

Time-Dependent Axial Velocity Effects on Mhd Free Convection Flow and Heat Transfer of a Viscous Fluid over a Vertical Moving Cylinder with Thermal Conductivity and Viscosity Depending on Temperature

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Abstract: In this present work the effects of time-dependent axial velocity Effects on MHD free convection flow of viscous fluid over a vertical moving cylinder with thermal conductivity and viscosity depending on temperature, the surface of which is exposed to a constant wall temperatures. Numerical solutions of the Navier-Stokes equations, concentration equation and energy equation are derived in this problem. A reduction of these equations is obtained by use of appropriate transformations. The semi-similar solution of the Navier-Stokes equations, concentration equation and energy equation has been obtained numerically using by the implicit finite difference scheme of Crank-Nicolson's type. The velocity, concentration and temperature profiles for different values of cylinder axial velocity are plotted. The influence of the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, viscosity-variation parameter, thermal conductivity variation parameter and magnetic parameter for different values of cylinder axial velocity on free convection flow and heat transfer are discussed. It is observed that, when Prandtl number increases the velocity and temperature decrease in the boundary layer. Also, it is found that as increase in the magnetic parameter leads to decrease in the velocity field and rise in the thermal boundary thickness.

Key words: mhd free convection • Time-dependent axial velocity • Moving cylinder • Temperature-dependent viscosity • Temperature-dependent thermal conductivity

INTRODUCTION

A study of the flow of electrically conducting fluid in presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. Natural convection flow of a dissipative fluid over a vertical moving cylinder with time-dependent axial velocity, thermal conductivity and viscosity depending on temperature under the influence of a uniform transverse magnetic field is an important problem relevant to many engineering applications. In the glass and polymer industries, hot filaments, which are considered as vertical cylinders, are cooled as they pass through the surrounding environment. Free convection flows are of great interest in a number of industrial applications such as fiber and

granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magnetohydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its application in MHD pumps, MHD bearings etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomena of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection

flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy.

Sparrow and Gregg [1] first studied the heat transfer from vertical cylinder. Goldstein and Briggs [2] presented an analysis of the transient free convective flow past vertical flat plate and circular cylinder to a surrounding initially quiescent fluid by employing Laplace transform technique. Nagendra *et al.* [3] presented a boundary layer analysis of free convection heat transfer from a vertical cylinder with uniform heat flux at its surface. An experimental and analytical study is reported by Evas *et al.* [4] for transient natural convection in a vertical cylinder. Velusamy and Grag [5], given a numerical solution for the transient natural convection over a heat generating vertical cylinder. The study of flow problems, which involve the interaction of several phenomena, has a wide range of applications in the field of science and technology. One such study is related to the effects of MHD free convection flow, which plays an important role in geophysics, astrophysics and petroleum industries. Michiyoshi *et al.* [6] considered natural convection heat transfer from a horizontal cylinder to mercury under a magnetic field. Magnetic field effect on a moving vertical cylinder with constant heat flux was given by Ganesan and Loganathan [7]. Free convection flow involving coupled heat and mass transfer occurs frequently in nature. It occurs not only due to temperature differences, but also due to concentration differences or a combination of these two, for example, in atmospheric flows there exist differences in the H₂O concentration. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing and environmental pollution. The effects of heat and mass transfer on natural convection flow over a vertical cylinder was studied by Chen and Yuh [8]. Combined heat and mass transfer effects on moving vertical cylinder for steady and unsteady flows were analyzed by Takhar *et al.* [9] and Ganesan and Loganathan [10] respectively. Gebhart & Pera [11] analysed the steady combined buoyancy effects of thermal and mass diffusion on vertical natural convection flows. Bottemanne [12] studied the combined effect of heat and mass transfer in the steady laminar boundary layer of a vertical cylinder placed in still air. Elgazery and

Hassan [13] presented a numerical study of radiation effect on MHD transient mixed convection flow over a moving vertical cylinder with constant heat flux through a porous medium. Reddy and Reddy [14, 15] presented a numerical study of the interaction of radiation and mass transfer effects on unsteady MHD free convection flow past a semi-infinite moving vertical cylinder by employing finite-difference scheme of Crank-Nicolson type. Recently R. K. Deka and A. Paul [16] studied the unsteady free convection flow past a moving vertical cylinder with constant temperature by employing Laplace transform technique. A numerical solution for the transient natural convection flow over a vertical cylinder under the combined buoyancy effect of heat and mass transfer was given by Ganesan and Rani [17], by employing an implicit finite-difference scheme. Shanker and Kishan [18] presented the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate. Ganesan and Rani [19] studied the MHD unsteady free convection flow past a vertical cylinder with heat and mass transfer. In the context of space technology and in processes involving high temperatures, the effects of radiation are of vital importance. On assuming that the viscosity of the fluid is linear functions of temperature, a semi-empirical formula was proposed by Charraudeau [20] which is appropriate for small Prandtl number. Studies of free convection flow along a vertical cylinder or horizontal cylinder are important in the field of geothermal power generation and drilling operations where the free stream and buoyancy induced fluid velocities are of roughly the same order of magnitude. Many researchers such as Arpacı [21], Cess [22], Cheng and Ozisik [23], Raptis [24], Hossain and Takhar [25, 26] have investigated the interaction of thermal radiation and free convection for different geometries, by considering the flow to be steady. The unsteady flow past a moving vertical plate in the presence of free convection and radiation were studied by Das *et al.* [27]. Radiation and mass transfer effects on two-dimensional flow past an impulsively started isothermal vertical plate were studied by Ramachandra Prasad *et al.* [28]. The combined radiation and free convection flow over a vertical cylinder was studied by Yih [29]. Radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving vertical cylinder was studied by Ganesan and Loganathan [30]. MHD natural convection flow from an isothermal horizontal circular cylinder under consideration of temperature dependent viscosity were studied by Molla *et al.* [31]. Rahman *et al.* [32] have investigated the effects

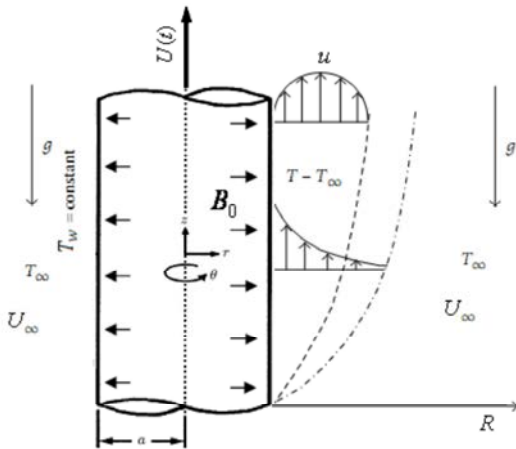


Fig. 1: Sketch of the physical model.

of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat conduction. MHD free convection flow along a vertical plate with thermal conductivity and viscosity depending on temperature was studied by Rehena and Alim [33].

The object of the present paper is to study the free convection flow of a viscous fluid on a vertical moving cylinder with time-dependent axial velocity, thermal conductivity and viscosity depending on temperature, under the influence of a uniform transverse magnetic field in the presence of constant wall temperatures. The dimensionless governing equations are solved by using an implicit finite difference scheme of Crank-Nicolson's type.

Mathematical Analysis: Consider the free convection flow of a viscous fluid on a vertical moving cylinder with time-dependent axial velocity, thermal conductivity and viscosity depending on temperature in constant wall temperatures (Fig. 1) under the action of a transverse magnetic field. Under these assumptions and Boussinesq's approximation, the flow is governed by the following system of equations:

Continuity Equation:

$$\frac{\partial(ru)}{\partial z} + \frac{\partial(rv)}{\partial r} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} = \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + \beta^* g (c - c_\infty) + \beta g (T - T_\infty) - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

Energy Equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} = \frac{1}{\rho C_p} \frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{\sigma B_0^2}{\rho C_p} u^2 + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial r} \right)^2 \tag{3}$$

Mass Equation:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial z} + v \frac{\partial c}{\partial r} = D \left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} \right) \tag{4}$$

where u and v are components of the velocity in z and r directions, respectively, t is the time, ν is the kinematic viscosity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric expansion coefficient for mass transfer, g is the acceleration due to gravity, ρ is the density, σ fluid electrical conductivity, B_0 is magnetic induction, α is fluid thermal diffusivity, c_p is specific heat at constant pressure, T is the temperature, T_∞ is the temperature of the fluid far away from the cylinder, C is the concentration, C_∞ is the concentration far away from the cylinder and D is the molecular diffusivity.

The necessary initial and boundary conditions are:

$$\begin{aligned} t \leq 0 : u = 0, v = 0, T = T_\infty, c = c_\infty \\ t > 0 : u = 0, v = 0, T = T_\infty, c = c_\infty, \text{ at } z = 0 \\ t > 0 : u = u_0 U(t), v = 0, T = T_w, c = c_w, \text{ at } r = a \\ t > 0 : u = 0, T \rightarrow T_\infty, c \rightarrow c_\infty, \text{ at } r \rightarrow \infty \end{aligned} \tag{5}$$

The variable thermal conductivity, which is used by Rahman *et al.* [32], as follows

$$k_f = k_\infty \left[1 + \delta^* (T - T_\infty) \right] \tag{6}$$

Where k_∞ is the thermal conductivity of the ambient fluid and δ^* is defined as follows

$$\delta^* = \frac{1}{k_f} \left(\frac{\partial k}{\partial T} \right)_f \tag{7}$$

Out of the many forms of viscosity variation, which are available in the literature, we will consider only following form proposed by Charraudeau [20]

$$\mu = \mu_\infty \left[1 + \gamma^* (T - T_\infty) \right] \tag{8}$$

Where

μ_∞ is the viscosity of the ambient fluid and γ^* is defined as follows

$$\gamma^* = \frac{1}{\mu_f} \left(\frac{\partial \mu}{\partial T} \right)_f \quad (9)$$

Here f denotes the film temperature of the fluid.

Now introduce the following non dimensional quantities:

$$\begin{aligned} \tau &= \frac{t u_\infty}{a^2}, U = \frac{u}{u_0}, V = \frac{v a}{u_\infty}, Z = \frac{z u_\infty}{u_0 a^2}, R = \frac{r}{a}, \gamma = \frac{1}{\mu_f} \left(\frac{\partial \mu}{\partial T} \right)_f (T_w - T_\infty) \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty}, C = \frac{c - c_\infty}{c_w - c_\infty}, Gr = \frac{g \beta a^2 (T_w - T_\infty)}{u_\infty \mu_0}, \delta = \frac{1}{k_f} \left(\frac{\partial k}{\partial T} \right)_f (T_w - T_\infty), \\ Sc &= \frac{u_\infty}{D} Gc = \frac{g \beta^* a^2 (C_w - C_\infty)}{u_\infty \mu_0}, M = \frac{\sigma B_0^2 a^2}{\mu_\infty}, Br = \frac{\mu_\infty u_0^2}{k(T_w - T_\infty)}, \\ Pr &= \frac{\mu_\infty C_p}{k}, u_\infty = \frac{\mu_\infty}{\rho} \end{aligned} \quad (10)$$

where a is cylinder radius, Z is the dimensionless axial coordinate, r is the dimensionless radial coordinate perpendicular to Z , U, V is the dimensionless velocities, θ is the dimensionless time, C is the dimensionless temperature, T_w is the non-dimensional species concentration, u_∞ is the temperature at the surface, Gc is the reference kinematic viscosity, Sc is the Schmidt number, Gc is the mass Grashof number, Gr is the thermal Grashof number, M is the magnetic parameter, Pr is the prandtl number, γ is the viscosity-variation parameter, δ is the thermal conductivity variation parameter and Br is the Brinkman number.

Continuity Equation:

$$\frac{\partial(RU)}{\partial Z} + \frac{\partial(RV)}{\partial R} = 0 \quad (11)$$

Momentum Equation:

$$\begin{aligned} \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial Z} + V \frac{\partial U}{\partial R} &= (1 + \gamma \theta) \left(\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right) \\ + \gamma \frac{\partial U}{\partial R} \frac{\partial \theta}{\partial R} &+ Gc.C + Gr.\theta - M.U \end{aligned} \quad (12)$$

Energy Equation:

$$\begin{aligned} \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial Z} + V \frac{\partial \theta}{\partial R} &= \frac{1}{Pr} (1 + \delta \theta) \left(\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R} \frac{\partial \theta}{\partial R} \right) \\ + \frac{\delta}{Pr} \left(\frac{\partial \theta}{\partial R} \right)^2 &+ \frac{Br}{Pr} M.U^2 + (1 + \gamma \theta) \frac{Br}{Pr} \left(\frac{\partial U}{\partial R} \right)^2 \end{aligned} \quad (13)$$

Mass Equation:

$$\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial Z} + V \frac{\partial C}{\partial R} = \frac{1}{Sc} \left(\frac{\partial^2 C}{\partial R^2} + \frac{1}{R} \frac{\partial C}{\partial R} \right) \quad (14)$$

The dimensionless boundary conditions become:

$$\begin{aligned} \tau \leq 0: U &= 0, V = 0, \theta = 0, C = 0 \\ \tau > 0: U &= 0, V = 0, \theta = 0, C = 0, \text{ at } Z = 0 \\ \tau > 0: U &= u_0, V = 0, \theta = 1, C = 1, \text{ at } R = 1 \\ \tau > 0: U &= 0, \theta = 0, C = 0, \text{ at } R \rightarrow \infty \end{aligned} \quad (15)$$

Numerical Solution of the Problem: The governing equations (11-14) are steady, coupled and non-linear with boundary conditions. An implicit finite-difference technique of Crank-Nicolson has been employed to solve the nonlinear coupled equations, as described (Thomas algorithm) in Carnahan *et al* [35]. The finite difference equations corresponding to equations (11-14) are as follows :

$$\frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n}{2\Delta Z} + \frac{V_{i,j+1}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j+1}^n - V_{i,j-1}^n}{4\Delta R} + \frac{V_{i,j}^{n+1} + V_{i,j}^n}{R} = 0 \quad (16)$$

$$\begin{aligned} \frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta \tau} + U_{i,j}^n \frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n}{2\Delta Z} + V_{i,j}^n \frac{U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^n - U_{i,j-1}^n}{4\Delta R} \\ = (1 + \gamma \theta_{i,j}^n) \left(\frac{U_{i,j+1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j-1}^{n+1} + U_{i,j+1}^n - 2U_{i,j}^n + U_{i,j-1}^n}{2\Delta R^2} + \frac{1}{R} \frac{U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^n - U_{i,j-1}^n}{4\Delta R} \right) \\ + \gamma \left(\frac{\theta_{i,j+1}^{n+1} - \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^n - \theta_{i,j-1}^n}{4\Delta R} \right) \left(\frac{U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^n - U_{i,j-1}^n}{4\Delta R} \right) + Gr \frac{\theta_{i,j}^{n+1} + \theta_{i,j}^n}{2} + Gc \frac{C_{i,j}^{n+1} + C_{i,j}^n}{2} - M \frac{U_{i,j}^{n+1} + U_{i,j}^n}{2} \end{aligned} \quad (17)$$

$$\begin{aligned} & \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta \tau} + U_{i,j}^n \frac{\theta_{i,j}^{n+1} - \theta_{i-1,j}^{n+1} + \theta_{i,j}^n - \theta_{i-1,j}^n}{2\Delta Z} + V_{i,j}^n \frac{\theta_{i,j+1}^{n+1} - \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^n - \theta_{i,j-1}^n}{4\Delta R} \\ & = \frac{(1 + \delta \theta_{i,j}^n)}{\text{Pr}} \left(\frac{\theta_{i,j+1}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^n - 2\theta_{i,j}^n + \theta_{i,j-1}^n}{2\Delta R^2} + \frac{1}{R} \frac{\theta_{i,j+1}^{n+1} - \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^n - \theta_{i,j-1}^n}{4\Delta R} \right) \\ & + \frac{\delta}{\text{Pr}} \left(\frac{\theta_{i,j+1}^{n+1} - \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^n - \theta_{i,j-1}^n}{4\Delta R} \right)^2 + \frac{Br}{\text{Pr}} M \left(\frac{U_{i,j}^{n+1} + U_{i,j}^n}{2} \right)^2 + (1 + \gamma \theta_{i,j}^n) \frac{Br}{\text{Pr}} \left(\frac{U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^n - U_{i,j-1}^n}{4\Delta R} \right)^2 \end{aligned} \tag{18}$$

$$\begin{aligned} & \frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta \tau} + U_{i,j}^n \frac{C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^n - C_{i-1,j}^n}{2\Delta Z} + V_{i,j}^n \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^n - C_{i,j-1}^n}{4\Delta R} = \\ & \frac{1}{\text{Sc}} \frac{C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^{n+1} + C_{i,j+1}^n - 2C_{i,j}^n + C_{i,j-1}^n}{2\Delta R^2} \end{aligned} \tag{19}$$

The region of integration is considered as a rectangle with sides $Z_{\max}(=1)$ and $R_{\max}(=10)$, where corresponding to $R \rightarrow \infty$ which lies far from the momentum, energy and concentration boundary layers. An appropriate mesh sizes considered for the calculation are $\Delta Z = 0.01$, $\Delta R = 0.05$ and $\Delta \tau = 0.005$.

RESULTS AND DISCUSSION

The velocity, temperature and concentration profiles have been computed by using implicit finite difference scheme of Crank-Nicolson's type. The numerical calculations are carried out for the effect of the flow parameters such as time-dependent axial velocity $U(\tau)$, Prandtl number (Pr), Schmidt number (Sc), thermal Grashof number (Gr), mass Grashof number (Gc), viscosity-variation parameter (γ), thermal conductivity variation parameter (δ), magnetic parameter (M) and Brinkman number (Br) on the velocity, temperature and concentration distribution of the flow fields are presented graphically in Figure 2-14.

The effects of Grashof number (Gr) on the velocity and temperature profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ are shown in Figs. 2-3. It is observed that the velocity and temperature increases with increase in Grashof number.

Fig 4 display the influence of mass Grashof number (Gc) on the transient velocity profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$. It is clear that increasing the mass Grashof number tends to increases the velocity. The hydrodynamics boundary layer become thick as the mass Grashof number increases.

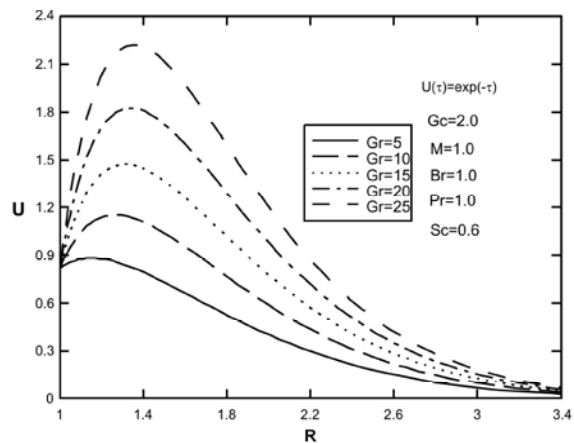


Fig. 2: Effect of Grashof number (Gr) on dimensionless velocity Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

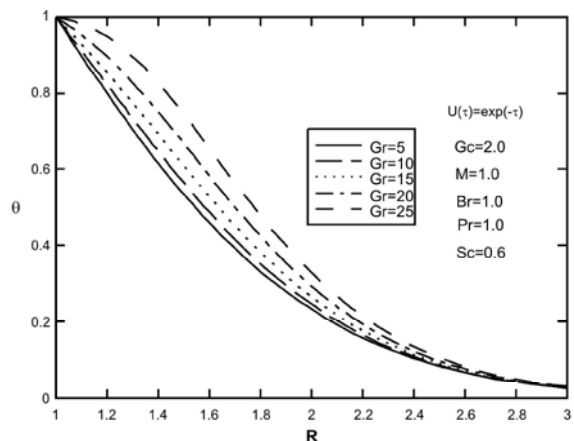


Fig. 3: Effect of Grashof number (Gr) on dimensionless temperature Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

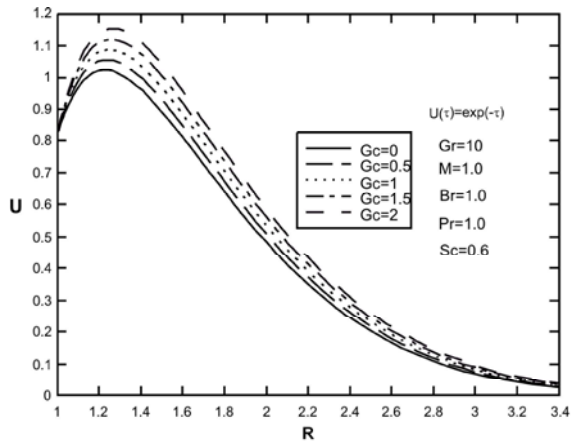


Fig. 4: Effect of mass Grashof number (G_c) on dimensionless velocity Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

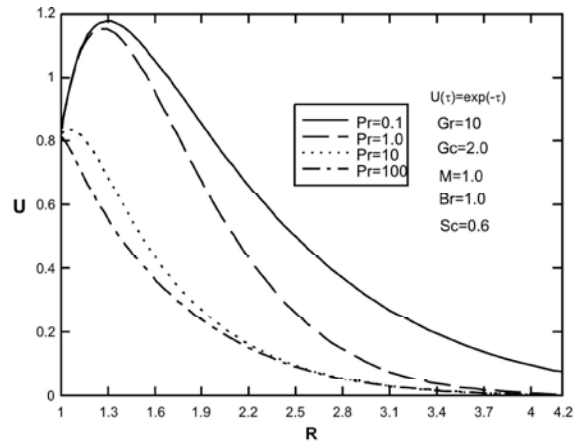


Fig. 7: Effect of Prandtl number on dimensionless velocity Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

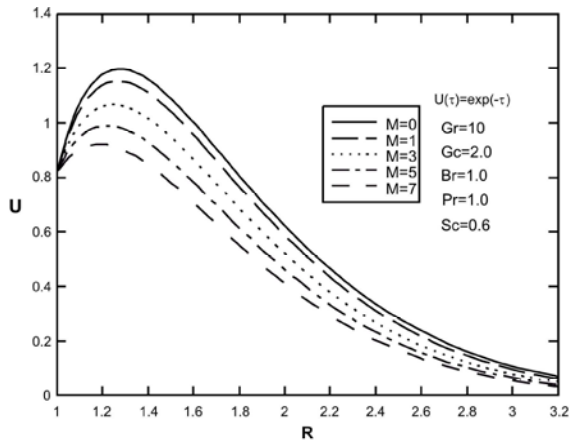


Fig. 5: Effect of magnetic parameter (M) on dimensionless velocity Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

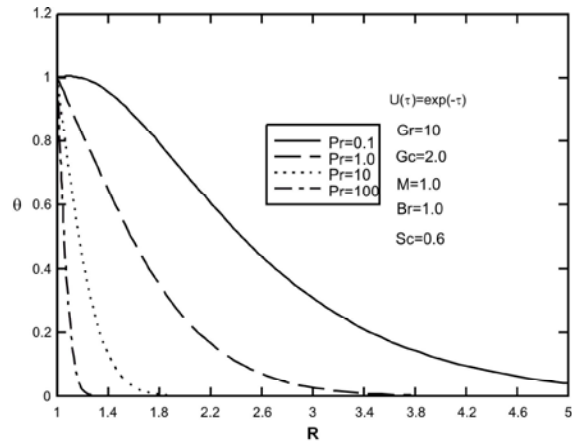


Fig. 8: Effect of Prandtl number on dimensionless temperature Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

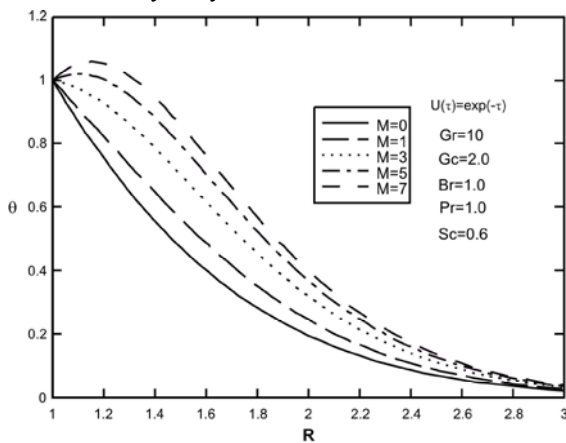


Fig. 6: Effect of magnetic parameter (M) on dimensionless temperature Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

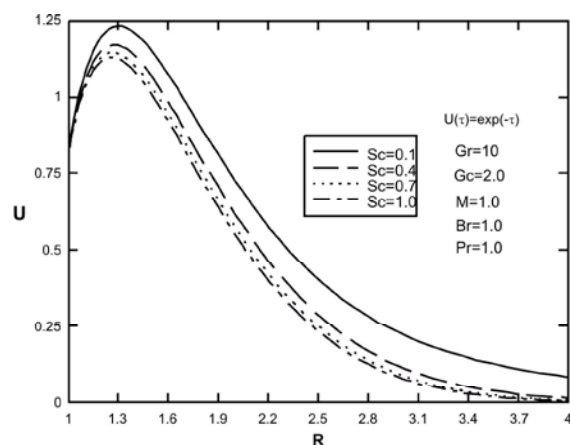


Fig. 9: Effect of Schmidt number (Sc) on dimensionless velocity Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

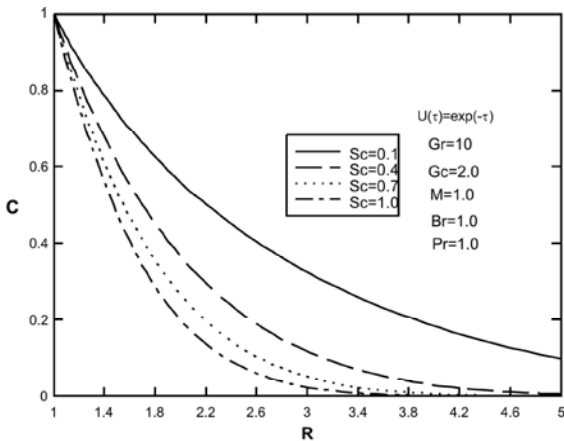


Fig. 10: Effect of Schmidt number (Sc) on dimensionless concentration Profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

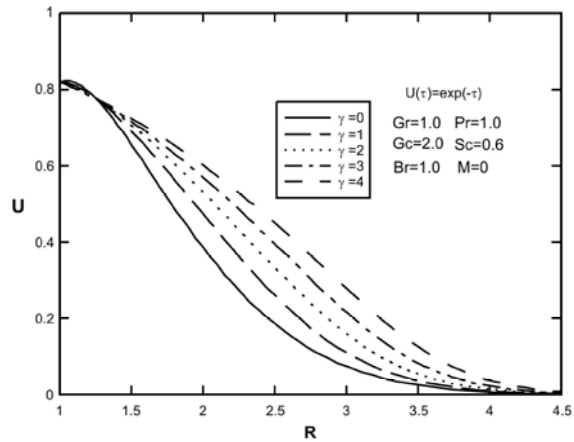


Fig. 13: Effect of viscosity-variation parameter (γ) on dimensionless velocity Profiles for time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

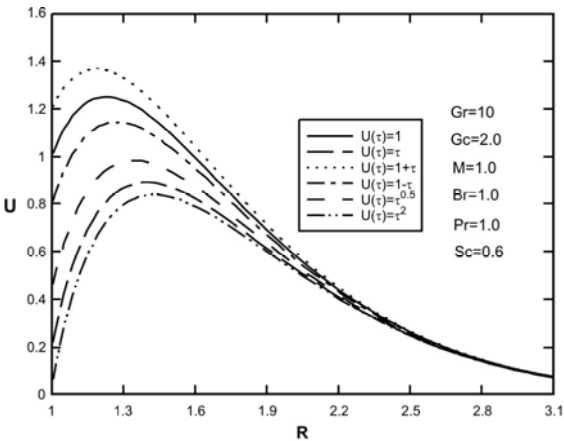


Fig. 11: Effect of different values time-dependent axial velocity of cylinder $U(\tau)$ on dimensionless velocity Profiles for $\gamma = 0$

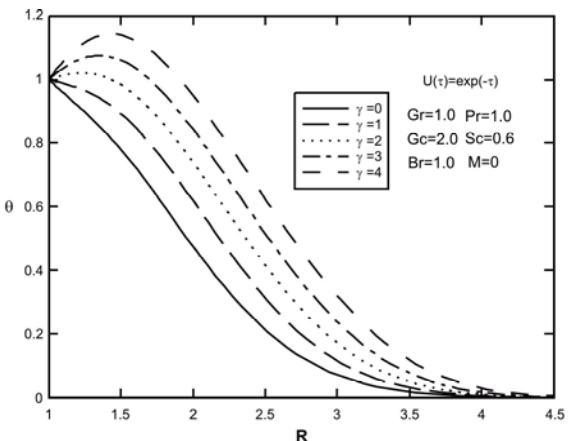


Fig. 14: Effect of viscosity-variation parameter (δ) on dimensionless temperature Profiles for time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

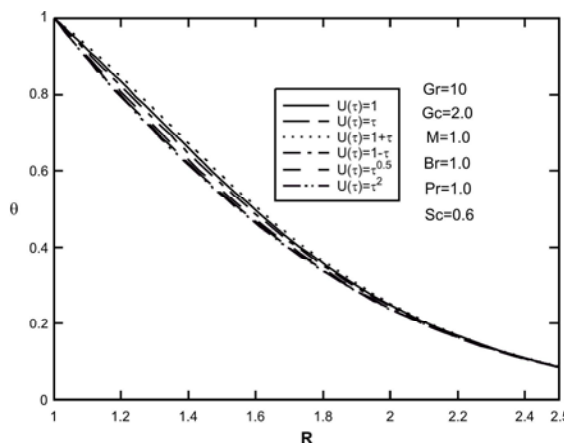


Fig. 12: Effect of different values time-dependent axial velocity of cylinder $U(\tau)$ on dimensionless temperature Profiles for $U(\tau) = \exp(-\tau)$

Figs 5-6 illustrates the dimensionless velocity and temperature profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ and for magnetic parameter (M). It is obvious that, the velocity and temperature decreases with increases in magnetic parameter. The presence of the transverse magnetic field produces a resistive force the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of electrically conducting fluid, which tends to increase the temperature.

Figs.7-8 depicts the velocity and temperature profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ and for different values of Prandtl number (Pr). It is observed that the velocity and temperature decreases with increase in the Prandtl number.

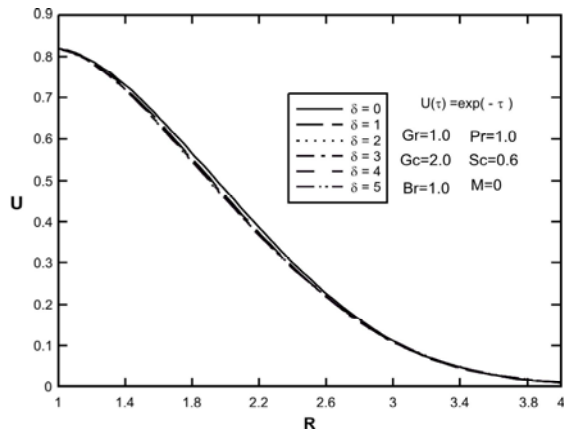


Fig. 15: Effect of thermal conductivity variation parameter (δ) on dimensionless velocity Profiles for $\gamma=0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

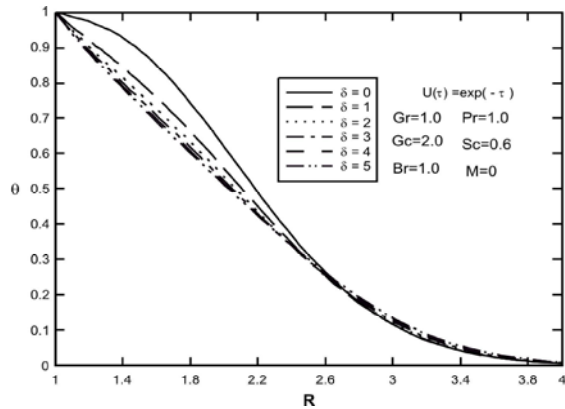


Fig. 16: Effect of thermal conductivity variation parameter (δ) on dimensionless temperature Profiles for $\gamma=0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

The influence of Schmidt number (Sc) on the velocity and concentration profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ are shown in Figs. 9-10. It is observed that the concentration and velocity decreases with increase in Schmidt number.

The effects of different values time-dependent axial velocity of cylinder $U(\tau)$ on the velocity and temperature profiles are shown in Figs. 11-12.

Figs.13-14 depicts the velocity and temperature profiles for time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ and for different values of viscosity-variation parameter (γ). It is observed that the velocity and temperature increases with increase in the viscosity-variation parameter (γ).

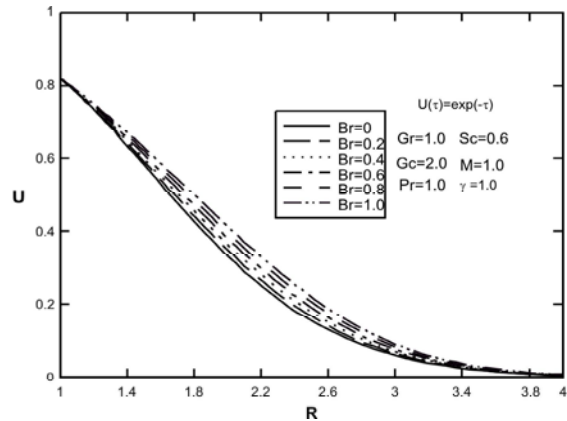


Fig. 17: Effect of Brinkman number (Br) on dimensionless velocity Profiles for time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

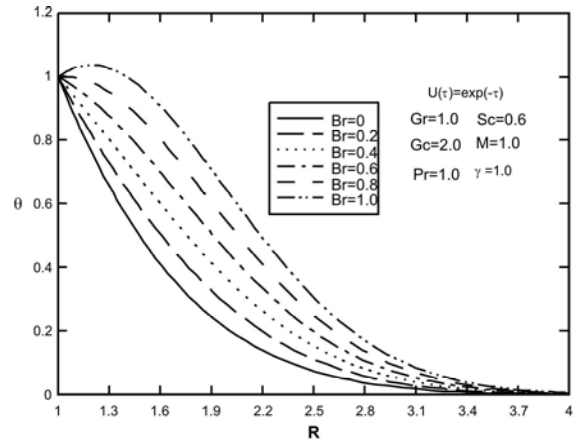


Fig. 18: Effect of Brinkman number (Br) on dimensionless temperature Profiles for time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$

The influence of thermal conductivity variation parameter (δ) on the velocity and temperature profiles for time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ and $\gamma = 0$ are shown in Figs. 15-16. It is observed that the velocity and temperature decreases with increase in the thermal conductivity variation parameter (δ).

Figs 17-18 illustrates the dimensionless velocity and temperature profiles for $\gamma = 0$ and time-dependent axial velocity of cylinder $U(\tau) = \exp(-\tau)$ and for different values of Brinkman number (Br). It is obvious that, the velocity and temperature increases with increases in Brinkman number.

CONCLUSIONS

A numerical study has been carried out to study the MHD free convection flow of a viscous fluid on a vertical

moving cylinder with time-dependent axial velocity, thermal conductivity and viscosity depending on temperature, in the presence of constant wall temperatures. The semi-similar solution of the Navier-Stokes equations, concentration equation and energy equation has been obtained numerically using by the implicit finite difference scheme of Crank-Nicolson's type. From the present numerical investigation, following conclusions have been drawn:

- Velocity increases, temperature decreases with an increase in Grashof number (Gr).
- Velocity and temperature decreases with an increase in magnetic parameter (M).
- Increase in mass Grashof number (Gc), velocity increases.
- Velocity and temperature decreases with an increase in Grashof number (Gr).
- Velocity and concentration decreases with an increase in Schmidt number (Sc).
- Velocity and temperature increases with an increase in viscosity-variation parameter (γ).
- Velocity and temperature decreases with an increase in thermal conductivity variation parameter (δ).
- Velocity and temperature increases with an increase in Brinkman number (Br).

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