TIME-FREQUENCY ANALYSIS APPLIED TO SIGNATURING OF UNDERWATER ACOUSTIC SIGNALS

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1. ABSTRACT

The Wigner-Ville Distribution (WVD) and the Cross Wigner-Ville Distribution (XWVD) are applied to signal detection and classification, and Signal to Noise Ratio (SNR) performance is discussed. The method is applied to signaturing the individual cylinders of a diesel engine. A comparison with the Short-Time-Fourier-Transform is provided.

1.1 INTRODUCTION

The problem of automatically detecting a signal from background noise is a common one and has been widely investigated. Most methods rely on the use of a matched filter, or variations of it, such as the Ambiguity Function (AF) when the signal to be detected may be subject to a doppler effect.

Since the AF of a signal r(t) is complex, its magnitude squared is used and the phase information is discarded. On the other hand, the WVD is real and can be shown to be the 2D Fourier Transform (FT) of the AF. Since the FT is a unitary transform, the WVD and AF provide essentially the same information, and the WVD can therefore be used as a basis for a detection scheme.

Previous work has demonstrated that for monocomponent asymptotic FM signals, the instantaneous frequency law $f_i(t)$ is the optimal characteristic of the signal for detection. Determination of $f_i(t)$ from a 2D WVD image is the practical option [4]. Since the signal contribution is concentrated around $f_i(t)$ [3], the application of a 2D windowing which will preserve all the points (t,f) in the neighbourhood of $f_i(t)$ and filter out the others, should preserve the useful information in the signal and indeed increase by an order of magnitude the SNR. The subsequent application of a pattern recognition algorithm, or a 2D crosscorrelation should enhance any detection scheme [4].

The purpose of this paper is to extend the method to a less restrictive class of signals, and present a practical application. For these more general types of signals, (eg. transient signals), the concept of instantaneous frequency is less meaningful. In this case, we propose to estimate the WVD of the signal as a 2D image which characterises the signal, even though there is generally no easy pattern to recognise. The proposed detection method will then compare the WVD of the received signal with the WVD of the reference signal. The performance of the method will be compared with that of the matched filter, and finally a modification to the method will be proposed, by using the XWVD with an effective time-varying filtering.

2. WVD DETECTION OF A SIGNAL IN NOISE

Theoretical results for detection of a signal using the Wigner Distribution (WD) were derived in [1] and [2]. The results are extended for application to the WVD, which uses the analytic signal, and therefore provides practical advantages over the WD [3].

The WVD of a signal s(t) is defined as:

$$W_{z_1}(t,f) = \int z_1(t+\tau/2)\, z_1^*(t-\tau/2) e^{-j2\pi f\tau} d\tau \quad (1)$$

where z(t) is the analytic signal corresponding to s(t). (All integrals are from $-\infty$ to $+\infty$).

The XWVD of two signals s(t) and r(t) is:

$$W_{rs}(t,f)=\int z_1(t+\tau/2)\, z_2^*(t-\tau/2) e^{-j2\pi f\tau} d\tau \quad (2)$$

where $z_1(t)$ and $z_2(t)$ are the analytic signals corresponding to s(t) and r(t) respectively.

2.1 DETECTION WITH A MATCHED FILTER

Consider that a signal s(t) is sent through a noisy transmission channel, so that the output of the channel is r(t) = s(t) + n(t), where n(t) is a zero mean, complex, gaussian white noise process of variance N_o. The detection problem is to determine whether or not the signal is present. A detection statistic (η) is formed, s.t. if η exceeds a certain threshhold, the signal is considered to be present. If not, it is decided that no signal is present. The SNR, which provides a measure for comparison between WVD, XWVD and the matched filter detection methods, is defined as:

$$SNR=|E(\eta|r)-E(\eta|n)|/\{1/2[var(\eta|r)+var(\eta|n)]\}^{1/2} \quad (3)$$

$E(\eta|r)$ is the expected value of η given r(t),
$var(\eta|r)$ is the variance of η given r(t),
$E(\eta|n)$ is the expected value of η given n(t),
and $var(\eta|r)$ is the variance of η given n(t).

For a matched filter we can easily verify that:

$$\eta = \int r(t)s^*(t)dt, \quad (4)$$

$$E(\eta|n) = 0 \text{ and } E(\eta|r) = A = \text{Signal Energy}, \quad (5)$$

$$\text{and } var(\eta|n) = var(\eta|r) = N_0A. \quad (6)$$

$$\text{Substit. in (3) yields: } SNR = \sqrt{A/N_0} \quad (7)$$

2.2 DETECTION WITH THE WVD

The detection statistic is a two dimensional correlation between the WVD's of the sent and received signals, r(t) and s(t):

$$\eta_{wvd} = \int \int W_{z_1}(t,f)\, W_{z_2}^*(t,f)dtdf \quad (8)$$

Applying Moyal's formula [5] gives:

$$\eta_{wvd} = \int \int W_{z_1}(t,f)\, W_{z_2}^*(t,f)dtdf \quad (9a)$$

$$= |\int R(f)S^*(f)df|^2 = |\int r(t)s^*(t)dt|^2 \quad (9b)$$

where R(f) and S(f) are the FT's of r(t) and s(t).

$$E(\eta_{wvd}|n) = N_0A,\ E(\eta_{wvd}|r) = N_0A + A^2 \quad (10)$$

$$var(\eta_{wvd}|n) = N_0^2A^2 \quad (11)$$

$$var(\eta_{wvd}|r) = N_0^2A^2 + 2N_0A^3. \quad (12)$$

$$SNR = \sqrt{A/N_0}\ .\ 1/\sqrt{1 + N_0/A}. \quad (13)$$

This expression equals the SNR for a matched filter scaled by a factor $1/\sqrt{1 + N_0/A}$. For large values of A/N_0 the scaling factor approaches 1, and the SNR is not significantly degraded. For small values, however, the SNR is substantially reduced. This is explained by the non-linearity of the WVD which accentuates the effects of noise by producing artifacts. However, even for low SNR, estimators based on the WVD can be more useful than matched filter based estimators. This will be seen in the practical example (Section 3).

2.3 DETECTION WITH THE XWVD

The XWVD formed between the sent and received signals, being a linear operator, should perform better than the WVD because it exhibits no interaction between signal and noise. The XWVD based detection statistic is:

$$\eta_{xwvd} = \int \int W_{z_1z_2}(t,f)\, W_{z_1}^*(t,f)dtdf \quad (14)$$

Using Moyal's formula (14) becomes:

$$\eta_{xwvd} = \int r(t)s^*(t)dt.\ \int s(t)s^*(t)dt \quad (15)$$

$$= A\int r(t)s^*(t)dt \quad (16)$$

(16) illustrates two interesting results. Firstly, if r(t) and s(t) are real then the detection statistic is real - this makes the task of interpretation easier. Secondly, the detection statistic is the matched filter detection statistic multiplied by a constant, A. Hence, the SNR for XWVD detection will be identical to that of the matched filter. i.e.:

$$E(\eta_{xwvd}|n) = 0 \text{ and } E(\eta_{xwvd}|r) = A^2, \quad (17)$$

$$var(\eta_{xwvd}|n) = var(\eta_{xwvd}|r) = N_0A^3, \quad (18)$$

$$\text{and } SNR = \sqrt{A/N_0} \text{ (Same as matched filter)} \quad (19)$$

For practical analysis, finite length signals must be used. If T is the duration of the emitted signal, then the limits of the integral over time become $-T/2$ and $+T/2$ instead of $-\infty$ and $+\infty$. The same results apply.

2.4 DIGITAL IMPLEMENTATION

Degradation in SNR (worst case 0.5 dB) was reported in [2] when using the discrete WD for detection, as opposed to the continuous time WD. It is shown here that in using the WVD there is no SNR degradation.

The discrete XWD is defined as:

$$\rho_{x_1x_3}(n,k) = 2 \sum_{l=-L}^{+L} x_1(n+l)x_3^*(n-l)e^{-j4\pi kl/N} \quad (20)$$

where $x_1(t)$ and $x_3(t)$ are time-limited to $(-T/2,T/2)$, n and k are the discrete time and frequency variables respectively, and $N=T+1$. A rectangular window is used, so that $L=T/2-|n|$.

The inner product of 2 discrete XWD's derived in [2] is:

$$\eta = \sum_{n=-T/2}^{+T/2} \sum_{k=0}^{N-1} \rho_{x_1x_3}(n,k)\rho^*_{x_2x_4}(n,k). \quad (21)$$

$$= 2N\langle x_1,x_2\rangle\langle x_3,x_4\rangle^* + 2N\langle(-1)^n x_1,x_2\rangle\langle(-1)^n x_3,x_4\rangle^* \quad (22)$$

$$\text{where } \langle x_1,x_2\rangle = \sum_{n=-T/2}^{+T/2} x_1(n)x_2^*(n)$$

Comparison with [5] reveals that discretising the XWD inner product introduces an extra set of inner products in the time domain. i.e.

$$2N\langle(-1)^n x_1,x_2\rangle\langle(-1)^n x_3,x_4\rangle^*.$$

It was shown in [2] that for XWD based detection this extra set of inner products contributes to degraded SNR (worst case 0.5 dB) as compared with the continuous case. Consider now the WVD. Given that X_1, X_2, X_3, and X_4 represent the DFT's of $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$ respectively, and that $-1 = e^{j\pi}$, (22) becomes:

$$\eta = 2N\langle X_1,X_2\rangle\langle X_3,X_4\rangle^* + 2N\langle e^{j\pi n}x_1,x_2\rangle\langle e^{j\pi n}x_3,x_4\rangle^* \quad (23)$$

Defining FT as the Discrete Fourier Transform operator, (23) modifies to:

$$\eta = 2N\langle X_1,X_2\rangle\langle X_3,X_4\rangle^* + 2N\langle FT[e^{j\pi n}x_1],X_2\rangle\langle FT[e^{j\pi n}x_3],X_4\rangle^* \quad (24)$$

$$= 2N\langle X_1,X_2\rangle\langle X_3,X_4\rangle^* + 2N\langle X_1^S,X_2\rangle\langle X_3^S,X_4\rangle^*, \quad (25)$$

with X_1^S and X_3^S representing the DFT's of $x_1(t)$ and $x_3(t)$, normalised frequency shifted by 0.5. Now if x_1 is an analytic signal sampled at greater than the Nyquist rate then X_1 will be zero in the normalised frequency region -0.5 to 0. When X_1 is shifted by 0.5, it will be zero in the region 0 to 0.5. Therefore $\langle X_1^S,X_2\rangle$ will be zero in the region 0 to 0.5. $\langle X_1^S,X_2\rangle$ will also be zero in the normalised frequency region 0.5 to 1.0 since X_2 is zero here (if $x_2(t)$ is analytic). Then $\langle X_1^S,X_2\rangle$ will be zero everywhere, and the 'extra' set of inner products in (22) reduces to zero. The equivalence of the continuous and discrete XWVD inner products is then achieved, so that no SNR loss occurs with discrete XWVD's.

2.5 DETECTION OF TIME AND FREQUENCY SHIFTED SIGNALS

If shifts in time ($=t_0$) and frequency ($=f_0$) are introduced into r(t) then by the shift-invariance property of the WVD [8], the resulting WVD is:

$$W_Z(t-t_0,f-f_0)$$

Using Moyal's formula, and letting first $f_0=0$, and then $t_0=0$, gives:

$$\int W_{Z_1}(t-t_0,f)W_{Z_2}^*(t,f)\,dtdf = |\int r(t-t_0)s^*(t)dt|^2 \quad (26)$$

$$\int W_{Z_1}(t,f-f_0)W_{Z_2}^*(t,f)\,dtdf = |\int R(f-f_0)S^*(f)df|^2 \quad (27)$$

Eqns. (26) and (27) show that time-domain (or frequency domain) correlation of two signals is essentially equivalent to the two-dimensional correlation of the corresponding WVD's evaluated at the frequency (or time) origin - the two types of correlation differ only by a squaring factor. More generally, the rows of the 2D WVD correlation matrix correspond to lagged time correlations, and the columns of the matrix correspond to lagged frequency correlations. Thus WVD correlation can detect time and frequency lags equally as well as time and frequency correlation can individually.

3. APPLICATION OF WVD TO SIGNAL SIGNATURING

The WVD has been used to signature the individual cylinders of a diesel engine, and has been shown to reliably discriminate one cylinder from the other. At this stage the WVD only, (and not the XWVD) has been used, yet good results have been achieved.

The signal from a diesel engine, produced predominantly by cylinder firing activity, was chosen for analysis. In order to signature these non-stationary transient type signals, time-frequency analysis using the WVD was used. The signal was first segmented, with each segment length corresponding to one 'cylinder length'. This was done by determining the approximate time span for one cylinder firing, and then matching similar characteristics in the time domain to obtain a more accurate estimate of the cylinder length. Wigner-Ville analysis was then performed on 8 segments of signal, (numbered 1-8) with a view to reliably discriminating the signatures of each cylinder. Segments 1 and 5 corresponded to consecutive firings of cylinder 1, segments 2 and 6 corresponded to consecutive firings of cylinder 2, etc.

In order to produce the WVD's a windowing procedure had to be applied [6]. It was found that in order to produce good quality signatures, there had to be a constraint on the window length. If the length was made very small, insufficient resolution was obtained, whilst a very large choice resulted in a loss of some of the time varying information from the signal. Despite these limits, however, there was still a wide range in window length over which reliable signatures could be obtained. An appropriate window length was selected, and WVD plots were produced for all 8 signal segments. Each of these segments contained 400 sample points (generated with a sample frequency of 5KHz), and the WVD's were produced by applying a 400 point rectangular window (with 50% overlap) at varying positions along the segment. It was observed that good correspondence existed in the low frequency region of the WVD for segments corresponding to the same cylinder. This result is illustrated in Fig. 1, which shows side by side, the low frequency region of the WVD for segments 1 and 5, both of which correspond to Cylinder 1. (Rectangles have been used to highlight the relevant parts of the WVD's).

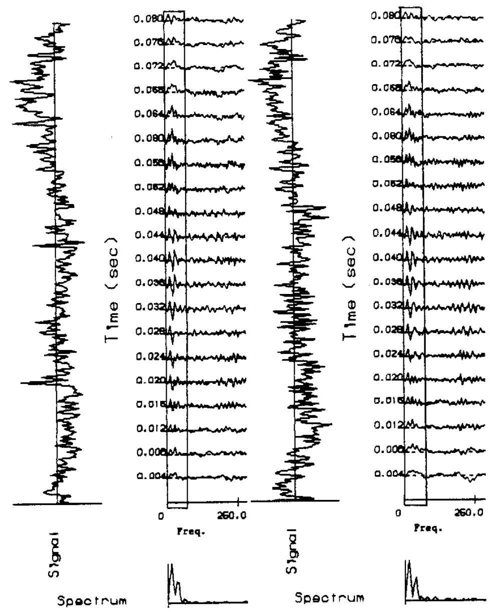

FIG 1 TWO WVD's OF FIRING FROM CYLINDER 1

Similar correspondence was observed for Cylinders 2-4 [7]. In order to quantify the similarities described above, WVD cross-correlation techniques were employed. The 2D normalised cross-correlation between two WVD's, R, is:

$$R(t_0,f_0) = \frac{\iint W_{z_1}(t,f) W_{z_2}(t-t_0,f-f_0)dtdf}{\left[\iint W_{z_1}{}^2(t,f)dtdf \iint W_{z_2}{}^2(t,f)dtdf\right]^{0.5}} \quad (28)$$

This translates into the discrete-time domain in a straightforward way. For this practical example the 2D crosscorrelation is performed not over an infinite frequency range, but over the low frequency region shown bounded by rectangles in the time-frequency plots (See Fig. 1). This is equivalent to estimating the quantity:

$$\sum_{n=-T/2}^{+T/2} \sum_{k=0}^{N_0-1} W_{z_1z_3}(n,k)W^*_{z_2z_4}(n,k) \text{ instead of (21).}$$

This corresponds to an effective prefiltering with a lowpass filter. Note that it could in fact be obtained in the time domain in this simple case. However, if the signal to be detected has nonstationary spectral characteristics, then we apply an effective time-varying prefiltering by multiplying the WVD of the reference signal by a time-varying function $W_h(t,f)$ which preserves and enhances the important features of the signal. Note that this procedure (which cannot be achieved in the time domain) has potential to significantly improve the SNR, and hence justifies the use of WVD correlations in preference to matched filters. This approach suffers from problems due to the WVD's non-linearity, but as discussed previously, these can be overcome by using the XWVD instead.

These low frequency parts of the WVD's from all 8 signal segments were taken, and the cross-correlation table shown in Fig. 2 was constructed. For this table, the rows correspond to segment i, (i=5 to 8), the columns correspond to segment j, (j=1 to 4), and the elements of the table are the peak cross-correlation values between the WVD's for segments i and j.

The table reveals comparatively high correlation values along the diagonal and lower values elsewhere. Since the diagonal elements represent cross-correlations from signals which correspond to the same cylinder, it is possible to make an objective claim that the cylinders have been signatured. Furthermore, the correlation table itself may prove useful in the process of machine identification, since it provides quantitative information as to the similarity relationships between each of the cylinders.

	Seg. 1	Seg. 2	Seg. 3	Seg. 4
Seg. 5	<u>0.813</u>	0.490	0.496	0.531
Seg. 6	0.487	<u>0.747</u>	0.468	0.579
Seg. 7	0.628	0.515	<u>0.737</u>	0.501
Seg. 8	0.393	0.604	0.428	<u>0.784</u>

FIG. 2 WVD CROSS-CORRELATION TABLE

The eight signal segments were also analysed using the Short-Time-Fourier-Transform (STFT). It was found that discriminating between the cylinders using the STFT was not possible in the same way that it was with the WVD. Furthermore, it was found that the process of signaturing the individual cylinders was much more sensitive to the selection of window length. This supports the claim discussed in [8] that the optimum choice of window length δ for the STFT is signal dependent.

4. CONCLUSIONS

The WVD and XWVD correlation provide alternative detection methods to the matched filter. No SNR degradation occurs for digital implementation.

The effectiveness of the WVD in detection is illustrated by signaturing 'noise transients' produced by individual cylinders of a diesel engine. The methods outlined could be extended for application to many areas of signal source identification, and could usefully supplement existing spectral techniques for Underwater Acoustic analysis [9].

5. ACKNOWLEDGEMENTS

This work is funded by the Defence Science and Technical Organisation, (Project No. CDS 10.86 - DST 86/726). The authors wish to thank Dr. Ross Barrett from WSRL, Salisbury, S.A. for helpful discussions.

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