi-spherical symmetry and ideal magnetohydrodynamics treatment, the dynamics of a magnetized accretion flow is described by the following equations: the continuity equation,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_r) = 0, \tag{1}$$

the radial force equation,

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{GM_*}{r^2} = r\Omega^2 - \frac{B_\varphi}{4\pi r \rho}\frac{\partial}{\partial r}(rB_\varphi), \tag{2}$$

the azimuthal force equation,

$$\rho\left[\frac{\partial}{\partial t}(r^2\Omega) + v_r \frac{\partial}{\partial r}(r^2\Omega)\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\nu\rho r^4 \frac{\partial \Omega}{\partial r}\right], \tag{3}$$

the energy equation,

$$\frac{1}{\gamma - 1}\left[\frac{\partial p}{\partial t} + v_r \frac{\partial p}{\partial r}\right] + \frac{\gamma}{\gamma - 1}\frac{p}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) = f\nu\rho r^2 \left(\frac{\partial \Omega}{\partial r}\right)^2, \tag{4}$$

and the field-freezing equation,

$$\frac{\partial B_\varphi}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r v_r B_\varphi) = 0. \tag{5}$$

Here ρ is the density, v_r the radial velocity, Ω the angular velocity, M_* the mass of the central object, p the gas pressure, B_φ the toroidal component of the magnetic field and ν the kinematic viscosity coefficient, which is given, as in Narayan & Yi (1995), by an α-model:

$$\nu = \alpha \frac{p_{\rm gas}}{\rho \Omega_{\rm K}}, \tag{6}$$

where $\Omega_{\rm K} = (GM_*/r^3)^{1/2}$ is the Keplerian angular velocity. The parameters γ and α are assumed to be constant; f measures the degree to which the flow is advection-dominated (Narayan & Yi 1994) and is assumed to be constant.

3 SELF-SIMILAR SOLUTIONS

3.1 Analysis

Self-similar models have proved very useful in astrophysics because the similarity assumption reduces the complexity of the partial differential equations. Even greater simplification is achieved in the case of spherical symmetry, since the governing equations then reduce to comparatively simple ordinary differential equations. We introduce a similarity variable η and assume that each physical quantity is given by the following form:

$$r = r_0(t)\eta, \tag{7}$$

$$\rho(r, t) = \rho_0(t)R(\eta), \tag{8}$$

$$p(r, t) = p_0(t)P(\eta), \tag{9}$$

$$v_r(r, t) = v_0(t)V(\eta), \tag{10}$$

$$\Omega(r, t) = \Omega_0(t)\omega(\eta), \tag{11}$$

$$B_\varphi(r, t) = b_0(t)B(\eta). \tag{12}$$

By assuming a time-dependent power-law relation, $r_0(t) = at^n$, where $n = 2/3$, we find the following relations:

$$r_0(t) = at^{2/3}, \tag{13}$$

$$p_0(t)/\rho_0(t) = \frac{GM_*}{a}t^{-2/3}, \tag{14}$$

$$v_0(t) = \sqrt{\frac{GM_*}{a}}t^{-1/3}, \tag{15}$$

$$\Omega_0(t) = \sqrt{\frac{GM_*}{a^3}}t^{-1}, \tag{16}$$

$$b_0^2(t)/8\pi\rho_0(t) = \frac{GM_*}{a}t^{-2/3}. \tag{17}$$

The above results imply that $p_0(t)$ and $b_0(t)$ are dependent on the time behaviour of $\rho_0(t)$. Therefore, for specifying the time dependence of $\rho_0(t)$, and then $p_0(t)$ and $b_0(t)$, we introduce the mass accretion rate $\dot{M}$:

$$\dot{M} = -4\pi r^2 \rho v_r. \tag{18}$$

Similarly to equations (7)–(12) for the mass accretion rate, we can write

$$\dot{M}(r, t) = \dot{M}_0(t)\dot{m}(\eta). \tag{19}$$

Under transformations of equations (7), (8) and (10), equation (19) becomes

$$\dot{M}(r, t) = \left[r_0^2(t)\rho_0(t)v_0(t)\right] \times \left[-4\pi\eta^2 R(\eta)V(\eta)\right], \tag{20}$$

which implies

$$\dot{M}_0(t) = r_0^2(t)\rho_0(t)v_0(t), \tag{21}$$

$$\dot{m}(\eta) = -4\pi\eta^2 R(\eta)V(\eta). \tag{22}$$

We now consider a set of solutions for which $\dot{M}_0(t)$ is a constant (KF08), thus we can write

$$\rho_0(t) = \left(\dot{M}_0/\sqrt{GM_* a^3}\right) t^{-1}, \tag{23}$$

which implies

$$p_0(t) = \left(\dot{M}_0\sqrt{GM_*/a^5}\right) t^{-5/3} \tag{24}$$

and

$$b_0^2(t)/8\pi = \left(\dot{M}_0\sqrt{GM_*/a^5}\right) t^{-5/3}. \tag{25}$$

Substituting equations (6)–(12) and (13)–(17) into the basic equations (1)–(6), the similarity equations are obtained as

$$-R + \left(V - \frac{2\eta}{3}\right)\frac{\mathrm{d}R}{\mathrm{d}\eta} + \frac{R}{\eta^2}\frac{\mathrm{d}}{\mathrm{d}\eta}(\eta^2 V) = 0, \tag{26}$$

$$-\frac{V}{3} + \left(V - \frac{2\eta}{3}\right)\frac{\mathrm{d}V}{\mathrm{d}\eta} + \frac{1}{R}\frac{\mathrm{d}P}{\mathrm{d}\eta} + \frac{1}{\eta^2} = \eta\omega^2 - \frac{2B}{\eta R}\frac{\mathrm{d}(\eta B)}{\mathrm{d}\eta}, \tag{27}$$

$$R\left[\frac{1}{3}\left(\eta^2\omega\right) + \left(V - \frac{2\eta}{3}\right)\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta^2\omega\right)\right] = \frac{\alpha}{\eta^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left[P\eta^{11/2}\frac{\mathrm{d}\omega}{\mathrm{d}\eta}\right], \tag{28}$$

$$\begin{aligned}\frac{1}{\gamma-1}\left[-\frac{5}{4}P + \left(V - \frac{2\eta}{3}\right)\frac{\mathrm{d}P}{\mathrm{d}\eta}\right] + \frac{\gamma}{\gamma-1}\frac{P}{\eta^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta^2 V\right) \\ = \alpha f P\eta^{7/2}\left(\frac{\mathrm{d}\omega}{\mathrm{d}\eta}\right)^2,\end{aligned} \tag{29}$$

$$-\frac{5}{4}B + \left(V - \frac{2\eta}{3}\right)\frac{\mathrm{d}B}{\mathrm{d}\eta} + \frac{B}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}(\eta V) = 0. \tag{30}$$

To investigate the existence of the transonic point, the square of the sound velocity is introduced, which subsequently can be expressed as

$$v_\mathrm{s}^2 \equiv \frac{p}{\rho} = \frac{GM_*}{a}\frac{P}{R}t^{-2/3}. \tag{31}$$

Here, $S = (P/R)^{1/2}$ is the *sound velocity* in a self-similar flow, which is rescaled in the course of time. The *Mach number* referred to the reference frame is defined as (Fukue 1984; Gaffet & Fukue 1983)

$$\mu \equiv \frac{v_r - v_\mathrm{F}}{v_\mathrm{s}} = \frac{V - n\eta}{S}, \tag{32}$$

where

$$v_\mathrm{F} = \frac{\mathrm{d}r}{\mathrm{d}t} = n\frac{r}{t} \tag{33}$$

is the velocity of the reference frame, which is moving outward as time goes by. The Mach number introduced so far represents the *instantaneous* and *local* Mach number of an unsteady self-similar flow. We will consider the transonic points of an accretion flow in the next subsection.

In order to consider the strength of the magnetic field in a plasma, the β parameter, which is the ratio of the magnetic to the gas pressures, is introduced:

$$\beta(r,t) = \frac{B_\varphi^2(r,t)/8\pi}{p(r,t)} = \frac{B^2(\eta)}{P(\eta)}. \tag{34}$$

To complete this section, we summarize our main results here. Solving equations (1), (10), (11) and (19) under transformations (12)–(15) in a non-magnetic state makes it clear that the time behaviour of the physical quantities in non-magnetic and magnetic discs is the same. This result is one of the strictures of the time-dependent self-similar solution. On the other hand, the fact that the time-dependent behaviour of the magnetic and gas pressures is similar limits the self-similarity solution. In contrast, physical quantities with the same physical dimension have similar behaviour.

3.2 Asymptotic behaviour

In this subsection, the asymptotic behaviour of the equations (22), (26)–(30) and (34) at $\eta \to 0$ and $\gamma < 5/3$ is investigated. The asymptotic solutions are given by

$$R(\eta) \sim R_0\eta^{-3/2}, \tag{35}$$

$$P(\eta) \sim P_0\eta^{-5/2}, \tag{36}$$

$$V(\eta) \sim V_0\eta^{-1/2}, \tag{37}$$

$$\omega(\eta) \sim \omega_0\eta^{-3/2}, \tag{38}$$

$$B(\eta) \sim B_0\eta^{-1/2}, \tag{39}$$

$$\dot{m}(\eta) \sim -4\pi R_0 V_0, \tag{40}$$

$$\beta(\eta) \sim \left(B_0^2/P_0\right)\eta^{3/2}, \tag{41}$$

in which

$$R_0 = -\frac{3}{8\pi}\alpha f \dot{m}_\mathrm{in}\left(\frac{\gamma-1}{\gamma-5/3}\right)\left(\frac{g_1}{g_3}\right), \tag{42}$$

$$P_0 = \frac{\dot{m}_\mathrm{in}}{6\pi\alpha}, \tag{43}$$

$$V_0 = \frac{2}{3\alpha f}\left(\frac{\gamma-5/3}{\gamma-1}\right)\left(\frac{g_3}{g_1}\right), \tag{44}$$

$$\omega_0 = -\frac{2}{3\alpha f}\left(\frac{\gamma-5/3}{\gamma-1}\right)\left(\frac{g_3}{g_1}\right)^{1/2}, \tag{45}$$

$$B_0^2 = \beta_0\frac{\dot{m}_\mathrm{in}}{6\pi\alpha}, \tag{46}$$

where

$$\frac{1}{g_1} = 1 - \frac{5f}{2}\left(\frac{\gamma-1}{\gamma-5/3}\right), \tag{47}$$

$$g_2 = \frac{3}{2}\alpha f\left(\frac{\gamma-1}{\gamma-5/3}\right), \tag{48}$$

$$g_3 = -1 + \sqrt{1 + 2g_1^2 g_2^2}, \tag{49}$$

$$\beta_0 = \beta_\mathrm{in}/\eta_\mathrm{in}^{3/2}. \tag{50}$$

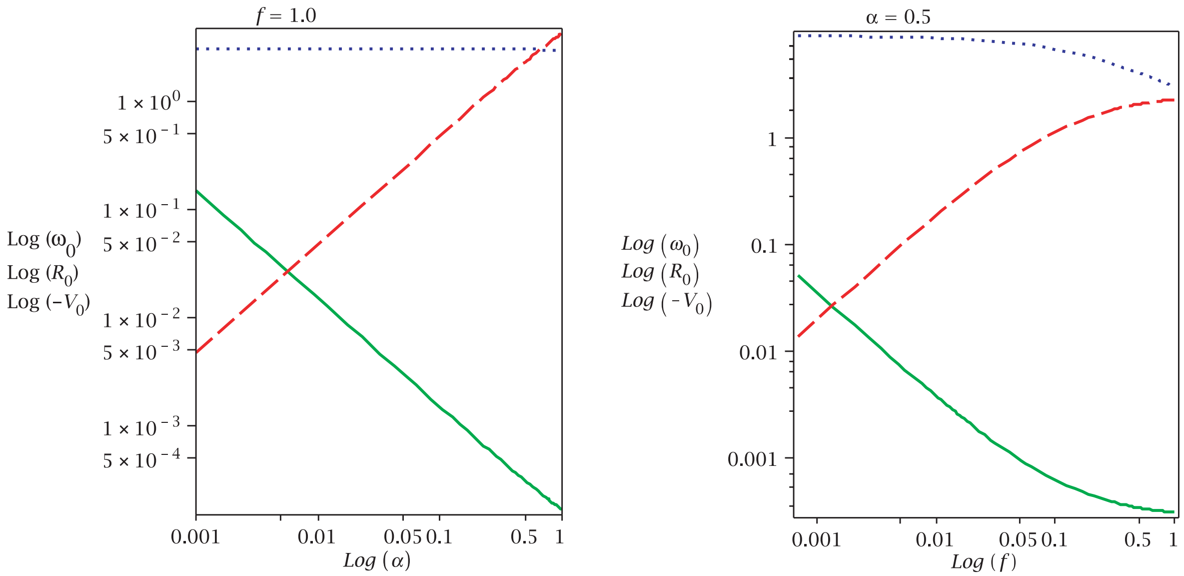

Figure 1. Numerical coefficient ω_0 (dotted lines), R_0 (solid lines) and V_0 (dashed lines) as functions of the advection parameter f or the viscous parameter α. The ratio of specific heats is set to be $\gamma = 1.5$ and the inner mass accretion rate is $\dot{m}_{\rm in} = 0.001$.

The results achieved for asymptotic behaviour of physical quantities show that the physical quantities of accretion flow are very sensitive to parameters α, γ, f, $\beta_{\rm in}$ and $\dot{m}_{\rm in}$.$\beta_{\rm in}$ and $\dot{m}_{\rm in}$ are the values of β and $\dot{m}$ at $\eta_{\rm in}$, where $\eta_{\rm in}$ is a point near to the centre. The effects of the viscous parameter α and the advection parameter f on accretion flow are plotted in Fig. 1. The angular velocity profiles indicate that by increasing the viscous parameter α, the angular velocity of the accretion flow decreases, because we increase the viscous torque by increasing parameter α. Also, increasing the advection parameter f decreases the angular velocity, which is qualitatively consistent with AF06. Fig. 1 shows radial infall velocity increases resulting from adding α and f that are similar to the results of AF06 and KF08. Also the density profiles represent density decreases caused by adding f and α.

3.3 Numerical solutions

If the value of $\eta_{\rm in}$ is guessed, i.e. we take a point very near to the centre, the equations can be integrated from this point outward through the use of the above expansion. Examples of such solutions are presented in Figs 2, 3 and 4. The profiles in Fig. 2 are plotted for different $\beta_{\rm in}$, the profiles in Fig. 3 are plotted for different f and in Fig. 4 the transonic behaviour of the accreting gas for different values of f and $\beta_{\rm in}$ is considered. The delineated quantities ($\log(\eta^{3/2} R)$, $\log(-\eta^{1/2}\,V)$, ...) in Figs 2, 3 and 4 are constant in steady self-similar solutions (Narayan & Yi 1994, 1995; Shadmehri 2004; AF06; Ghanbari et al. 2007; Abbassi et al. 2008), while here they vary with position.

Fig. 2 informs us that the density and radial thickness of the disc decrease on increasing the strength of the toroidal magnetic field; these results are consistent with KF08. Also, by decreasing the amount of the magnetic field, the behaviour of the density becomes similar to the non-magnetic case (Ogilvie 1999). In KF08 the gas pressure displayed polytropic behaviour, causing it to follow the density behaviour, while here we see that the behaviour of the gas pressure does not follow the density behaviour. Also, by adding the β parameter the radial infall velocity increases; such a property is qualitatively consistent with AF07 and KF08. This is due to the magnetic tension terms, which dominate the magnetic pressure term in the radial momentum equation and assist the radial infall motion. The profiles of the angular velocity imply that the disc is sub-Keplerican in the inner part and super-Keplerian in the outer part, while for a polytropic accreting flow (KF08) or a non-magnetic accretion flow (Ogilvie 1999) the angular velocity is sub-Keplerian at all radii (KF08). Similarly to the results for the β parameter in KF08, the ratio of the magnetic pressure to the gas pressure is a function of position and increases when moving outward, a result that is consistent with the observational evidence obtained by some authors (Aitken et al. 1993; Wright et al. 1993; Greaves et al. 1997). While the β parameter in a steady self-similar solution becomes constant at all radii (AF06), that is one of the restrictions of such a solution. Fig. 3 is plotted for different values of the advection parameter f. The advection parameter f has a slight effect on the toroidal magnetic field, the β parameter and the Mach number, and a significant effect on the density, gas pressure, radial infall velocity and angular velocity. The density and the radial thickness of the disc decrease with greater advection of accreting gas, an effect that is the same at all parts of the disc; this result can be achieved by assuming that the value of f is constant. Also we see that by increasing the value of the advection parameter f the gas pressure decreases, the radial infall velocity increases and the angular velocity decreases. These results are qualitatively consistent with the results of AF06.

The Mach number profiles in Fig. 4 imply that the flow of the outer part of the disc for all selected values of the magnetic field becomes supersonic. We can see this result for polytropic accretion flow in KF08. The advection parameter decreases the value of the Mach number slightly.

The profiles of the physical quantities in Fig. 2 imply that they have a power-law dependence on η in the magnetic domain ($\beta_{\rm in} > 1$). Therefore, by fitting a power-law function to data in the magnetic domain ($\beta_{\rm in} = 10$), we can write

$$R(\eta) \propto \eta^{-1.66}, \tag{51}$$

$$P(\eta) \propto \eta^{-2.58}, \tag{52}$$

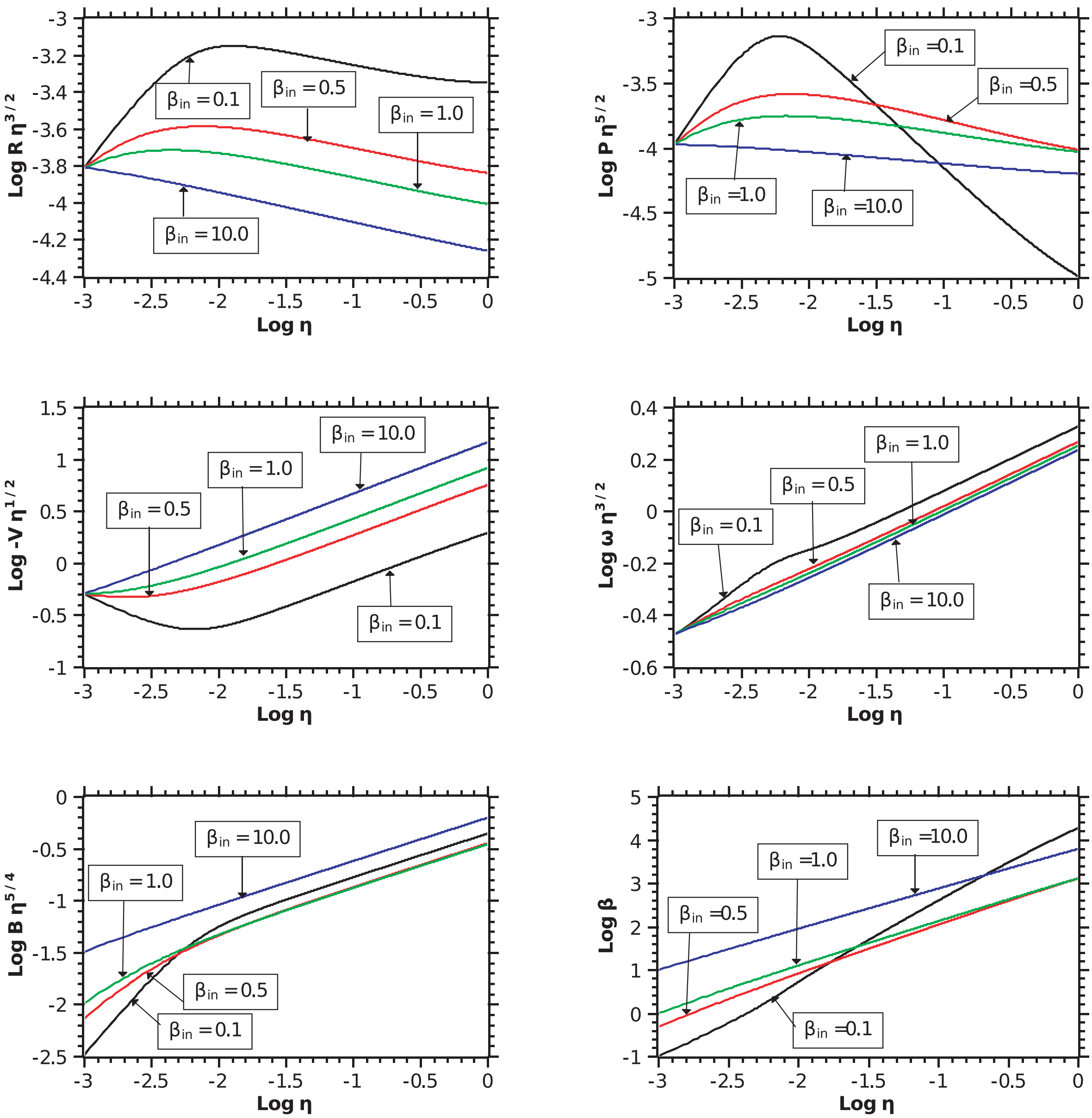

Figure 2. Time-dependent self-similar solution for $\gamma = 1.5$, $\alpha = 0.5$, $f = 1.0$ and $\dot{m}_{\rm in} = 0.001$. The lines represent $\beta_{\rm in} = 1.0 = 0.1, 0.5, 1.0, 10$, where $\beta_{\rm in}$ is the value of β at $\eta_{\rm in}$.

$$V(\eta) \propto \eta^{-0.01}, \tag{53}$$

$$\omega(\eta) \propto \eta^{-1.25}, \tag{54}$$

$$B(\eta) \propto \eta^{-0.83}, \tag{55}$$

$$\beta(\eta) \propto \eta^{-0.92}, \tag{56}$$

$$\mu(\eta) \propto \eta^{0.93}, \tag{57}$$

$$\dot{m}(\eta) \propto \eta^{-0.33}. \tag{58}$$

The results achieved are different from those for steady magnetic-dominated accretion flow (Meier 2005; Shadmehri & Khajenabi 2005).

4 SUMMARY AND DISCUSSION

In this paper, the equations of a time-dependent advection-dominated accretion flow with a toroidal magnetic field have been solved by semi-analytical similarity methods. The flow is not able to radiate efficiently, so we substituted the energy equation instead of the polytropic equation used by KF08. A solution was found for the case $\gamma < 5/3$, which has differential rotation and viscous dissipation. The flow avoids many of the strictures of steady self-similar solutions (Narayan & Yi 1994; AF06; Ghanbari et al. 2007; Abbassi et al. 2008). Thus, the radial dependence of the calculated physical quantities in this approach is different from that for a steady self-similar solution.

Increasing the advection parameter f and the parameter $\beta_{\rm in}$ will separately increase the infall radial velocity and decrease the angular velocity. The flow has differential rotation and is sub-Keplerian in the inner part and super-Keplerian at large radii, a behaviour seen in

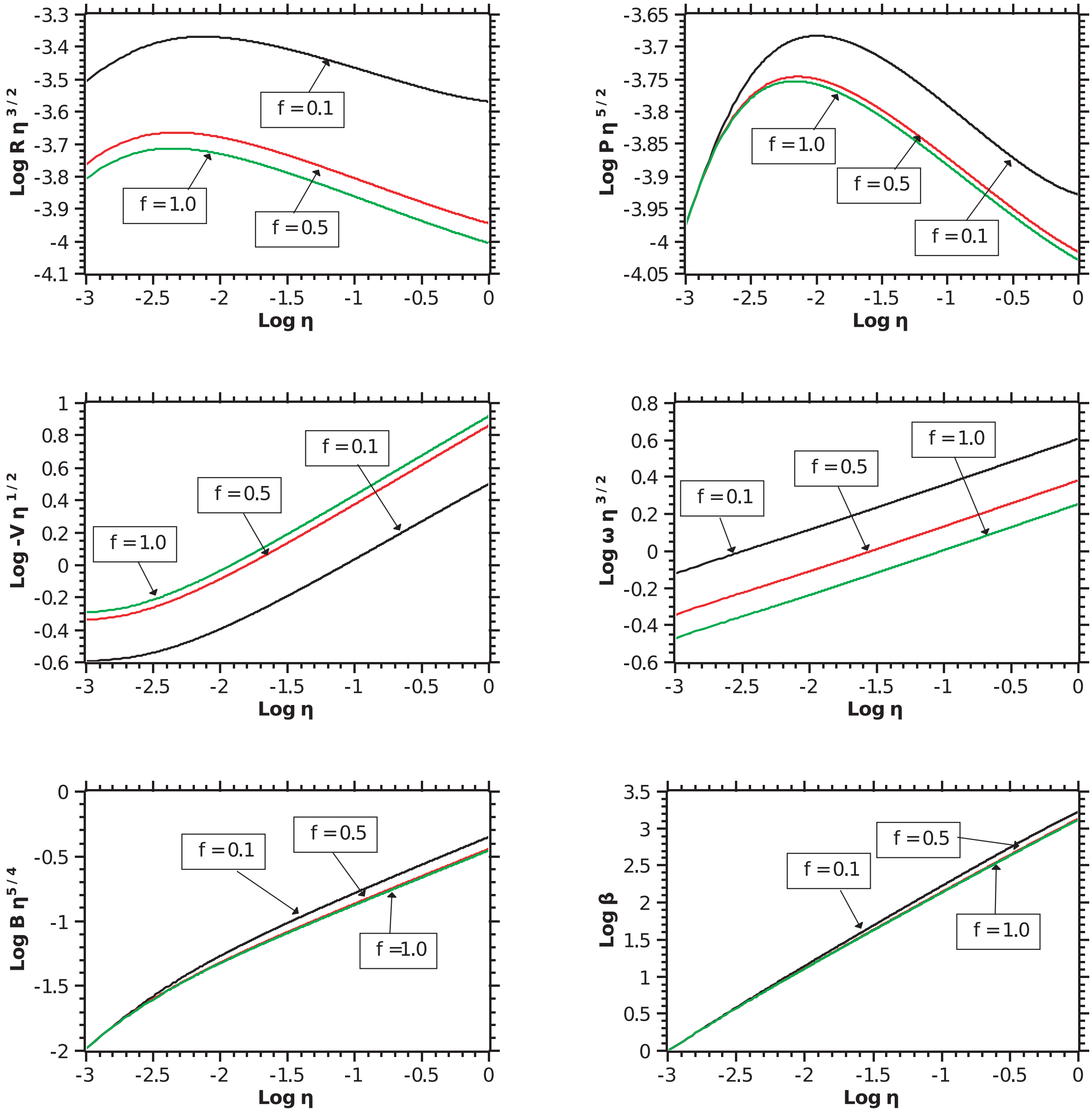

Figure 3. Time-dependent self-similar solution for $\gamma = 1.5$, $\alpha = 0.5$, $\beta_{\rm in} = 1.0$ and $\dot{m}_{\rm in} = 0.001$. The lines represent $f = 0.1, 0.5, 1.0$.

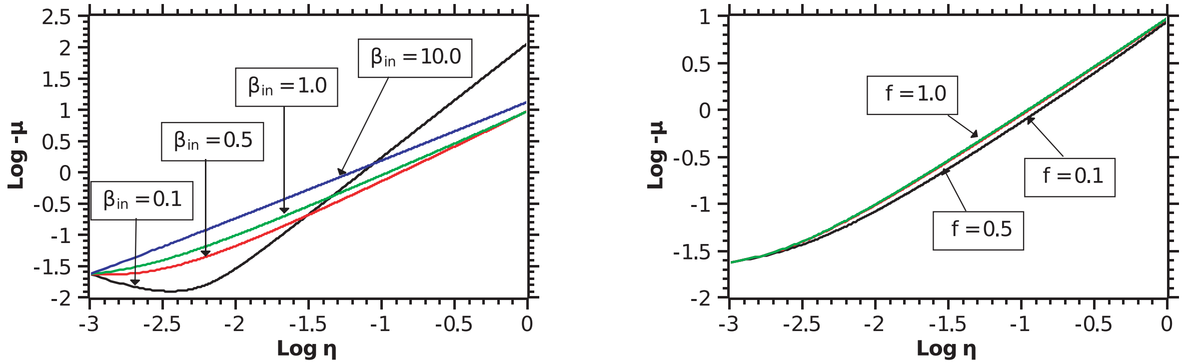

Figure 4. Left panel: Mach number profiles for $\gamma = 1.5$, $\alpha = 0.5$, $f = 1.0$ and $\dot{m}_{\rm in} = 0.001$. Right panel: Mach number profiles for $\gamma = 1.5$, $\alpha = 0.5$, $\beta_{\rm in} = 1.0$ and $\dot{m}_{\rm in} = 0.001$.

some astrophysical objects such as M81, M87 and the Milky Way (Sofue 1998; Ford & Tsvetanov 1999). The solution showed that the flow for all selected values of f and $\beta_{\rm in}$ becomes supersonic at large radii and subsonic at small radii, which is qualitatively consistent with the results of KF08. The parameter β is a function of position that increases moving outward, and shows that the magnetic field is more important at large radii. It is also consistent with observational evidence in the outer regions of YSO discs (Aitken et al. 1993; Wright et al. 1993; Greaves et al. 1997) and in the Galactic Centre (Chuss et al. 2003; Novak et al. 2003).

Here, the latitudinal dependence of physical quantities is ignored, although some authors have shown that latitudinal dependence is important for the structure of a disc (Narayan & Yi 1995; Ghanbari et al. 2007). The latitudinal behaviour of such discs can be investigated in other studies. Also, we did not consider relativistic effects. If the central object is relativistic, the gravitational field should be altered. Furthermore, in a realistic model the advection parameter f is a function of position and time; other researchers can take this into account.

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