Time-Domain Finite-Difference and Finite-Element Methods for Maxwell Equations in Complex Media

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***Abstract*—Extensions of finite-difference time-domain (FDTD) and finite-element time-domain (FETD) algorithms are reviewed for solving transient Maxwell equations in complex media. Also provided are a few representative examples to illustrate the modeling capabilities of FDTD and FETD for complex media. The term complex media refers here to media with dispersive, (bi)anisotropic, inhomogeneous, and/or nonlinear properties present in the constitutive tensors.**

***Index Terms*—Anisotropic media, dispersive media, finite-difference time-domain (FDTD) methods, finite elements (FEs), Maxwell equations, nonlinear media.**

I. Introduction

A. Background and Motivation

ANALYSIS of electromagnetic (EM) wave propagation and scattering in complex media has been a topic of continual interest. We can cite three areas of application, among others, that have served as important catalysts for this interest in recent years.

- Biological tissues are complex media with inhomogeneous and frequency dispersive properties. Analysis of EM wave interaction with biological media are fundamental in many medical applications such as noninvasive diagnosis and techniques, in understanding the effects of long term exposure of humans to (low intensity) EM fields, and for advancing the quality of medical imaging and telemedicine in general.
- Characterization of EM wave interaction with earth media is of great importance for environmental remote sensing and global climate assessment. Many earth media such as rocks, soils, snow, and vegetation have complex constitutive properties. For instance, porous rocks filled with salt water behave similarly to a metal-dielectric composite with insulator (rock matrix) and conductor (salt water) phases. In addition, many earth media are anisotropic.
- In recent years, there has been an upsurge in the design and development of new materials with tailored EM properties under the conceptual umbrella of "metamaterials." These include, but are not limited to, ferroelectric materials with (comparatively) small dielectric constants and losses at room temperature, ultra-high dielectric materials (e.g., carbon nanotubes), EM (or photonic) bandgap materials, antiferroelectrics, (low-loss) magnetodielectrics, left-handed or double-negative (DNG) media, low-k dielectrics, and surface plasmon devices. Engineered metamaterials have shown great promise as building blocks for devices with unique EM responses, from the microwave to the optical frequency range.

For the solution of Maxwell equations in complex and inhomogeneous media, it is often not possible to obtain the associated Green's function. As a result, the numerical solution is most often sought by methods that discretize Maxwell equations directly on a volumetric mesh as opposed to integral-equation-based boundary element methods. The former can be classified into finite methods [1]–[9] and (pseudo-)spectral methods [10], [11]. For problems where they are applicable, spectral methods exhibit faster convergence. Finite methods are more adequate for partial differential equations (PDEs) with variable and possibly discontinuous coefficients such as Maxwell equations in complex inhomogeneous media.

Finite methods for PDEs can be roughly classified into finite difference (FD), finite element (FE), and finite volume (FV) methods. FD methods are based on the approximation of partial derivatives by finite differences, and most often rely on regular structured grids. Although originally developed for elliptical equations (boundary value problems), FE methods have been later extended for hyperbolic equations (initial boundary value problems). Being naturally constructed for unstructured grids, FE is quite suited for numerical solution of PDEs in complex geometries [12], [13]. This is augmented by the fact that a-posteriori error estimates make FE suited for mesh generators with adaptive p- and h-refinement capabilities. However, FE methods in irregular grids require the solution of a (sparse) linear system (in the time domain, this is necessary at each time step). For hyperbolic PDEs in the time-domain, it is possible to obtain "matrix-free" (explicit) FE methods using, for instance, mass (matrix) lumping, but not without shortcomings [13]–[15]. The resulting explicit FE method can resemble FD and FV methods

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in unstructured grids. Indeed, from a geometric viewpoint [7], [16]–[18], the conceptual distinction among FE, FD, and FV becomes blurred. The FE framework can be also seen as a convenient way to generate high-order FD schemes [19], [20] and obtain accurate error estimates. The term FV has a somewhat loose meaning, and often refers to different discretization methods. In broad terms, FV relies on the evaluation of integrals over volume elements and facets surrounding each grid node and on the enforcement of local conservation laws. Although FD discretizations can be developed for unstructured grids—where they can retrace some FV schemes—it is more customary to restrict the FD classification to discretizations that rely on structured (or tensor product) grids. An overview of finite-volume time-domain techniques for Maxwell equations is presented in [21]

B. Time-Domain Finite Differences and Finite Elements

Time-domain simulations can produce wideband data with a single code execution. Moreover, nonlinear or time-varying media/components are more easily tackled in time-domain [22], [23]. In general, a time discretization strategy may involve either implicit or explicit updates. Some implicit updates are able to overcome the maximum time step bound imposed by the Courant stability limit [24]. However, they require the solution of a linear system at every time step, as alluded to above. On the other hand, even though explicit updates are bound by the Courant stability limit, they can be made matrix-free and with low computational complexity, down to $O(N^{1.5})$ in 2-D and $O(N^{1.33})$ in 3-D. Indeed, matrix-free updates such as the finite-difference time-domain (FDTD) method lead to optimal algorithms in the sense that $O(N)$ numbers are produced in $O(N)$ operations.

In its basic form as introduced by Yee [25] and pioneered by Taflove [2], the FDTD method is conceptually very simple and relies on the approximation of time and space derivatives of Maxwell curl equations by central differences on staggered grids, leading to a scheme which is second-order accurate in both space and time. The conceptual simplicity of FDTD should not belittle its power, though. Because FDTD is matrix-free, its memory requirements scale only linearly with the number of unknowns. This, added to the fact that FDTD is massively parallelizable, makes FDTD quite suited for petascale computing and beyond. Higher-order versions of FDTD exist, which trade accuracy by sparsity [26], [27].

There are two basic approaches for constructing finite element time-domain (FETD) methods for Maxwell equations. The first one is based on the second-order vector Helmholtz wave equation (for either the electric or magnetic field) [4], [13], [28], [29], which is discretized by expanding the unknown field in terms of local basis functions, most often (curl-conforming) Whitney edge elements [30], followed by application of the method of weighted residuals using an inner product with test functions. In order to produce symmetric matrices (in reciprocal media), the set of test functions is commonly chosen to be identical to the set of basis functions (Galerkin testing) [4]–[8]. The second FETD approach is based on the discretization of the first-order coupled Maxwell curl equations by expanding the electric and magnetic fields in terms of mixed elements, most often Whitney edge elements for the electric field and (div-conforming) Whitney face elements for the magnetic flux density [19], [30]–[34]. This choice satisfies a discrete version of the de Rham diagram and avoid spurious modes. This is followed by either (*a*) application of the method of weighted residuals with appropriate (also mixed) test functions [19] or (*b*) by the use of incidence matrices and construction of discrete Hodge operators.[1]

The present review will focus exclusively on FDTD and FETD methods as described above. Therefore, it will not include other popular volumetric for Maxwell equations such as the transmission-line modeling (TLM) [36], the discontinuous Galerkin method [37], the finite integration technique (FIT) [38], [39], "mimetic" finite-differences [40], and other related techniques. Note that these other methods are amenable to extensions to complex media as well [41]–[43].

C. Complex Media

This paper provides a brief overview of extensions of FDTD and FETD to solve transient Maxwell equations in complex media. The term "complex media" refers here to media with any of the following properties incorporated in the (bulk) constitutive equations:

- (frequency) dispersive;
- inhomogeneous;
- (bi-)anisotropic;
- nonlinear;

as opposed to, for example, media where the complexity is essentially geometric (at the macroscale) [44], [45].

Following the thematic focus of this journal, we restrict ourselves to problems where EM wave propagation and scattering (and any modification thereof induced by the macroscopic constitutive relations) is the phenomena of primary importance. This excludes media that introduces interesting dynamics of their own, such as carrier transport in semicondutor devices for example. Likewise, we do not discuss algorithm developments specific to modeling of plasmas or ionized media.

II. Dispersive Media

A. FDTD Implementations

In dispersive media, the permittivity is a function of frequency. In linear, time-invariant, isotropic media, the time-domain constitutive equation relating the electric field and the electric flux density $\mathbf{D}$ is cast as a convolution between $\mathbf{E}$ and the permittivity as follows:

$$\mathbf{D}(\mathbf{r},t) = \int_{-\infty}^{t} \epsilon(\mathbf{r},t-\tau)\mathbf{E}(\mathbf{r},\tau)d\tau \quad (1)$$

$$\epsilon(\mathbf{r},t) = \epsilon_0\delta(t) + \epsilon_0\chi_e(\mathbf{r},t) \quad (2)$$

where $\epsilon(\mathbf{r},t) = \mathcal{F}^{-1}[\epsilon(\mathbf{r},\omega)]$ is the time-domain permittivity function, $\mathcal{F}$ stands for Fourier transformation, ϵ_0 is the vacuum permittivity, $\delta(t)$ is the Dirac delta function, and $\chi_e(\mathbf{r},t)$ is the (time-domain) electric susceptibility function. In FDTD or FETD, the time variable is discretized as $t = n\Delta t$ with $n =$

[1]When Hodge operators are expressed as inner product integrals of Whitney elements (*Galerkin Hodges*), it can be shown that alternatives (*a*) and (*b*) lead to equivalent algorithms [35].

$0, 1, 2 \ldots$. We denote $\mathbf{E}(n\Delta t) = \mathbf{E}^n$, and similarly for the other fields. A direct implementation of the convolution above would require storage of the entire past time series of $\mathbf{E}^n$, which is obviously not practical. Due to the exponential nature of the susceptibility kernel (a consequence of the time-invariant and linear properties of the underlying differential equation), a *recursive* convolution can be implemented instead [46], whereby recursive accumulators are introduced and storage of only a few previous time step values is necessary (the actual number depends on the order of accuracy sought, with two or three steps being sufficient in typical FDTD simulations). Examples of the implementation of such recursive convolution (RC) in FDTD can be found, e.g., in [46]–[54]. The terminology RC is usually reserved for the low-order implementation that assumes a piecewise constant electric field between each time step [47]. Other implementations for the convolution also exist [52], [53], [55], with the *piecewise linear recursive convolution* (PLRC) being a popular choice. Instead of implementing a convolution for the $\mathbf{E}$ and $\mathbf{D}$ constitutive relation, it is more advantageous in some cases to implement the convolution for an equation involving $\mathbf{E}$, and the electric current density $\mathbf{J}$. This is exemplified, e.g., in [56], [57]. A review and comparison of some approaches for modeling of dispersive media can be found in [58] and [59]. A stability and numerical dispersion analysis of the recursive convolution FDTD approach for Drude and Lorentz dielectrics and also for anisotropic dielectrics is presented in [60]. Since conventional FDTD is second-order accurate in time, it is usually not advantageous to implement a high-order accurate recursive convolution unless the time integration in the core FDTD update (of Maxwell curl equations) itself is of high order.

In linear time-invariant media, the constitutive equation in dispersive media can also be cast as ordinary differential equation (ODE) in time involving $\mathbf{D}$ and $\mathbf{E}$ [61], [62] or, alternatively, involving $\mathbf{E}$ and some induced macroscopic polarization field such as $\mathbf{P}$ [63], [64]. In linear time-invariant media, this ODE is linear with constant coefficients (in time) with the generic form

$$\sum_{p=0}^{N_1} a_p(\mathbf{r}) \frac{\partial^p \mathbf{D}(\mathbf{r},t)}{\partial t^p} = \sum_{p=0}^{N_2} b_p(\mathbf{r}) \frac{\partial^p \mathbf{E}(\mathbf{r},t)}{\partial t^p}. \quad (3)$$

or an analogous ODE involving $\mathbf{E}$ and polarization fields [63]. In some cases, additional dynamic fields are present in the physical model so that an ODE *system* ensues [65], [66].

The order N_1, N_2 and the coefficients $a_p(\mathbf{r})$, $b_p(\mathbf{r})$ above depend on the particular dispersion model considered for $\epsilon(\mathbf{r}, \omega)$. In FDTD, the above ODE can be discretized by, for example, recasting it as an equivalent *system* of ODEs involving only first- and/or second-order differential equations [67], followed by a FD approximation in time of each differential equation. This is commonly referred to as the *auxiliary differential equation* (ADE) approach. Similarly to the recursive convolution approach, different orders of accuracy in time can be implemented. An analysis of the stability and numerical dispersion error of some ADE schemes is presented in [68] and a *numerical* dispersion analysis of both ADE and convolutional approaches appears in [69]. A comprehensive analysis and comparison of both convolutional and ADE approaches in the case of Debye and Lorentz models is presented in [59]. Furthermore, the formulation and evaluation of a memory-efficient, full-synchronous ADE for various dispersion models is carried out in [70]. The underlying connection between the (PL)RC and the ADE approaches is explored in [71] to derive a systematic approach for FDTD modeling dispersive media that is in some sense a generalization of PLRC and ADE.

Since a discrete time series has a natural representation in terms of z-transforms, it is also possible to cast the (time) discrete equations in dispersive media using a z-transform domain approach. This was first suggested for FDTD in [72] and further developed in [73]–[75]. A comparison between z-transform and ADE approaches for a monospecies Debye model describing biological tissues is provided in [76]. Another, but less frequently used, approach for frequency dispersion modeling is the so-called *frequency approximation method* described in [77], [78] and based on a substitution of the frequency variable in the (frequency-domain) dispersion models by linear backward differences.

Spatially dispersive media occurs when the constitutive parameters are nonlocal or depend on the spatial derivatives of the field, or, equivalently, their Fourier transform (in the wavenumber space) depend on the wavenumber components [79]. An FDTD formulation for "wire media", a type of metamaterial with spatial dispersion, is presented in [80]. Special cases of spatially dispersive media are bianisotropic and biisotropic media, which are considered in Section V ahead.

B. Frequency-Dispersive $\epsilon(\omega)$ and $\sigma(\omega)$ Models

The most frequently used dispersion models for $\epsilon(\omega)$ in linear dispersive media are the Drude, Debye, Lorentz, and Cole-Cole models. Debye models are commonly used to approximate the frequency behavior of biological tissues and soil permittivities in wideband frequency ranges. Lorentz models describe the frequency behavior of some metamaterials such as DNG media close to resonances. Drude models are useful in describing the behavior of metals at optical frequencies, where they can sometimes also be augmented by Lorentz terms. Cole-Cole models find applications in modeling of biological media, polymers, and some dispersive dielectrics. These models are somewhat interrelated: The Cole-Cole model can be viewed as a generalization of the Debye model for broader and flatter relaxation spectra [81] and the Debye model can be viewed as a special case of the Lorentz model under a particular choice of parameters [53]. In some cases (such as for Drude model description of metals at optical frequencies), it is customary to treat the (frequency-dependent) conductivity $\sigma(\omega)$ as the primary function of frequency. Of course, any functional dependence of $\sigma(\omega)$ can always be incorporated into complex permittivity dispersion models. Depending on the particular application, these models may be complemented by a static conductivity term (i.e., a simple pole at $\omega = 0$) as well.

It is beyond our objective here to describe all these dispersion models in detail. Suffice it to say that they can be expressed as (a sum of) rational functions in the variable ω except for the Cole-Cole model which involves fractional exponents and can be *approximately* described by a sum of rational functions. Furthermore, all these models obey Kramers-Kronig relations, a fundamental requirement from causality [82]. In passive media, this is equivalent to having $\epsilon(\omega)$ analytic over the entire complex ω upper half-plane. Because it can be difficult to extract exact macroscopic dispersion model parameters from first principles, they are frequently determined by fitting of experimental data over the frequency range of interest. If the frequency range is not too large, monospecies dispersion models are sufficient. For wide frequency bands, a multispecies model or combination of different models can be necessary.

It should be pointed out that dispersive media have inherent time scales determined by complex resonance(s) and relaxation time(s) that may need to be properly captured in a time-domain simulation. In this case, these time scales impose additional constraints on the maximum time step Δt of the time-domain simulation based on accuracy considerations. These constraints may supersede the Courant stability limit or accuracy considerations based on the frequency of the excitation signal [83].

C. FDTD for Dispersive Media: Selected Examples

1) Microwave Subsurface Radar: For applications related to microwave remote sensing of underground objects using subsurface radar, a monospecies Debye model is often adequate for typical dispersive soils. Examples of FDTD simulations of microwave subsurface radar in dispersive soils modeled by monospecies Debye models using the RC approach can be found in [84] and [85] and using the ADE approach in [86]. Furthermore a single-pole conductivity model is considered in [87]. For simulations of ultrawideband subsurface radar, a multispecies Debye model may become necessary to capture the frequency-dispersion behavior of typical soils. Multispecies Debye models are incorporated by the RC approach in [88], by the ADE approach in [89]–[92], and by the PLRC approach together with perfectly matched layers (PMLs) in [53], [54]. An approach that incorporates dispersion via ADE coupled with a fourth-order accurate FD in space is described in [93]. The incorporation of dispersive media properties into subcell FDTD techniques for the modeling of electrically thin dispersive layers is described in [94]. In the case of borehole subsurface radars, FDTD modeling is best done utilizing cylindrical grids because they conform to the borehole geometry and avoid staircasing errors [95]–[97]. Cylindrical FDTD algorithms that incorporate dispersive media modeling can be found, for example, in [54] and [98].

2) Biological Media: Examples of FDTD modeling of biological media using monospecies Debye models can be found in [99] and [100]. The modeling of monospecies or multispecies Debye models in biological applications was considered in [62], [101]–[103] using the ADE approach and in [51] using the RC approach. Reference [104] utilizes FDTD to assess capabilities of narrowband versus ultrawideband microwaves in hyperthermia breast cancer treatment. A FDTD calculation that employs a two-species Debye model for the calculation of SAR in a heterogeneous model of the human body is presented in [105]. Cole-Cole dispersion models are frequently used for biological media since they cab provide a better representation than Debye models for broad frequency ranges including frequencies below tens of MHz [106]–[108] and, as alluded before, involve fractional derivatives. These can be approximated in terms of a finite series and incorporated into the FDTD update by, for example, applying a conventional ADE approach for each term of the series [64] or by the z-transform approach [109].

3) Metallodielectric Nanostructures: Nanoscale metallodielectric structures can support surface plasmon resonances at optical frequencies and can be used for a variety of applications. These include subwavelength (nano)optical waveguiding for optical/electronic integration at CMOS scale. Examples of FDTD modeling of metallodielectric structures incorporating Drude models for noble metals at optical frequencies can found in [110]–[115] using the ADE approach or in [116] using the z-transform approach. For wide frequency bands, a multispecies Drude-Lorentz model is necessary [117]. More recent examples of FDTD implementation of Drude-Lorentz models for nanophotonics applications are found using RC in [118], using PLRC in [119], and using ADE in [120], [121]. A very wideband model for Ag using six complex conjugate pairs is described in [122].

Fig. 1 shows an example from the FDTD simulation of a chain of 50-nm diameter Au nanospheres forming an L-junction waveguide. The nanospheres are modeled by a monospecies Drude model [115]. The center-to-center spacing along the chain is 75 nm. The excitation wavelength is centered at 516.67 nm, which is much larger than the diameter or spacing of the nanospheres. The spatial grid cell size is equal to $\Delta_s = 1.5625$ nm, for a total of $448 \times 544 \times 256$ grid points. Subwavelength guiding along the chain is clearly visible. This is achieved through near-field coupling among successive plasmon resonances in each nanosphere [115].

D. Magnetic Dispersion and Doubly-Dispersive Media

For materials that exhibit frequency dispersive (effective) permeability $\mu(\omega)$, the magnetic constitutive equation can be expressed in terms of either a convolution integral or an ODE in time relating the magnetic field intensity $\mathbf{H}$ and either the magnetic flux density $\mathbf{B}$ or some magnetic polarization vector(s), similarly to the electric constitutive equation. As a result, the implementation of dispersive permeability in FDTD follows analogous steps to the above.

For *doubly* dispersive media, i.e., media that exhibit dispersion in *both* the permittivity and permeability, the FDTD update can be constructed by including a sequence of either two RC or two ADE implementations within each time step of the FDTD update, viz., implementing a permittivity model following the electric field update from Ampere's curl equation and implementing a permeability model following the magnetic field update from Faraday's curl equation.

An important example of doubly-dispersive media is DNG media [123], [124], where both $\Re e[\epsilon(\omega)] < 0$ and $\Re e[\mu(\omega)] < 0$ in some frequency range. Examples of FDTD modeling of bulk DNG media with electric and magnetic Lorentz susceptibilities can be found, for example, using the ADE approach in [124] and [125], using the PLRC approach in [126], and using a

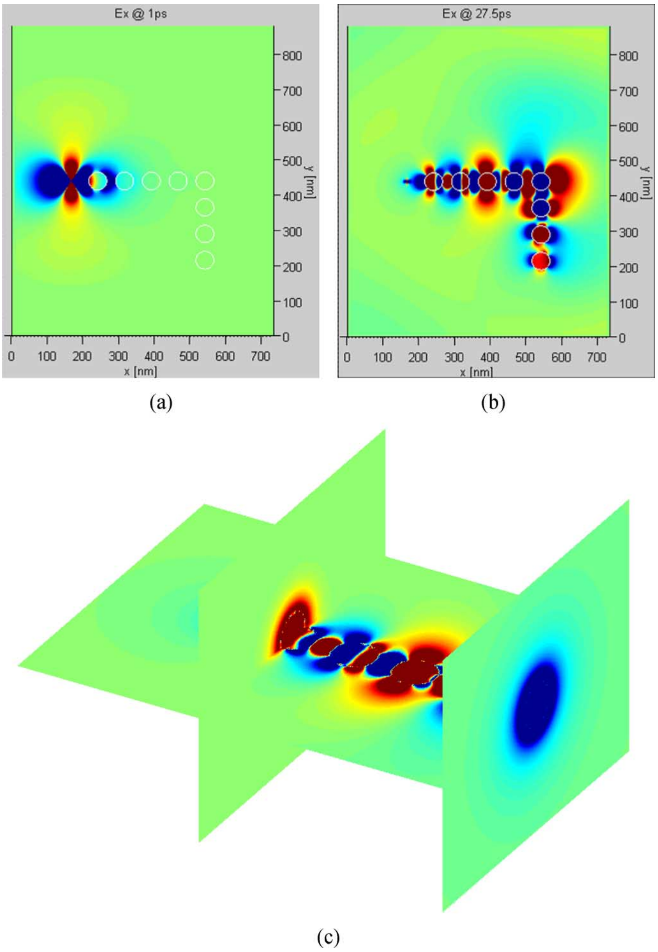

Fig. 1. Snapshots of the electric field distribution along a chain (L-junction) of spherical Au nanoparticles obtained by FDTD. The FDTD algorithm uses a Drude model to model of the frequency dispersion of Au in the optical band considered [115]. This figure illustrates the waveguiding along the L-junction produced by near-field coupling between plasmon resonances in each sphere. The sub-wavelength lateral confinement of the field is apparent in the 3-D sliced section view. (a) Field distribution at $t = 1$ ps. (b) Field distribution at $t = 27.5$ ps. (c) 3-D sliced section view.

combination of ADE and z-transform approaches in [77]. Reference [127] describes an FDTD algorithm for double-negative media with arbitrary number of poles using the z-transform approach. Reference [128] shows that the approaches for Lorentzian DNG media described using different approximation principles in [77] and [129] are actually equivalent to the algorithm developed in [130]. Reference [131] points out that a spatial averaging is required at DNG interfaces to yield accurate results in FDTD.

E. FETD Implementations

Since the main distinction between FDTD and FETD resides on the *spatial* discretization, the same approaches used for implementing frequency dispersive media in FDTD can be also employed for FETD. Reference [132], for example, utilizes RC to implement Lorentz dispersion models in FETD, while [133] utilizes a similar RC approach to implement multispecies Debye dispersion models. The use of ADE to implement either Lorentz or Debye models in FETD is considered in [134].

Hybridization of FDTD and FETD is of interest for exploiting the inherent strengths of each method, viz., geometrical flexibility in FETD and computational efficiency in FDTD. With this objective in mind, [135] discusses the implementation of a hybrid FDTD/FETD scheme to dispersive media. For narrowband problems, it can be advantageous to work with the complex envelope representation of the fields instead of the real time-domain fields. This reduces the numerical dispersion error and increases the overall computational efficiency of the simulation for a given accuracy. An extension of the complex envelope FETD to dispersive media is described in [136].

The FETD implementations above are based on Helmholtz equation and on the use of edge elements to expand the electric field. For doubly-dispersive media, the curl-curl operator in the Helmholtz equation includes the inverse permeability, which is a convolutional operator in the time domain. This causes FETD implementations for doubly dispersive media based on the Helmholtz equation to be somewhat contrived. A more natural approach in this case is to use a mixed FETD based on the system of coupled first-order Maxwell curl equations, whereby both permeability and permittivity operators are naturally factored out from the (curl) equations involving spatial derivatives [19], [31], [33], [34]. [137] presents an extension of the mixed FETD method to general multispecies doubly-dispersive media (where both the permittivity and the permeability have arbitrary number of poles) using the ADE approach.

Fig. 2 shows the example of a simulation of doubly dispersive media using a mixed FETD. The structure consists of a slanted plane-concave lens made of a zero-index metamaterial [138], which is a special case of a doubly dispersive material. In this case, $\epsilon(\omega)$ and $\mu(\omega)$ are described by identical Lorentz models that become zero (simultaneously) at a critical frequency ω_c. At ω_c, the *spatial* distribution of the field inside such material has a static character corresponding to an infinitely long wavelength. At the same time, the field retains an ordinary oscillatory behavior in time.

III. Inhomogeneous Media

Being volumetric methods, FDTD and FETD are particularly suited to study transient wave propagation in inhomogeneous media. Ideally, the (discrete) representation of the EM fields should automatically observe the correct jump conditions across material discontinuities, viz., tangential continuity for $\mathbf{E}$ and $\mathbf{H}$ and normal continuity for $\mathbf{B}$ and $\mathbf{D}$. In FETD, this condition simply requires the grid to be aligned with any material discontinuities and the use of proper basis functions, *viz.*, edge elements with tangential continuity for $\mathbf{E}$ and face elements with normal continuity for $\mathbf{B}$. In FDTD, the basic difficulty is the limitation imposed by the use of a Cartesian mesh, which leads to staircasing error along slanted or curved interfaces. This can be mitigated by subgridding [139]–[142], nonorthogonal FDTD meshes [143], [144], locally conformal FDTD cells [145], [146], heterogeneous grid components [147], or a combination thereof. Hybridization of FDTD with FETD is another attractive alternative [148], [149]. These are extensions to increase the geometrical flexibility of FDTD, which is beyond the scope of this review.

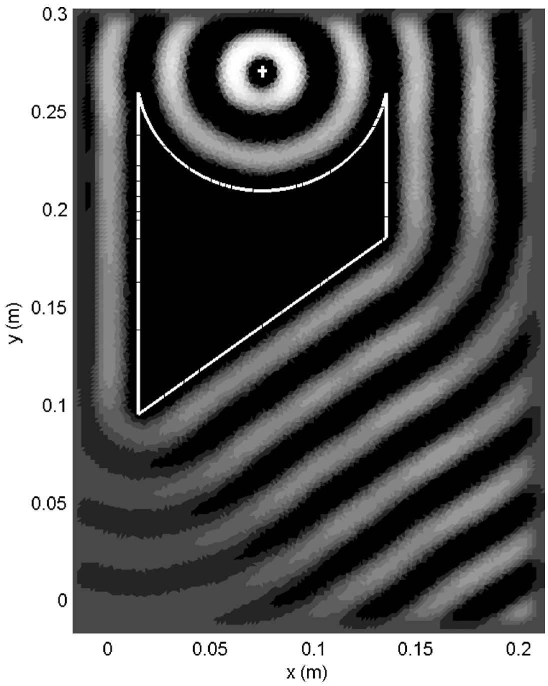

Fig. 2. FETD simulation of a slanted plane-concave lens made of zero-index metamaterial. The FETD algorithm uses a mixed (basis function) formulation and incorporates doubly dispersive (ϵ and μ) material modeling [137]. This figure shows a snapshot of H_z due to a line source located close to the lens and surrounded by air, at the central frequency ω_c where $\epsilon(\omega_c) = \mu(\omega_c) = 0$. The wavelength of the source is 3 cm in air. The lateral width of the lens is 12 cm. The concave side is circular with 6.1 cm radius and slanted side makes an angle of 53° with the y-axis. The point source is located at the center of curvature of the concave side. The field inside the zero-index lens has a static character in space with zero-phase variation among all points (corresponding to an infinite wavelength) while being dynamic in time [138]. This property allows for the seamless wavefront "re-shaping" observed in this figure.

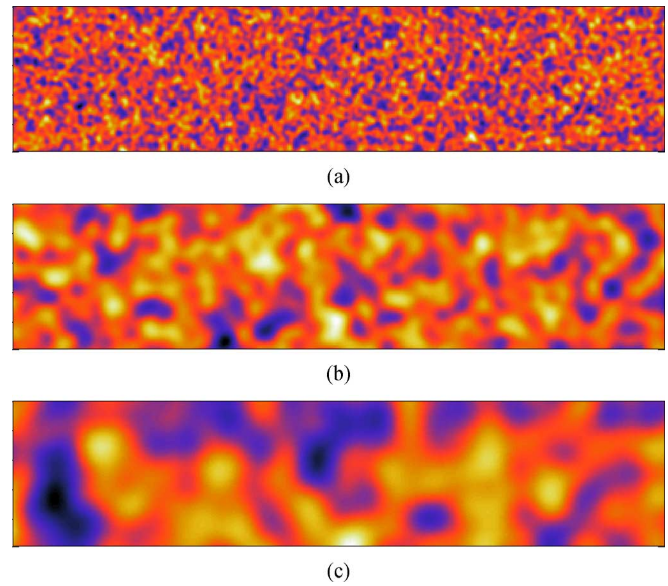

Fig. 3. Three FDTD grid realizations of the permittivity distribution of inhomogeneous, continuous (clipped) Gaussian random media with (spatially invariant) Gaussian correlation function. The three realizations have same average and variance but different correlation lengths l_s. (a) $l_s = 3.0\Delta_s$. (b) $l_s = 9.0\Delta_s$. (c) $l_s = 20.0\Delta_s$.

A. Homogenization Techniques

For (possibly multiscale) materials having microstructures which are at least one order of magnitude smaller than characteristic excitation lengths such as the wavelength or the skin depth, a homogenization [79], [150]–[156] of the microstructural inhomogeneities is of interest to avoid the need for (excessive) grid refinement. Classical homogenization approaches include the Maxwell-Garnett (MG) approximation [79], [151], [155] (Clausius-Mossotti formalism) for static metallodielectric mixtures [153], and the effective medium approximation (EFA) [150], [157] (Bruggeman formalism). Rigorously speaking, both MG and EFA are applicable only to statics; however, they can be extrapolated to finite frequencies in the long-wavelength limit under hypothesis such as of tenuous and/or sparse media that allow for the solution of the problem assuming independent scattering. In dense or nontenuous media – or in media where one phase occupy a low fractional volume but can be locally dense due to clustering effects – the independent scattering assumption fails [158] and Foldy's approximation [159] or the quasicrystalline approximation (QCA) [160] are frequently employed. In QCA, second-order statistics such as a pair correlation function (for discrete scatterers) must be specified. Since such statistics are rarely available, approximations such as the hole correction or the Percus-Yevick function are employed [161]. For multiscale media, homogenization techniques can be coupled to finite methods such as FDTD and FETD via down-scaling techniques [162] in order to obtain a (coarser) description where the scale of inhomogeneities is above the discretization scale. Examples of FDTD applications to actually validate effective permittivity models can be found in [163] and [164].

B. Random Inhomogeneous Media

In some practical scenarios, only partial information is available about the constitutive properties of the medium. In this case, a deterministic description is not appropriate. Instead, a random model where the constituive properties are described statistically should be employed. Examples of random medium implementations in FDTD can be found, e.g., in [165] and [166], where simulations of subsurface problems are considered with continuous Gaussian distributions and (spatially invariant) Gaussian correlation functions assumed for the permittivity. Reference [167] assumes a discrete random model for the permittivity in the same type of problem. For random media, second-order statistics such as the correlation length l_s influence the wavenumber spectrum. If $l_s < \lambda_{\min}$, where $\lambda_{\min}$ is the smallest wavelength of operation, l_s would set the criterion for the spatial discretization scale (note that if $l_s \ll \lambda$, a homogenization technique can be applied instead). Fig. 3 shows realizations of a (continuum) random medium model where the permittivity is a clipped Gaussian random variable and the correlation function is Gaussian [168]. The three permittivity realizations have same average and variance but different correlation lengths. Such distributions can be implemented in FDTD by using a random number generator at each grid point followed by application of a spatial filter with spectrum dictated by the correlation function.

IV. Anisotropic Media

A. FDTD Implementations

In anisotropic media, the permittivity, permeability and/or conductivity are represented by tensors $\bar{\epsilon}$, $\bar{\boldsymbol{\mu}}$, and $\bar{\boldsymbol{\sigma}}$. The implementation of *diagonal* anisotropic-medium tensors in FDTD is very similar to isotropic media because the spatial finite-difference stencil does not change [169], [170]. This is not the case for *nondiagonal* anisotropic media because of coupling between nonparallel components which are also noncollocated in the staggered FDTD grid. Using a leap-frog time integration, the FDTD update in anisotropic media can be written in the generic form

$$\mathbf{H}^{n+1/2} = -\Delta t \bar{\boldsymbol{\mu}}^{-1} \cdot \nabla \times \mathbf{E}^n + \mathbf{H}^{n-1/2} \quad (4)$$

$$\mathbf{E}^{n+1} = \left(\frac{1}{\Delta t}\bar{\epsilon} + \frac{1}{2}\bar{\boldsymbol{\sigma}}\right)^{-1} \cdot \left[\nabla \times \mathbf{H}^{n+1/2} + \left(\frac{1}{\Delta t}\bar{\epsilon} - \frac{1}{2}\bar{\boldsymbol{\sigma}}\right)^{-1} \cdot \mathbf{E}^n\right]. \quad (5)$$

The extension of Yee's FDTD scheme to nondiagonal anisotropic tensors for both the permittivity and the conductivity was considered in [171] using a second-order spatial interpolation for noncollocated field components in the FDTD grid. Reference [172] describes a systematic procedure to develop FDTD schemes in lossless anisotropic media using either second-order or fourth-order approximations for the (spatial) derivatives. An extension to anisotropic dielectric media that includes the treatment of PEC boundaries is provided in [173] and FDTD extensions to materials with anisotropy in both permittivity and permeability are considered in [174] and [175]. Furthermore, [176] presents an extension of cylindrical FDTD to fully anisotropic conductive earth media models.

Examples of FD approaches for anisotropic media that have been developed for frequency-domain simulations but can be adapted to the time-domain are found, e.g., in [177] and [178] based on a volumetric averaging, in [179] based on a two-term spatial interpolation scheme, and in [180] based on the use of Lebedev's staggered grid.

B. FETD Implementations

Finite-element methods for anisotropic media are relatively less developed in the time-domain than in the frequency domain, but since our focus here is restricted to time-domain, we will not dwell on the frequency-domain finite-element literature for anisotropic media in much detail. Some earlier works considering variational aspects of finite-element implementation in general anisotropic media can be found, for example, in [181]–[184]. A finite-element implementation for anisotropic media using hexahedral meshes and amenable to mass lumping is presented in [185]. Examples of finite-element implementations in anisotropic media using edge elements and tetrahedral meshes are described in [186]–[188].

A particular example of anisotropic media which is of great importance in time-domain simulations is the "uniaxial" perfectly matched medium (PML), that represents one of the possible strategies for implementing the PML absorbing boundary condition (ABC) to truncate FD, FE, or FV grids in open-domain problems. Because of its special usage as artificial absorber, the uniaxial PML will be considered separately in Section VII.

In a FETD implementation based on the vector Helmholtz wave equation and the use of Whitney edge elements $\mathbf{W}_i^e$ (1-form proxys) as basis and test functions for the electric field, the semi-discrete equation in a source-free region writes as

$$[M]\frac{\partial^2[\mathbf{E}]}{\partial t^2} + [T]\frac{\partial[\mathbf{E}]}{\partial t} + [S][\mathbf{E}] = 0 \quad (6)$$

where $[\mathbf{E}]$ above represents the array (column vector) of unknowns, and $[M]$, $[T]$, $[S]$ are mass (capacitance), conductance, and stiffness (inductance) matrices whose elements in general (linear, nondispersive) anisotropic media $\bar{\epsilon}$, $\bar{\boldsymbol{\sigma}}$, $\bar{\boldsymbol{\mu}}$ are given by the following volume integrals (in 3-D) [35]

$$M_{ij} = \int_\Omega \mathbf{W}_i^e \cdot \bar{\epsilon} \cdot \mathbf{W}_j^e dV$$
$$T_{ij} = \int_\Omega \mathbf{W}_i^e \cdot \bar{\boldsymbol{\sigma}} \cdot \mathbf{W}_j^e dV$$
$$S_{ij} = \int_\Omega \nabla \times \mathbf{W}_i^e \cdot \bar{\boldsymbol{\mu}}^{-1} \cdot \nabla \times \mathbf{W}_j^e dV \quad (7)$$

where Ω denotes the computational domain (since the basis function are local, each integral above effectively spans only a few elements). The time-discretization of (6) can be done in a number of ways [13]. To achieve unconditionally stability, popular choices are the Wilson-θ [13] and the Newmark-β [29] methods.

It is common practice to assume $\bar{\epsilon}$, $\bar{\boldsymbol{\sigma}}$, and $\bar{\boldsymbol{\mu}}$ uniform over each volumetric cell (in 3-D) to calculate the integrals in (7). In inhomogeneous media, this assumption requires material averaging over adjacent cells to retain the symmetry of $[M]$, $[T]$, $[S]$ in reciprocal media (where the underlying $\bar{\epsilon}$, $\bar{\boldsymbol{\sigma}}$, $\bar{\boldsymbol{\mu}}$ are themselves symmetric[2]). Using a geometric discretization approach, it is easy to show [35], [189] that $[S]$ can be decomposed as

$$[S] = [C]^t[\star_\mu^{-1}][C] \quad (8)$$

where $[C]$ is the (curl) *incidence matrix* and the superscript denotes transpose. The matrix $[C]$ has only entries $\{-1, 0, 1\}$ and represents the discrete curl operator distilled from its metric structure (or, equivalently, the discrete exterior derivative applied to 1-forms [35], [190]). The elements of $[\star_\mu^{-1}]$ (discrete Hodge operator) are given by

$$(\star_{\mu^{-1}})_{ij} = \int_\Omega \mathbf{W}_i^f \cdot \bar{\boldsymbol{\mu}}^{-1} \cdot \mathbf{W}_j^f dV \quad (9)$$

where $\mathbf{W}_i^f$ are Whitney face elements (2-form proxys). In this context, the mass matrix is also equivalent to a discrete Hodge operator and henceforth denoted as $[M] = [\star_\epsilon]$.

For a FETD based upon the first-order Maxwell curl equation, the semi-discrete equations in general (nondispersive) anisotropic media can be written as

$$\frac{\partial [\star_\epsilon]\mathbf{E}}{\partial t} + [\star_\sigma]\mathbf{E} = [C]^t\left[\star_{\mu^{-1}}\right]\mathbf{B}$$
$$\frac{\partial \mathbf{B}}{\partial t} = -[C]\mathbf{E}, \quad (10)$$

[2]This is also the case in inhomogeneous *isotropic* media.

where again $\mathbf{E}$ and $\mathbf{B}$ denote arrays of unknowns, and $[\star_\sigma] = [T]$. The time-discretization of (10) can be done, for example, by means of a leap-frog scheme akin to the FDTD update in (5) [33]. Note that by eliminating $\mathbf{B}$ in (10), we recover (6).

C. Duality Between Metric and Material Tensor Properties

Using a discretization based on differential forms [16], anisotropies (either electric or magnetic) in the background media are incorporated in the discrete Hodge operators $\star_\epsilon$ and $\star_{\mu^{-1}}$ above. However, Hodge operators also incorporate *all* metric information [191]–[195]. In other words, Maxwell equations—written in terms of differential forms—factor out into two parts: one part encoding only *metric* and *material* information and the other part encoding only *topological* information[3] (at the discrete level, topological information corresponds to mesh connectivity information). As a result, there is a duality between material and metric (coordinate transformation) properties. Namely, the simulation of Maxwell equations in a *irregular* grid and in *homogeneous isotropic* media is dual to the simulation in a *regular* grid and in *inhomogeneous anisotropic* media—where the permittivity and permeability tensors are proportional to each other [16], [193], [196]. Interestingly, this property was recently explored in a different context to obtain metamaterial tensors for "cloaking" and masking of scatterers [197]–[199].

D. Combined Effects

Frequency-dispersive properties and anisotropies can appear simultaneously. For example, ferromagnetic materials such as magnetized (saturated) ferrite have constitutive parameters of the form $\overline{\boldsymbol{\epsilon}}(\omega) = \epsilon_0\epsilon_r\overline{\mathbf{I}}$ and $\overline{\boldsymbol{\mu}}(\omega) = \mu_0\overline{\boldsymbol{\chi}}(\omega)$ with [200]–[202]

$$\overline{\boldsymbol{\chi}}(\omega) = \begin{bmatrix} 1+\chi_{xx}(\omega) & \chi_{xy}(\omega) & 0 \\ \chi_{yx}(\omega) & 1+\chi_{yy}(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11a)$$

where $\overline{\mathbf{I}}$ is the identity matrix and

$$\chi_{xx}(\omega) = \chi_{yy}(\omega) = \frac{(\omega_0 + j\omega\alpha)\omega_m}{(\omega_0 + j\omega\alpha)^2 + (j\omega)^2} \quad (12a)$$

$$\chi_{xy}(\omega) = -\chi_{yx}(\omega) = \frac{j\omega\omega_m}{(\omega_0 + j\omega\alpha)^2 + (j\omega)^2}. \quad (12b)$$

where α is the damping (loss) constant, $\omega_0 = \gamma H_0$, $\omega_m = \gamma 4\pi M_s$, γ is the gyromagnetic ratio, H_0 is the (z-directed) DC magnetic bias, and M_s is the DC saturation magnetization. Earlier FDTD algorithms for ferrites are described in [1], [203]–[211] based on either a frequency-dependent (Polder) $\overline{\boldsymbol{\mu}}(\omega)$ tensor as above or on the Gilbert's equation of motion, describing the interaction between the magnetic intensity $\mathbf{H}$ and the magnetization $\mathbf{M}$. An improved FDTD algorithm for ferrite based on the Gilbert's equation of motion and space synchronism is discussed in [212]. FDTD treatment of nonsaturated or partially magnetized ferrites is considered in [208] using the Green-Sandy model and in [213] using the Gelin-Berthou model. Modeling of lumped ferrites (subcell lumping) is considered in [214]. A modified approach to incorporate Polder-model ferrites in FDTD based on equivalent resistor-capacitor-inductor circuit models is formulated in [215]. An analysis of the numerical errors due to losses in the FDTD modeling of ferrites is presented in [216]. The inclusion of PML absorbing boundary conditions in a FDTD algorithm for the modeling of saturated ferrites was recently considered in [201].

We note that the same basic techniques used in the FDTD modeling of isotropic dispersive isotropic media apply to the anisotropic-dispersive case as well, with the main differences being the need to account for the extra coupling of field components and the need to interpolate field components at grid locations where they are not directly available as in the dispersionless anisotropic case. An FDTD algorithm for anisotropic and dispersive media based upon TLM concepts was proposed in [217]. Moreover, an FDTD algorithm with fourth-order Runge-Kutta time integration scheme to model dielectric anisotropic-dispersive media for ground penetrating radar applications was described in [89]. A comparison between various FDTD approaches to model anisotropic dispersive media is present [218]. An FDTD algorithm for anisotropic-dispersive media that incorporates both the PLRC technique for dispersive media and the convolutional (C)PML approach is described in [219].

One interesting class of artificially engineered material that exploits the combined effects of dispersion and anisotropy are magnetic photonic crystals (MPhCs) [220], [221], as illustrated in Fig. 4. MPhCs are made by periodic stacks composed of misaligned anisotropic dielectric layers (A-layers) and ferromagnetic layers (F-layers). Under a proper choice of geometry and tensor parameters, they can be designed to display an asymmetric dispersion relation $\omega(k)$ with a *stationary inflection point* (SIP) in a forward direction (for example, from left to right) and no SIP in a backward direction (for example, from right to left). Spectrally asymmetric MPhCs yield dramatic pulse slow-down (frozen modes), amplitude increase (via pulse compression), and unidirectionality [201]. Since group velocities become extremely low near the SIP, EM pulses seem to be "frozen" inside MPhCs when propagating in the forward direction [202]. At the same time, the pulses exhibit a giant growth in amplitude despite the passive nature of the material. In the backward direction, EM waves inside the MPhCs propagate in an ordinary fashion [221], with no wave slowdown or amplitude growth.

V. Bi-Anisotropic, Bi-Isotropic, and Chiral Media

FDTD algorithms for bianisotropic media, in which the constitutive relations exhibit magnetoelectric coupling of the form $\mathbf{D} = \overline{\boldsymbol{\epsilon}} \cdot \mathbf{E} + \overline{\boldsymbol{\xi}} \cdot \mathbf{H}$ and $\mathbf{B} = \overline{\boldsymbol{\mu}} \cdot \mathbf{H} + \overline{\boldsymbol{\zeta}} \cdot \mathbf{E}$ have been developed in [222] and, for the uniaxial case, in [223]. FDTD algorithms for bi-isotropic media where there exists an isotropic magnetoelectric coupling of the form $\mathbf{D} = \epsilon\mathbf{E} + \xi\mathbf{H}$, $\mathbf{B} = \mu\mathbf{H} + \zeta\mathbf{E}$ have been presented in [224] and [225]. An important case of bi-isotropic media is chiral media, where the frequency-domain constitutive relations can be written as $\mathbf{D} = \epsilon\mathbf{E} - (j\kappa/c)\mathbf{H}$ and $\mathbf{B} = \mu\mathbf{H} + (j\kappa/c)\mathbf{E}$, where κ is the chirality parameter, and c is the speed of light in vacuum. Various extensions of FDTD for chiral media have been developed over the years [226]–[230]. In particular, the FDTD algorithms formulated in [224], [229], [230] incorporate frequency-dispersion

[3]In a numerical context, this property was first recognized by Weiland and co-workers [38], [39].

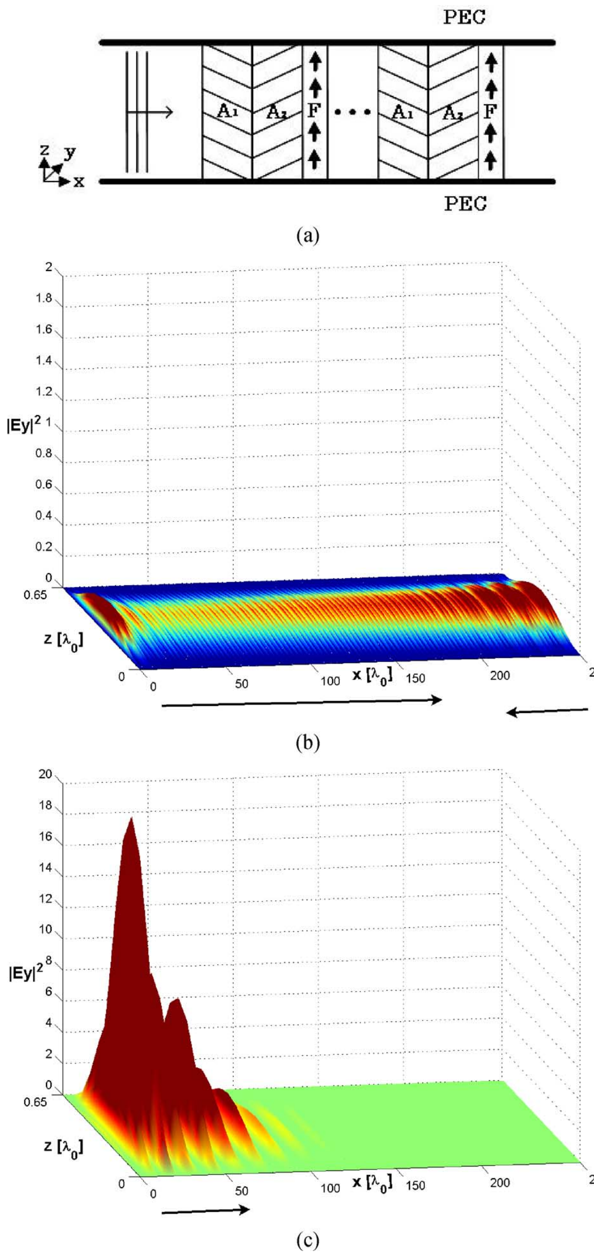

Fig. 4. FDTD simulation of EM pulse propagation in a magnetic photonic crystal (MPhC) [201]. (a) Simplest MPhC geometry consists of a periodic structure with two misaligned anisotropic layers, A_1 and A_2, and one ferromagnetic layer, F, in each unit cell as shown in (a) [220], [221]. The MPhC is designed to exhibit a stationary inflection point (SIP) in the dispersion diagram [220]. Plots (b) and (c) show E_y^2 field snapshots along the MPhC waveguide for a pulse with center frequency at the SIP. The pulse is seen propagating in the (b) backward and (c) forward directions. In the backward direction, the SIP is not present so that the pulse propagates at a normal speed and it is seen reflecting in the far end of the MPhC. In the forward direction, the group velocity is drastically reduced and a frozen mode is observed. Due to the decrease in the group velocity, the pulse is spatially compressed and its amplitude is greatly increased [201], [202]. (a) Magnetic photonic crystal arrangement. (b) Backward propagation. (c) Forward propagation.

in the bi-isotropic/chiral media models including the Condon model (obeying Kramers-Kronig relations) for the chirality parameter and either constant [224] or Lorentz models for the permittivity and permeability [229], [230]. FDTD applications to the scattering from chiral objects can be found in [226] and [228] and to the analysis of slabs and chiral discontinuities in waveguides in [227] and slabs [231].

VI. Nonlinear Media

In nonlinear media, the constitutive parameters themselves may depend on the electric or magnetic field strengths, often up to multiple orders. For example, the constitutive relation for a (local, instantaneous) nonlinear medium can be expressed as $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, with each component of $\mathbf{P}$ given by

$$P_i = \sum_j \chi_{ij}^{(1)} E_j + 2\sum_{j,k} \chi_{ijk}^{(2)} E_j E_k + 4\sum_{j,k,l} \chi_{ijkl}^{(3)} E_j E_k E_l + \cdots \quad (13)$$

where $\chi^{(1)}$ is the linear susceptibility and $\chi^{(2)}, \chi^{(3)}, \ldots$ are nonlinear susceptibilities. Important special cases of the above are isotropic models of the form [232], [233]

$$\mathbf{D} \approx \epsilon \left(1 + \chi_n^{(2)} |\mathbf{E}|^2 + \chi_n^{(3)} |\mathbf{E}|^4\right) \mathbf{E} \quad (14)$$

often denoted as cubic-quintic Kerr model when $\chi_n^{(3)} \neq 0$ and cubic Kerr model when $\chi_n^{(3)} = 0$. Nonlinear constitutive equation can be incorporated in the FDTD update by means, e.g., of an iterative procedure [234]–[236] or a root finding algorithm such as Newton-Raphson's algorithm [237]–[239] applied within each time step. For FETD, an approach involving only a standard linear system solution is presented in [240]. Sometimes (frequency-dispersive) memory effects cannot be neglected in nonlinear media. Examples of FDTD approaches for dispersive nonlinear media can be found in [235], [241]–[243].

An alternative FDTD implementation for nonlinear media that is based upon z-transforms and a recursive approach is formulated in [244]. The FDTD method has been used in a number of nonlinear media applications over the years [241], [242], [245]–[247], including optical gain media involving Maxwell-Bloch equations [248]–[250], soliton propagation [234], [241], [245], [251]–[253], nonlinear effects in absorbing and gain media including electron/atomic population rate equations [254], [255], distributed Bragg reflector microlasers [256], eletrooptic modulators [257], and photonic crystals lasers [258]. Optical parametric four-wave mixing (FWM) in Kerr media has also been studied by means of FDTD in [259] and incorporated in the analysis of microresonators in [260]. Similarly, FDTD algorithms to model nonlinear bistability has been considered in [261] for saturable Kerr media and in [262] (where it is combined with coupled mode theory) for microresonator studies.

VII. PML for Complex Media

For problems in unbounded regions, an absorbing boundary condition (ABC) needs to be imposed at the outer edges of the computational domain to suppress spurious reflection from the grid truncation. The most versatile ABC for complex media is the perfectly matched layer (PML) [263]–[268], which can be implemented for either FDTD [263], [264], [269]–[274], or FETD [275]–[279]. Apart from its numerical efficiency, a major advantage of PML over other ABCs is that its reflectionless absorption properties hold independently of the frequency of the incident wave (in the continuum limit). Most previously proposed ABCs are not suited for dispersive media because they require knowledge of the wave velocity near the grid boundary, a quantity that is not well defined for dispersive media in the time-domain. Another advantage of PML is that it preserves the nearest-neighbor-interaction property of FDTD and FETD, hence retaining their suitability for parallelization.

A. *(Doubly) Dispersive Media*

Extensions of the PML for FDTD simulations in dispersive media are considered in [53], [280], [281]. The extension to dispersive media proposed in [280] is based on the anisotropic PML formulation [265] with single-term Lorentz dispersive media. On the other hand, the extension to dispersive media in [281] is based on a modification of the original PML formulation of Berenger [263]. In that case, different sets of frequency-dependent parameters for the PML media need to be derived for each dispersion model in order to achieve perfect matching at all frequencies. The extension of the PML to dispersive media proposed in [53] is based on the complex coordinate stretching PML approach [264], which is equivalent to an analytic continuation of Maxwell equations to complex space [82], [194]. This approach is transparent to the dispersion model assumed for the background medium. Moreover, the use of complex coordinate stretching makes the implementation of the PML at corner grid regions straigthforward. A similar complex stretching-based approach to extend the cylindrical PML [266], [270] to model dispersive media in cylindrical FDTD grids is detailed in [54], where an anisotropic PML (uniaxial PML) approach is also considered. In DNG dispersive media, special care is need in implementing the PML to ensure that analytical instabilities do not arise. Reference [282] discusses this issue and presents a stable PML implementation for DNG media.

B. *Inhomogeneous Media*

In order to match inhomogeneous interior media, the PML region is defined as the region where the analytic continuation of the spatial coordinates is enforced. Specifically, the constitutive parameters at the PML interface are *locally* matched to those of the interior inhomogeneous media and the PML region is setup through the enforcement of the analytical continuation of the spatial coordinates [283]. Note that a windowing effect may result since the PML is intended to suppress all reflections from the outer domain – while for some inhomogeneous media problems reflections from the outer domain may exist (unbounded inhomogeneous domain). Mathematically, this is expressed by the fact that a Sommerfeld-like radiation condition cannot be invoked in those cases. Still, for many inhomogeneous problems of practical interest (e.g., layered media problems), the truncated PML-FDTD solution and the infinite-domain solution do coincide [283].

C. *(Bi)Anisotropic and Chiral Media*

Extension of the PML to anisotropic media have been formulated in [284], [285] based on a impedance matching approach, in [286] based on a material-independent approach as suggested in [287], and in [288] based on the complex coordinate stretching approach.

Reference [290] derives PML tensors (uniaxial PML) matched to arbitrary linear (bi-)anisotropic interior media. Other extensions to of the PML to bi-anisotropic (and bi-isotropic) media are considered in [291] and [292]. An extension of the UPML to chiral media is presented in [293].

D. *Nonlinear Media*

An extension of the PML to nonlinear media based on the z-transform approach [244] is provided in [294] while an extension implemented using the TLM-based FDTD scheme [217] is considered in [295] and [296]. A provably well-posed PML formulation for nonlinear media is considered in [297]. A comparative study of various PML configurations for nonlinear media is presented in [298]. A PML-FDTD algorithm for media with both dispersion and anisotropy is developed in [201] and an algorithm for media with both dispersion and nonlinearities is formulated in [299] and applied to optical soliton propagation in [300]. One particularly simple formulation of a *nearly* PML to truncate general media in FDTD is presented in [301]. Extensions of this nearly PML to Kerr-Raman nonlinear media and Lorentz-dispersive media are considered in [302].

E. *CFS-PML for Complex Media*

One useful implementation of the PML for low frequency problems and to reduce late-time reflections is the complex frequency-shifted (CFS) PML, where the zero-frequency pole in the complex stretching parameter is shifted to a nonzero frequency [303]. An extension of the CFS-PML for arbitrary media based on a CPML approach is presented in [289] and for dispersive media based on a z-transform approach in [304]. Another extension of the PML for general linear media including anisotropic media based on the CPML with multiple convolutional terms is carried out in [305].

VIII. Further Remarks

We have provided a survey on various extensions of FDTD and FETD to media with complex constitutive parameters. Among the many topics not covered here in detail were incorporation of micromaterial models into FDTD and FETD [306]–[308] (as opposed to bulk parameters), multiphysics models (for example, electrothermal coupling), and subcell or lumped elements modeling for complex media [309]. Additionally, time-domain unconditionally stable $O(N)$ methods based on operator splitting such as the alternating-direction-implicit (ADI) scheme [310]–[313] and the locally-one-dimensional (LOD) scheme [314]–[316] have attracted much interest in recent years. Extensions of these methods to complex media have been developed recently but have not been considered here.

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