## Time-dependent Dalitz plot analysis of $B^{0} \rightarrow D^{\mp} K^{0} \boldsymbol{\pi}^{ \pm}$decays

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We present for the first time a measurement of the weak phase $2 \beta+\gamma$ obtained from a time-dependent Dalitz plot analysis of $B^{0} \rightarrow D^{\mp} K^{0} \pi^{ \pm}$decays. Using a sample of approximately $347 \times 10^{6} B \bar{B}$ pairs

[^1]collected by the BABAR detector at the PEP-II asymmetric-energy storage rings and assuming the ratio $r$ of the $b \rightarrow u$ and $b \rightarrow c$ decay amplitudes to be 0.3 , we obtain $2 \beta+\gamma=(83 \pm 53 \pm 20)^{\circ}$ and the equivalent solution at $+180^{\circ}$. The magnitudes and phases for the resonances associated with the $b \rightarrow c$ transitions are also extracted from the fit.

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The weak phase $\gamma \equiv \arg \left(-\frac{V_{u d} V_{u b}^{*}}{V_{c d} V_{c b}^{*}}\right.$, where $V_{i j}$ are elements of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1], is the least constrained angle of the unitarity triangle [2]. Over the past few years, several methods [3] have been employed to measure $\gamma$ directly in charged $B \rightarrow$ $D^{(*)} K^{(*)}$ decays [4], where sensitivity to the weak phase arises from interference between the $b \rightarrow c$ (favored) and $b \rightarrow u$ (suppressed) transitions. In addition, decays to twobody final states containing charm have been studied, such as $B^{0} \rightarrow D^{(*) \mp} \pi^{ \pm}$and $B^{0} \rightarrow D^{\mp} \rho^{ \pm}[5]$ which are sensitive to the weak phase $2 \beta+\gamma$ due to $B^{0}-\bar{B}^{0}$ mixing. The phase $\beta \equiv \arg \left(-\frac{V_{c d} V_{c b}^{*}}{V_{t d} V_{t b}^{*}}\right)$ is well measured in neutral $B$ decays to charmonium final states [6]. The sensitivity of this method is limited by the ratio $r$ between the $b \rightarrow u$ and $b \rightarrow c$ transitions, which is expected to be very small $(\sim 0.02)$. Three-body $B$ decays have been suggested [7] as a way to avoid this limitation, since $r$ in these decays could be as large as 0.4 in some regions of the Dalitz plot.

In this paper we report on the first measurement of the weak phase $2 \beta+\gamma$ obtained from a time-dependent Dalitz plot analysis of the decay $B^{0} \rightarrow D^{\mp} K^{0} \pi^{ \pm}$[8] (charge conjugation is implied throughout the paper). In the decay $B^{0} \rightarrow D^{\mp} K^{0} \pi^{ \pm}$the three-body final state is reached predominantly through intermediate $B^{0} \rightarrow \stackrel{(-)}{\mathrm{D}^{* * 0}} K_{S}^{0}$ and $B^{0} \rightarrow$ $D^{-} K^{*+}$ decays. In the first case, $\mathrm{D}^{* * *}$ indicates a $D_{0}^{*}(2400)$ or a $D_{2}^{*}(2460)$ state produced through $b \rightarrow u$ and $b \rightarrow c$ color-suppressed transitions. In the second case, $K^{*}(892)$, $K_{0}^{*}(1430), K_{2}^{*}(1430)$, and $K^{*}(1680)$ are produced through $b \rightarrow c$ tree-level transitions. A small contribution from the $b \rightarrow u$ decay $B^{0} \rightarrow D_{s}^{*+}(2573) \pi^{-}$is also expected.

Defining $\vec{x}$ as the vector of the two invariant masses squared $m^{2}\left(K_{S}^{0} \pi^{ \pm}\right)$and $m^{2}\left(D^{ \pm} \pi^{\mp}\right)$, the amplitude $A$ at each point $\vec{x}$ of the Dalitz plot can be parameterized as a coherent sum of two-body decay matrix elements according to the isobar model [9]:

$$
\begin{equation*}
A_{c(u)}(\vec{x}) e^{i \phi_{c(u)}(\vec{x})}=\sum_{j} a_{j} e^{i \delta_{j}} B W\left(\vec{x} ; M_{j}, \Gamma_{j}, s_{j}\right), \tag{1}
\end{equation*}
$$

where $c(u)$ indicates the $b \rightarrow c(b \rightarrow u)$ transition and $\phi$ is the total strong phase. Each resonance $j$ is parameterized by a magnitude $a_{j}$, a phase $\delta_{j}$, and a factor $B W\left(\vec{x} ; M_{j}, \Gamma_{j}, s_{j}\right)$ giving the Lorentz invariant expression for the matrix element of the resonance as a function of the position $\vec{x}$, the spin $s$, the mass $M$, and the decay width $\Gamma$.

The time-dependent probability of a $B^{0}$ or $\bar{B}^{0}$ initial state to decay to a final state with a $D^{+}$or $D^{-}$can be expressed
as [7]

$$
\begin{align*}
P(\vec{x}, \Delta t, \xi, \eta)= & \frac{A_{c}(\vec{x})^{2}+A_{u}(\vec{x})^{2}}{2} \times \frac{e^{-\left(|\Delta t| / \tau_{B}\right)}}{4 \tau_{B}} \\
& \times\left\{1-\eta \xi C(\vec{x}) \cos \left(\Delta m_{d} \Delta t\right)\right. \\
& \left.+\xi S_{\eta}(\vec{x}) \sin \left(\Delta m_{d} \Delta t\right)\right\} . \tag{2}
\end{align*}
$$

Here:

$$
\begin{align*}
S_{\eta}(\vec{x}) & =\frac{2 \operatorname{Im}\left(A_{c}(\vec{x}) A_{u}(\vec{x}) e^{i(2 \beta+\gamma)+\eta i\left(\phi_{c}(\vec{x})-\phi_{u}(\vec{x})\right)}\right)}{A_{c}(\vec{x})^{2}+A_{u}(\vec{x})^{2}} \\
C(\vec{x}) & =\frac{A_{c}(\vec{x})^{2}-A_{u}(\vec{x})^{2}}{A_{c}(\vec{x})^{2}+A_{u}(\vec{x})^{2}} \tag{3}
\end{align*}
$$

where $\Delta t$ is the difference in proper decay times of the reconstructed meson $B_{\text {rec }}$ and the flavor-tagging meson $B_{\text {tag }}, \xi=+1(-1)$ if the flavor of the $B_{\text {rec }}$ is a $B^{0}\left(\bar{B}^{0}\right)$, and $\eta=+1(-1)$ if the final state contains a $D^{+}\left(D^{-}\right)$. We use the world averages for the $B^{0}$ lifetime $\tau_{B}$ and the masseigenstate difference $\Delta m_{d}$ [10].

Because Eq. (2) contains the terms $B W^{j}(\vec{x}, m, \Gamma, s)$, which vary over the Dalitz plot, we can fit the magnitudes $a_{j}$ and the phases $\delta_{j}$ of Eq. (1) to determine $2 \beta+\gamma$ with only a twofold ambiguity [7]. Most of the sensitivity to $2 \beta+\gamma$ is expected to come from the interference between $b \rightarrow u$ and $b \rightarrow c$ transitions leading to $D^{* * 0} K_{S}^{0}$ final states (with expected $r \sim 0.4$ ), and from the interference of the former with the $b \rightarrow c$ transition of the decay $B^{0} \rightarrow$ $D^{-} K^{*+}$.

The analysis is based on $347 \times 10^{6} B \bar{B}$ pairs collected at the $\Upsilon(4 S)$ resonance by the $B A B A R$ detector at the PEP-II storage rings. The detector is described in detail elsewhere [11]. In order to estimate signal selection efficiencies and to study physics backgrounds, a Monte Carlo simulation based on GEANT4 [12] is used.

We reconstruct $D^{+}$mesons in the decay mode $K^{-} \pi^{+} \pi^{+}$. The tracks from $D^{+}$decay are required to originate from a common vertex, and the kaon is selected using a likelihood based particle identification algorithm. The $D^{+}$candidates are required to have a mass within $\pm 12$ $\mathrm{MeV} / c^{2}(2 \sigma)$ of the nominal $D^{+}$mass [10], where $\sigma$ is the experimental resolution. Oppositely charged tracks from a common vertex are recognized as $K_{S}^{0}$ candidates if they have an invariant mass within $\pm 7 \mathrm{MeV} / c^{2}(3 \sigma)$ of the nominal $K_{S}^{0}$ mass [10] and a transverse flight-length significance $4 \sigma$ greater than zero. The $\pi^{-}$candidate is a track for which the particle identification is inconsistent with its being a kaon or an electron.

To form $B^{0}$ candidates, each $D^{+}$candidate is combined with a $K_{S}^{0}$ candidate and a $\pi^{-}$candidate requiring that the three particles originate from a common vertex. We reject $B^{0}$ candidates with $m^{2}\left(K_{S}^{0} \pi^{ \pm}\right)$in the window [3.40, 3.95] $\mathrm{GeV}^{2} / c^{4}$ in order to remove backgrounds with nonzero $C P$ content arising from $B^{0} \rightarrow D^{\mp} D_{(s)}^{ \pm}$decays. Using the beam energy in the $e^{+} e^{-}$center-of-mass (CM) frame, two kinematic variables are constructed: the beam-energy substituted mass $\mathrm{m}_{\mathrm{ES}} \equiv \sqrt{s / 4-p_{B}^{* 2}}$, and the difference between the measured $B^{0}$ candidate energy and the beam energy, $\Delta \mathrm{E} \equiv E_{B}^{*}-\sqrt{s} / 2$. Here $p_{B}^{*}$ and $E_{B}^{*}$ are the momentum and the energy of the $B_{\text {rec }}$ in the CM frame, respectively. Candidates with $\Delta \mathrm{E}$ in the range $[-0.1,0.1] \mathrm{GeV}$ and $m_{\text {ES }}$ in the range $[5.24,5.29] \mathrm{GeV} / c^{2}$ are selected. We require $\left|\cos \theta_{\mathrm{B}}\right|$, the absolute value of the cosine of the angle between the $B_{\text {rec }}$ momentum and the beam axis, be less than 0.85 , and $\left|\cos \theta_{\mathrm{T}}\right|$, the absolute value of the cosine of the angle between the thrust axis of the $B_{\text {rec }}$ decay products and the thrust axis of the rest of the event (ROE), be less than 0.95, both in the CM frame [13].

The difference of proper-time $\Delta t$ of the two $B \mathrm{~s}$ in the event is calculated from the measured separation $\Delta z$ between the vertices of the $B_{\text {rec }}$ and the $B_{\text {tag }}$ along the beam direction [6]. We accept events with calculated $\Delta t$ uncertainty less than 2.5 ps and $|\Delta t|<20 \mathrm{ps}$. The average $\Delta t$ resolution is approximately 1.1 ps . The flavor of the $B_{\mathrm{tag}}$ is identified from particles that do not belong to the $B_{\text {rec }}$ using a multivariate algorithm [6]. The effective efficiency of the tagging algorithm, defined as $Q=\Sigma_{k} \epsilon_{k}\left(1-2 w_{k}\right)^{2}$, is $(30.1 \pm 0.5) \%$, where $\epsilon_{k}$ and $w_{k}$ are the efficiency and the mistag probability, respectively, for each of the six tagging categories $k$. A separate seventh category contains the untagged events, about $38 \%$ of the sample, which contribute to the determination of magnitudes and phases of the resonances and correspond to the case $\xi=0$ in Eq. (2).

To further suppress the dominant continuum background, which has a more jetlike shape than $B \bar{B}$ events, we use a linear combination $\mathcal{F}$ of five variables: $L_{0}=$ $\sum_{i} p_{i}, L_{2}=\sum_{i} p_{i}\left|\cos \theta_{i}\right|^{2}$, the global thrust of the event, $\left|\cos \theta_{\mathrm{T}}\right|$, and $\left|\cos \theta_{\mathrm{B}}\right|$. Here, $p_{i}$ is the momentum and $\theta_{i}$ is the angle, with respect to the thrust axis of the $B_{\mathrm{rec}}$, of the tracks and clusters of the ROE in the CM frame. The coefficients of $\mathcal{F}$ are chosen to maximize the separation between the distributions obtained from Monte Carlo simulated signal events and $28 \mathrm{fb}^{-1}$ of continuum events collected at a CM energy 40 MeV below that of the $\mathrm{Y}(4 S)$ resonance (off resonance), whose energy is rescaled to the energy of the beams. The correlations among the set of measured values of the variables ( $m_{\mathrm{ES}}, \Delta \mathrm{E}, \mathcal{F}$ ) are negligible. Since both $\mathcal{F}$ and the flavor tagging utilize the ROE information, the distribution of $\mathcal{F}$ is correlated with the tagging category. To take into account this correlation, we parameterize the $\mathcal{F}$ distribution for each tagging category separately.

Approximately $7 \%$ of selected events contain more than one reconstructed signal candidate, arising primarily from multiple $D^{+}$candidates. We select the one having the $D$-candidate mass nearest to the nominal value [10]. For simulated signal events, the entire selection chain has an efficiency of $(9.9 \pm 0.1) \%$, where the error is statistical only.

To separate signal from background and to determine their yields, we first perform an unbinned extended maximum likelihood fit to the selected on-resonance data sample in the variables $m_{\mathrm{ES}}, \Delta \mathrm{E}$, and $\mathcal{F}$. The role of this first step fit is to extract all the shape parameters, the fractions of events by tagging category, and the overall yields, which will then be fixed in the subsequent time-dependent fit to the Dalitz plot. We define the logarithm of the likelihood:

$$
\begin{equation*}
\ln \mathcal{Y}=\sum_{k=1}^{7}\left(\sum_{i=1}^{N_{\mathrm{tot}}} \ln \left(\sum_{j} N_{j k} Y_{j k}^{i}\right)-\sum_{j} N_{j k}\right) \tag{4}
\end{equation*}
$$

where $Y_{j k}^{i}$ is the product of the probability density functions (PDFs) of $m_{\mathrm{ES}}, \Delta \mathrm{E}$, and $\mathcal{F}_{k}$ for the event $i$ in the tagging category $k . N_{\text {tot }}$ is the total number of events and $N_{j k}$ is the number of events of each sample component $j$ : signal (Sig), continuum (Cont), combinatoric $B \bar{B}$ decays $(b \bar{b})$, and $B \bar{B}$ events that peak in $m_{\mathrm{ES}}$ but not in the $\Delta \mathrm{E}$ signal region (Peak).

The signal is described by a Gaussian function for the $m_{\text {ES }}$ distribution, two Gaussian functions with common mean for the $\Delta \mathrm{E}$ distribution, and a Gaussian function with different widths on each side of the mean ("bifurcated Gaussian function'") for the $\mathcal{F}$ distribution. The signal model parameters are obtained from a fit to a high-statistics data control sample of $B^{0} \rightarrow D^{\mp} a_{1}^{ \pm}$decays. The selection of these events is similar to that of the signal, except that no $K_{S}^{0}$ candidate is required. The decay chain $a_{1}^{ \pm} \rightarrow \rho^{0} \pi^{ \pm}$ with $\rho^{0} \rightarrow \pi^{ \pm} \pi^{\mp}$ is reconstructed requiring the dipion invariant mass be within $\pm 150 \mathrm{MeV} / c^{2}$ of the nominal $\rho^{0}$ mass [10], and the invariant mass of the $\rho$ candidate with the third pion be within $\pm 250 \mathrm{MeV} / c^{2}$ of the nominal $a_{1}^{ \pm}$mass [10].

The $m_{\mathrm{ES}}$ distributions of the continuum and combinatoric $B \bar{B}$ backgrounds are described by empirical threshold functions [14], while for $\Delta \mathrm{E}$ distributions linear functions are used. The $\mathcal{F}$ distributions are parameterized by a bifurcated Gaussian function for the continuum background and a sum of two Gaussian functions for the $B \bar{B}$ combinatoric background. For the latter the parameters are determined by $B \bar{B}$ Monte Carlo simulation. All the shape parameters of the continuum background are taken from fitting the off-resonance data.

The $m_{\mathrm{ES}}$ distribution of the Peak background is parameterized by a Gaussian function with the same mean as the signal and a width fixed to the value obtained from Monte Carlo simulation. The $\Delta \mathrm{E}$ distribution is described
by an exponential function. The $\mathcal{F}$ distribution of Peak is described using the same PDF as for $B \bar{B}$ background.

The yields and the fraction of events for each tagging category are fitted together with the free shape parameters. The yields obtained for each component are $N_{\text {Sig }}=558 \pm$ $34, \quad N_{\text {Cont }}=13222 \pm 226, \quad N_{b \bar{b}}=5647 \pm 213, \quad$ and $N_{\text {Peak }}=183 \pm 41$, in agreement with the previous result [15].

The second stage of the analysis is the time-dependent fit to the Dalitz plot. For each background component, the $\Delta t$ distribution is modeled as an exponential, with an effective lifetime parameter. To model the detector resolution, it is convolved with the sum of three Gaussians, the sum of two Gaussians, and one Gaussian in the case of continuum background, $B \bar{B}$ combinatorial background, and Peak, respectively. The widths of the Gaussians, the relative fraction of them, the effective dilution parameters, and the effective lifetimes are determined independently from fits to the control samples: the off-resonance data sample for the continuum background, the $B \bar{B}$ Monte Carlo sample for $B \bar{B}$ combinatorial background and the Peak component. In the case of Peak the lifetime is fixed to the $B^{0}$ lifetime [10]. The $\Delta t$ parameterizations described above for each background component are combined in a global time-dependent PDF, $\mathcal{T}^{i}{ }_{ \pm, \mathrm{Bkgd}}$, obtained as a weighted average based on the fitted yields, where $+(-)$ indicates $B_{\mathrm{tag}}=B^{0}\left(B_{\mathrm{tag}}=\bar{B}^{0}\right)$.

To obtain the PDF describing the Dalitz plot of the background in the tagging category $k$, we use the results of the yields fit and calculate for each event a background weight [16]:

$$
\begin{equation*}
W_{\mathrm{Bkgd}}^{k}=1-W_{\mathrm{Sig}}^{k} \equiv 1-\frac{\sum_{j} \mathbf{V}_{\mathrm{Sig}, j} Y_{j k}\left(m_{E S}, \Delta E \mathcal{F}_{k}\right)}{\sum_{j} N_{j} Y_{j k}\left(m_{E S}, \Delta E, \mathcal{F}_{k}\right)}, \tag{5}
\end{equation*}
$$

where $N_{j}$ is the number of events in a given sample component $j$ [see Eq. (4)], $Y_{j k}$ is defined as in Eq. (4), and $\mathbf{V}_{\mathrm{Sig}, j}$ is the signal row of the covariance matrix of the component yields obtained from the likelihood fit. In the absence of correlations, $W_{\text {Bkgd }}$ are the background probabilities $P_{\text {Bkgd }} / P_{\text {total }}$. Applying these weights to the Dalitz
plot of on-resonance data we obtain the observed background Dalitz plot PDF, $\mathcal{D}_{\mathrm{Bkgd}}$.

For the signal the effect of finite $\Delta t$ resolution is described by convolving Eq. (2) with a resolution function composed of three Gaussian distributions. Incorrect tagging dilutes the coefficient of $\cos \left(\Delta m_{d} \Delta t\right)$ in Eq. (2) by a factor $\left(1-2 w_{i}\right)$. The parameters of the resolution function and those associated with flavor tagging are fixed to the values obtained in [6].

The expression for the time-dependent Dalitz plot likelihood function is then

$$
\begin{equation*}
\ln \mathcal{L}=\sum_{\mathrm{k}=1}^{7}\left[\sum_{B^{0} \mathrm{tag}} \ln \mathcal{L}_{+, k}+\sum_{\bar{B}^{0} \mathrm{tag}} \ln \mathcal{L}_{-, k}\right], \tag{6}
\end{equation*}
$$

The likelihood function $\mathcal{L}_{+, k}\left(\mathcal{L}_{-, k}\right)$ for an event in the tagging category $k$ with $B_{\text {tag }}=B^{0}\left(B_{\text {tag }}=\bar{B}^{0}\right)$ is

$$
\begin{equation*}
\mathcal{L}_{ \pm, k}=N_{\mathrm{Sig}}^{k} \mathcal{P}_{ \pm, \mathrm{Sig}}^{k} Y_{\mathrm{Sig}}^{k}+N_{\mathrm{Bkgd}}^{k} \mathcal{D}_{\mathrm{Bkgd}} \mathcal{T}_{ \pm, \mathrm{Bkgd}}^{k} Y_{\mathrm{Bkgd}}^{k} \tag{7}
\end{equation*}
$$

Here $Y$ indicates the product of PDFs for $m_{\mathrm{ES}}, \Delta \mathrm{E}$, and $\mathcal{F}_{k}$; $\mathcal{P}_{ \pm, \text {Sig }}$ is the time-dependent Dalitz plot PDF for signal. The $Y_{\text {Bkgd }}$ parameterization is obtained from a weighted average, using the fitted yields, of the shapes obtained from the first step fit.

With the current data set we are unable to determine the magnitudes for the suppressed $b \rightarrow u$ decays. We therefore fix the ratio $r=\frac{A(b \rightarrow u)}{A(b \rightarrow c)}=0.3$ for each resonance in the PDF, which is compatible with the limit $r<0.4$ ( $90 \%$ C.L.) reported in Ref. [17]. The $D_{s}^{*+}(2573)$ magnitude and phase are fixed to the values given in [8]. Despite the fact that the $b \rightarrow u$ phases cannot be precisely determined, they are left free in the fit. All the $b \rightarrow c$ magnitudes and phases together with $2 \beta+\gamma$ are free parameters. The whole fitting procedure has been validated using high statistic parameterized (toy) Monte Carlo samples.

The fit is performed on events satisfying $m_{\mathrm{ES}}>$ $5.27 \mathrm{GeV} / c^{2},|\Delta \mathrm{E}|<50 \mathrm{MeV}$, and $\mathcal{F}>-2$. Results are shown in Table I. Figure 1 shows the projections of the onresonance data sample on the two Dalitz plot variables $m^{2}\left(K_{S}^{0} \pi^{ \pm}\right)$and $m^{2}\left(D^{ \pm} \pi^{\mp}\right)$ with the fitted PDFs superimposed. Figure 2(a) shows the $m_{\text {ES }}$ distribution and the

TABLE I. Results for $2 \beta+\gamma$ and the $b \rightarrow c$ transitions magnitudes $a_{c}$ and phases $\delta_{c}$ assuming $r=0.3$. The first uncertainty is statistical; the second is systematic.

| Resonance | Bias correction for the magnitude | $a_{c}$ magnitude after bias correction | Phase $\delta_{c}\left({ }^{\circ}\right)$ |
| :--- | :---: | :---: | :---: |
| $K^{*}(892)$ | $\ldots$ | 1 | 0 |
| $D_{0}^{*}(2400)$ | +0.003 | $0.290 \pm 0.048 \pm 0.067$ | $267 \pm 22 \pm 35$ |
| $D_{2}^{*}(2460)$ | -0.033 | $0.042 \pm 0.050 \pm 0.048$ | $325 \pm 46 \pm 20$ |
| $K_{0}^{*}(1430)$ | -0.025 | $0.135 \pm 0.058 \pm 0.099$ | $284 \pm 30 \pm 11$ |
| $K_{2}^{*}(1430)$ | -0.017 | $0.108 \pm 0.056 \pm 0.051$ | $221 \pm 30 \pm 14$ |
| $K^{*}(1680)$ | -0.011 | $0.404 \pm 0.047 \pm 0.046$ | $128 \pm 22 \pm 24$ |
|  | $2 \beta+\gamma=(83 \pm 53 \pm 20)^{\circ}$ and $(263 \pm 53 \pm 20)^{\circ}$ |  |  |



FIG. 1. The distributions of $m^{2}\left(K_{S}^{0} \pi\right)$ and $m^{2}(D \pi)$ in data (solid points). The overall PDF is superimposed (black line). The gray full line is the signal PDF; the dashed black line is the background PDF.
fitted PDFs for each component, after applying additional requirements on $\Delta \mathrm{E}$ and $\mathcal{F}$. Besides the value of $2 \beta+\gamma$, an important outcome of the analysis is the fit of the resonance contributions to the $b \rightarrow c$ part of the Dalitz plot. Biases related to the small sample size are observed in the measurement of the magnitudes. They are estimated using a large number of toy Monte Carlo experiments generated with the magnitudes values obtained in the fit to the on-resonance data sample.

The systematic errors are summarized in Table II. The main contribution is related to the parameterization of the background Dalitz plot. This effect has been estimated by repeating the fit with a parameterization obtained from offresonance data and $B \bar{B}$ generic Monte Carlo simulation. The systematic uncertainty due to the efficiency variation
over the Dalitz plot has been evaluated assuming a flat efficiency. The effect of potential $C P$ content of the $B \bar{B}$ peaking background is taken into account assuming the same $C P$ violation structure as in the signal with a value $r_{\text {eff }}=0.4$. The systematic uncertainties on the signal Dalitz plot come from the variation of the $r$ factor $(0.3 \pm$ $0.1)$ and of the $D_{s}^{*+}(2573)$ magnitude $(0.02 \pm 0.01)$ and from the introduction of up to $7 \%$ of a nonresonant component. In addition, the masses and widths of the resonances have been varied by 1 standard deviation [10]. We obtain the systematic uncertainty arising from imperfect knowledge of the $Y$ shape parameters and the yields by varying all fixed parameters within their uncertainties. Similar variations are applied to the signal and background fractions in each tagging category as well as for the $\Delta t$


FIG. 2. (a) The $m_{\text {ES }}$ distribution of on-resonance data (solid points) for the global fitted PDF (black line) with the contribution of the PDF for each component superimposed: signal (gray line), continuum (small dashed line), combinatoric $B \bar{B}$ (large dashed line), and Peak (dotted line). To enhance the signal $|\Delta \mathrm{E}|<0.025 \mathrm{GeV}$ and $\mathcal{F}>0.2$ have been required. (b) Distribution of the values of $2 \beta+\gamma$ fitted on data for different hypotheses on the $r$ value. (c) Variation of the logarithm of the likelihood with $2 \beta+\gamma$.

TABLE II. Sources and sizes of systematic errors.

|  |  | $D_{0}^{*}(2400)$ |  | $D_{2}^{*}(2460)$ |  | $K_{0}^{*}(1430)$ |  | $K_{2}^{*}(1430)$ |  | $K^{*}(1680)$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Systematic | $2 \beta+\gamma$ | $a_{c}$ | $\delta_{c}$ | $a_{c}$ | $\delta_{c}$ | $a_{c}$ | $\delta_{c}$ | $a_{c}$ | $\delta_{c}$ | $a_{c}$ | $\delta_{c}$ |
| Background Dalitz plot param. | $16.0^{\circ}$ | 0.058 | $3.2^{\circ}$ | 0.034 | $12.1^{\circ}$ | 0.088 | $9.5^{\circ}$ | 0.005 | $12.0^{\circ}$ | 0.015 | $10.3^{\circ}$ |
| Efficiency over the Dalitz plot | $5.8^{\circ}$ | 0.014 | $17.5^{\circ}$ | 0.028 | $10.8^{\circ}$ | 0.005 | $1.9^{\circ}$ | 0.036 | $0.8^{\circ}$ | 0.017 | $19.4^{\circ}$ |
| $C P$ content of bkgd. | $1.0^{\circ}$ | 0.021 | $6.9^{\circ}$ | 0.003 | $8.4^{\circ}$ | 0.005 | $1.4^{\circ}$ | 0.007 | $3.9^{\circ}$ | 0.003 | $1.0^{\circ}$ |
| $r$ | $1.0^{\circ}$ | 0.013 | $8.6^{\circ}$ | 0.013 | $2.2^{\circ}$ | 0.039 | $3.0^{\circ}$ | 0.012 | $0.7^{\circ}$ | 0.016 | $0.3^{\circ}$ |
| $a\left(D_{s}^{*+}(2573)\right)$ | $0.7^{\circ}$ | $\ldots$ | $\cdots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $m, \Gamma$ | $9.5^{\circ}$ | 0.012 | $28.0^{\circ}$ | 0.011 | $6.9^{\circ}$ | 0.018 | $2.8^{\circ}$ | 0.032 | $5.9^{\circ}$ | 0.036 | $9.3^{\circ}$ |
| Y PDF param. | $3.0^{\circ}$ | 0.005 | $1.4^{\circ}$ | 0.002 | $0.4^{\circ}$ | 0.007 | $0.6^{\circ}$ | 0.003 | $0.1^{\circ}$ | 0.002 | $0.5^{\circ}$ |
| Signal and bkgd. frac. | $1.4^{\circ}$ | 0.012 | $2.9^{\circ}$ | 0.004 | $1.2^{\circ}$ | 0.013 | $1.4^{\circ}$ | 0.008 | $0.7^{\circ}$ | 0.004 | $1.4^{\circ}$ |
| Yields | $0.1^{\circ}$ | 0.003 | $1.3^{\circ}$ | 0.001 | $0.3^{\circ}$ | 0.005 | $0.4^{\circ}$ | 0.002 | $0.1^{\circ}$ | 0.002 | $0.1^{\circ}$ |
| Tagging and time param. | $2.6^{\circ}$ | 0.003 | $1.4^{\circ}$ | 0.001 | $0.3^{\circ}$ | 0.004 | $0.4^{\circ}$ | 0.002 | $0.2^{\circ}$ | 0.002 | $0.2^{\circ}$ |

resolution parameters, the effective lifetimes, the $B$ lifetime $\tau_{B}$, and the mixing frequency $\Delta m_{d}$. The systematic uncertainties due to the dependence of the tagging efficiency on the $B$ flavor, the beam spot position, and the SVT alignment have been obtained following the procedure described in [6].

Figure 2(b) shows the dependence of the measurement of $2 \beta+\gamma$ on $r$. For each fixed value of $r$, a point in the plot represents the result of the fit on $2 \beta+\gamma$ with its statistical error. The error decreases, as expected, for increasing $r$, and the central value remains stable. The projection on $2 \beta+\gamma$ of the negative logarithm of the likelihood in Fig. 2(c) clearly shows the minimum corresponding to the result of the fit and the expected mirror solution at $+\pi$ rad. Having fixed some magnitudes and strong phases, the second solution is disfavored, but it should be regarded as equivalent.

In summary, we present the first results of a timedependent Dalitz plot analysis of the decay $B^{0} \rightarrow$ $D^{ \pm} K^{0} \pi^{\mp}$ to determine the Dalitz plot model parameters and the weak phase $2 \beta+\gamma$. Assuming $r=0.3$ we find $2 \beta+\gamma=(83 \pm 53 \pm 20)^{\circ}$ and the equivalent solution at $+180^{\circ}$, where the first error is statistical and the second is systematic.

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