# Time in Fundamental Physics

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The first three sections of this article contain a broad brush summary of the profound changes in the notion of time in fundamental physics that were brought about by three revolutions: the foundations of mechanics distilled by Newton in his *Principia*, the discovery of special relativity by Einstein and its reformulation by Minkowski, and, finally, the fusion of geometry and gravity in Einstein's general relativity. The fourth section discusses two aspects of yet another deep revision that waits in the wings as we attempt to unify general relativity with quantum physics.

## I. NEWTON'S ABSTRACTION AND ITS SUCCESS

We perceive the passage of time through change. However the existence of change by itself does not establish the reality of an *objective* time. Indeed, our direct observations refer to a *relational* notion of time. For example, routine measurements only tell us that while the earth goes around the sun once, the moon goes around the earth approximately 13 times. Or, while the second hand completes one round on one's wristwatch, one's pulse beats 70 times. What we directly experience is change and observations only let us compare durations involved in one change with those involved in another. This led to what is often referred to as the Leibnizian space-time view in which the only meaningful questions about motion refer to relative motion.

This notion of relational time has a curious similarity with the barter system people used before the advent of the abstract concept of money. A sheep was worth $n$ chickens, a chicken was worth $m$ bottles of oil, and so on. People only *compared* values of objects. Money — particularly in the form of banknotes— is an abstract concept, a mental creation, that simplifies trade. Money is not essential for survival. One can imagine abolishing money and using just the barter system that only assigns relative values to pairs of necessary objects. But the notion of money is extremely powerful: it *streamlines* all commercial transactions by giving each item an absolute value in place of the pairwise relational values used in the barter system. It is difficult to imagine a flourishing trade without money, let alone the more abstract monetary instruments that are now used.

Through his *Principia*, Newton streamlined time in the same fashion. He postulated that *absolute time* in itself has a direct physical meaning, without reference to any physical systems or phenomena. To distinguish this notion from other subjective or psychological measures of the passage of time, he represented the absolute physical time by a 1-dimensional mathematical continuum.[1] All durations were to be measured against this absolute time. Newton taught us that we need not be satisfied with *relational time*, e.g., just with thinking

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[1] In the contemporary terminology used in the general relativity literature, what Newton called absolute space provides a canonical foliation of space-time and each slice is labeled by a value of the time parameter

of how many rotations of the moon around the earth correspond to one rotation of the earth around the sun. Rather, the moon's orbit around earth marks an interval on the *physical* one dimensional continuum of time by itself, and so does the earth's orbit around the sun. Any inertial observer can measure these intervals and Newtonian mechanics asserts that they will all agree in spite of the uniform relative motion. Thus, the flow of time is absolute in spite of Galilean relativity. *If we wish*, we can compare the lengths of these intervals and conclude that there are approximately 13 moon-orbit intervals in one earth-orbit interval. And we can also compare these intervals with those marked on the absolute time continuum by the rotation of the second hand on one's watch or one's pulse. But the comparisons and the resulting relational notion of time are all secondary. Absolute time is the primary, physical notion.

This notion lies at the foundation of Newton's laws of mechanics. The resulting celestial mechanics were astonishingly successful. Already in the 1750s, papers appeared in the Philosophical Transactions of the Royal Society, calculating sophisticated consequences of Newton's laws, such as of the effect of the gravitational pull of Jupiter and Saturn on earth's motion (see, e.g.,[2])! Had one continued to use the relational time of direct experience —as Leibniz, for example, advocated— such calculations would have become very cumbersome. Celestial mechanics could be developed and its predictions could be compared against observations so quickly largely because it was based on Newton's absolute time.

The success of celestial mechanics brought home the message that the heavens are not so mysterious after all; they were brought within the grasp of the human mind. Thus, the *Principia* shattered Aristotelian orthodoxy by abolishing the distinction between heaven and earth. For the first time, there were truly universal principles. An apple falling on earth and the planets orbiting around the sun were now subject to the same laws, formulated using the same absolute time continuum. No wonder then that *Principia* literally sculpted human consciousness, providing the mental images that people commonly use. Most people now think of time as a part of physical reality, flowing serenely, all by itself, untouched by the external world.

In spite of this success, Newton was well aware of the fact that there was no objective basis for his postulates on absolute time and absolute space. He had to invoke theological arguments in support of their absolute character. Not surprisingly, Leibniz criticized these arguments as untenable.

## II. SPECIAL RELATIVITY: ABOLISHING ABSOLUTE TIME

The *Principia* quickly replaced Aristotle's four books on physics and became the new orthodoxy. It reigned supreme for over 150 years. However, a challenge to the Newtonian world view then emerged from totally unexpected quarters: advances in the understanding of electromagnetic phenomena. In the middle of the 19th century, the Scottish physicist James Clerk Maxwell achieved an astonishing synthesis of all the accumulated knowledge concerning these phenomena in just four vectorial equations. These equations further pro-

(taking values in the one-dimensional affine space $\mathbb{R}$). The preferred family of observers are the Galilean ones, in uniform motion with respect to one another. They all agree on this parametrization. What I call Newtonian space-time in this article is generally referred to as *neo-Newtonian* or *Galilean* space-time in the history and philosophy of science circles following [1]. I thank Tom Pashby for pointing this out.

vided a specific value of the velocity $c$ of light. But this velocity did not refer to a reference frame; *it appeared as an absolute constant of Nature.* Now, the notion of an absolute velocity blatantly contradicts Galilean relativity, a cornerstone on which Newtonian mechanics rests. This tension between Maxwell's electrodynamics and Newtonian mechanics dismayed natural philosophers. But by then learned men had developed deep trust in the Newtonian world and therefore concluded that Maxwell's equations can only hold in a specific reference frame, called the *ether.* The value of the speed of light $c$ that emerged from Maxwell's equations, they concluded, is relative to this ether. But by doing so, they in fact reverted back to the Aristotelian view that Nature specifies an absolute rest frame. A state of confusion remained for some 50 years.

There were several leading figures such as Henri Poincaré and Hendrik Lorentz who attempted to resolve this tension through mathematical modifications of the Galilean transformations. However, it was Albert Einstein who grasped the deep physical implications of this quandary: *It was asking us to abolish Newton's absolute time.* In 1905 Einstein accepted the implications of Maxwell's equations at their face value and used simple but ingenious thought experiments to argue that, since the speed $c$ of light is a universal constant, the same for all inertial observers, Newton's notion of absolute simultaneity is physically untenable. Spatially separated events which appear as simultaneous to one observer can not be so for another observer, moving uniformly with respect to the first. The Newtonian model of space-time can only be an approximation that holds when speeds involved are all much smaller than $c$. A new, better model of space-time structure emerged and with it a new kinematics, called *special relativity.*

In special relativity the elementary notion is that of an event —e.g., the explosion of a firecracker— which is completely localized in space and in time. These events constitute the space-time continuum. Strictly, this is also the case in Newtonian mechanics. What changes in special relativity is that *time no longer has a privileged standing.* Already in Newtonian mechanics, the spatial distance between two *events* separated by time is not absolute; it depends on the state of motion of the Galilean observer. In special relativity, time joins space: time intervals between two distant events also depend on the state of motion of the observer. Minkowski realized and emphasized the profound implication of this change: in special relativity, only the *4-dimensional* space-time continuum and its geometry are observer-independent.[2]

This new paradigm immediately led to some dramatic predictions. Energy and mass lost their identity and could be transformed into one another, subject to the famous equation $E = Mc^2$. Since the velocity of light $c$ is so large in conventional units, the energy contained in a gram of matter can therefore illuminate a town for a year. It predicted that a twin who leaves her brother behind on earth and goes on a trip in a spaceship traveling at a speed near the speed of light for a year would return to find that her brother had aged

[2] In special relativity, space-time is still represented by the 4-dimensional affine space $\mathbb{R}^4$. However, there is no longer a preferred foliation of this continuum into space and time. Each inertial observer introduces her own foliation and can speak of space and time intervals between any two events. But as we change the observer, the values of these intervals change, leaving only the space-time interval invariant. In the Newtonian space-time of footnote 1, there is a preferred spatial slice through each event, labeled by the value of absolute time. In special relativity, through each event one has instead a light cone, trajectories of light rays emanating (and converging) at that event. In the limit $c \to \infty$, these cones become the spatial slices of absolute Newtonian time.

several decades. The origin of these astonishing consequences lies in the replacement of Newton's absolute time by the special relativistic *observer dependent* time. But they are so counter-intuitive that, as late as the 1930s, there were debates in prominent western universities whether special relativity could be philosophically viable. But we know that these misgivings were all completely misplaced. Nuclear reactors function on earth and stars shine in the heavens, converting mass into energy, obeying $E = mc^2$. In high energy laboratories, particles routinely reach velocities close to that of light and are known to live orders of magnitude longer than their twins at rest on earth.

More generally, we now routinely encounter billions of applications of special relativity both in frontier science and in gadgets used in everyday life. They provide convincing evidence that the absolute time of Newton's does *not* correspond to physical reality. It is an approximate concept that is very useful in situations in which all speeds involved are low compared to that of light (as is the case in celestial mechanics). Special relativity led us to a more sophisticated notion in which the absolute distinction between space and time intervals is lost. Only the *space-time* intervals between two events have an absolute meaning.

## III. GENERAL RELATIVITY: ELASTICITY OF TIME

In 1907, while writing a review article on special relativity, Einstein realized that while his 1905 space-time model successfully reconciled the predictions of Maxwell's electrodynamics, it was in deep conflict with Newton's theory of gravity. For Newton's law of universal gravitational attraction required absolute time: the gravitational force between two bodies is inversely proportional to the square of the distance between them, measured at an instant of this absolute time. As the earth moves around the sun, the Newtonian gravitational force it exerts on the moon changes instantaneously. Special relativity, on the other hand, had abolished absolute time. So Newton's law cannot even be stated in Einstein's 1905 framework. In 1907, then, Einstein set himself the task of finding a new, better theory of gravity. After eight years of concentrated effort, he finally completed it in November 1915. The notions of space and time changed again, and even more dramatically than ever before.

In general relativity, space and time continue to form a 4-dimensional continuum. But now the geometry of this continuum is *curved* and the amount of curvature in a region encodes the strength of gravity there. Space-time is not an inert stage on which things happen. Rather, space-time geometry acts on matter and can be acted upon. There are no longer any spectators in the cosmic dance. The stage itself joins the troupe of actors. This is a profound paradigm shift.

Einstein was motivated in this work by two seemingly simple observations. First, as Galileo demonstrated through his famous experiments at the leaning tower of Pisa, the effect of gravity is universal: all bodies fall the same way if the only force on them is gravitational. Second, gravity is *always* attractive. This is in striking contrast with, say, the electric force where unlike charges attract while like charges repel. As a result, while one can easily create regions in which the electric field vanishes, one can not build gravity shields. Thus, gravity is omnipresent and non-discriminating; it is everywhere and acts on everything the same way. These two facts make gravity unlike any other fundamental force and suggest that gravity is a manifestation of something deeper and universal. Since space-time is also omnipresent and the same for all physical systems, Einstein was led to regard gravity not as a force but a manifestation of space-time geometry. Space-time of special relativity is rigid, flat, and fixed once and for all. Space-time of general relativity is supple and can be 'bent' by massive bodies. The sun for example, being heavy, bends space-time significantly. Planets like earth move in this curved geometry. In a precise mathematical sense, they follow the simplest trajectories called geodesics —generalizations of straight lines of the flat geometry of Euclid to the curved geometry of Riemann. So, when viewed from the *curved space-time* perspective, earth takes the straightest possible path. But since space-time itself is curved, the trajectory appears elliptical from the *flat space and absolute time* perspective of Euclid and Newton.[3]

As a consequence of this paradigm shift, time in general relativity is less rigid, more 'elastic', than even in special relativity. In special relativity each inertial observer assigns a well-defined time interval between any two space-time events. In general relativity, because the space-time geometry is curved and therefore inhomogeneous, we no longer have the three-parameter family of global inertial observers. What about the absolute *space-time* intervals between distant events of special relativity? In general relativity they are no longer absolute but depend on the *path joining them.* These contrasts can be summarized (in a semi-heuristic fashion) as follows. Newton postulated that time flows in an absolute manner, indifferent to the observers carrying out measurements or to the physical content of the universe. Special relativity taught us that time intervals between distant events depend on the state of motion of the observers who measure them. General relativity taught us that they depend, in addition, on the location of the observer. Gravity curves geometry, making clocks tick at different rates at different locations: All accurate clocks tick a little faster at the summit of Mont Blanc than they do in Paris. And these are *real, physical effects.* The Global Positioning Satellites (GPS) system that we all use, for example, has to take into account the special relativistic effects because the clocks on various satellites are moving relative to us, on earth, and relative to each other. They have to take into account the general relativistic effect because space-time is bent a little less at the location of the satellite than it is on earth. (For details, see, e.g., [3].) If these effects were ignored, the GPS system would fail in its navigational functions within about 2 minutes!

To summarize, with each generalization the notion of time has become less rigid, less absolute, more elastic, more fluid. Time continues to retain its physical reality that Newton first attributed to it. However, contrary to the mental image that most people have, physical time does not march steadily, uniformly indifferent to everything else.

General relativity also has a profound implication for the global nature of time. In Newtonian mechanics and special relativity, time runs from eternal past to the eternal future. In general relativity, space-time geometry is dynamical. And the 'singularity theorems' of general relativity inform us that so long as matter satisfies local positive energy conditions, the evolving geometry would develop singularities in a finite amount of time in generic cosmolog-

[3] In general relativity, space-time is again represented by a 4-dimensional continuum but it need not be topologically $\mathbb{R}^4$. In cosmology, the actual topology of the entire universe is directly correlated to the total matter density. Because the metric is allowed to be curved, subject only to Einstein's equations, entirely new possibilities can open up and some of them are known to be realized in Nature: the universe can be expanding (or contracting); black holes can form through a dynamical collapse of matter under gravitational attraction; and ripples in the geometry of space-time can propagate as gravitational waves, carrying physical attributes such as energy, momentum and angular momentum! Since the focus of this article is on time, for simplicity I have restricted myself to globally hyperbolic space-times so that the initial value problem of dynamics is well posed.

ical situations (see, e.g., [4]). Of particular interest is the truncation of general relativity to spatially homogeneous and isotropic situations with flat spatial metric; at large cosmological scales, it describes the observed universe to an excellent degree of approximation. Because of the assumed symmetries, irrespective of the dynamical equations, the space-time metric is captured entirely by a single function $a(t)$ of time, called the scale factor:

$$ds^2 = -dt^2 + a^2 d\vec{x}^2 \tag{3.1}$$

where $t$ is called the proper or cosmic time and $\vec{x}$ the co-moving spatial coordinates. Now, the dynamics provided by Einstein's equations imply that there must come a time $t$ at which the scale factor vanishes and space-time curvature, which goes as $a^{-n}(t)$, diverges. (The positive number $n$ depends on the matter content, i.e., the right hand side of Einstein's equations.) According to general relativity, this event occurs at a finite value of $t$ and represents the end of space-time; all physics comes to an abrupt halt. If the event lies in the past, it is called the *Big Bang* singularity and if it lies in the future, the *Big Crunch* singularity. Of special interest to this discussion is the Big Bang singularity. It is not a beginning in the sense that time was just ticking along and matter suddenly burst into existence with a bang at a certain event. Rather, space-time geometry itself was born at the Big Bang together with matter. And, since it is this geometry that enables us to speak of time intervals, *within general relativity* it is simply meaningless to ask what was there before the Big Bang. Thus, thanks to the dynamical nature of space-time geometry, the global nature of time has also undergone a profound transformation in the passage from special to general relativity.

## IV. BEYOND GENERAL RELATIVITY?

This section is divided into two parts. In the first, I will discuss certain limitations of general relativity and the necessity of including quantum physics in the description of space-time geometry near the putative Big Bang. In the second, I will indicate why and how the notion of time is bound to undergo another major revision in the resulting theory of quantum gravity.

### A. The Issue of the Beginning

According to general relativity, then, the observed large scale homogeneity and isotropy of the universe implies that our universe had a well-defined, finite beginning. However, it is commonly accepted that general relativity is incomplete because it ignores quantum physics. In solutions to Einstein's equations, if we run the movie of the history of the universe backwards in time and approach the Big Bang, we would find that the density of matter increases continuously and the universe enters the Planck regime in which the characteristic scales are set by the *fundamental* constants of Nature, $G, \hbar$ and $c$. To arrive at the Big Bang we have to keep going backwards in time, further increasing matter density and curvature until they become infinite. Now, already in the Planck regime the matter density is approximately $10^{92}$gm/cc, which is some eighty(!) orders of magnitude larger than the nuclear density encountered, for example, in neutron stars. Even at the nuclear density quantum mechanics is absolutely vital e.g., for the very existence of the neutron stars. Therefore, it is completely inappropriate to ignore the quantum properties of matter in the Planck regime. Quantum matter is described by quantum fields. Since matter appears on the right hand side of Einstein's equations, the description of the left hand side representing geometry should also incorporate the quantum paradigm. So, the use of classical Einstein's equations in (and beyond) the Planck regime is an approximation that has no justification at all. Consequently, *singularities such as the Big Bang are predictions of general relativity in a domain in which it is simply invalid.*[4] In spite of the standard rhetoric, it is not at all clear that our universe really originated in the Big Bang or had any singularity in the past.

It is sometimes claimed that the observed cosmic microwave background (CMB) provides a 'proof' that the universe began with a Big Bang. But in the standard scenario used to make this claim, in the CMB epoch the matter density and curvature were some $10^{-114}$ times Planck scale! Therefore, the current CMB observations are quite inadequate for deciphering what exactly transpired at the Planck scale. Neither does the success of inflationary scenarios provide us insights on this issue because it refers to an era when the matter density and curvature were approximately $10^{-11}$ times the Planck scale. Thus, to use general relativity to conclude that the universe began with a Big Bang would be as grossly incorrect as using Newtonian mechanics to conclude that there is no time dilation for particles in the CERN accelerator, or, using the flat space-time of special relativity to conclude that black holes cannot exist because there are no trapped surfaces in Minkowski space-time.

To find what *really* happened in the Planck era of the very early universe, one needs a unification of general relativity and quantum physics, at least in the setting of cosmology. Attempts at constructing quantum cosmology theories date back at least to the early 1970s, when Misner, DeWitt and Wheeler introduced influential ideas to probe the effects of quantum physics in the very early universe [6, 7]. By now there are detailed calculations, particularly in loop quantum cosmology (LQC) [8, 9], that strongly suggest that the Big Bang singularity *is* resolved once general relativity is replaced by a more fundamental theory based on *quantum* Riemannian geometry.[5] In LQC, all physical observables —including those that diverge at the Big Bang in general relativity— remain manifestly finite throughout evolution. Quantum dynamics of LQC does not break down; quantum physics does not come to an abrupt halt. The conceptual underpinnings of this resolution have been analyzed in detail using the dynamics of physical observables (see, e.g.,[8]) as well as in the 'consistent histories approach' to quantum physics which is well suited to cosmology [10].

What about the issue of the Beginning, then? As has happened repeatedly in the past —e.g. when Newtonian mechanics was replaced by special relativity, or the flat, inert space time of special relativity was replaced by the curved, dynamical space-time of general relativity— the new paradigm brings out the necessity of sharpening the questions since answers can vary greatly depending on which precise version of the question is chosen. As for the issue of the Beginning, we have at least two possibilities. One can inquire if, as we evolve backwards in time, the equations break down and the evolution can no longer be

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[4] Einstein was well aware of this. On the issue of whether the prediction of the Big Bang singularity of general relativity is trustworthy, he wrote [5]: "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense."

[5] Singularity resolution is a sharp and definitive result within LQC. However, LQC refers only to that sector of full quantum gravity in which one restricts oneself, from the start, to symmetry reductions of general relativity normally used in cosmology. Therefore, from the viewpoint of full Loop Quantum gravity (LQG), the LQC results can only be taken as indications of what one would expect in complete LQG.

continued. If this occurs, one can say that space-time and matter originated at that event; this is the Beginning. In general relativity, of course, this event is the Big Bang. In LQC, on the other hand, as I have explained, the *quantum* evolution never breaks down; the quantum state never encounters infinities; none of the physical observables ever become singular [8]. In this sense, then, time does *not* have a finite beginning in LQC. But, alternatively, one could also ask whether there is an epoch in which the quantum fluctuations in geometry cannot be ignored.[6] If there is, then the space-time continuum obeying Einstein's equations would become a trustable approximation only at the end of that epoch. Then one would say that the notion of time à la general relativity 'emerged' at the end of that epoch; in this sense, time has a finite beginning. In LQC there is such an epoch. Therefore, in this second sense, time *did* have a finite beginning in LQC.

### B. Relational Time and Quantum Gravity

So far, in this section I have focused only on the global question of whether time is likely to have a finite beginning in quantum gravity theories that go beyond general relativity. What about the more local issue of time and space-time intervals, discussed in the last three sections? In quantum gravity theories, the smooth space-times of general relativity would not be fundamental. The fundamental objects of the theory —'degrees of freedom' in the physics literature— may be quite different from the space-time metric of general relativity. In loop quantum gravity (LQG), for example, the fundamental excitations of gravity/geometry are polymer-like with one space and one time dimension, encoded in 'spin networks' and 'spin foams' [12–15]. Consequently, at the Planck scale, there is discreteness. This is very similar to what one encounters in, say, atomic physics. For the hydrogen atom, for example, the angular momentum and energy of the electron can assume continuous values in the classical theory but the eigenvalues of the corresponding quantum operators are quantized. Similarly, while geometrical observables such as the area of a physically defined surface and volume of a physically defined region assume continuous values in general relativity, in LQG the corresponding quantum operators have discrete eigenvalues.

Thus, in LQG, geometry has an 'atomic structure' with fundamental building blocks. The space-time continuum emerges only in the classical limit (perhaps as a condensate [16] of these building blocks) if one coarse grains, i.e., ignores physics below, say, $10^{-17}$cm, the length scale used in particle physics. Therefore the space-time intervals between two events along specified curves, used in general relativity, are also approximate concepts. But in general relativity, we need the notion of such intervals to speak of dynamics and make predictions. Without access to these notions, how would one discuss dynamics in quantum gravity? This is a difficult question because, to face it adequately, one must treat the clocks themselves using quantum mechanics [17]. The 'issue of time in quantum gravity' is still very far from being fully understood and therefore a subject of active discussion (for a summary, see, e.g., [18]).

I will now summarize a strategy that has so far proved to be most successful. Interestingly, it calls on us to return to the notion of relational time I began with. The primary emphasis is on *interdependence*, advocated by Ernst Mach. The idea is to treat one of the dynamical

[6] In loop quantum cosmology, there is such an epoch in which there are large deviations from general relativity leading to potentially observable effects [11].

variables —e.g., a scalar field $\phi$, or the total matter density $\rho$, or the anisotropy parameters $\beta_i$, or a suitable curvature scalar $R$, or, ...— as the 'clock variable' with respect to which all other physical observables change. Thus, the questions one poses are of the following type. Suppose that the quantum state $\Psi(R, \beta_i, \rho, \ldots, \phi)$ is such that, when the scalar field $\phi$ (treated as the clock variable) equals $\phi_1$, the expectation value and quantum uncertainties of observables $R, \beta_i, \rho \ldots$ of interest have such and such values. What would their values and uncertainties be when the scalar field $\phi$ equals $\phi_2$? This change is interpreted as the 'evolution' of chosen observables, $R, \beta_i, \rho, \ldots$, with respect to the chosen relational time variable $\phi$. For this qualitative idea to work in quantitative detail, the clock variable has to be chosen astutely (see, e.g. [17]). In simple cosmological models, there are natural choices, the strategy has been implemented in detail, and, as one would hope, the results reduce to those from general relativity once one is well outside the Planck regime (see, e.g., [8]). However, at a fundamental level one now faces the following obvious issue: How do you compare predictions from two distinct choices of the clock variables in the full quantum gravity regime?

To probe this question, it is useful to begin by discussing the issue of time within general relativity in greater detail than I have done so far. Let us, for a moment, return to special relativity. Recall that, although we do not have an absolute time in this framework, we can associate a notion of time with each inertial observer. This corresponds to a 3 parameter freedom in the choice of time. In practice, one typically fixes an inertial observer, describes dynamics using the corresponding time variable, and at the end verifies that the physical results are insensitive to the initial choice of the inertial observer. In general relativity we do not have global inertial observers. Therefore now the ambiguity becomes *infinite* dimensional. To discuss dynamics, one first slices space-time into space and time using *any* family of space-like (Cauchy) surfaces. Each of these slices now represents a possible 'instant of time' and time evolution now refers to the change of physical fields with respect to the time coordinate that labels these slices. Thus, given initial (or, Cauchy) data for, say, the electromagnetic field at an initial time, one can evolve them using Maxwell's equations on the given curved space-time and obtain a solution on the entire space-time. This procedure makes heavy use of the underlying space-time metric. But the setup guarantees that the final solution is independent of the choice of slicing in the following precise sense. Suppose we have two slicings, leading to two time variables $t$ and $t'$. For simplicity let us suppose that they agree for, say $t < t_1$ and $t > t_2$ for some $t_1, t_2$ (with $t_2 > t_1$), but differ over the intermediate times between $t_1$ and $t_2$. Then, the two evolutions carried out using $t$ and $t'$ from any given Cauchy data on any slice $t = \text{const} < t_1$ will lead to the *same* Cauchy data on any slice with $t = \text{const} > t_2$. Furthermore, even in the intermediate region where the two slices are distinct, if we were to evaluate the Cauchy data of the solution obtained using the $t$ evolution on any $t' = \text{const}$ slice, we would find a complete agreement with the Cauchy data on that slice provided by the $t'$ evolution. Thus, although the notion of time evolution in general relativity is nowhere as unique or canonical as that in Newtonian mechanics or special relativity —time in general relativity is *'multi-fingered'*— the coherence we just discussed ensures that all permissible evolutions are physically equivalent. In the language of the barter system I began this article with, the coherence guarantees that if I started out with $n$ sheep and traded them for chickens and then traded chickens for oil and then traded oil back for sheep, (in an ideal world without profits) I would again obtain the same number $n$ of sheep I started with. There is a global consistency to the scheme. In this commercial example, it is this coherence that enables one to introduce an abstract notion of money and supplant the barter system by assigning a well-defined monetary value to each of our goods. Returning to general relativity, the coherence lets one attribute a physical reality to the notion of time and evolution although, being 'multi-fingered', the notion is now much more sophisticated than that in special relativity.

With relational observables in quantum gravity, the situation is not so simple. First, we no longer have a space-time continuum that can be foliated by space-like slices because there are regimes in which there would be no adequate coarse graining leading to a smooth space-time metric. Second, each choice of a clock variable naturally leads one to focus on a subset of observables whose evolution can be studied in a natural manner (e.g., without having to make ad-hoc factor ordering choices for operators), and the subset can vary from one choice to another. A related but distinct problem is that each procedure endows the space of solutions to the quantum Einstein equations with a specific and distinct Hilbert space structure with which one computes expectation values and fluctuations of observables. As a result, in general it is quite difficult to compare dynamical predictions in quantum theories arising from two different choices. In the simplest cases where this is feasible, one does find consistency [19]. However, the general problem is open. As a result, it is not clear whether, when the dust settles, there will be the appropriate 'coherence' between different choices to say that there is an underlying, objective (but vastly generalized) notion of time that the quantum dynamics refer to. It could well happen that, for each (admissible) choice of relational time, we have well-defined and internally consistent dynamics. But rather than fusing together to give us a single, globally coherent paradigm, these theories could remain distinct. Furthermore, it may be that each of them passes its own observational tests. This could happen because the observables measured to test one of these theories fail to be well-defined in another! In that case, time would be relational *in its core*, i.e., at the most fundamental level, and a coherent notion of time and evolution would arise only as an approximation, i.e., in regimes in which general relativity is adequate to describe reality to the desired degree of accuracy. In that case, in the fundamental description the primary focus would be on correlations between physical quantities. Leibniz's 'relational' notions and Mach's 'interdependence' would then constitute the very essence of fundamental physics. An adequate discussion of the nature of time in quantum gravity awaits the resolution the question: Does there there is a canonical class of 'preferred' relational times for which the quantum evolution has the required coherence/covaraince?

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overlap with another of my articles [20].[7]


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