Time Dependent Reliability Model Incorporating Continuum Damage Mechanics for High-Temperature Ceramics

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SUMMARY

Presently there are many opportunities for the application of ceramic materials at elevated temperatures. In the near future, ceramic materials are expected to supplant high temperature metal alloys in a number of applications. It thus becomes essential to develop a capability to predict the time-dependent response of these materials. This paper focuses on the creep rupture phenomenon and outlines a time-dependent reliability model that integrates continuum damage mechanics principles and Weibull analysis. Several features of the model are presented in a qualitative fashion, including predictions of both reliability and hazard rate. In addition, a comparison of the continuum and the microstructural kinetic equations highlights a strong resemblance in the two approaches.

INTRODUCTION

The utilization of structural ceramic components in high temperature environments requires thoughtful consideration of fast fracture as well as strength degradation due to time dependent phenomenon such as subcritical crack growth, creep rupture, and stress corrosion. In all cases this can be accomplished by specifying an acceptable reliability level for a component. Here reliability is defined as the probability that a component performs its required function adequately for a specified period of time under predetermined (design) conditions. Methods of analysis exist that

1

capture the variability in strength of ceramics as it relates to fast fracture (see Gyekenyesi (1986)). However the calculation of an expected lifetime of a ceramic component has been limited to a statistical analysis based on subcritical crack growth (see Wiederhorn and Fuller (1985) for a detailed development). The subcritical crack growth approach establishes relationships among reliability, stress, and time to failure based on principles of fracture mechanics. The analysis combines the Griffith (1921) equation and an empirical crack velocity equation with the underlying assumption that steady growth of a preexisting flaw is the driving failure mechanism.

Several authors including Quinn (1987), Ritter et al. (1980) and Dalgleish et al. (1985) have emphasized that time dependent failure of ceramics is not limited to subcritical crack growth and may occur by either stress corrosion or creep rupture. Stress corrosion involves nucleation and growth of flaws by environmental/oxidation attack. Creep rupture typically entails the nucleation, growth, and coalescence of voids dispersed along grain boundaries. This paper highlights creep rupture with the intent to provide the design engineer with a method that determines an allowable stress for a given component lifetime and reliability. This is accomplished by combining Weibull analysis with the principles of continuum damage mechanics, which was originally developed by Kachanov (1958) to account for tertiary creep and creep fracture of ductile metal alloys.

This effort does not represent the first application of continuum damage mechanics to brittle materials. The observed differences of creep behavior in tension and compression have been addressed through the use of damage mechanics by Krajcinovic (1979) for concrete, and Rosenfield et al. (1985) for ceramics. In addition, Krajcinovic and Silva (1982) explored several fundamental aspects of combining damage mechanics with statistical strength theories for perfectly brittle materials. What is novel here is that the incorporation of damage mechanics within the framework of a weakest link theory allows the computation of reliability for intermediate times less than a component's given life.

Ideally, any theory that predicts the behavior of a material should incorporate parameters that are relevant to its microstructure (grain size, void spacing, etc.). However this would require a determination of volume averaged effects of microstructural phenomena reflecting nucleation, growth, and coalescence of microdefects that in many instances interact. This approach is difficult

2

even under strongly simplifying assumptions. In this respect Leckie (1981) points out the difference between the materials scientist and engineer is one of scale. He notes the materials scientist is interested in mechanisms of deformation and failure at the microstructural level and the engineer focuses on these issues at the component level. Thus the former designs the material and the latter designs the component. We adopt the engineer's viewpoint and note from the outset that continuum damage mechanics does not focus attention on microstructural events, yet it does provide a practical model which macroscopically captures the changes induced by the evolution of voids and defects. As outlined in the section Comparison With Void Growth Mechanisms, a comparison of the continuum and microstructural kinetic equations bear strong resemblance. Thus adopting a continuum theory of damage with its attendant phenomenological view would appear to be a logical first approach.

THEORETICAL DEVELOPMENT

It is assumed that the evolution of the microdefects represents an irreversible thermodynamic process. On the continuum level this requires the introduction of an internal state variable that serves as a measure of accumulated damage. Consider a uniaxial test specimen and let A_o represent the cross-sectional area in an undamaged (or reference) state. Denote A as the current cross-sectional area in a damaged state where material defects exist in the cross section (i.e., $A < A_o$). Microstructurally this can be represented by figure 1. The macroscopic damage associated with this specimen is represented by the scalar

$$\omega = (A_o - A)/A_o \qquad (1)$$

or alternatively by $\psi = 1 - \omega$, which is referred to as "continuity". The variable ψ represents the fraction of cross-sectional area not occupied by voids. A material is undamaged if $\omega = 0$ or $\psi = 1$.

For time dependent analysis the rate of change of continuity $\dot{\psi}$ (or the damage rate $\dot{\omega}$) must be specified. This rate is functionally dependent on stress and the current state of continuity, that is

$$\dot{\psi} = \dot{\psi}(\sigma, \psi) \qquad (2)$$

and is monotonically decreasing ($\dot{\psi} < 0$). For a uniaxial specimen the dependence of $\dot{\psi}$ on stress is taken through a net stress defined as

3

$$\tilde{\sigma} = P/A = \sigma_o/\psi \qquad (3)$$

where P is the applied tensile load and $\sigma_o = P/A_o$. A power law form of the kinetic equation is adopted, that is

$$\dot{\psi} = -B\,(\tilde{\sigma})^n = -B(\sigma_o/\psi)^n \qquad (4)$$

where B>0 and n≥1 are material constants determined from creep rupture data as discussed below. The authors recognize this form of evolutionary law is simplistic, stipulated a priori, and that experimental data may indicate some inconsistencies and/or inadequacies. Modification would be guided by experiment and material science models of creep damage outlined in a later section. For example, physical processes that involve void growth mechanisms along grain boundaries typically exhibit threshold behavior. This is illustrated in a schematic plot of log of stress as a function of log of time to failure in figure 2. Marion et al. (1983) suggest that for stress levels along grain boundaries below a threshold value, liquid phase sintered ceramics deform by a solution/precipitation mechanism without damage accumulation. Experimental data generated by Wiederhorn et al. (1988) supports the existence of this threshold for silicon nitride. Tsai and Raj (1982) suggest methods of estimating values of a threshold stress for ceramics, and the above form of the kinetic equation could easily accommodate a threshold, that is

$$\dot{\psi} \begin{cases} = 0 & \sigma_0 \leq \sigma_{th} \\ = -B(\sigma_0/\psi)^n & \sigma_0 > \sigma_{th} \end{cases} \qquad (5)$$

Dalgleish et al. (1985) have presented experimental data that suggest the existence of a second threshold that delineates regions where subcritical crack growth and creep rupture failure mechanisms are operative. Chuang et al. (1986) predict the value of this threshold stress by using principles of irreversible thermodynamics within the framework of several well accepted models for crack growth. If this threshold (σ_{th}^*) exists, one can construct a composite reliability model such that

$$R \begin{cases} = R(\text{subcritical crack growth}) & \sigma_0 > \sigma_{th}^* \\ = R(\text{creep rupture}) & \sigma_{th} < \sigma_0 \leq \sigma_{th}^* \end{cases} \qquad (6)$$

where R is the reliability of a component. In this paper the aforementioned thresholds are recognized as a possibility. However, a lack of quality experimental data leaves the authors unsure

4

as to whether or not these thresholds are a universal phenomenon, and therefore to ignore the thresholds is expedient at this time.

It is postulated that during a creep rupture experiment σ_o is abruptly applied and held at a fixed value (see the inset of figure 2). With $\psi=1$ at $t=0$, equation (4) can be integrated to yield an expression for ψ as a function of time, stress, and model parameters as follows:

$$\psi = \left[\, 1 - B(\sigma_o)^n (n+1)t \,\right]^{1/(n+1)} \tag{7}$$

An expression for the time to failure (t_f) can be obtained from equation (7) by noting $t=t_f$ when $\psi=0$. Hence

$$t_f = 1/B(\sigma_o)^n (n+1) \tag{8}$$

and the equation for ψ is simplified to

$$\psi = \left[\, 1 - (t/t_f) \,\right]^{1/n+1} \tag{9}$$

This is consistent with the strong dependence of failure times on stress, as suggested by Johnson et al. (1984). As noted by Jones (1987), the distribution of failure times for a given stress level σ_o may be probabilistic or deterministic. Currently the data is insufficient to postulate either case. In this paper t_f is treated in a deterministic fashion noting that a probability distribution function for t_f could be introduced to the analysis in a manner similar to that suggested by Bolotin (1979).

To generate meaningful data, great care must be taken to determine the operative failure mechanism (i.e., subcritical crack growth or creep rupture). Dalgleish et al. (1985) proposed using the Monkman–Grant constant to separate experimental rupture life data. However, the creep–damage tolerance parameter, defined by Leckie (1986) as the total creep strain divided by the Monkman–Grant constant, may prove more suitable. After the data has been carefully screened, the model parameters n and B would be easily determined from creep rupture data. Taking the natural log of equation (8) yields

$$\ln(t_f) + n \ln(\sigma_o) = -\ln[B(n+1)] \tag{10}$$

The value $1/n$ corresponds to the slope of $\ln(\sigma_o)$ plotted against $\ln(t_f)$ and B would be computed from the intercept.

Now consider that the uniaxial test specimen is a monolithic ceramic with its inherent large scatter in strength. The variation in strength can be suitably characterized by the weakest link

5

theory by using Weibull's (1939) statistical distribution function. This is often referred to as Weibull analysis. With Weibull analysis the reliability of a uniaxial specimen is

$$R = \exp\left[-V\, (\sigma/\beta)^{\alpha} \right] \qquad (11)$$

This assumes that the stress state is homogeneous and that the two–parameter Weibull distribution sufficiently characterizes the specimen in the failure probability range of interest. Taking σ equal to the net stress defined above and, for simplicity, assuming a unit volume yield the following expression for reliability

$$R = \exp\left[-(\sigma_o/\psi\beta)^{\alpha} \right] \qquad (12)$$

Substituting for ψ by using equation (7) yields

$$R = \exp\left\{ -\left[\frac{\sigma_o}{\left[1 - B\,(\sigma_o)^n\,(n+1)\,t \right]^{1/(n+1)} \beta} \right]^{\alpha} \right\} \qquad (13)$$

Alternatively, substituting for ψ by using equation (9) yields

$$R = \exp\left\{ -\left[\frac{\sigma_0}{\beta} \right]^{\alpha}\left[1 - \left[\frac{t}{t_f} \right] \right]^{-\alpha/(n+1)} \right\} \qquad (14)$$

Here it is clearly evident that in the limit as t approaches t_f, R approaches zero. Examples of reliability curves and their dependence upon time and model parameters are presented in the section Fundamental Implications of the Model.

Next, the hazard rate function is considered. By definition the hazard rate (or mortality rate) is the instantaneous probability of failure of a component in the time interval $(t, t+\Delta t)$, given that the component has survived to time t. In more general terms, this function yields the failure rate normalized to the number of components left in the surviving population. This function can be expressed in terms of R or the probability of failure P_f as

$$h(t) = -\frac{dR}{dt}\left[\frac{1}{R} \right] = \frac{dP_f}{dt}\left[\frac{1}{1 - P_f} \right] \qquad (15)$$

With equation (14) used to define R, the hazard rate becomes

$$h(t) = \left[\frac{\alpha}{n+1} \right]\left[\frac{1}{t_f} \right]\left[\frac{\sigma_0}{\beta} \right]^{\alpha}\left[1 - \left[\frac{t}{t_f} \right] \right]^{-(\alpha+n+1)/(n+1)} \qquad (16)$$

The hazard function can be utilized from a modeling standpoint in one of two ways. First it can be used graphically as a goodness–of–fit test. If any of the underlying assumptions or distributions

6

used to construct equation (14) are invalid, one would obtain a poor correlation between model prediction of the hazard rate and experimental data. On the other hand, experimental data can be used to construct the functional form of the hazard rate and R can be determined from equation (15). In a sense this represents an oversimplified curve fitting technique. Since it was assumed that the creep rupture failure mechanism can be modeled by continuum damage mechanics, this effort has followed the first approach. In this spirit the hazard rate function would be used to assess the accuracy of the model in comparison to experiment.

The hazard rate is interpreted as follows:

(1) A decreasing hazard rate indicates component failure has been caused by defective processing.

(2) A constant hazard rate indicates failure is caused by random factors.

(3) An increasing hazard rate denotes wear–out of the component.

Here we note that negative values of α and n are physically absurd, hence

$$-(\alpha + n + 1)/(n + 1) < 0 \qquad (17)$$

and equation (16) yields an increasing hazard rate. This is compatible with the underlying assumption that creep rupture is the operative failure mechanism if one recognizes creep rupture as strictly a wear–out mechanism. Examples of hazard rates and their dependence on material parameters are presented in the section Fundamental Implications of the Model.

COMPARISON WITH VOID GROWTH MECHANISMS

When a ceramic undergoes high temperature deformation $(T \geq 0.5T_m)$ its microstructure changes. Voids nucleate and grow at the grain boundary, and grain size may increase to a point where it is possible for dislocations to move through grains. These changes in microstructure are referred to as creep damage and typically accelerate creep strain rates and the rate of damage until the specimen fails. This type of microstructural damage is similarly found in metals subjected to creep at elevated temperatures. The role of continuum damage mechanics in predicting the behavior of metals under creep loading has been discussed by a number of authors. Dyson and Leckie (1988) summarize the different metallurgical mechanisms and reconcile them with the

7

continuum damage mechanics approach. A more modest, yet similar effort is attempted here for ceramics. The mechanisms that promote damage in ceramics include growth of cavities in the second phase of sintered ceramics, void growth along grain boundaries attributed to vacancy diffusion in reaction bonded ceramics, and intragranular motion of dislocations (although this is not a typical mechanism in ceramics). Each mechanism is briefly discussed and a comparison is made with continuum damage theory.

Before discussing specific mechanisms, void growth on a grain boundary is considered in general terms. In a fashion similar to Cocks and Ashby (1980), figure 3 illustrates equally spaced spherical voids along the boundary between two grains subjected to a far field uniaxial stress. A cylindrical element of material surrounding a single void is isolated. It is assumed the voids are equally spaced at a distance 2l, with a void diameter 2r. The area fraction of holes η is defined as the ratio of the projected area of the void to the area of the cylindrical element, that is

$$\eta = \frac{\pi r^2}{\pi l^2} = \left[\frac{r}{l}\right]^2 \tag{18}$$

We now identify the parameter η with the continuum damage parameter ω. In the same sense $1-\eta$ would be associated with ψ. This association makes fundamental sense, and the two parameters would be directly related by some general volume integral relationship, that is

$$\omega = \left[\int_V \eta(x_1,x_2,x_3) \, dV\right](1/V) \tag{19}$$

where V is the volume of a continuum element located at spatial coordinates x_1, x_2, and x_3. Specifically, Murakami and Ohno (1981) developed an integral relation that resembles equation (19) by assuming the principal effect of net area reduction convoids along the grain boundary. If the volume of the continuum element is independent of time, then by our analogy $\dot{\omega} \sim \dot{\eta}$, and the assumed form of $\dot{\psi}(=-\dot{\omega})$ should compare favorably with material science evolutionary laws for $\dot{\eta}$.

Cannon and Langdon (1988) report power law creep behavior (dislocation climb and glide) in ceramics with large grain sizes. Cocks and Ashby (1980) analyzed this mechanism and developed an expression for an approximate growth rate by using an upper bound theorem from Martin (1966). The expression is given as

$$\frac{d\eta}{dt} = P\left[\frac{1}{(1-\eta)^\nu} - (1-\eta)\right]\sigma_\infty^\nu \tag{20}$$

8

where P and ν are constants and σ_∞ is the far–field uniaxial stress. By identifying $1-\eta$ with ψ, this expression resembles equation (4), and as η approaches 1, the two equations become quite similar. Note that this mechanism requires large grain sizes and that a very fine grain structure is easily maintained in ceramics. This is the microstructure of choice since it increases fracture toughness. Hence the power law creep mechanism is highly undesirable and, by using well controlled processing techniques, not likely to occur.

A second mechanism that promotes the growth of voids is vacancy diffusion. This mechanism is controlled by diffusive motion through the grain (termed Nabarro–Herring creep (1948), (1950)) or motion along the grain boundary (termed Coble creep (1963)), and it plays a more enhanced role in creep of ceramics. The growth of voids by grain–boundary diffusion was analyzed by Cocks (1985). He assumed that the grain boundaries serve as perfect sources and sinks for vacancies. Under the far–field stress σ_∞, voids grow by the plating of material from around the void onto the grain boundary. The rate of growth increases as the voids enlarge and is expressed as

$$\frac{d\eta}{dt} = \left(\frac{2 D_b \delta_b \Omega}{kT l^3 (\eta)^{0.5} \ln(1/\eta)} \right) \sigma_\infty \qquad (21)$$

where D_b is the grain–boundary diffusivity, δ_b is the grain boundary thickness, Ω is the atomic volume, k is Boltzman's constant, and T is absolute temperature. As η approaches 1 (effectively when $\eta > 0.1$) then

$$(\eta)^{0.5}\ln(1/\eta) \simeq (1 - \eta) \qquad (22)$$

and for isothermal conditions equation (21) can be expressed as

$$\frac{d\eta}{dt} = H \left[\frac{1}{1 - \eta} \right] \sigma_\infty \qquad (23)$$

where H is a constant that incorporates k, T, l, D_b, δ_b, and Ω. Again identifying $1-\eta$ with ψ, this equation is similar to equation (4) with n=1. If this analogy holds, one implication is that the constant B in equation (4) has information pertaining to grain size and diffusional properties embedded in it.

A fundamental difference between the growth rates for η and ψ must be pointed out. The two previous models for growth in η tacitly assumes that $\dot{\eta}=0$ when $\eta=0$; that is, nonexistent holes cannot grow. This is not assumed in the continuum damage approach. This may be viewed as a

9

strength or a weakness. It is a weakness in the sense that for small values of η, the two growth rates will be quite dissimilar. However the models for void growth must include some mechanism to account for the nucleation of voids to accurately predict time to failure. Models have been proposed (e.g., Page and Chan (1987), Tsai and Raj(1982)) for cavity nucleation in ceramics. With this approach time to failure has two components, time for nucleation and a second component for growth. Alternatively, the rate of growth in ψ is finite even in the absence of damage. Thus $\dot{\psi}$ includes both the nucleation and growth phases of damage accumulation and a special accounting for nucleation is unnecessary.

A large percentage of technologically important structural ceramics (e.g., β–Si$_3$N$_4$) are processed using liquid phase sintering techniques. This leaves a viscous second phase material along the grain boundary which has an adverse effect on mechanical properties at high temperature. Tsai and Raj (1982) proposed a method to compute the growth of a microcrack along a two–grain boundary. This approach is complex in that it allows an inhomogeneous stress field to be approximated by a Mode I stress field in a material that is assumed to behave as a viscous Newtonian fluid. A far simpler approach proposed by Raj and Dang (1975) models the growth of penny–shaped bubbles sandwiched between two rigid plates with a uniform normal traction applied to the plates. In this work the rate of growth of the penny–shaped bubbles is given in a general form as

$$\frac{d\eta}{dt} = S\left[\frac{\sigma_\infty}{f(\eta)}\right] \tag{24}$$

where

$$f(\eta) = (1.1 - \eta)[\ln(1/\sqrt{\eta}) + 0.96\eta - 0.72 - 0.23\eta^2] \tag{25}$$

Note that $\dot{\eta}$ remains finite as η approaches 1, however the two previous expressions for $\dot{\eta}$ as well as $\dot{\omega}$ become infinitely large as η approaches 1. The structure of equation (24) resembles equation (23), but the form of $f(\eta)$ in the denominator of the two equaare distinctly different. Even though equation (23) is an approximation of the physical process occurring along the grain boundary, one can still argue that the structure of this expression resembles the structure of the damage rates for continuum damage mechanics.

FUNDAMENTAL IMPLICATIONS OF THE MODEL

Unfortunately, at the present time we lack the experimental data to properly estimate model parameters. Thus an assessment of the model in comparison to experimental data is reserved for a later date, and for the examples that follow, model parameters are arbitrarily chosen for the purpose of illustration. The Weibull plots represent uniaxially stressed continuum elements (or links) of unit volume. For dimensionless R, the Weibull parameter β has units of stress$\cdot$(volume)$^{1/\alpha}$ with stress measured in megapascals. In all the examples β=400 and α=10 (where α, the Weibull modulus, is a unitless exponent). The damage parameter B has units of $1/(\text{time}\cdot\text{stress}^n)$, where the exponent n is unitless.

Figures 4(a) through 5(b) depict several Weibull reliability plots that include fast fracture (t=0) and three curves representing constant time (or damage). Note that the fast fracture curve is linear, however, the reliability curves become nonlinear with accumulated damage. Increasing the damage parameter B widens the spacing between the reliability curves and generally tends to increase the slope of the curves especially for smaller values of reliability. These trends can be seen in figures 4(a) through 5(a). Figures 4(b) and 5(a) strongly suggest the existence of vertical asymptotes. This is consistent with the assumption that t_f is deterministic. Hence t must always be less than t_f for a given applied stress. Figure 5(b) depicts the effect of changing the damage parameter n. As n is decreased the spacing between the curves diminishes. In this figure the reliability curves tend to collapse to the fast fracture curve indicating a material that is damage tolerant.

Figures 6(a) and 6(b) represent families of hazard rate curves. In each figure four curves representing applied stresses of σ=250, 300, 350, and 400 MPa are shown. In both figures the damage parameter n is taken equal to 2, and the Weibull parameters α and β are the same values as in the previous Weibull reliability plots. In figure 6(a) B=5×10^{-9} and in 6(b) B=1×10^{-9}. Lowering the value of damage parameter B decreases the spacing between the family of curves and also decreases the initial hazard rates. Decreasing the exponent n has the same relative effect. More importantly, these hazard rate curves would serve as a useful guide for the design engineer. Note that there tends to be a well–defined knee in the curves of both families where values of the

11

hazard rate increases rapidly with small increments in normalized time (t/t_f). The design engineer would use these curves to remove a component from service at a normalized time (t/t_f) less than the value at the breakpoint of a given curve.

CONCLUDING REMARKS

In this paper a time dependent reliability model for ceramics used in high temperature applications was developed by integrating the principles of continuum damage mechanics within the framework of Weibull analysis. It was assumed that the failure processes of subcritical crack growth and creep rupture are separable in that failure due to the former is a result of preexisting flaws, whereas creep rupture is characterized by the nucleation, growth, and coalescence of a new population of flaws. The nucleation of new flaws is a grain boundary phenomenon and an attempt was made to reconcile the continuum damage formulation to existing material science models that predict void growth along the grain boundary of polycrystalline ceramics. However the main objective of this work was to provide the design engineer with a reliability theory that incorporates the expected lifetime of a ceramic component undergoing damage in the creep rupture regime.

Several features of the model were presented in a qualitative fashion, including predictions of both reliability and hazard rate. The predictive capability of this approach depends on how well the macroscopic scalar state variable ψ captures the growth of these grain boundary microdefects. The influence of microdefects can be measured and used to quantify damage. Density change, acoustic attenuation, and change in the reliability of the material by using the concept of effective stress are methods that can be used to quantify damage. The authors are exploring the use of nondestructive evaluation techniques to accomplish this.

Finally, the kinetics of damage also depend significantly on the direction of the applied stress. Here it was expedient from a theoretical and computational standpoint to use a scalar state variable for damage since only uniaxial loading conditions were considered. The incorporation of a continuum damage approach within a multiaxial Weibull analysis necessitates the description of oriented damage by a second–order tensor. The authors are currently pursuing this task following the approach of Murakami and Ohno (1981).

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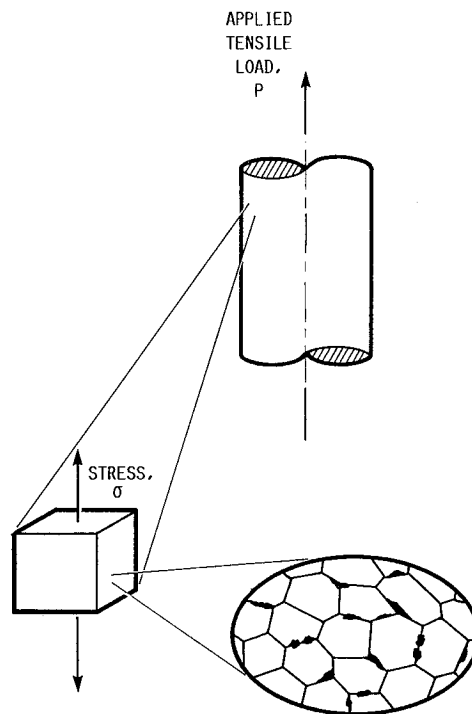
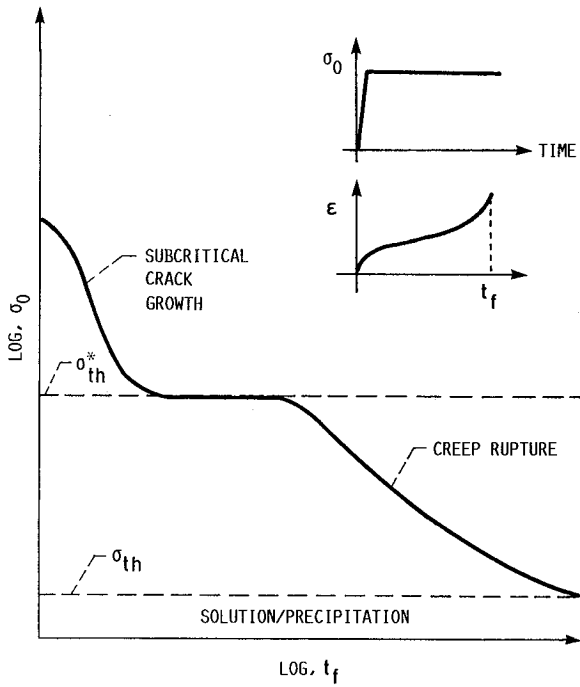FIGURE 1. − UNIAXIAL TEST SPECIMEN WITH MICRO-STRUCTURAL DEFECTS ALONG GRAIN BOUNDARIES.

FIGURE 2. - SCHEMATIC PLOT OF LOG OF STRESSES AS A FUNC-
TION OF LOG OF TIME TO FAILURE DELINEATING THRESHOLD
STRESSES, AND OUTLINING DISTINCT REGIONS WHERE EXPECTED
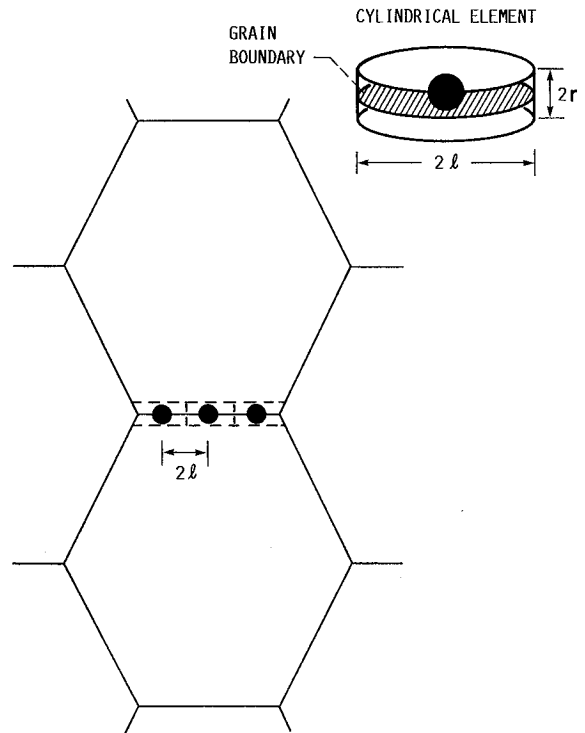FAILURE MECHANISMS WOULD BE OPERATIVE.

FIGURE 3. - SCHEMATIC OF SPHERICAL VOIDS GROWING ALONG TWO
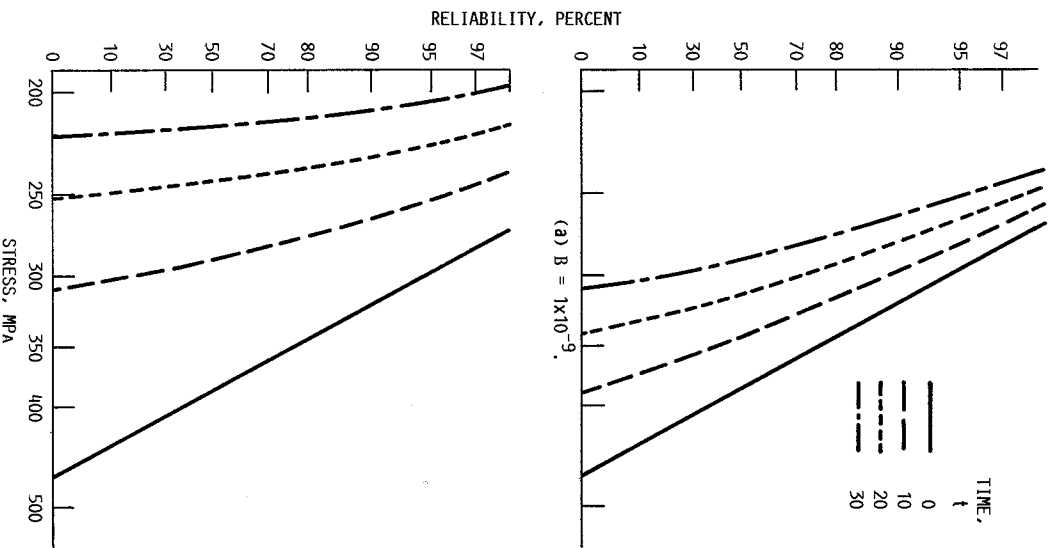GRAIN BOUNDARIES.

16

RELIABILITY, PERCENT

(a) B = 1×10^{-9}.

TIME,
t
0
10
20
30

(b) B = 3×10^{-9}.

STRESS, MPA

FIGURE 4. - FAMILIES OF WEIBULL RELIABILITY PLOTS DEPICTING THE EFFECTS OF INCREASING THE DAMAGE PARAMETER B. HERE α = 10, β = 400, AND n = 2.75.
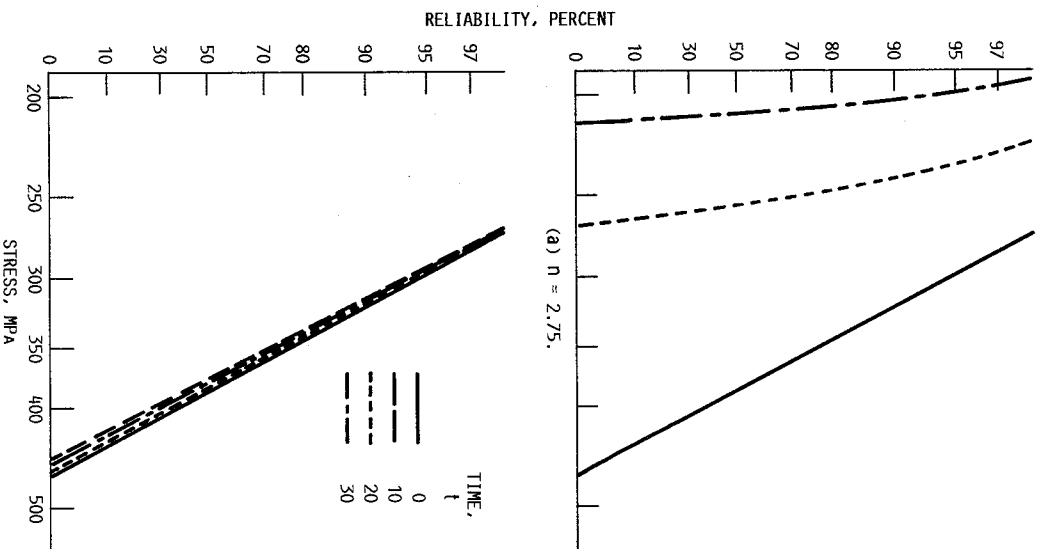
RELIABILITY, PERCENT

(a) n = 2.75.

TIME,
t
0
10
20
30

(b) n = 2.0.

STRESS, MPA

FIGURE 5. - FAMILIES OF WEIBULL RELIABILITY PLOTS DEPICTING THE EFFECT OF DECREASING THE DAMAGE PARAMETER n. HERE α = 10, β = 400, AND B = 5×10^{-9}.
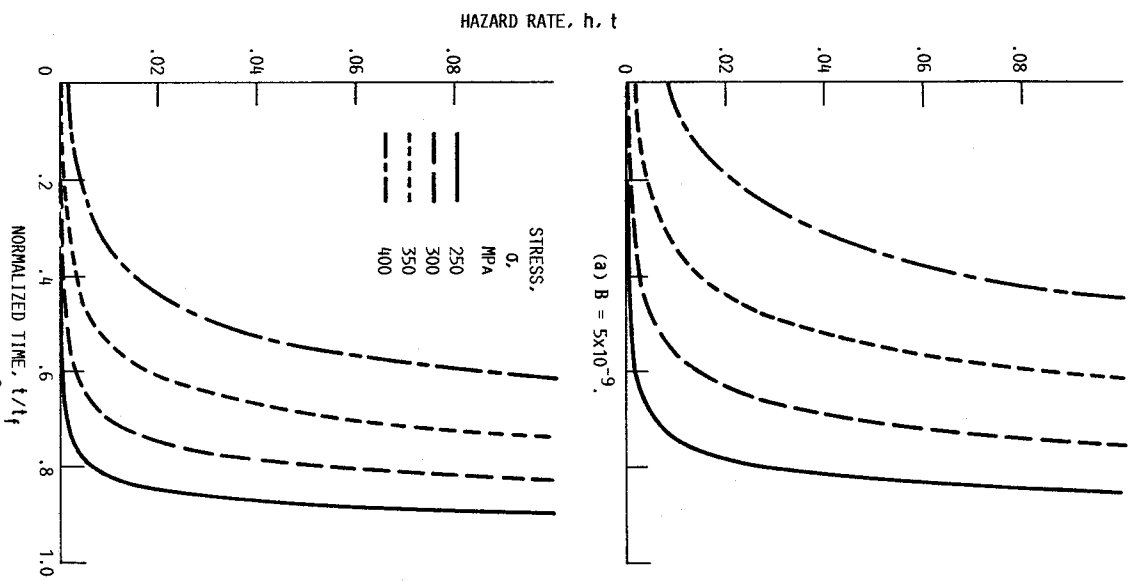
18

HAZARD RATE, h, t

STRESS,
σ,
MPA
250
300
350
400

(a) B = 5x10⁻⁹.

(b) B = 1x10⁻⁹.

NORMALIZED TIME, t/t_f

FIGURE 6. - FAMILIES OF HAZARD RATE CURVES DEPICTING
THE EFFECT OF DECREASING THE DAMAGE PARAMETER n.
HERE α = 10, β = 400, AND n = 2.0.

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16. Abstract

Presently there are many opportunities for the application of ceramic materials at elevated temperatures. In the near future ceramic materials are expected to supplant high temperature metal alloys in a number of applications. It thus becomes essential to develop a capability to predict the time-dependent response of these materials. This paper focuses on the creep rupture phenomenon and outlines a time-dependent reliability model that integrates continuum damage mechanics principles and Weibull analysis. Several features of the model are presented in a qualitative fashion, including predictions of both reliability and hazard rate. In addition, a comparison of the continuum and the microstructural kinetic equations highlights a strong resemblance in the two approaches.

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