Time-Frequency Spectral Estimation of Multichannel EEG Using the Auto-SLEX Method

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***Abstract*—In this paper, we apply a new time-frequency spectral estimation method for multichannel data to epileptiform electroencephalography (EEG). The method is based on the smooth localized complex exponentials (SLEX) functions which are time-frequency localized versions of the Fourier functions and, hence, are ideal for analyzing nonstationary signals whose spectral properties evolve over time. The SLEX functions are simultaneously orthogonal and localized in time and frequency because they are obtained by applying a projection operator rather than a window or taper. In this paper, we present the Auto-SLEX method which is a statistical method that 1) computes the periodogram using the SLEX transform, 2) automatically segments the signal into approximately stationary segments using an objective criterion that is based on log energy, and 3) automatically selects the optimal bandwidth of the spectral smoothing window. The method is applied to the intracranial EEG from a patient with temporal lobe epilepsy. This analysis reveals a reduction in average duration of stationarity in preseizure epochs of data compared to baseline. These changes begin up to hours prior to electrical seizure onset in this patient.**

***Index Terms*—Electroencephalography, spectral analysis, stochastic processes, time-frequency analysis.**

I. Introduction

ELECTROENCEPHALOGRAPHY (EEG) has been used as a clinical diagnostic tool for more than 70 years, since its introduction by Hans Berger in 1929. To record EEG, electrodes are placed either on the scalp or implanted intracranially, and the electrical activity of the brain, which results from the summed activity of thousands of neurons in the vicinity of the electrode, is monitored. The electroencephalographer makes diagnoses of brain dysfunction against empirical observations of normal electrographic recording using, in part, the language of frequency content of the electrical signal. It is for this reason that spectral analysis is a natural tool for analytically studying the time varying activity of EEG.

The state of the brain is in continual change, with the EEG having different spectral properties depending on the behavioral state of the organizm (i.e., sleep/wake) and cognitive tasks being undertaken. The EEG signal then is taken to be nonstationary, its spectrum changing with time. In fact, as is the case with most biological processes, we assume that the spectral characteristics of the EEG are changing continuously and slowly. Such a signal can be approximated as piecewise stationary, a series or sequence of independent stationary signals [1]. The spectrum of piecewise stationary processes, however, change abruptly over time, presenting problems when analyzing signals that change slowly and continuously. Ombao *et al.* [2] introduced blended stationary processes that are more general than piecewise stationary processes. Under this model, one can form an interval around each time point that is approximately stationary.

The field of spectral analysis has been dominated by use of the Fourier transform. Because they are perfectly localized in frequency and are periodic, the Fourier basis functions are ideal for representing stationary signals, i.e., signals whose spectral properties do not change with time. For such stationary signals, the Fourier transform has the additional property, through Parseval's relation, that the signal energy can be completely recovered from the transform coefficients. This property gives the spectrum a desirable physical interpretation and makes spectral analysis easy to relate to that part of the electroencephalographer's interpretive language that deals with frequency content of the EEG. Energy conservation, however, is dependent upon the transform being an expansion of an orthonormal and complete set of basis functions. The Fourier functions do not adequately represent nonstationary signals. To alleviate this time localization problem, smooth windows with compact support have been applied to the Fourier functions [3] giving the short time Fourier transform (STFT). Windowed Fourier functions, however, are generally no longer orthogonal. In fact, the Balian–Low theorem states that there does not exist a smooth function such that the windowed Fourier basis functions are simultaneously 1) orthogonal and 2) localized in time and frequency [3], [4]. Therefore, the STFT coefficients lose the important physical interpretation of representing the energy in the original time series.

In this paper, we present the Auto-SLEX method that uses the smooth localized complex exponentials (SLEX) basis functions,

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which are derived from the Fourier complex exponentials [2], [4]. The SLEX transform is complex valued and, hence, is preferred over real valued transforms because they retain important phase information. In addition, unlike wavelets and time-frequency distributions, they exhibit linear phase behavior under time shift, which makes the estimation of time lags between components of a multichannel EEG (or multivariate signal) a straightforward calculation. Unlike the windowed Fourier functions used in STFT, the SLEX basis consists of functions that are simultaneously orthogonal and localized in time and frequency. They evade the Balian–Low obstruction because they are constructed by applying a projection operator, rather than a window, on the Fourier functions. Thus, the SLEX basis can be considered as localized generalizations of the Fourier basis. The details on the construction of projection operators are given in [4], which also shows that applying a projection operator is identical to applying two special smooth and compactly supported windows to the Fourier basis functions. The general form of these two windows is given in [2], and it is this double-window method we use here in constructing the basis.

Time frequency methods were developed to study deterministic signals. The EEG is either not deterministic, or its determinism is not completely known. It, therefore, is usually treated as a realization of an underlying stochastic process whose spectrum we want to estimate. The standard approach to estimating the spectrum is to compute the periodograms, which are simply the square of the modulus of the Fourier coefficients. The periodogram is, approximately, an unbiased estimator of the spectrum. That is, if we repeatedly observe many signals that are realizations from the same underlying process, then the average of the periodograms computed from each signal is close to the true unknown spectrum of the underlying process. The periodograms, however, are not consistent estimators. The periodogram estimates computed from many realized signals tend to vary and fluctuate with high degree and the variability cannot be ignored even for long signals. To form a consistent estimator of the spectrum, one can smooth the periodograms over frequency, by applying a moving average to them [5]. In smoothing periodograms, the choice of the span of the smoothing window (also called bandwidth) is more crucial than the choice of the shape of the smoothing window. The Auto-SLEX method includes an automatic criterion for span selection.

The Auto-SLEX method also automatically divides the nonstationary signal into segments that are approximately stationary. It is necessary to use a moving time window that is small enough to approximate the stationarity of the underlying random process. However, if the time window is too small, the resolution in the frequency domain diminishes and components of the signal that oscillate at similar frequencies will be indistinguishable. In addition, smaller time windows have fewer observations, thus increasing the bias in the spectral estimates. The Auto-SLEX method for segmenting the nonstationary signal is similar to a method described by Adak [1] in which the original signal is divided dyadically into possibly overlapping segments, and segments are combined whose spectral estimates are similar based on a distance metric. This provides an improved segmentation in which each segment is as long as possible while remaining approximately stationary. One limitation of the Adak method is that it is not easily extended to multivariate data, thus, it cannot estimate coherencies between channels of EEG signals. The method presented here also divides the signal in a dyadic manner. To determine the optimal segmentation, we use an objective cost function that is a log energy function with an added complexity penalty to prevent over-segmentation. The penalized log energy cost function can be derived as a Kullback–Leibler distance for random processes with Gaussian innovations [6]. The best segmentation is then obtained by applying the best-basis algorithm of Coifman and Wickerhauser [7].

II. METHODS

A. The SLEX Transform

The SLEX transform utilizes a double-window procedure, which is equivalent to a projection operation on the Fourier functions [4]. The two windows, denoted Ψ_+ and Ψ_-, are constructed as follows on the discrete interval $[\alpha_0 - \varepsilon, \alpha_1 + \varepsilon]$

$$\begin{aligned}\Psi_+(n) =& r^2\left(\frac{n-\alpha_0}{\varepsilon}\right) r^2\left(\frac{\alpha_1 - n}{\varepsilon}\right)\\ \Psi_-(n) =& r\left(\frac{n-\alpha_0}{\varepsilon}\right) r\left(\frac{\alpha_0 - n}{\varepsilon}\right)\\ &- r\left(\frac{n-\alpha_1}{\varepsilon}\right) r\left(\frac{\alpha_1 - n}{\varepsilon}\right)\end{aligned} \tag{1}$$

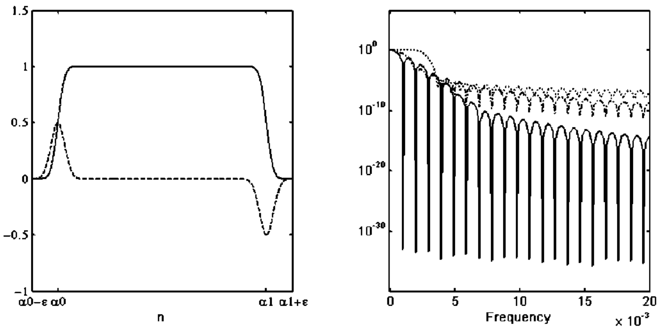
where ε is the number of points overlapping into a neighboring block and should not exceed 1/2 the block size, $\alpha_1 - \alpha_0$, to retain statistical independence of neighboring blocks. $r(\cdot)$ is the iterated sine function which controls the steepness of the windows Ψ_+ and Ψ_-, and is given by [4]

$$\begin{aligned}r_{[0]}(n) =& \sin\left[\frac{\pi}{4}(1+n)\right]\\ r_{[d+1]}(n) =& r_{[d]}\left(\sin\frac{\pi}{2}n\right).\end{aligned} \tag{2}$$

The number of iterates d controls the steepness in the rising and falling phases of r and, hence, Ψ, which in turn controls the time-frequency localization of the transform. A steeper rising function preferentially selects time localization over frequency localization and *vice versa*. Fig. 1(a) shows the Ψ_+ and Ψ_- functions and Fig. 1(b) the log modulus squared of the Fourier transform of the sum $\Psi_+ + \Psi_-$. Fig. 1(b) also shows spectra of windows used in popular time-frequency localized Fourier methods, the short-time Fourier transform (STFT) and Thomson's multiple window method (MWM).

Given the time series $x(n)$ for $n = 0, 1, \ldots, N-1$, the SLEX transform, of $x(n)$, denoted $G^{(x)}(f)$ is

$$G^{(x)}(f) = \sum_{n=0}^{N-1} \Psi_+(n)x(n)e^{-j2\pi fn} + \sum_{n=0}^{N-1} \Psi_-(n)x(n)e^{j2\pi fn}. \tag{3}$$



(a) (b)

Fig. 1. (a) The window functions Ψ_+ (solid) and Ψ_- (dashed) using a 1,024 point window for $d = 1$ and $\varepsilon = 128$ (see text for an explanation of the role of d and ε in the construction of the window functions) and (b) modulus squared of the Fourier transform of the sum $\Psi_+ + \Psi_-$ (solid). In addition, (b) contains the Fourier transform of windows used in two other popular Fourier based methods: Hanning window (dot-dash) and Thomson's MWM (dotted).

Each term on the right-hand side of (3) should be recognized as a windowed Fourier transform of the sequence $x(n)$ using the windows Ψ_+ and Ψ_-, respectively. In addition, the second term is the complex conjugate of the Fourier transform. This second term, using the window Ψ_-, is necessary to restore the orthogonality to the transform. The periodogram, $I^{(x)}(f)$, is defined to be $|G^{(x)}(f)|^2$ where $|\cdot|$ denotes the modulus of the complex terms of the SLEX transform.

B. The Auto-SLEX Algorithm

If it is assumed that the spectral density function $h^{(x)}(f)$ of the sequence $x(n)$ is continuous and smooth, the periodogram is an asymptotically unbiased estimate of $h^{(x)}(f)$. It is also, however, an inconsistent estimator of $h^{(x)}(f)$ [5]. That is as $N \to \infty, \mathrm{var}\{I^{(x)}(f)\} \not\to 0$. In addition, as $N \to \infty$ the correlation between neighboring frequencies actually decreases, giving the periodogram wildly fluctuating behavior different from that expected from a continuously smooth $h^{(x)}(f)$ where the neighboring frequencies would be correlated. Spectral smoothing (over frequency) can be applied to the periodogram, which alleviates the consistency problem. The method presented here for estimating the time-varying spectra and cross spectra includes a method for selecting the optimal span of the spectral smoothing window and a procedure which segments the time series into approximately stationary segments based on the best basis algorithm (BBA) of Coifman and Wickerhauser [7].

The optimal span of the smoothing window is selected by generalized cross validation (GCV) [8]. The GCV function is a measure of the deviation of the smoothed SLEX periodograms to the raw periodograms, and the bandwidth where the GCV function is a minimum is the optimum. The GCV function is appropriate for statistical quantities like the periodogram that are asymptotically distributed as gamma random variables. The GCV span selector attempts to balance precision and accuracy. It gives estimates that are smooth (precise) and at the same time smoothed estimates that are close to the raw observed periodograms (accurate).

The GCV function of the bth block in the jth level of the initial segmentation for the time series $x(n)$, for $n = 0, 1, \ldots, N-1$, is

$$\mathrm{GCV}_{j,b}(\nu) = \frac{2}{(df_{j,b,\upsilon})^2} \times \sum_{k=0}^{M_j/2} \left\{ \frac{\hat{I}_{j,b}(f_k)}{\tilde{I}_{j,b,\nu}(f_k)} - \log \frac{\hat{I}_{j,b}(f_k)}{\tilde{I}_{j,b,\nu}(f_k)} - 1 \right\} \quad (4)$$

where M_j is the block size of the jth level, $M_j = N/2^j$, ν is the bandwidth of the smoothing window, $df_{j,b,\nu}$ the "error degrees of freedom" and is $df_{j,b,\nu} = 1 - \mathrm{trace}\{\mathbf{H}_\nu\}/(M_j/2+1)$ where $\mathbf{H}_\nu$ is the smoothing window convolution matrix, $\hat{I}_{j,b}(f_k)$ the estimated raw periodogram and $\tilde{I}_{j,b,\nu}(f_k)$ the smoothed periodogram using bandwidth ν. The optimum bandwidth is that which minimizes the GCV function

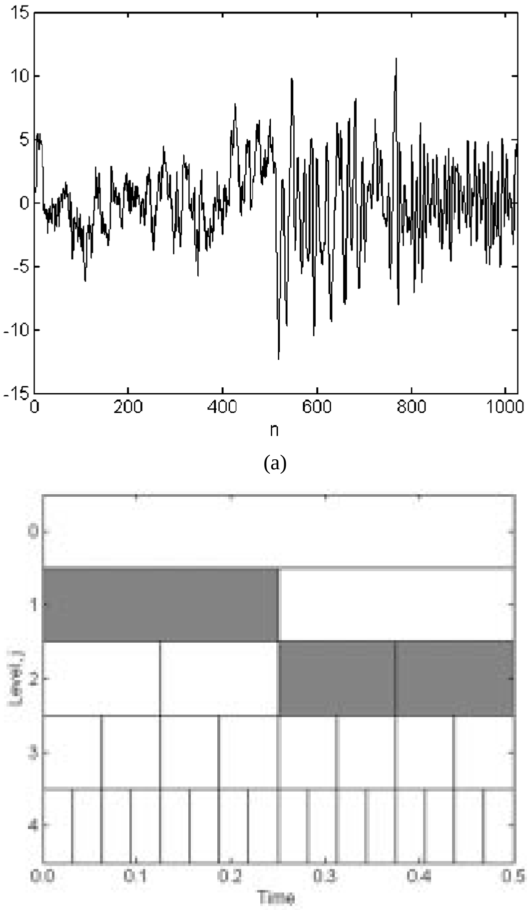
$$\nu_{\mathrm{opt},j,b} = \arg\min_\nu \mathrm{GCV}_{j,b}(\nu). \quad (5)$$

The SLEX transform forms a library of orthornormal transforms, corresponding to each block of the initial segmentation. To each smoothed periodogram, an objective cost function is applied and the goal is to find the segmentation that minimizes the overall cost. In our implementation, we use the total log-energy cost plus a complexity penalty term as motivated in [6]. The cost of the bth block of the jth level (j, b) is defined as

$$\mathrm{Cost}(j,b) = \left\{ \sum_{k=-M_j/2+1}^{M_j/2} \log \tilde{I}_{j,b}(f_k) \right\} + \beta\sqrt{M_j} \quad (6)$$

where β is a complexity penalty parameter and $\tilde{I}_{j,b}(f_k)$ is now the periodogram smoothed using the optimal bandwidth. The penalty parameter safeguards the procedure from obtaining a segmentation that either has too many or too few blocks. A small value of β tends to select a segmentation that has too many blocks while a larger value chooses segmentations with too few blocks. If the segmentation has too many blocks, stationary segments will be split resulting in inflated variances. If too few blocks are selected, bias is introduced by computing the spectrum on a nonstationary segment. Therefore, careful consideration in the choice of β is required. Donoho *et al.* [6] give a theoretical argument for the choice of $\beta = 1$, which is supported by Gao [9]. The optimal value, however, may be data dependent. For data with rapidly changing spectra a small value of β will give segmentations with many blocks that will faithfully follow rapid transitions in the spectra. The same small value used on data with slowly changing spectra may over-split the data. It is unclear at this time just how data dependent this parameter is and we are currently working on a data driven automatic method for selecting the optimal value. The choice of this parameter used in the EEG analysis in Section III-B was based on simulation results in Section III-A.

The next step is to apply the BBA to the smoothed SLEX periodograms, which searches for the segmentation with the overall lowest cost. Fig. 2 shows a realization of a piecewise stationary



(a)

(b)

Fig. 2. (a) Realization of a piecewise stationary process and (b) segmentation table following Auto-SLEX processing. Gray blocks indicate those chosen by BBA for final segmentation.

process used in the simulations and a segmentation chart depicting all blocks of the dyadic segmentation and the blocks selected by BBA (gray) as minimizing the overall cost. The chart shows that BBA divided the time series into three stationary blocks with break points at 1/2 and 3/4. The final periodograms, $\tilde{I}_{j,b}^{(x)}(f)$ are constructed from the blocks (j, b) which define the best segmentation as selected by BBA.

The complete Auto-SLEX procedure can be summarized as follows: 1) divide the time series into blocks in a dyadic manner; 2) compute the SLEX transform (3) on each data block and create the periodogram; 3) smooth the periodograms with a smoothing window of optimal span by minimizing the GCV function [(4) and (5)]; 4) assign to each block an objective cost measure (6); and 5) select the blocks using BBA that minimize the overall cost of the segmentation.

C. Bivariate Time Series

It is easy to extend the method presented above to a bivariate nonstationary time series $\{x(n), y(n)\}$. Estimates of the periodograms of $x(n)$ and $y(n)$ proceeds as for the univariate case described above. The estimate of the cross periodogram is $\hat{I}^{(xy)}(f) = \left|G^{(x)}(f)G^{(y)*}(f)\right|$ where $*$ denotes complex conjugation. To ensure that the estimate of the spectral density matrix is nonnegative, the spectral smoothing bandwidth of the periodograms and cross periodograms must be the same [2]. The best bandwidth for block (j, b) is that which minimizes

$$P_{j,b}(\nu) = \mathrm{GCV}_{j,b}^{(x)}(\nu) + \mathrm{GCV}_{j,b}^{(y)}(\nu)$$

where $\mathrm{GCV}_{j,b}^{(x)}(\nu)$ and $\mathrm{GCV}_{j,b}^{(y)}(\nu)$ are computed from (4). The spectra and cross spectrum are smoothed using this bandwidth and a modified cost function is applied

$$\mathrm{Cost}(j,b) = \left\{ \sum_{k=-M_j/2+1}^{M_j/2} \log \tilde{I}_{j,b}^{(x)}(f_k) + \log \tilde{I}_{j,b}^{(y)}(f_k) \right\} + \beta\sqrt{M_j}.$$

As in the univariate case, BBA selects the blocks, (j, b) which minimize the overall cost. The phase and coherence spectra are obtained from

$$\tilde{\Phi}_{j,b}^{(xy)}(f) = \tan^{-1}\left\{ \frac{\mathrm{imag}\left(\tilde{I}_{j,b}^{xy}(f)\right)}{\mathrm{real}\left(\tilde{I}_{j,b}^{xy}(f)\right)} \right\}$$

$$\tilde{R}_{j,b}^{(xy)}(f) = \frac{\left|\tilde{I}_{j,b}^{(xy)}(f)\right|}{\sqrt{\tilde{I}_{j,b}^{(x)}(f)\tilde{I}_{j,b}^{(y)}(f)}}.$$

D. Computational Notes

The Auto-SLEX algorithm was implemented using MATLAB release 12 (The Mathworks, Natick, MA), and the m-files will be made available upon request to the authors. If the time series is divided such that the size of each block is 2^L where L is an integer, then the SLEX transform can be computed in a computationally efficient manner. Since the SLEX transform is a form of the windowed Fourier transform, the fast Fourier transform (FFT) can be used to compute the SLEX transform. Use of non-Radix-2 data lengths should generally be avoided not only for computational efficiency but also because there are a limited number of splits of the data that is possible and, therefore, a limited number of segmentation levels. Radix-2 FFT algorithms require $(N/2)\log_2 N$ complex multiplications and since the SLEX transform requires two FFT computations, the number of multiplications is $N\log_2 N$ at the first level (full data length, level $J = 0$). At the second level, there are two blocks and $2(N/2)\log_2(N/2) = N\log_2(N/2)$ multiplications. This continues to the Jth level where there are 2^J blocks of $N/2^J$ length and $N\log_2(N/2^J)$ multiplications. The total complex multiplications needed to compute the transform on the entire tree then becomes $\sum_{j=0}^{j} N\log_2(N/2^j)$. For the EEG data presented here, epochs of $N = 2^{20}$ data points were analyzed to level $J = 12$. This required greater than 190 million multiplications, but completed in less than 1 min on a 1.7-GHz processor PC.

The most time consuming stage of the algorithm is the selection of the optimal smoothing bandwidth. Since the GCV function cannot be explicitly minimized, the minimum must be found through a search algorithm. We currently use a simple gradient search, which can find the minimum in less than 40 iterations, however when smoothing each periodogram in the entire tree, as is done prior to applying the log-energy cost, the time required for each epoch was approximately 30 min. During the course of the simulations presented below we noted that the segmentation accuracy was not too greatly affected by computing the cost on the raw rather than optimally smoothed periodograms. This generally led to segmentations with a greater number of blocks, and with it inflated variances, but greatly reduced computation time so that larger data sets could be analyzed very quickly. Of course, smoothing of the final spectrogram is necessary to give consistent and low variance estimates.

E. Simulated Data

The data sets used for the simulations are piecewise stationary autoregressive (AR) processes. An $\mathrm{AR}(p)$ process is of the form

$$x(n) = \theta_1 x(n-1) + \cdots + \theta_p x(n-p) + \xi(n)$$

where $\xi(n)$ are independent random variables with zero mean and variance σ^2.

For each realization the averaged squared error (ASE) of the log-spectrum was computed

$$\mathrm{ASE} = \left(\frac{1}{T(M_j+1)}\right) \times \sum_{n=0}^{N-1} \sum_{k=0}^{M_j/2} \left\{ \log \hat{S}\left(\frac{n}{N}, f_k\right) - \log S\left(\frac{n}{N}, f_k\right) \right\}^2$$

where $\hat{S}(u,f)$ is the estimate of the true time-dependent spectrum, $S(u,f)$. ASE combines errors due to both estimate bias and variance. For each simulation, we created 200 data sets of $N = 1024$ points, and the highest level of segmentation for SLEX was $J = 4$, which gives a block size of 64 points. The piecewise stationary simulated data used had stationary segments of at least 256 points, which corresponds to the smallest block size used in the EEG data described in the next section. This allowed us to use the simulations to find a value of the complexity parameter, β in (6), which is approximately optimal for EEG.

F. Experimental Data: Subjects and Data Acquisition

We analyzed EEG recordings from a patient with presumed mesial temporal lobe epilepsy undergoing localization of seizures with intracranial electrodes during evaluation for epilepsy surgery. Continuous EEG was recorded for 14 days using the 64-channel, 12-bit Nicolet BMSI 5000 epilepsy monitoring system (Nicolet Biomedical, Madison, WI) which digitizes the signal at 200 Hz. Apart from using a bipolar montage to remove common mode signals and artifact, no preprocessing was applied to the digitized EEG data. During the recording period this patient had four seizures of unilateral mesial temporal onsets, three seizures localized to the left and one to the right anterior hippocampus. Approximately six hours of continuous EEG (2^{22} data points) prior to each seizure was extracted for SLEX analysis. Only the channel determined to be the seizure lead channel was processed. Because of the size of the data set, analysis was performed on four contiguous segments of 2^{20} data points each to reduce computation time. For each of the four segments, the best segmentation was computed to a maximum level of $J = 12$, corresponding to a block size of 256 points (~1.25 s). At this level, the EEG is assumed to be approximately stationary based on the work of Qin [10].

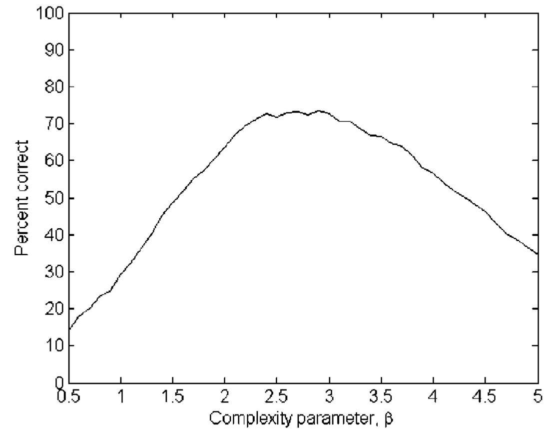


Fig. 3. Effect of the complexity parameter, β, on segmentation accuracy.

III. Results

A. Simulations

Any method that gives a good spectral estimate of a nonstationary process must be able to accurately segment the data and give spectral estimates with low bias and variance. In this section, we compute the time-varying SLEX periodogram on simulated data to assess the accuracy of the segmentation and then compare the spectral estimate to those of two other Fourier based time-frequency methods: the STFT and Thomson's MWM using four windows [11], [12]. Optimal spectral smoothing, by using the GCV method described above, is applied to the STFT spectrogram to give consistent and low variance estimates for fair comparison. MWM is designed to simultaneously minimize bias and variance through specially designed window functions.

Segmentation on a Dyadic Boundary: Selection of the penalty parameter, β, in (5) may be crucial to the accuracy of the segmentation. As explained in the Section II, too small a value of β will give segmentations with too many blocks, whereas a value of β too large, gives segmentations with too few blocks. We tested the segmentation on the following piecewise stationary process with known break points on the dyadic boundaries 1/2 and 3/4 ($n = 512$ and 768, respectively) and computed the percentage that the method was able to correctly identify the break points

$$x(n) = \left\{ \begin{array}{ll} x_1(n), & \text{if } 0 \le n \le 511 \\ x_2(n), & \text{if } 512 \le n \le 767 \\ x_3(n), & \text{if } 768 \le n \le 1023 \end{array} \right\} + \xi(n)$$

where $x_1(n)$ is an $\mathrm{AR}(1)$ process with $\theta = 0.91$, $x_2(n)$ is an $\mathrm{AR}(2)$ process with $\theta_1 = 1.69$ and $\theta_2 = -0.81$, and $x_3(n)$ is an $\mathrm{AR}(2)$ process with $\theta_1 = 1.32$, $\theta_2 = -0.81$, and $\xi(n) \sim N(0,1)$. Fig. 3 shows an accuracy rate of 72% with $\beta = 2.7$.

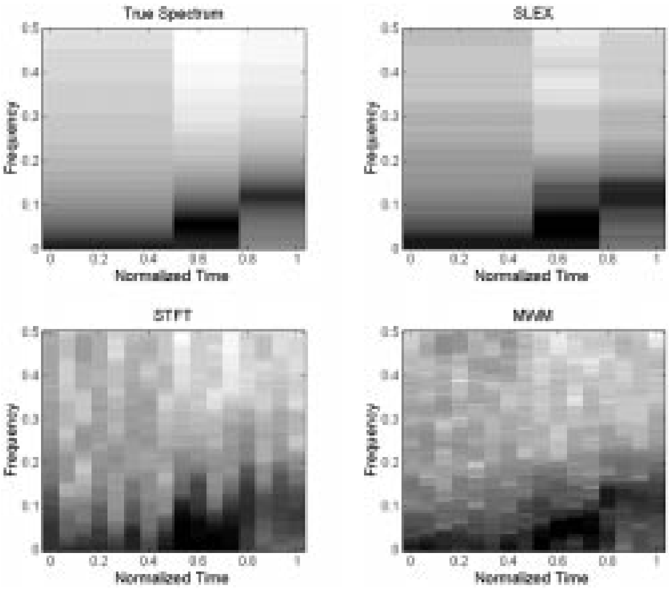


Fig. 4. Time-varying spectra of a piecewise stationary process segmented on a dyadic boundary.

We proceed with the remaining simulations, and analysis of the EEG data set, using this value of β.

Fig. 4 shows the resulting spectrograms from a realization of this process. The spectrograms shown are those that produced the lowest ASE for each of the three methods and Table I compiles the average ASE for all simulations. SLEX divided the time series into an average of 4.5 segments which indicates that the method errs on the side of too many blocks (oversplitting) rather than too few, undersplitting in less than 5% of the cases. This is preferable since under-segmenting will cause a bias due to nonstationarity. However the method should also guard against too much oversplitting since this will reduce the frequency resolution and increase the estimate variance.

Segmentation on Nondyadic Boundary: Rarely will real signals change system parameters exactly on dyadic boundaries, where SLEX segmentation is optimum. In this simulation, we generated the following piecewise stationary process with a single break at $n = 196$

$$x(n) = \begin{cases} x_1(n), & \text{if } 0 \le n \le 196 \\ x_2(n), & \text{if } 197 \le n \le 1023 \end{cases}$$

where $x_1(n)$ is an $\mathrm{AR}(1)$ process with $\theta = 0.91$, $x_2(n)$ and $\mathrm{AR}(1)$ process with $\theta_1 = -0.91$. An ideal segmentation would be one that gives very small blocks around the break point, which is dependent on the length of the time series and the level of initial segmentation, J. Fig. 5 shows the resulting spectrograms from one realization. As in Fig. 4, the spectrograms shown are those that showed the best fit to the true spectrum. The average number of segments was 7.8 with a minimum of four again indicating the method favored over-splitting the time series. The ASE reveals that the Auto-SLEX fit to the true spectrum was not as good as in the previous case, which shows the drawback of using a dyadic transform on a nondyadic process, however the mean ASE is still below any values of both STFT and MWM.

Slowly Varying Process: It is unlikely that biological processes such as EEG abruptly change parameters as the piecewise stationary processes above do. Rather their parameters most likely will evolve relatively slowly over time. It is for this reason that the SLEX transform is based on blended stationary processes instead of piecewise stationary. To investigate how Auto-SLEX performs on such data we constructed 200 realizations of the following slowly varying $\mathrm{AR}(2)$ process

$$x(n) = \theta_1(n)x(n-1) + \theta_2 x(n-2) + \xi(n)$$

$\theta_1(n) = 0.8[1 - 0.5\cos(\pi n/N)]$ and $\theta_2 = 0.81$. Fig. 6 shows the spectrograms from one realization. Note that Auto-SLEX gave segmentations of relatively large blocks, which is reasonable since the process parameter θ_1 changes slowly. The average number of segments was 5.1 with a range from 2 to 10 and the number of segments was more evenly distributed across the range than in the previous cases. The mean ASE of 0.037 indicates that Auto-SLEX performs quite well on this type of data even though the process is not piecewise.

B. *Stationarity of Preseizure EEG*

Segmentation of the times series is important for high quality spectral estimates in Fourier based methods since the Fourier transform assumes stationarity of the data. Also, consistent estimators of the spectrum benefit from data of increasing length as this reduces the estimate variance. The stationarity of EEG prior to a seizure may itself be an interesting phenomenon to study since it may be suggestive of changes in the state of the system. It is already known that there are changes in the EEG, which takes place from minutes to several hours prior to seizure onset [13]–[16]. To our knowledge there have been no comprehensive studies examining stationarity of EEG for large data sets.

In this study, we looked at the segmentation provided by Auto-SLEX over approximately six hours of continuous EEG prior to seizures in one patient with four partial seizures arising from the temporal lobes, three from the left and one from the right hemisphere. The segmentation chart for one such seizure is shown in Fig. 7. Gray regions depict blocks selected by Auto-SLEX as minimizing the overall cost of the segmentation. Block size decreases dyadically to the bottom of the chart. It is clear that the EEG has frequent runs of stationarity as long as 10 min in duration until approximately 40 min prior to a seizure (time 0 in Fig. 7) when the EEG becomes more heavily segmented. This time frame is interesting in that recent evidence [15] has shown that EEG energy increases occur approximately one hour prior to seizure onset. Since segmentation is based on log-spectral energy, rapid changes in stationarity are a reflection of changes in energy of the time series.

In total, three of the four seizures analyzed showed changes to stationarity preceding a seizure. Fig. 8 shows the segment duration, in minutes, averaged in a 5-min moving window for each of the four preseizure records. In seizures 1, 2, and 4, all of which were generated in the left temporal lobe, step-like drops in stationarity can be seen at 300, 40, and 185 min prior to seizure onset, respectively. Of interest, seizure 3, which arose from the right temporal lobe, did not demonstrate any of these changes prior to seizure onset, suggesting the possibility of a different

TABLE I
ESTIMATION RESULTS

Method	Dyadic	Nondyadic	Slowly varying
SLEX	0.038 ± 0.025	0.055 ± 0.019	0.037 ± 0.011
STFT	2.98 ± 0.21	2.95 ± 0.19	2.90 ± 0.09
MWM	0.071 ± 0.01	0.085 ± 0.012	0.063 ± 0.065

Each value is the mean ASE ± standard deviation from 200 simulations.

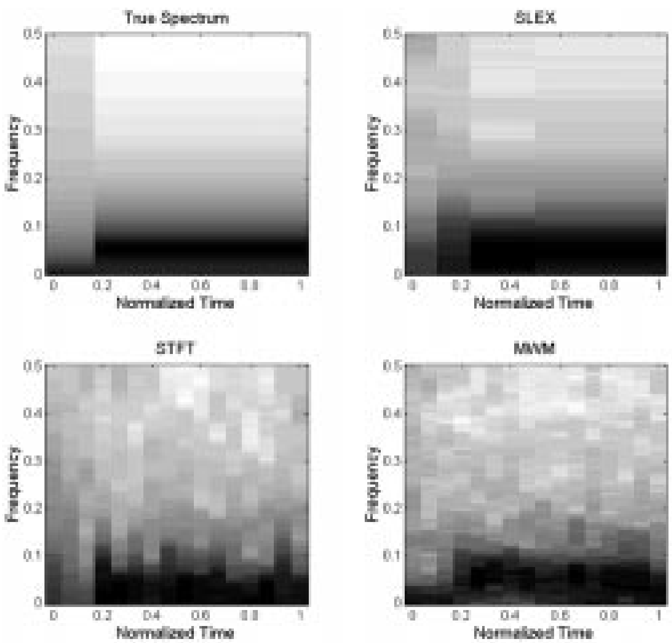


Fig. 5. Time-varying spectra of a piecewise stationary process segmented on a nondyadic boundary.

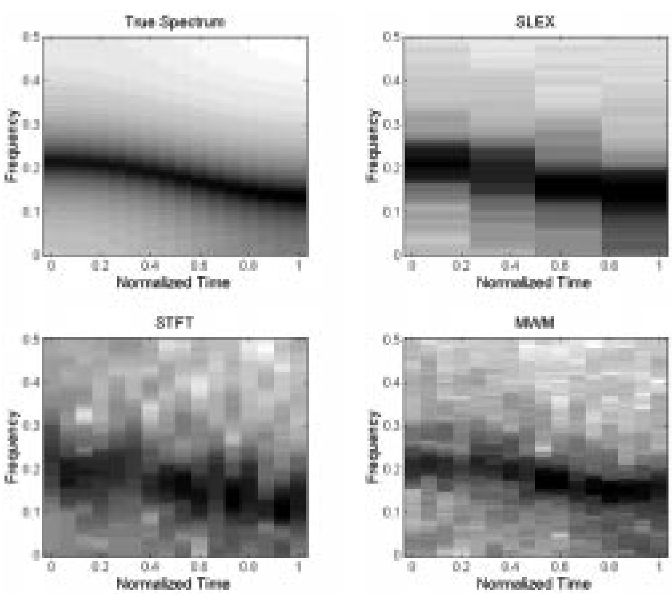


Fig. 6. Time-varying spectra of a slowly varying AR process.

mechanism of generation for this seizure. It is important to note that while this analysis was restricted to a single patient, there

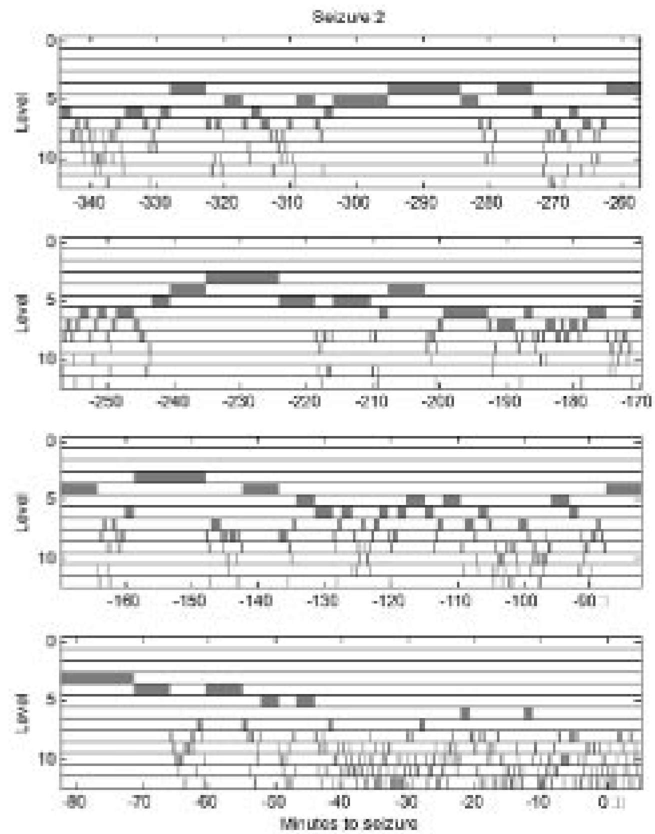


Fig. 7. Segmentation table of preseizure EEG from a patient with temporal lobe epilepsy. The four tables represent contiguous segments from one continuous EEG record, approximately six hours in duration.

are relatively few studies in the literature that analyze preseizure data epochs of this duration, and none which focus on the stationarity of EEG signals as seizures approach.

IV. DISCUSSION

We have demonstrated the Auto-SLEX method for analyzing nonstationary time series in the context of Fourier analysis. It uses the SLEX transform, which is localized in both time and frequency, while maintaining orthogonality that allows us to use the powerful best-basis algorithm for selecting an optimal segmentation. The method is computationally efficient, as it utilizes the FFT algorithm for data sets whose length is an integer power of two. The method was developed for bivariate time series and can give estimates of time-varying coherence and linear phase relationships.

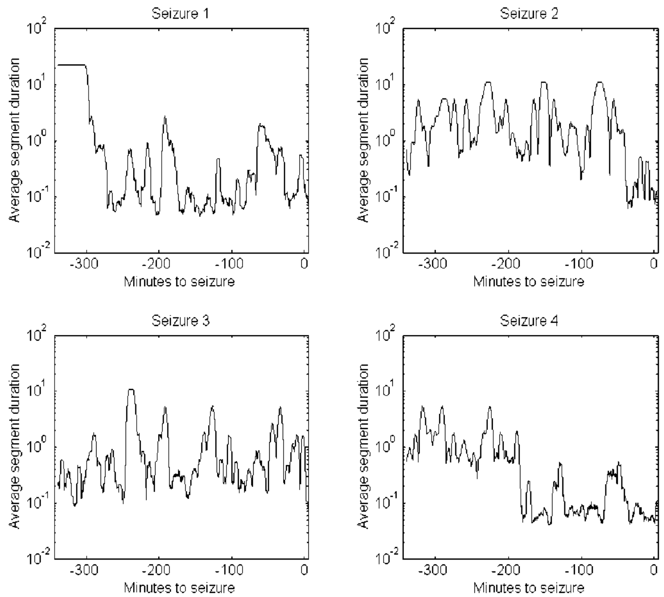


Fig. 8. Time-varying average segmentation duration of preseizure records from a patient with temporal lobe epilepsy. The segmentation duration was averaged using a 5-min moving window.

In this paper, we have demonstrated the segmentation capability on simulated data sets and have shown that the Auto-SLEX spectral estimates outperform two existing Fourier based time-frequency methods by providing an estimate with consistently lower ASE. A more theoretical treatment of the Auto-SLEX method is given in [2] where the authors prove the statistical consistency of the Auto-SLEX estimator. Moreover, a model of nonstationary random processes that is based on the SLEX basis is given in [17].

We have demonstrated the promise of the Auto-SLEX method in clinical practice by applying it to prolonged EEG recordings from a patient with temporal lobe epilepsy during presurgical evaluation. Reproducible, periodic fluctuations in the segmentation of the signal prior to seizures, marked by a significant drop in segment length over time were observed. The timing of these findings agrees with reports of signal changes that occur prior to seizures in such recordings [14], [15].

In summary, the Auto-SLEX method holds great promise for analyzing biological data, particularly EEG signals. Using simulated data, the method has significant advantages over other methods for segmenting experimental data into stationary segments. In a clinical example, preseizure data epochs demonstrated reproducible decreases in signal stationarity up to several hours prior to seizure onset in a patient with temporal lobe epilepsy. The predictive value of these results will need to be assessed, and validated in a larger number of patients over longer periods of time. The method may prove to be extremely useful in improving the accuracy of algorithms that require signal stationarity in the EEG, and perhaps in elucidating mechanisms underlying human epilepsy.

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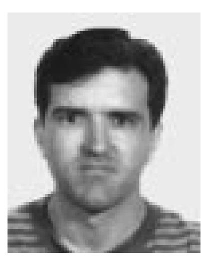

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