Sādhanā, Vol. 18, Part 6, December 1993, pp. 911–925.

Time-domain-finite-wave analysis of the engine exhaust system by means of the stationary-frame method of characteristics. Part I. Theory

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MS received 19 August 1992; revised 19 January 1993

Abstract. Time-domain-finite-wave analysis of the engine exhaust system is usually done using the method of characteristics. This makes use of either the moving frame method, or the stationary frame method. The stationary frame method is more convenient than its counterpart inasmuch as it avoids the tedium of graphical computations. In this paper (part I), the stationary-frame computational scheme along with the boundary conditions has been implemented. The analysis of a uniform tube, cavity-pipe junction including the engine and the radiation ends, and also the simple area discontinuities has been presented. The analysis has been done accounting for wall friction and heat-transfer for a one-dimensional unsteady flow. In the process, a few inconsistencies in the formulations reported in the literature have been pointed out and corrected. In the accompanying paper (part II) results obtained from the simulation are shown to be in good agreement with the experimental observations.

Keywords. Noise control; engine simulation; method of characteristics.

1. Introduction

The investigation of finite amplitude waves in engine intake and exhaust systems is well established; see for example Benson (1982), and Annand & Roe (1974). The analysis is done using the method of characteristics. Generally, the method of characteristics is implemented in two ways. One is the method of wave diagrams (also called 'graphical method') and the other is the mesh-method of Benson (1982). These have been referred to here as the 'moving frame' and the 'stationary frame' methods respectively. Jones & Brown (1982) computed all the three variables (two modified Riemann variables, and the entropy variable) using the moving frame method. Benson computed the modified Riemann variables (λ_{I} and λ_{II}) by the

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stationary frame method, and the entropy variable by a graphical method (see for example Benson 1982). A similar approach was followed by Deshpande & Chandra (1989) to compute the noise radiated from the exhaust pipe tail end.

In this paper a modification in Benson's method has been presented wherein all the three variables have been evaluated using the stationary frame method. Low & Baruah (1981), and Ferrari & Castelli (1985) have reported similar results for pipes. However this paper also presents the extension to sudden area discontinuities. In the accompanying paper (part II), typical computed results and experimental verification of these predictions have been provided.

2. Basic formulation

The basic formulation is well known and documented in literature, and here it has been adopted from Munjal (1987). The three nondimensional variables are

$$P \equiv A + [(\gamma - 1)/2]U, \quad \text{moving along the path } dX/dZ = U + A, \tag{1}$$

$$Q \equiv A - [(\gamma - 1)/2]U, \quad \text{moving along the path } dX/dZ = U - A, \tag{2}$$

$$A_0 \equiv a_0/a_{ref}, \quad \text{moving along the path } dX/dZ = U. \tag{3}$$

where A_0 is the nondimensional acoustic speed for gas originally at a particular entropy and pressure, when the gas is isentropically brought to a reference pressure p_{ref} and where

$$A = a/a_{ref}, \quad U = u/a_{ref}, \quad X = x/L, \quad Z = a_{ref}t/L,$$

a_{ref}, is the (arbitrary) reference sound speed, and L is (arbitrary) reference length.

Here, P and Q may be recognized as strengths (to some scale) of the forward pressure wave and reflected pressure wave, respectively, and A_0 as the strength (to some scale) of the entropy wave (see for example Munjal 1987). Variations of P, Q and A_0 with respect to the nondimensional time variable Z in terms of friction and heat transfer are also given in the same reference. All the other variables like Mach number M, pressure p, density ρ, temperature T, mass flux $\dot{m}$ etc. can be written in terms of the wave variables P, Q, the entropy variable A_o, reference values p_o, a_{ref}, and L by certain relations, as given in Munjal (1987).

The interaction of these three variables with one another at the $Z - X$ (time-space) mesh intersections, and the interpolation thereof is given in the following section for completeness, see for example Munjal (1987), Low & Baruah (1981), and Ferrari & Castelli (1985).

3. Interpolation in uniform pipe

The stationary-frame method consists in dividing the tube into a number of equal segments and the values of P, Q and A_0 at the next instant are determined at these fixed points by an interpolation scheme described hereunder.

Let P, Q and A_0 be known at all junctions ($i = 1, 2, \ldots, n$) at nondimensional time Z (see figure 1). In order to evaluate these variables at the junction i at time $Z + \Delta Z$ (next instant, or the point O'), the following interpolation technique is made use of.

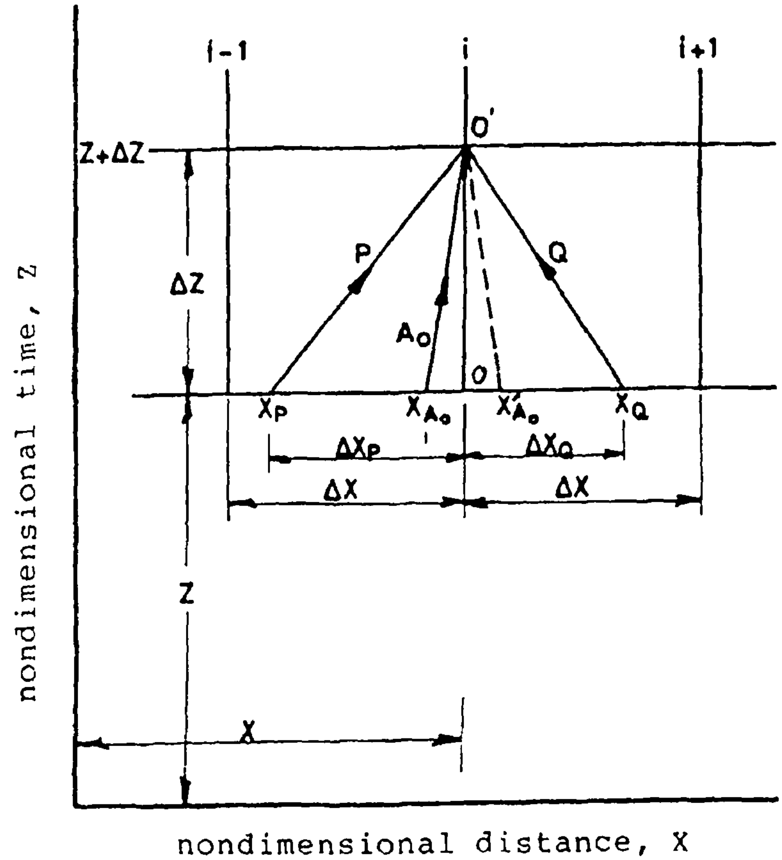

Figure 1. Interpolation by the stationary-frame method.

The characteristics for P, Q and A_0 reaching point O' start respectively from points X_P, X_Q and X_{A_0}, the locations of which are not known *a priori*. If the particle velocity at point O (junction i) is negative, then the A_0 characteristic will be the dotted line starting from X'_{A_0}. For convenience, the formulae for dP, dQ and dA_0 are written as follows:

$$\mathrm{d}P = (A/A_0)\mathrm{d}A_0 + \mathrm{d}P_{fh}, \quad \text{along } \mathrm{d}X/\mathrm{d}Z = U + A, \tag{4}$$

$$\mathrm{d}Q = (A/A_0)\mathrm{d}A_0 + \mathrm{d}Q_{fh}, \quad \text{along } \mathrm{d}X/\mathrm{d}Z = U - A, \tag{5}$$

$$\mathrm{d}A_0 = \mathrm{d}A_{0fh}, \quad \text{along } \mathrm{d}X/\mathrm{d}Z = U, \tag{6}$$

where

$$\mathrm{d}P_{fh} = -\frac{\gamma-1}{2}\frac{2fL}{D}U^2\frac{U}{|U|}\left\{1-(\gamma-1)\frac{U}{A}\right\} + \frac{(\gamma-1)^2}{2}\frac{qL}{Aa_{\mathrm{ref}}^3}, \tag{7}$$

$$\mathrm{d}Q_{fh} = \frac{\gamma-1}{2}\frac{2fL}{D}U^2\frac{U}{|U|}\left\{1+(\gamma-1)\frac{U}{A}\right\} + \frac{(\gamma-1)^2}{2}\frac{qL}{Aa_{\mathrm{ref}}^3}, \tag{8}$$

and

$$\mathrm{d}A_{0fh} = \frac{\gamma-1}{2}\frac{A_0}{A^2}\left\{\frac{qL}{a_{\mathrm{ref}}^3} + \frac{2fL}{D}U^3\frac{U}{|U|}\right\}, \tag{9}$$

where the subscript fh indicates that the term involves friction and heat transfer and will be zero in the absence thereof. In other words, dP and dQ would not be zero even in the absence of friction and heat transfer. The terms dP_{fh}, dQ_{fh} and dA_{0fh}

are evaluated using the values of the constituent variables at the starting point of the characteristics (P, Q, or A_0, indicated by subscripts P, Q or A_0 as the case may be), as an initial approximation.

The slope of P characteristic at the starting point, X_P, can be expressed in terms of the nondimensional variables at that point. The same variables can be written in terms of ΔX_P; also, ΔX_P is equal to the product of the slope and the nondimensional time-step, ΔZ. These three simultaneous linear algebraic equations can be solved to obtain ΔX_P.

Similarly, three equations each can be written for positions X_Q, and X_{A_0} (if $U_i \geqslant 0$), and X'_{A_0} (if $U_i < 0$), which on solution yield ΔX_Q and ΔX_{A_0} (where subscripts Q and A_0 indicate evaluation at X_Q and X_{A_0} respectively).

This stationary frame interpolation for all the three variables is easier to program without much loss of accuracy (see Low & Baruah 1981, and Ferrari & Castelli 1985).

Using the values of ΔX_P, ΔX_Q and ΔX_{A_0}, it is possible to evaluate A_{0P}, A_{0Q} and A_{0A_0}, by means of linear interpolations.

From the knowledge of the values of the variables at their corresponding starting points, their values at the next time instant $Z + \Delta Z$ can be evaluated as follows.

Using the values of P, Q and A_0 at X_{A_0} (or X'_{A_0}), $\mathrm{d}A_{0fh}$ can readily be evaluated. Then, as per (6), the value of A_0 at O' (denoted by $A_0(O')$), is given by

$$A_0(O') = A_{0A_0} + \mathrm{d}A_{0fh} \tag{10}$$

Similarly P_P, Q_P, and A_{0P} are used to evaluate $\mathrm{d}P_{fh}$ required in (4). The value of $\mathrm{d}A_0$ required in (4) is given by

$$\mathrm{d}A_0 = A_0(O') - A_{0P}. \tag{11}$$

It is clear now, as also mentioned elsewhere (see for example Benson 1982), why it is necessary to interpolate for A_0 before P and Q. Following (4), P at O' (denoted by $P(O')$) can be written as

$$P(O') = P_P + (A_P/A_{0P})(A_0(O') - A_{0P}) + \mathrm{d}P_{fh}. \tag{12}$$

In an identical fashion, the interpolation for Q can be accomplished. First, making use of P_Q, Q_Q and A_{0Q}, $\mathrm{d}Q_{fh}$ is evaluated, and then $\mathrm{d}A_0$ required in (5) is obtained by

$$\mathrm{d}A_0 = A_0(O') - A_{0Q}. \tag{13}$$

Then Q at $O'(Q(O'))$ is evaluated using

$$Q(O') = Q_Q + (A_Q/A_{0Q})(A_0(O') - A_{0Q}) + \mathrm{d}Q_{fh}. \tag{14}$$

In the preceding interpolation procedure, it is implied that the slope of a characteristic is the same as that at the starting point. For instance, the slope of the P characteristic $X_P \to O'$ has been taken to be equal to the slope at X_P. Obviously, there is need for a refinement by means of some iteration. To accomplish this, after the values at point O' are computed, the slope for P is recalculated as the mean of the slopes at X_P and O', and this slope is used to relocate the point X_P; i.e., a new value of ΔX_P is computed. Finally, new values of variables at X_P and hence at O' are computed. The characteristic variables A_0 and Q also are refined in the same fashion. However, as pointed out earlier, the refinement for A_0 should be done before P and Q. This refinement is then repeated. Up to two or three such iterations have been found to be necessary for evaluating P, Q, and A_0 at the next instant with an accuracy of 0·01% (see Gupta 1991).

At this juncture, it should be pointed out that the aforementioned details have not been given by Low & Baruah (1981), and Ferrari & Castelli (1985). A small conceptual error at this stage results in rather large errors in the overall performance. Munjal (1987, chap. 4), for instance, made use of (9) to evaluate dA_0 which is correct, but used the same value of dA_0 in (4), instead of using (11). Equation (9) is valid only along the path defined by (6) and hence such a procedure produced erroneous results. Besides, he also modified P and Q in proportion to A_0 and this modification violated the mass continuity equation when applied in the stationary-frame method.

Accuracy and stability are two important performance criteria for any numerical scheme. These considerations lead to certain relations which can be used to obtain the various step sizes in terms of other known engine and exhaust system parameters. These are given in detail in many references (see for example Munjal 1987).

4. Boundary conditions

It is quite evident from the foregoing procedure and figure 1 that the interpolation procedure will not be applicable at the left end ($i = 1$), and at the right end ($i = n$). Therefore, suitable end conditions (or boundary conditions) with the cylinder, the atmosphere or the neighbouring element, as the case may be, have to be worked out. In fact, this is how various parts of the system are dynamically coupled. These are discussed in the following subsection.

4.1 *The cavity pipe junction*

Let there be a junction as shown in figure 2. There are two possible flow processes, one being the flow from the cavity to the pipe which henceforth is referred to as the forward flow, and the other being the flow from the pipe to the cavity referred to hereafter as the reverse flow. At the interface of the two lies the trivial no-flow case. In the case of the forward flow, A_0 is not known, whereas in the case of the reverse flow A_0 can be obtained by interpolation. Also, in the case of the forward flow, P and A_0 are unknown and even Q is unknown inasmuch as although the starting value of Q (corresponding to Q_s) is known yet Q will depend upon A_0 as required by the scheme of computation. Thus, three equations are required. In the case of reverse flow, however, only one equation is required.

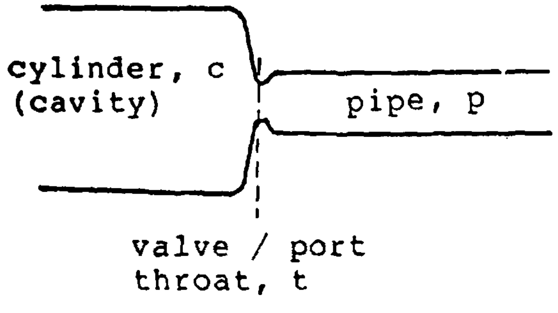

Figure 2. A schematic sketch of flow through the exhaust valve/port.

4.1a *Forward flow*: The first equation is the adiabaticity condition between the cylinder and the pipe. This can be symbolically written in terms of the non-dimensional variables as

$$A_c^2 = A^2 + [(\gamma - 1)/2]U^2, \tag{15}$$

where subscript c indicates cavity and the unsubscripted variables correspond to the pipe.

The mass conservation equation, isentropicity condition from the cavity to the throat, and the experimentally known fact that the pressure at the throat equals that in the pipe for subsonic flow, yield (see Munjal 1987)

$$\frac{\psi}{A_c}\frac{p_c}{p_{ref}}\left(\frac{2}{2(\gamma+1)}\right)^{(\gamma+1)/2(\gamma-1)} - \left[\left(\frac{P+Q}{2A_0}\right)^{2\gamma/(\gamma-1)}\left(\frac{P-Q}{\gamma-1}\right)\Big/\left(\frac{P+Q}{2}\right)^2\right] = 0, \tag{16}$$

for sonic choked flow, and

$$\frac{\psi}{A_c}\frac{1}{[2(\gamma-1)]^{1/2}}\left(\frac{p_c}{p_{ref}}\right)^{(\gamma-1)/\gamma}\left(\frac{P+Q}{2A_0}\right)^{2/(\gamma-1)} \times \left[1 - \left(\frac{p_{ref}}{p_c}\right)^{(\gamma-1)/\gamma}\left(\frac{P+Q}{2A_0}\right)^2\right]^{1/2}$$
$$- \left[\left(\frac{P+Q}{2A_0}\right)^{2\gamma/(\gamma-1)}\left(\frac{P-Q}{\gamma-1}\right)\Big/\left(\frac{P+Q}{2}\right)^2\right] = 0, \tag{17}$$

for the subsonic flow case, where ψ is the effective area ratio defined by

$$\psi \equiv C_d S_t / S, \tag{18}$$

and C_d is the coefficient of discharge.

Here it must be known as to which flow case is valid at a particular instant of time. The decision of the case depends upon the ratio p/p_c but p is not known *a priori*. This difficulty is overcome by deducing a critical value of Q as follows.

In the limiting case both conditions will be simultaneously valid and hence the pressure in the pipe would be equal to the pressure required for sonic flow; that is

$$\frac{p}{p_c} = \left(\frac{P+Q}{2A_0}\right)^{2\gamma/(\gamma-1)}\frac{p_{ref}}{p_c} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}. \tag{19}$$

Making use of this in (16) one gets

$$\frac{\psi}{A_c}\left(\frac{2}{2(\gamma+1)}\right)^{-1/2} - \left[\left(\frac{P-Q}{\gamma-1}\right)\Big/\left(\frac{P+Q}{2}\right)^2\right] = 0. \tag{20}$$

Equation (20) can be solved with (15) to obtain a critical value of $Q_{cr,ss}$, the subscript ss denoting the interface between sonic and subsonic flow. The expression for $Q_{cr,ss}$ is given by

$$Q_{cr,ss} = A_{cr,ss}\left[1 - \frac{\gamma-1}{2}\frac{\psi}{A_c}\left(\frac{2}{2(\gamma+1)}\right)^{-1/2} A_{cr,ss}\right], \tag{21}$$

where $A_{cr,ss}$ is computed by

$$A_{cr,ss} = \sqrt{2}A_c\left[\frac{(1+(\gamma^2-1)\psi^2)^{1/2}-1}{(\gamma^2-1)\psi^2}\right]^{1/2}. \tag{22}$$

If $Q \leqslant Q_{cr,ss}$, it is a case of choked sonic flow and if $Q > Q_{cr,ss}$, it is a case of subsonic flow.

The third equation for the forward flow is the compatibility equation, similar to the interpolation (14)

$$Q = Q_Q + (A_Q/A_{0Q})(A_0 - A_{0Q}) + (\mathrm{d}Q_{fh})_Q, \tag{23}$$

where $(\mathrm{d}Q_{fh})_Q$ indicates $\mathrm{d}Q_{fh}$ evaluated from the value of P_Q, Q_Q etc.

Thus, in the event of forward flow, (15), (16) and (23) are solved simultaneously for the case of $Q \leqslant Q_{cr,ss}$, and (15), (17) and (23) are solved for the case $Q > Q_{cr,ss}$ to obtain the next-instant values of the unknowns P, Q and A_0. The simultaneous solution of these three equations can be done by several numerical methods. The one used by the author is the Newton–Raphson method. The previous instant values of the three variables are used as the starting values for the iteration which often converges in 2 to 4 steps.

4.2a *Reverse flow*: If Q, the strength of the incoming wave is so large that p, the pressure in the pipe exceeds the cylinder pressure, the flow will be reversed, that is, it would move from the pipe into the cavity. This is the third case and the critical value at which this reversal happens is denoted by $Q_{cr,fr}$ (subscript fr indicating the interface between forward and reverse flow). This critical value is computed from the interface conditions

$$p = p_c \quad \text{and} \quad u = 0, \tag{24}$$

which yield the desired critical value of Q (see for example Munjal 1987)

$$Q_{cr,fr} = A_0(p_c/p_{ref})^{(\gamma-1)/2\gamma}. \tag{25}$$

One could also work out the critical value of Q at the interface of the choked inward flow and the subsonic inward flow. That, however, would be unnecessary as the flow from pipe to cavity is, invariably, subsonic in typical exhaust systems.

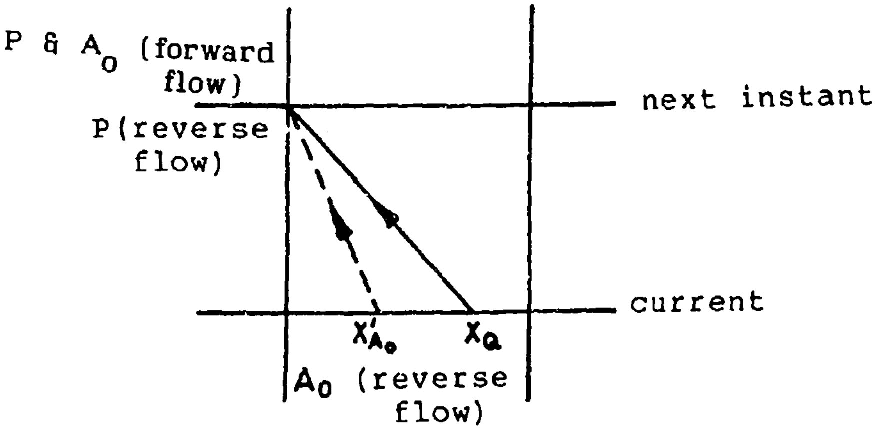

Figure 3. Cavity-pipe boundary computation illustrated.

The flow from pipe to throat is adiabatic and reversible and the pressure at the throat is equal to the pressure in the cavity. These, when combined with mass continuity between the pipe and the throat, yield (see Munjal 1987)

$$\frac{P-Q}{[2(\gamma-1)]^{1/2}}\left[\left(\frac{p_{\text{ref}}}{p_c}\right)^{2/\gamma}\left(\frac{P+Q}{2}\right)^{4/(\gamma-1)}\frac{1}{\psi^2}-A_0^{4/(\gamma-1)}\right]^{1/2}$$
$$-A_0^{2/(\gamma-1)}\left[\left(\frac{P+Q}{2}\right)^2-\left(\frac{p_c}{p_{\text{ref}}}\right)^{(\gamma-1)/\gamma}A_0^2\right]^{1/2}=0. \tag{26}$$

The transcendental (26) can be solved by the Newton–Raphson method for the only unknown, P; Q in the case of reverse flow is already known through interpolation.

However, if the throat area happens to be equal to that of the pipe, (26) cannot be applied. This can also happen if the reverse flow is from a simple pipe into a cavity without a throat. For such a case, the desired relation for P can be found from the fact that at the pipe cavity junction, pressure in the pipe must be equal to that in the cavity, that is, since

$$p=p_c, \tag{27}$$

we get

$$P=2A_0(p_c/p_{\text{ref}})^{(\gamma-1)/2\gamma}-Q. \tag{28}$$

Thus, one gets an explicit expression for P for the case of inward flow from a throatless pipe to cavity.

4.2 *Some remarks*

Equations (15), (16), (17), (23), (21), (26), (28) and (25) describe all cases of flow across the junction of a cavity (that includes cylinder and atmosphere) and a pipe with or without a throat. Of course, refinement similar to that described earlier in the scheme of interpolation for a uniform pipe, is needed in both the cases of the cavity-pipe junction as well. This presents a unified approach for all the three variables P, Q and A_0. Benson (1982) computed A_0 by a graphical method. However, the basic physical concepts involved are the same. Munjal (1987) makes use of his upgradation technique to modify P and Q in proportion to A_0 and solves an equation which is a combination of adiabaticity and mass continuity equation. In effect, his method makes use of just two equations, one being the combination of adiabaticity, and the mass continuity equation, and the other being the upgradation equation. This is basically incorrect inasmuch as the two most important physical relations, those of adiabaticity and mass conservation, are not explicitly and independently satisfied. In fact, a solution thus obtained has hardly any real physical significance. Low & Baruah (1981), and Ferrari & Castelli (1985) have not touched upon these intricacies at all.

Also the junction has been modelled with the assumption that there is no pressure recovery between the throat and the pipe. In the actual flow case, however, the possibility of pressure recovery cannot be precluded. This is due to the fact that the exhaust port (and the inlet port as well) is designed with smooth profiles causing a gradual increase in area. This makes the port act more as a divergent nozzle between the throat and the pipe and therefore a certain amount of pressure recovery should be anticipated.

4.3 *Simple area discontinuities*

As the flow is from the element number one to the element number two, in the corresponding mesh diagram (see figure 4a) it is clear that the known variables are P_1, A_{0_1} and Q_{2s}; and the unknown variables are P_2, Q_1, A_{0_2} and Q_2. This requires four equations for the four unknown variables. The first three equations are similar to those for the cavity pipe junction, namely, adiabaticity, mass continuity, and compatibility equations. These can be symbolically written as

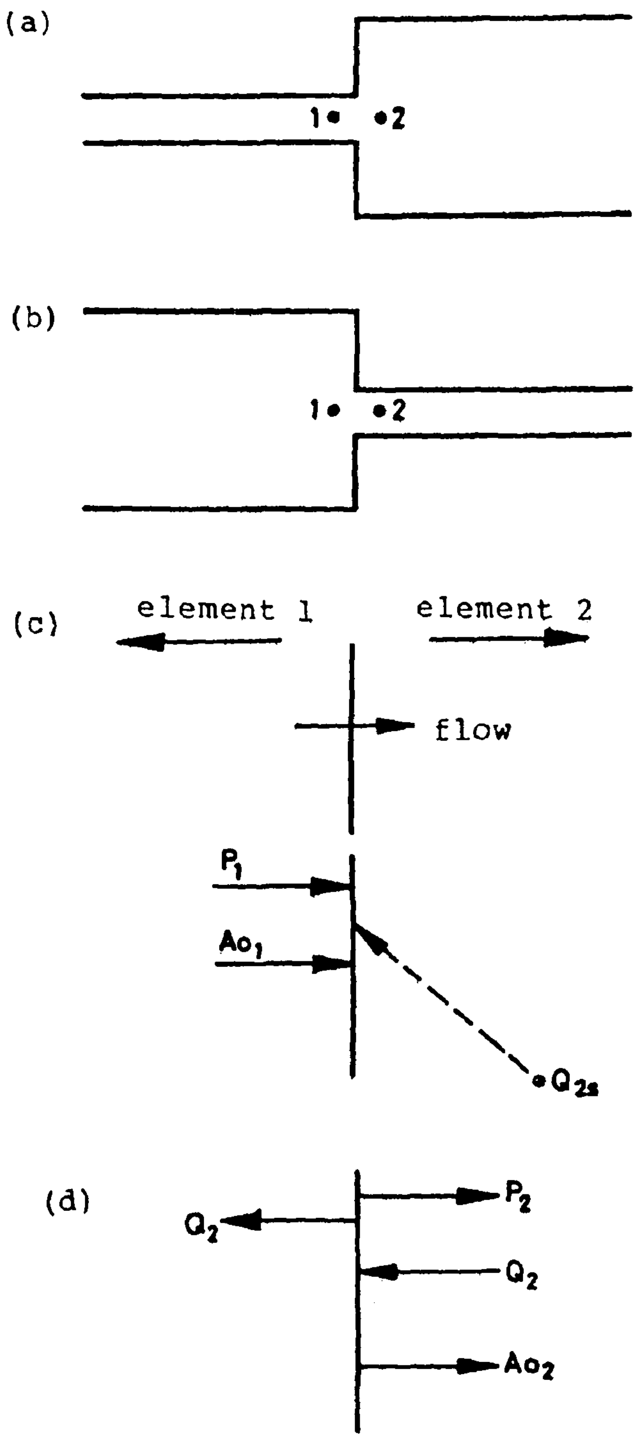

Figure 4. Computation at a simple area discontinuity illustrated. Sudden expansion (a) and contraction (b), known (c) and unknown (d) variables.

Adiabaticity:

$$\left(\frac{P_1+Q_1}{2}\right)^2+\frac{\gamma-1}{2}\left(\frac{P_1-Q_1}{\gamma-1}\right)^2=\left(\frac{P_2+Q_2}{2}\right)^2+\frac{\gamma-1}{2}\left(\frac{P_2-Q_2}{\gamma-1}\right)^2. \tag{29}$$

Mass continuity:

$$\left(\frac{2}{P_1+Q_1}\right)^2 S_1\left(\frac{P_1-Q_1}{\gamma-1}\right)\left(\frac{P_1+Q_1}{2A_{0_1}}\right)^{2\gamma/(\gamma-1)}$$
$$=\left(\frac{2}{P_2+Q_2}\right)^2 S_2\left(\frac{P_2-Q_2}{\gamma-1}\right)\left(\frac{P_2+Q_2}{2A_{0_2}}\right)^{2\gamma/(\gamma-1)}, \tag{30}$$

where S_1 and S_2 are the respective duct cross section areas, and the compatibility equation

$$Q_2=Q_{2s}+(\mathrm{d}Q_{fh})_s+(A_{2s}/A_{0_{2s}})(A_{0_2}-A_{0_{2s}}), \tag{31}$$

where subscript s indicates values computed at the starting point S shown in figure 4b.

The fourth equation is obtained by means of the application of entropy change relations given by

$$A_{0_2}/A_{0_1}=\exp((s_2-s_1)/2C_p), \tag{32}$$

$$s_2-s_1=(R/p_1)K(\tfrac{1}{2}\rho_0 u_1^2). \tag{33}$$

Equations (32) and (33) when applied to this discontinuity yield

$$A_{0_2}/A_{0_1}=\exp\{[(\gamma-1)/4]KM_1^2\}. \tag{34}$$

On substitution for M_1 in terms of the nondimensional variables, (34) yields

$$A_{0_2}/A_{0_1}=\exp\{[K/(\gamma-1)][(P_1-Q_1)/(P_1+Q_1)]^2\}, \tag{35}$$

where K is the stagnation pressure loss factor given by (see for example Munjal 1987)

$$K=0{\cdot}5(S_u/S_d)[(S_u/S_d)-1], \quad \text{for simple contraction,} \tag{36}$$

and

$$K=[1-(S_u/S_d)]^2, \quad \text{for simple expansion,} \tag{37}$$

where subscripts u and d indicate upstream and downstream, respectively. Equation (35) is the fourth equation. These four transcendental equations, namely, (29), (30), (31), and (35) can be solved simultaneously for the four unknowns P_2, Q_1, A_{0_2} and Q_2.

It is clear that the same analysis can be used to solve for other simple area discontinuities like an orific plate or an air-filter element, for which the loss coefficient K is known by measurement of stagnation pressure drop in steady flows.

Munjal (1987) makes use of the same adiabaticity and mass continuity relations. However, instead of the compatibility equations, he uses the upgradation scheme (that is, modifying P and Q in proportion to A_0). This again is inherently incorrect. Also, he approximates the entropy change equation by expanding the exponential term in power series, retaining only its first term. However, this does not lead to any significant errors as the approximation is valid in case of typical mufflers where $M_1^2 \ll 1$.

Benson (1982) makes use of the momentum-balance equation instead of the entropy-change equation. In case of sudden expansion, the two equations are equivalent inasmuch as the loss factor K can be computed analytically making use of the momentum equation. However, in general, it is not so. In fact, the analytical value for K, derived using the momentum equation, turns out to be double the experimentally obtained value for sudden contraction. Similar disparities are observed in other cases as well. Therefore, it seems better to make use of the loss factor K and entropy-change equation because these have experimental validation. Use of the momentum equation, though conceptually appealing, yields results which are in disagreement with those computed using experimentally obtained loss factors. This is because of the fact that, though it is certain that momentum is conserved, the equation that is written to describe such a conservation requires a guess of the pressure on the annular end plate. This is illustrated in appendix A.

5. Thermodynamics of the cavity

A cylinder is a variable-volume cavity from which there is efflux of gases and to which there is influx of gases. The rates of change of T_c, ρ_c, and p_c can be computed making use of the rate of volume change and influx of gases to, and exhaust of gases from, the cavity, as given in Munjal (1987). In fact, the same approach can also be applied to any lumped cavity in the exhaust and/or intake systems.

6. Considerations of friction and heat transfer

In the literature, there are a number of empirical relations for Froude's friction factor, $\mathrm{F}(f \equiv \mathrm{F}/4)$ for different ranges of Reynold's number. For the typical flow velocities in the exhaust mufflers, F is given by the well-known Lee's formula (see for example Munjal 1987)

$$\mathrm{F} = 0{\cdot}0072 + (0{\cdot}612/\mathrm{Re}^{0{\cdot}35}), \quad \mathrm{Re} < 4 \times 10^5, \tag{38}$$

where

Re is the Reynold's number $uD\rho/\mu$,

u is the flow velocity,

D is the diameter (or hydraulic diameter) of the duct, and

μ is the coefficient of dynamic viscosity.

The evaluation of friction as well as heat transfer coefficients is basically difficult and uncertain inasmuch as these coefficients are valid for steady flow whereas the typical exhaust muffler flow is largely unsteady (Wallace 1954). In this paper, Benson's (1982) approach has been followed. Nevertheless, other empirical relations mentioned in Munjal (1987) were also tried and they were found to be less appropriate. As pointed out earlier, there is an inherent error or contradiction in applying steady flow coefficients to unsteady flow computations. Therefore, the guiding principle for a suitable choice is for simplicity accompanied by reasonable accuracy.

Application of Reynold's analogy yields the heat transfer coefficient h as

$$h = \tfrac{1}{2}\rho C_p u f, \tag{39}$$

where ρ, u and C_p are density, particle velocity and constant pressure heat capacity, respectively. The instantaneous heat addition per unit mass q, can then be written as

$$q = -h(\Delta T)P_e(\Delta x)/[\rho S(\Delta x)] \tag{40}$$

where P_e is the perimeter of the wall, S is the area of the duct, and ΔT is the appropriate temperature difference. There have been different claims regarding appropriate temperature difference in the above references. Benson (1982) claimed that ΔT must be taken as $T - T_w$ (T being the instantaneous gas temperature, and T_w being the average wall temperature) instead of $T - T_a$ (T_a being the ambient temperature). This claim rests on the fact that the thermal conductivity of the muffler material (generally metal) is much higher (at least a few orders of magnitude) than that of the gas. Besides, the thermal inertia of the muffler material is much higher than that of the gas. Both these indicate that muffler wall rapidly attains a constant and almost uniform temperature under steady state engine operation. This thesis was verified in the experiments during the present investigation and therefore $\Delta T = T - T_w$ was used in preference to $\Delta T = T - T_a$. Of course, the value of h used in (40) is the one due to forced convection within the pipe.

Another factor that supports Reynold's analogy approach is as follows. In a single-cylinder four-stroke cycle engine exhaust system, the valve is open for only about one-fourth of the total cycle duration. For the remaining part of the cycle (about three fourths), when the valve remains closed, the flow velocities in the muffler are considerably smaller. If a constant heat transfer coefficient as mentioned in Munjal (1987, chap. 4) is used, it results in a sort of contraction of gases in the muffler. This, in turn, creates an artificial suction from the tail-pipe end of the muffler. This is basically incorrect as, when the gases slow down to rest, the heat transfer changes from forced convection to natural convection and hence the heat transfer coefficient undergoes drastic reduction (by a few orders of magnitude). This ensures that the numerically observed contraction actually does not take place in the real muffler. The other correlation involving Nusselt number and Prandtl number given in Munjal (1987), and Goyal *et al* (1967) is, on the one hand, valid only for forced convection and, on the other hand, tends to be much more cumbersome and tedious as compared to Reynold's analogy, and at the same time does not offer any substantial gain in accuracy.

7. Scheme of computation

In order to evaluate the performance of an engine with the given exhaust and intake systems by means of the method of characteristics discussed in the foregoing sections, general subroutines were written for

(1) evaluation of P, Q, and A_0 at all discrete points of a uniform pipe from their values at the previous instant, making use of the interpolation technique of § 3.
(2) evaluation of the variables at a cavity-pipe junction for all cases of flow, making use of the analysis of § 4.1 (this took care of the exhaust valve and the tail-pipe end);
(3) evaluation of the thermodynamic variables of a cavity, making use of the previous values thereof, and the incoming and the outgoing flow, as enunciated in § 5 (this will, however, generally not be appropriate for a cylinder, where one has to keep track of residual gases and fresh air retained for the estimation of scavenging efficiency

etc., nevertheless it sufficed in the present case because the simulation involved only the exhaust process when these complications are not present);
(4) evaluation of the variables at simple area discontinuities as explained in §4.3.

The main program was built around the cylinder. It also contained all the geometric details of the intake system and the exhaust system (intake system details are required to carry out the simulation during the valve overlap phase of the exhaust process). Also, functional subroutines to compute the other variables from the characterisic variables were written. The value of a_{ref} was computed assuming isentropic expansion from the blow-down conditions (denoted here by subscript bd) to the pressure of the atmosphere, thus

$$a_{ref} = a_{c,bd} \left(\frac{p_{atm}}{p_{c,bd}} \right)^{(\gamma - 1)/2\gamma} \tag{41}$$

The time interval was computed by dividing the complete cycle process into $1024 = 2^{10}$ equal parts, hence $\Delta\theta$ can be written as

$$\Delta\theta = 720/1024. \tag{42}$$

The reference length was assigned an arbitrary value of 1 m, and the values for Δx were determined using the stability and accuracy criteria given in Munjal (1987). The three variables P, Q and A_0 at all points of all the elements were initialized to unity.

With these assumed values, the numerical computations were started from the blow-down (when the exhaust valve or the port just opens) and continued in degree steps up to the start of the next blow-down. The conditions in the cylinder were reinitialized to the blow-down conditions and the condition in the pipe at the end of the preceeding cycle were taken as the initial values for the succeeding cycle.

The blow-down conditions, necessary as input to the computational scheme, were obtained from the experiments, the details of which are given in the accompanying paper.

8. Noise radiation

At all instants of time, the mass flux $\dot{m}(t)$ from the tail-pipe end was computed using the values of the nondimensional characteristic variables P, Q, A_0, and the dimensional reference values. This mass-flux history can readily be used to predict the noise radiated. The procedure for this is well described in literature (see, for example, Munjal 1987).

9. Concluding remarks

In the following paper (part II), the theory described above has been used to develop a FORTRAN program, which in turn has been made use of to compute typical results. These have been compared with the experimental results, and the two are shown to be in good agreement (Gupta & Munjal 1993).

Appendix A. Loss factor

For an area discontinuity as shown in figure 4a, writing the mass-balance and momentum-balance equations for an incompressible flow one obtains

$$u_1 S_1 = u_2 S_2, \tag{A1}$$

and

$$p_1 S_1 + \rho_0 S_1 u_1^2 = p_2 S_1 + \rho_0 S_2 u_2^2. \tag{A2}$$

Also defining the stagnation pressure p_s and the stagnation pressure loss coefficient K by the equations:

$$p_s = p_0 + \tfrac{1}{2}\rho_0 u^2, \tag{A3}$$

and

$$p_{s1} = p_{s2} + K(\tfrac{1}{2}\rho_0 u_1^2), \tag{A4}$$

one can obtain the necessary correlations as follows.

Substituting for u_2 from (A1) in (A2) and writing the loss coefficient K as in (A4) one obtains for K

$$K = [1 - (S_u/S_d)]^2 \tag{A5}$$

which is the same as in (37).

However, the same exercise for an area discontinuity as shown in figure 4b, yields

$$u_1 S_1 = u_2 S_2, \tag{A6}$$

and

$$p_1 S_1 + \rho_0 S_1 u_1^2 = p_2 S_2 + \rho_0 S_2 u_2^2 + (p_1 + \tfrac{1}{2}\rho_0 u_1^2) S_a, \tag{A7}$$

where S_a is the annular area on which the pressure is taken to be same as the upstream stagnation pressure. This yields the loss coefficient K as

$$K = (S_u/S_d)[(S_u/S_d) - 1] \tag{A8}$$

which is twice as much as given by (36).

Hence, solution of the momentum equation gives the correlation, corroborated experimentally, for a sudden expansion; therefore the formulations making use of the momentum equation are the same as that with the loss coefficient. However, for a sudden contraction the momentum equation formulation gives, for the loss factor, double the experimentally corroborated value; hence the two formulations, viz., the loss factor formulation and the momentum equation formulation, are not equivalent.

Here it may be noted that there is nothing wrong with the applicability of the momentum equation. The catch lies in one's having to assume a certain pressure on the annular end plate.

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