# Time-Frequency and Point Process Algorithms for Cardiac Arrhythmia Analysis and Cardiorespiratory Control

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### Declaration

The content of this thesis is a result of original research done and has not been submitted for a higher degree in any other university. Most of the work presented in this thesis have already been published in peer-reviewed journals and conference proceedings as listed below:

#### **Journal Publications**

- S. Kodituwakku, S. W. Lazar, P. Indic, Z. Chen, E. N. Brown, and R. Barbieri, "Point process time-frequency analysis of dynamic respiratory patterns during meditation practice," *Medical and Biological Engineering and Computing*, vol. 50, no. 3, pp. 261-275, 2012.
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#### Abstract

Cardiovascular diseases are major causes of disability and premature death globally. In particular, atrial fibrillation is the most common cardiac arrhythmia condition found in clinical practice, and is associated with an increased risk of stroke. Heart rate variability (HRV) and respiratory sinus arrhythmia (RSA) are important indicators of cardiovascular health, and provide useful information on autonomic nervous system inputs to cardiac cycle and cardiorespiratory coupling, respectively. New methods to support the treatment of cardiovascular diseases and identifying efficient ways of measuring cardiovascular health could yield significant benefits. In this thesis, we present a number of advanced algorithms for cardiorespiratory signal processing.

We present algorithms for analyzing atrial fibrillation arrhythmia from electrocardiograms (ECG). We propose an orthonormal basis function based representation for fibrillatory waveforms, and use a regularized least square solution for atrial activity extraction from ECG, suppressing more dominant ventricular components. Time-frequency analysis of atrial activity is used to identify and track fibrillatory frequencies from extracted atrial activity, which provides possible guidance to tailored treatments. In addressing the problem of tracking fibrillatory frequencies, we have developed a framework for generating new classes of time-frequency distributions with many desirable properties. This framework is based on multi-dimensional Fourier transform of a radially symmetric function, and can be used to generate new distributions with unique characteristics. A realization of this framework on a high-dimensional radial delta function results in a new class of time-frequency distributions, which we call radial- $\delta$  distributions. The class of radial- $\delta$  distributions unifies number of well known distributions, and further provides methods for high resolution time-frequency analysis of multi-component signals with low interference terms.

We present a maximum likelihood inverse Gaussian point process model for dynamic and instantaneous HRV and RSA estimation from heart beat interval series and respiration recordings. Unlike previous methods, we perform timefrequency analysis of heart beat interval series, respiration, as well as the coherence between the two, and dynamically evaluate RSA transfer function based on instantaneous respiration and maximum coherence frequencies. The point process algorithm and dynamic respiration based RSA estimation methods are applied on two experimental protocols, a meditation experiment and a pain experiment. These applications demonstrate the robustness of the point process model in estimating HRV and RSA under different psychophysiological states. Regardless of the significant variations in respiration during meditation practice, goodness-of-fit tests are still found to be well within the desired confidence bounds, which validate the proposed models. Results indicate a significant increase in RSA during meditation practice, which suggest positive influence of meditation on the cardiovascular health. In the second experiment, reduced RSA during pain indicates the ability of the method to differentiate between different acute pain levels.

Novel time-frequency distributions and orthonormal basis atrial activity representation based analysis provide accurate tracking of fibrillatory frequencies of atrial fibrillation arrhythmia from ECG. The point process model with timefrequency analysis provides accurate estimations of HRV and RSA, and is robust to dynamic changes in respiration and autonomic inputs. These algorithms provide useful tools for monitoring cardiovascular health and particular arrhythmia conditions.

# List of Acronyms

Average Beat Subtraction
Atrial Fibrillation
Akaike Information Criterion
Autoregression
Additive White Gaussian Noise
Blind Source Separation
Blood Volume Pressure
Choi-Williams Distribution
Electrocardiogram
Heart Rate
Heart Rate Variability
Kolmogorov-Smirnov
Pulse Interval
Reduced Interference Distribution
Root Mean Squared
Respiration Signal
R-peak to R-peak Interval on Electrocardiogram
Respiratory Sinus Arrhythmia
Signal to Noise Ratio
Short-Time Fourier Transform
Time-Frequency Distribution
Wigner-Ville Distribution

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