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Time perception and the filled-duration illusion*

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A reproduction design is used to show that temporal intervals containing brief tones appear longer than empty intervals of the same duration, the effect being independent of duration. These and previous data are discussed within a theoretical framework which allows for the interrelation of data from different time perception tasks; and a reversible encoding model is stated which accounts for much of the data obtained with empty intervals. A "chunking" model, in which tones occurring in an interval serve to segment the interval during encoding, can account for the filled-duration illusion if certain conditions are met. Finally, mechanisms that are consistent with these conditions are stated.

1. INTRODUCTION

The role of time in the perception of presented information seems to be much better understood, through, for example, detection and masking studies, than is the role of presented information in the perception of time. The interdependence of these two issues can be seen by considering a task in which S hears a periodic sequence of n clicks over a period of t sec. We can ask S to give an estimate, $\hat{n}$, of n and look at the dependence of $\hat{n}$ on t, n constant (e.g., White, 1963), or we can ask S to give an estimate, $\hat{t}$, of t and look at the dependence of $\hat{t}$ on n, t constant (e.g., Buffardi, 1971). Typically, both functions, $\hat{n}(t)$ and $\hat{t}(n)$, are increasing, though little is known about their interdependence. As a step in this direction, the present study will consider the latter issue, viz, the role of presented information in the perception of time, by providing more data and by attempting to place these and similar results in a general framework.

The result that $\hat{t}$ increases with n, t constant, that is, that a "filled" duration is perceived to be longer than an identical duration that is "empty," is referred to as the filled-duration illusion. Methods used to study the illusion include: (a) estimation or recognition—a single interval, filled or empty, is presented on each trial, and S estimates its length, orally or in writing; (b) paired-comparison—a filled interval and an empty interval of equal length are presented on each trial, and S has to say which is longer; (c) reproduction—an interval (filled or empty) is presented, and S has to produce an empty interval of the same length; and (d) production—a temporal interval is given, orally or in writing, to S who has to produce a filled interval of the stated length. The last two methods are similar since both require S to *produce* an interval, but they can be expected to yield different relationships between produced and presented intervals because, when the presented interval is filled in (c), S produces an empty interval to match a filled duration, whereas in (d) the opposite is true (if we assume that an orally presented interval is equivalent to an empty interval). Therefore, evidence of the filled-duration illusion consists of overestimation (produced greater than presented interval) in (c) and underestimation in (d). Differences between the first two and the last two methods will be discussed later.

Evidence of the illusion comes from studies using different types of "filler" material. Ornstein (1969) and Buffardi (1971) used intervals filled with discrete events, e.g., clicks, which intervals we shall refer to as *discrete* stimuli. Buffardi, using the paired-comparison method, showed that the illusion is independent of modality, and replicated findings that the illusion is stronger if the discrete events occur near the beginning than if they occur near the end of the interval (cf. Israeli, 1930), and if the events are regularly rather than irregularly spaced (cf. Grimm, 1934). Other studies have used continuous stimulation to define the temporal intervals (e.g., Oléron, 1952; Treisman, 1963; Craig, 1973), and still others have had Ss attend to more "complex" stimuli, such as line drawings (Ornstein, 1969), choice reaction stimuli (Michon, 1965), and mathematical tasks (Burnside, 1971).

In this paper, we will concentrate on the temporal judgments of discrete stimuli, because it seems reasonable to suppose that "amount of filler" is directly related to number of discrete events and, possibly, to the distribution of these events within the interval. With more complex stimuli, the specification of amount of filler can become arbitrary, as is seen

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when one tries to prove that solving multiplication problems is more "filling" of time than reading. We will first provide some more data on the illusion, using the method of reproduction, since this allows us to compare produced intervals with clock time. It should be noted that the forced-choice procedure of the method of paired-comparisons may, in fact, force S to base his judgment solely on nontemporal aspects of the stimulus on those trials when the perceived durations are equal. This possibility does not exist with the reproduction method, though one now has to consider response processes, such as muscular coordination, and their effects on produced intervals (see, e.g., Guilford, 1954, for a fuller comparison of the two methods).

In our experiment, the length of presented intervals (t) varies within a block of trials, with a range of 1 sec, and between Ss with a range of about 4 sec. This design allows us to test the expectation, derived by appealing to Weber's law, that the difference between a reproduction of 1.5 sec and a reproduction of 0.5 sec should, on the average, be more than the difference between reproductions of 5.5 and 4.5 sec. Further, we shall be able to see if the size of the illusion depends on time when the latter is varied "locally" (within blocks) and "globally" (between Ss).

After presenting the new data, we will present a theoretical framework which allows us to interrelate data from different time-perception tasks, and also to account for some of the previous data on the estimation of empty and filled intervals. Some impetus to interrelate data from different tasks comes from the desire to define the notion of *accuracy*. One can say that an accurate S is one who makes no errors in a recognition task; or, as Craig (1973) has suggested, one who, in a reproduction task, adjusts his average produced interval by exactly the same amount as the presented interval is changed. Additionally, one can relate accuracy to the absolute difference between produced and presented intervals, or to the variance of the produced intervals. We shall show that some comparisons can be made among these different measures of accuracy.

2. METHOD

Subjects

Twenty paid volunteers between the ages of 18 and 22 years were recruited from the Stanford University premises to comprise two groups of Ss. Half the Ss reproduced temporal intervals that varied around a mean of 1.25 sec, and the other half reproduced intervals that varied around a mean of 5.1 sec. These two sets of stimuli are hereafter referred to as 1- and 5-sec stimuli. All Ss participated individually.

Apparatus

Stimuli consisted of temporal intervals delimited by 10-msec (900-Hz) tones which were prerecorded on magnetic tape. Four interval lengths were used for each set of intervals, and these were measured with the aid of two Iconix timers (Models 6010 and 6255): 750, 1,000, 1,500, and 1,750 msec for the 1-sec set, and 4,500, 5,000, 5,300, and 5,500 msec for the 5-sec set. There were three stimulus types. *Filled-regular* intervals had three 10-msec tones (480 Hz) equally spaced between the begin and end markers. *Filled-irregular* intervals differed from filled-regular intervals in that the three intervening tones were unequally (randomly) spaced. *Empty intervals* were composed of only the begin and end tones. A two-channeled, 10-msec tone (1,000 Hz), which followed the end tone by 1 sec, served as the signal for S to begin the reproduction interval. Output through one of these channels triggered an electronic timer, which stopped when S pressed a button held in his/her hand. The delay between the 1,000-Hz "start-reproduction" tone and the begin tone of the next stimulus interval was 3 and 7 sec for the 1- and 5-sec intervals, respectively. The four interval lengths and three stimulus types yielded 12 different stimuli in each of the two sets of stimuli.

The experiment was conducted with S seated in an 8 x 8 ft soundproof chamber with a 2 x 2 ft window. E controlled the tape recorder and timing apparatus from outside the soundproof chamber in the surrounding experimental room.

Procedure

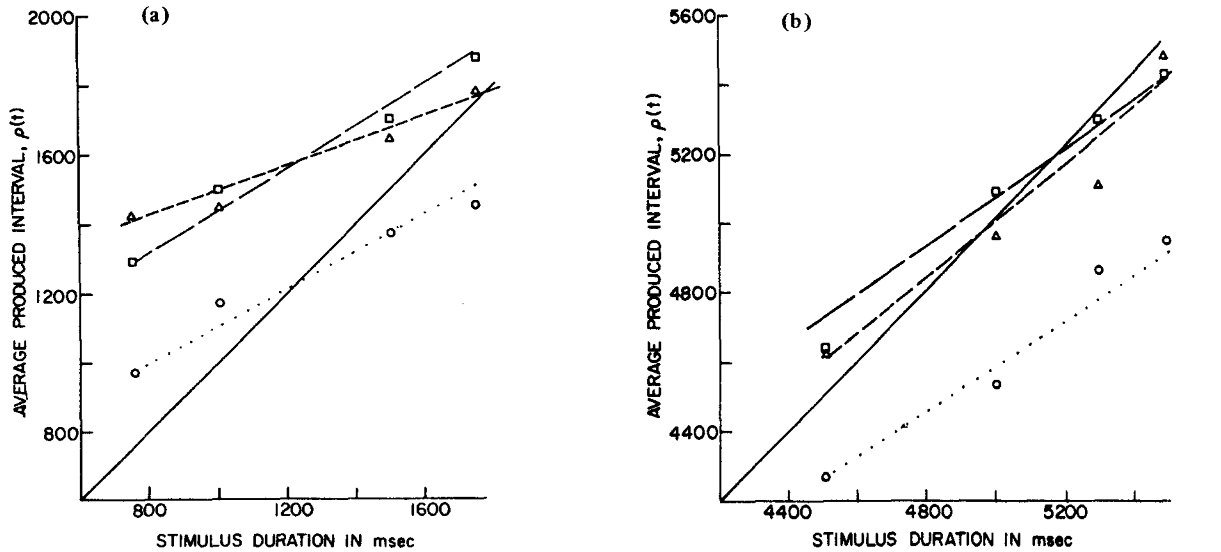
After S arrived at the experimental room, E explained that S would be participating in an experiment on time perception, and that the task would involve the reproduction of various temporal intervals. E explained the design of the stimuli and the apparatus to S and, in pointing out the three types of stimulus intervals, emphasized that in all cases the task would be the same. S was instructed to press a button when an amount of time had elapsed since the reproduction tone that was perceived as equivalent to the previous stimulus interval. E introduced S to a second aspect of the task, which involved monitoring the type of interval being heard—whether it was Type 1 (empty), Type 2 (filled-regular), or Type 3 (filled-irregular). S was told that he/she would be called upon at "several" points during the experiment to state what type of interval (empty, filled-regular, or filled-irregular) the immediately preceding interval was by responding 1, 2, or 3, respectively. This procedure was introduced to ensure that S attended to the stimulus.

Following explanation of the experimental procedure, S was led into the sound chamber and seated. First, S was instructed to listen to the stimuli until the various types of intervals and the various types of tones could be distinguished. This was followed by 50 practice trials to stabilize S's estimates and to further familiarize S with the apparatus.

During the experiment, S was given 10 presentations of each of the 12 different stimuli (type and length). The order of stimuli on the tape was quasi-random and each S received them in the same order. S was called upon eight times (randomly chosen) during the experiment to identify the type of stimulus reproduced.

3. RESULTS

Within each of the two sets of stimulus intervals (1- and 5-sec), each S gave 10 reproductions of each stimulus length by stimulus type combination. The means and standard deviations of these 10 observations were computed, and an analysis of variance was performed on the means, separately for 1- and 5-sec stimulus intervals.[1] Significant main effects were obtained for stimulus type and stimulus length for 1-sec intervals [$F(2,16) = 34.0$, $p < .001$, and $F(3,24) = 51.7$, $p < .001$, respectively] and for 5-sec intervals [$F(2,18) = 15.5$, $p < .001$, and $F(3,27 = 37.8$, $p < .001$, respectively]. In the analysis of the 1-sec intervals, a significant interaction was found between stimulus type and stimulus length [$F(6,48) =$



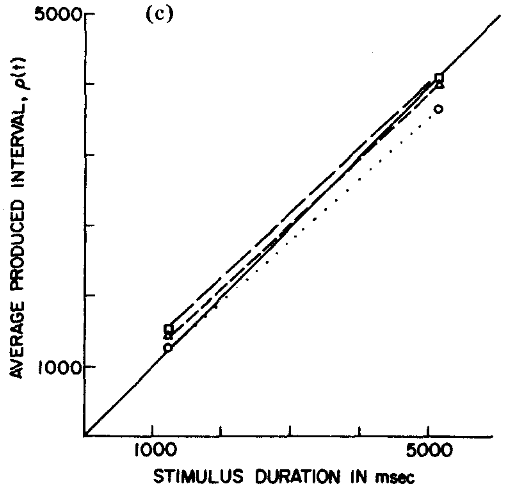


Fig. 1. Plot of mean produced interval, ϱ(t), against presented interval: (a) short intervals c.1 sec, (b) long intervals c.5 sec, (c) average short and average long intervals. ○ ..... ○ Empty intervals. □ ——— □ Filled regular intervals. △ ----- △ Filled irregular intervals.

2.3, $p \cong .05$]; however, no significant interaction was found in the analysis of the 5-sec intervals [$F(6,54) < 1$].

Although the means for regular intervals were larger than those for irregular intervals, this difference was not statistically significant for either the 1- or 5-sec intervals. Orthogonal contrasts showed filled intervals to be significantly greater than empty intervals for both 1- and 5-sec intervals ($p < .001$). These findings are thus consistent with previous studies employing different methods (e.g., Buffardi, 1971). Finally, when the sums of squares due to stimulus length were partitioned into linear, quadratic, and cubic components, only the linear components were statistically significant.

In Fig. 1, the group mean produced interval, ϱ(t), is plotted against stimulus duration for 1-sec intervals (Fig. 1a) and 5-sec intervals (Fig. 1b). It can be seen in Fig. 1a that the significant interaction between stimulus length and stimulus type is accounted for by the reversal of regular and irregular intervals at 750 msec. The straight lines shown in Figs. 1a and 1b were derived by the least-squares method, and their slopes and intercepts are shown in Table 1. It can be seen that the slopes are less than 1. However, as can be seen in Fig. 1c, the slopes of the lines are much closer to 1 when the data are pooled within the 1- and 5-sec stimulus intervals, in order to show the differences between the two groups of Ss. Figures 2a and 2b show the group average standard deviation of

Table 1
Intercepts (a) and Slopes (b) of Best Fitting Straight Lines of Average Produced Interval [ρ(t)] as a Function of Stimulus Duration

Intervals	1-Sec Intervals a	1-Sec Intervals b	5-Sec Intervals a	5-Sec Intervals b
Empty	618	.50	1188	.68
Filled-Regular	900	.55	1572	.70
Filled-Irregular	1125	.37	1112	.78

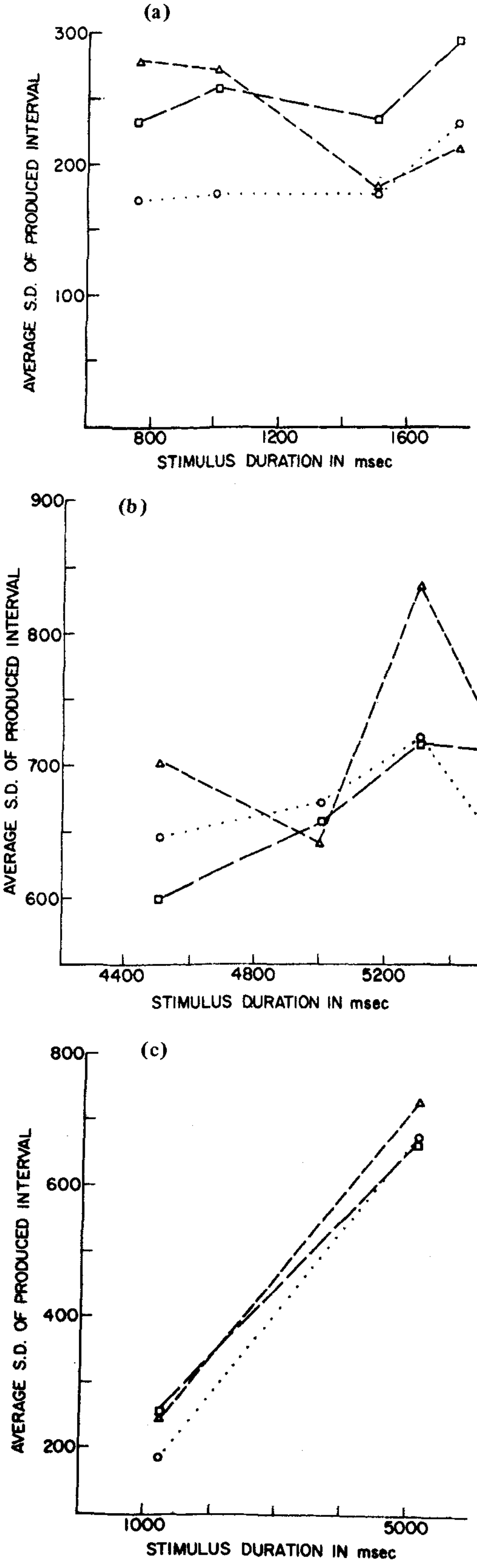


Fig. 2

produced intervals for each stimulus length and stimulus type. The standard deviations for the 1-sec intervals appear not to be related to the length of the interval but are generally greater for filled than for empty intervals. No statistical tests were performed on these differences. The standard deviations for the 5-sec intervals also show little dependence on stimulus length, and there are no clear differences among the stimulus types. When the data are pooled within the 1- and the 5-sec sets, standard deviation does depend on stimulus length, as shown in Fig. 2c.

4. SUMMARY OF THEORETICAL METHOD AND RESULTS

Most theorization about time perception contains the notion that the apprehension of time consists of the counting of events or "pulses" which occur at some rate, which is either constant (Treisman, 1963) or Poissonian (Creelman, 1962; Kinchla, 1972). In studies where S is told that he would have to estimate the length of a filled interval only after he had experienced the interval, the assumption has been that S's temporal estimate depends on the number and the "size" of the units of stored information (Ornstein, 1969; Burnside, 1971). The belief, that counting is a more primitive subjective process than estimating time qua time, is a reasonable one and accounts for many features of the data on time estimation, e.g., that variability of estimates increases with mean estimate (Treisman, 1963; Kinchla, 1972). However, it is not necessary to have a "counting" explanation of time estimation, and we shall leave this issue open while we try to specify conditions which every explanation has to satisfy.

The following is a summary of the points that will be established in later sections.

(i) The reproduction and production of a presented interval can be viewed as a two-stage process. In the first stage, the presented interval is encoded, and, in the second stage, this encoding is decoded. The outcome of the encoding stage is such that it is the basis for temporal judgments in estimation, recognition, and paired-comparison tasks.

(ii) By assuming that the encoding of an interval has a particular form and that decoding is the inverse of encoding, it can be shown that accuracy (d') in a recognition task is directly related to the absolute difference between produced and presented intervals, obtained from a reproduction task. Also, d' is inversely related to the variance of produced intervals, and unrelated to the slope of the curve, produced vs presented interval.

(iii) Overestimation of short intervals and underestimation of long intervals can be accounted for by appropriate choice of the encoding function.

In trying to account for the filled duration illusion, we will argue as follows.

(iv) The occurrence of clicks in a (filled) interval

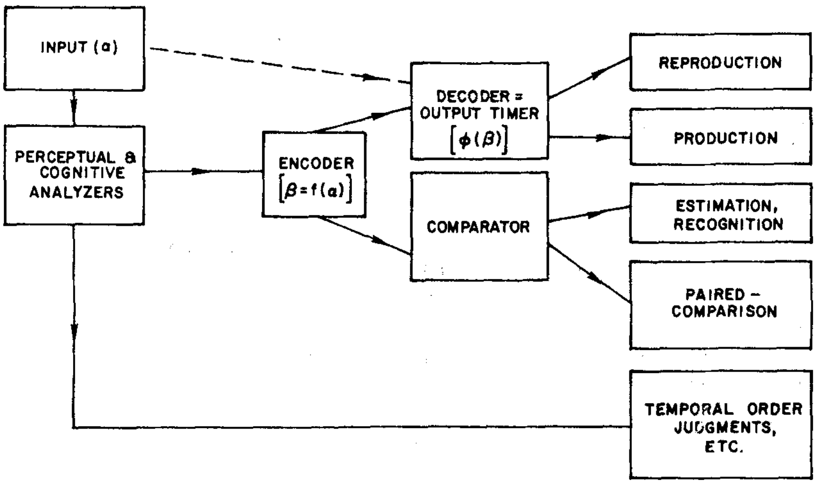


Fig. 3. Schema showing the components functional in time-estimation tasks.

causes the interval to be encoded in "chunks" or subintervals. The subintervals are then decoded serially, and the produced interval is the sum of the decodings. The illusion can then be accounted for by positing certain functional relationships between the length of a subinterval and the length of its decoding.

(v) A variety of models can be stated which yield the functional relations referred to in (iv). One class is of models in which the decoding function is assumed to be the inverse of the encoding function, as in (ii) and (iii). Another class is of models in which the encoding function is a *count* of "pulses," and the decoding function is linear. A third class is of "attention" models in which encoding is an intermittent process, and the decoding function is linear.

(vi) Some comparisons can be made between some of these theoretical results and the present experimental data. The resulting conclusions are tentative, because these data represent group averages while the theory is stated in terms of the individual S.

5. INTERRELATIONSHIPS AMONG TASKS

5.1 A General Model

Earlier it was pointed out that only the reproduction and production tasks require S to produce a temporal interval as a response. This distinction is stated formally in Fig. 3. The input, a, consisting of a duration, t, and an amount of information (number of clicks), n, is first analyzed at levels varying from the perceptual to the cognitive. The output of these analyzers is encoded as a vector, β, one component being the encoded duration, $f_n(t)$, plus an error term, and the other being the encoded information, $g_t(n)$, plus an error term.[2] The dependence of $f_n(t)$ on n is suggested by the filled-duration illusion, and that of $g_t(n)$ on t is the usual assumption that the amount of stimulus information encoded depends on the stimulus duration. This form of encoding has some redundancy in it, since both f and g contain information about t and n. It was chosen to reflect the assumption that S encodes duration as a separate component because he knows that the task requires judgments of duration. In the case where the true nature of the task is not revealed to S until after the duration is experienced, it is reasonable to assume that, on the average, $\beta = h(t,n)$, a scalar; and that when S is asked to estimate the duration, his *response* time would be related to the time to extract duration (t) information from the encoding function h(t,n).

In the reproduction task, the empty reproduced interval is a decoding, Φ, of $f_n(t)$ plus an error term. In the production task, the stimulus duration is given orally or visually so that $n = 0 = g_t(0)$ and the encoded duration is $f_0(t)$ plus an error term. The average produced interval is some decoding, Φ^*, of $f_0(t)$, where Φ^* is influenced by the amount of information presented during the produced interval. In the estimation task, we assume that there already are encodings $f_0(\bar{t})$ in the comparator of Intervals 1, 1.4, 2, . . . , sec, and that S estimates the presented interval t by comparing $f_n(t)$ with the $f_0(\bar{t})$. Similarly, in the recognition task, we assume that there are encodings $f_0(\bar{t})$ in the comparator of the intervals "short," "medium," "long," etc., and that S's categorical judgment is based on comparing $f_n(t)$ with $f_0(\bar{t})$. Finally, in the paired-comparison task, where two inputs, (t,n) and (t,n′) are presented on a trial, the judgment is assumed to be based on comparing $f_n(t)$ and $f_{n'}(t)$. The main assumption in this schema is that reproduction and production tasks involve a decoder, whereas estimation, recognition, and paired-comparison tasks involve a comparator. These assumptions are in too general a form to allow them to be tested. However, when the functions f(·) and $\Phi(\cdot)$ are specified, the model does make testable predictions. We now consider some predictions concerning the relationships between reproduction and recognition tasks.

5.2 Judgments on Empty Intervals

We assume that, when the presented duration is t, the (scalar) encoding β and the reproduced interval r(t) are random variables which can be expressed as, writing f(t) for $f_0(t)$,

$$\beta = f(t) + \varepsilon \quad (1)$$

$$r(t) = \Phi(\beta) + \eta \quad (2)$$

where $f(\cdot)$ and $\Phi(\cdot)$ are strictly increasing, at least twice differentiable functions, and ε and η are independent random variables with mean 0 and standard deviation σ and v, respectively. We assume that σ and v are independent of t.

In a recognition task in which a "short" interval, t, or a "long" interval, $t' = t + \delta$, is presented on each trial, one can assume that S sets a criterion, c, and responds "long" if and only if the encoded duration exceeds c (Kinchla, 1972). Ignoring terms containing δ^2 and writing $f'(t)$ for $d[f(t)]/dt$,

$$E(\beta') = f(t') \cong f(t) + \delta f'(t) = E(\beta) + \delta f'(t),$$

using Eq. 1. Therefore, the d' measure from this experiment is given approximately by

$$d' \cong \frac{\delta}{\sigma} f'(t). \quad (3)$$

That is, d' depends on t only if $f'(t)$ depends on t. The case where $f(t) = \sqrt{t}$ yields

$$d' \cong \frac{\delta}{2\sigma} \frac{1}{\sqrt{t}} \quad (3a)$$

which was also obtained by Kinchla (1972) on the assumption that $f(t) = t = K\sigma^2$.

For the reproduction task, the distribution of r(t) depends on $\Phi(\cdot)$. It seems reasonable to assume that if there were no uncertainty in the encoding and decoding processes, S's estimates would be veridical. That is, we assume that a constant error in time estimation occurs only when there is variability in the subjective processes. More formally, we assume that, when $\sigma = v = 0$,

$$r(t) = \phi[f(t)] = t,$$

$$\text{i.e., } \phi(t) = f^{-1}(t). \quad (4)$$

From Eq. 4 it follows that

$$\phi'[f(t)] = 1/f'(t),$$

and

$$\phi''[f(t)] = -f''(t)/[f'(t)]^3. \quad (5)$$

On expanding Eq. 2 in a Taylor series and ignoring the third and higher moments of ε,

$$r(t) = \phi[f(t) + \epsilon] + \eta$$

$$\cong \phi[f(t)] + \epsilon\phi'[f(t)] + \frac{1}{2}\epsilon^2\phi''[f(t)] + \eta.$$

So that, using Eq. 5,

$$\rho(t) \equiv E[r(t)] \cong t - \frac{\sigma^2}{2}\frac{f''(t)}{[f'(t)]^3}, \quad (6)$$

and

$$\text{var}\,[r(t)] \cong \left[\frac{\sigma}{f'(t)}\right]^2 + v^2. \quad (7)$$

It was noted in Eq. 3a that the choice, $f(t) = \sqrt{t}$, gave an expression for d' similar to that given by the Poisson counting model, When $f(t) = \sqrt{t}$,

$$\rho(t) = E[(\sqrt{t} + \epsilon)^2] = E(t + 2\sqrt{t}\epsilon + \epsilon^2)$$

$$= t + \sigma^2, \quad (6a)$$

and

$$\text{var}\,[r(t)] \cong 4\sigma^2 t + v^2. \quad (7a)$$

Therefore, the variance of reproduced interval increases linearly with t, even though encoding and decoding variability are independent of t. These last two equations are similar, apart from the positive intercepts, to the corresponding equations derived from the Poisson counting model. Equation 6a can be compared with an empirical relation obtained by Treisman (1963; Fig. 1)

$$\varrho(t) = .98t + .13,$$

and with those obtained by Craig (1973), using *filled* intervals.

$$\varrho(t) = t + e,$$

e depending on the modality of the stimulus information. Equation 7a predicts that var [r(t)] is approximately linear with t, the slope of the line being four times the intercept $\varrho(0)$. The data presented by Treisman do not support this prediction, since the slope of graph, var [r(t)] vs t, is *less* than $\varrho(0)$. In the data for empty intervals in the present experiment, var [r(t)] changes little as t varies from 750 to 1,750 msec or from 4,500 to 5,500 msec, but between these two sets of intervals its rate of change is about 110 msec^2 per msec, which is *less* than the intercept (130 msec) in Fig. 1c. Thus these data do not support the prediction.

From Eqs. 3a, 6a, and 7a, it can be seen that d$'$, $\varrho(t)$, and var [r(t)] are functionally related because of their predicted dependence on σ. We can say that recognition accuracy (d$'$) is monotonically related to the absolute difference between produced and presented intervals $[\varrho(t) - t]$, and to the variance of the produced intervals, but is independent of the rate at which S adjusts his average produced interval to changes in presented interval. This illustrates the possibility, within the present framework, of comparing data from different time-perception tasks. The outcomes of these comparisons depend on the choice of f(t), for if we put $f(t) = \log t$, so that $f^{-1}(t) = e^t$, then we would have

$$\rho(t) = E[\exp(\log t + \epsilon)] = E(e^{\epsilon})t$$

$$\cong \left[1 + \frac{1}{2}\sigma^2\right]t,$$

and

$$\text{var}\ [r(t)] \cong \sigma^2 t^2 + v^2.$$

Here d$'$ *is* related to the rate at which S adjusts his average produced interval to changes in presented interval.

Referring to Eq. 6, it can be seen that overestimation tends to occur if $f''(t) < 0$ and not if $f''(t) > 0$. For example, the choice, $f(t) = \sqrt{t} + t^2$, yields the classical pattern of overestimation at short intervals ($t < .25$) and underestimation at long intervals (Woodrow, 1934).

In sum, the model outlined here allows us to interrelate the data from different tasks, and, with the appropriate choice of encoding function, f(t), it can account for many features of time estimation data. We noted that some of the predictions, for example, Eq. 3a, are indistinguishable from the predictions of other models, in this case, models in which σ depends on t. The testing of this and other specific assumptions of the present model seems, therefore, to be a useful exercise for the future.

6. THE FILLED-DURATION ILLUSION

6.1 A "Chunking" Model

Let us consider a reproduction task in which the input (t,n) consists of clicks occurring at times 0, t_1, $t_1 + t_2, \ldots, t_1 + t_2 + \ldots + t_n = t$, and in which the average reproduction interval is denoted by $\varrho(t,n)$. We assume that the occurrence of clicks causes each subinterval, t_i, to be encoded separately as $f(t_i) + \varepsilon_i$, and the reproduced interval is the sum of the reproduced subintervals, $r(t_i)$. That is, we assume that

$$\rho(t,n) = \sum_{i=1}^{n} \rho(t_i), \tag{8}$$

where $\varrho(t_i) \equiv \varrho(t_i,1) = E[r(t_i)]$. We now account for certain aspects of the illusion by making different sets of assumption about $\varrho(t)$. We will show that, if $\varrho(t)$ is linear with a positive intercept, then the size of the illusion (t fixed) increases with n, and is the same for regular and irregular intervals. Next we show that, if $\varrho(t)$ is concave, increasing with $\varrho(0) = 0$, then the size of the illusion increases with n for regular intervals, and is greater for regular than for irregular intervals. Neither of these sets of assumptions accounts for the finding that the illusion is stronger when the clicks occur near the beginning of the interval than when they occur near the end, and we will offer an ad hoc assumption which seems to be sufficient.

Theorem 6.1.1. If $\varrho(t)$ is linear and $\varrho(0) > 0$, then, for fixed n, $\varrho(t,n)$ is linear in t with the same slope as $\varrho(t)$, and, for fixed t, $\varrho(t,n)$ is linear in n with slope $\varrho(0)$.
Proof. If $\varrho(t) = a + bt$, $a > 0$, then, from Eq. 8,

$$\rho(t,n) = \sum_{i=1}^{n} (a + bt_i) = an + bt,$$

from which the result follows.

Q.E.D.

Theorem 6.1.2. If $\varrho(t)$ is a concave increasing function with $\varrho(0) = 0$, then, for filled regular intervals, the size of the illusion increases with n, and, for fixed n, the size of the illusion is greater for regular than for irregular intervals.
Proof. For regular intervals, $t_i = t/n$ and

$$\varrho(t,n) = n\varrho(t/n).$$

It can be seen by drawing the straight line through the points (0,0) and $(t/n, \varrho(t/n))$, that, since $\varrho(0) = 0$, $\varrho'(t) > 0$ and $\varrho''(t) \leqslant 0$, $\varrho(t/n - 1) \leqslant (n/n - 1)\varrho(t/n)$, i.e., $n\varrho(t/n) - (n-1)\varrho(t/n-1) \geqslant 0$, i.e., $\varrho(t,n) - \varrho(t,n-1) \geqslant 0$, as asserted.

The second part is proved by noting that, by a well-known result on convex functions, the expression on the right-hand side of Eq. 8 is maximized when $t_1 = t_2 = \ldots = t_n = t/n$.

Q.E.D.

An additional assumption is needed to account for a third aspect of the illusion, viz, that the illusion is stronger when the clicks occur near the beginning of the interval than near the end. One that seems reasonable is that the end points of chunks do not always coincide with clicks, and the average length of a chunk is positively correlated with t_1. In other words, if the first chunk length is small, e.g., because t_1 is small, succeeding chunk lengths will tend to be small and the number of chunks will be relatively large. This implies that the illusion would be larger when t_1 is small than when it is large.

We can now consider what time-keeping mechanisms yield $\varrho(t)$ having the properties stated in the above theorems or properties suggested by the experimental data.

Example 6.1.3. Reversible Encoding Models. In the model discussed in Section 5.2, the properties of $\varrho(t)$ depend on the choice of the encoding function, f(t). For example, we saw that, when $f(t) = \sqrt{t}$, $\varrho(t)$ was linear with unit slope and positive intercept. To obtain $\varrho(t)$ such that $\varrho(0) = 0$ and $\varrho''(t) \leqslant 0$, we can choose $f(t) = t^{1/3}$. Then, assuming the distribution of ε is symmetric

$$\varrho(t) = E[(t^{1/3} + \varepsilon)^3) = E(t + 3t^{2/3}\varepsilon + 3t^{1/3}\varepsilon^2 + \varepsilon^3)$$
$$= t + 3\sigma^2 t^{1/3}, \tag{9}$$

giving $\varrho(0) = 0$, $\varrho'(t) \geqslant 1$, and $\varrho''(t) \leqslant 0$.

Example 6.1.4. Non-Poissonian Counting Models. We can describe a counting model as one in which an interval, t, is encoded as a number, f(t), which is the number of pulses occurring in t; and in which $\varrho(t)$ is proportional to the average value of f(t). Poissonian counting models lead to $\varrho(t)$ being proportional to t, but if we assume that the time between successive pulses is not exponentially distributed, $\varrho(t)$ has different properties. For example, if we assume that the interpulse times have a two-stage gamma distribution with parameter a/2, then

$$\varrho(t) = c(at + e^{-at} - 1) \tag{10}$$

where c is a constant (Cox, 1962, pp. 57-58). However, for this choice of distribution, we get $\varrho''(t) \geqslant 0$, which is incompatible with a "chunking" explanation of the illusion.

Another choice of interpulse time distribution is the mixed exponential, $(1/2)(\lambda e^{-\lambda t} + \mu e^{-\mu t})$. In Appendix 1, it is shown that this choice gives

$$\varrho(t) = c(At + 1 - e^{-Bt}), \tag{11}$$

where A and B are functions of μ and λ. It can be seen that $\varrho''(t)$ is now negative.

Example 6.1.5. "Attention" Models. In contrast to the previous two examples, we now assume that encoding is an intermittent process. We assume that S alternates between periods of "attention," during which time is encoded without error, and periods of inattention, during which the passage of time is not recorded. Then f(t) is simply the total length of attention periods occurring in the interval. Clearly, $f(t) \leqslant t$, but whether or not $\varrho(t) \leqslant t$ depends on the choice of the decoding function, $\Phi(\cdot)$. To obtain the distribution of f(t) for *empty* intervals, we assume that S is in the attentive state at the start of the interval, and that the lengths of attentive and inattentive periods are exponentially distributed with Parameters a and b, respectively. It is shown in Appendix 2 that

$$E[f(t)] = \frac{at}{a+b} + \frac{b}{(a+b)^2}[1 - e^{-(a+b)t}].$$

So that, if the decoding function is linear, this model gives the same $\varrho(t)$ as in Eq. 11.

For filled intervals, we assume that each click resets S to the attentive state, and it is this assumption which gives chunking. The average reproduced interval is then given by Eq. 8.

6.2 Comparisons Between Theory and Data

The foregoing examples serve to illustrate the variety of approaches that can be taken in order to account for time perception data. Each approach is concerned with processes of the individual S, and some of the more important theoretical results involve the comparison of data from two or more tasks. Experimental data, including the present results, consist of averages across Ss and, typically, do not contain comparisons across time estmation tasks. To the extent that models are linear, in some sense or other, comparisons between theoretical predictions and group data is a valid and useful exercise. However, discrimination among nonlinear models probably has to await the collection of stable data from individual Ss.

Linearity in the function $\varrho(t)$ is a useful property, both because it is predicted by some models (Eq. 6a) and because it can be used to account for one aspect of the filled-duration illusion (Theorem 6.1.1). Therefore, it is of interest to compare the group data of the present experiment with some of the theoretical results.

The properties of $\varrho(t)$ are obtained from the judgments on empty intervals. The data in Figs. 1a and 1b suggest that $\varrho(t)$ is linear, and that the size of the illusion is independent of t, within certain limits. The observation of parallel straight lines is consistent with Theorem 6.1.1. However, if Theorem 6.1.1 holds, we have another prediction, which is that $\varrho(0,4) = 4\varrho(0,1) \equiv 4\varrho(0)$, that is, the intercepts of the graphs for filled and empty intervals should be in the ratio 4:1. This prediction is not borne out by the data shown in Table 1. It is interesting, though, that if one *assumes* that $\varrho(t)$, as estimated in Fig. 1c, is linear, then the parallelism evident in this figure is consistent with Theorem 6.1.1, and the intercepts of the two graphs are nearly in the ratio 4:1.

These comparisons between theory and data raise the question as to whether the effect of varying t within a block of trials is the same as that of varying t between blocks (between Ss, in the present case). The data on empty intervals in Fig. 1a show the classical pattern of overestimation for (relatively) small t and underestimation for large t, and the corresponding data in Fig. 1b show less underestimation for small than for large t. As pointed out in Section 5, the reversible encoding model with $f(t) = \sqrt{t} + t^2$ could account for this feature. With this choice of f(t), $\varrho(t)$ is no longer linear and Theorem 6.1.1 no longer applies to the data. Therefore, it is possible that the same nonlinear function $\varrho(t)$ is estimated whether t is varied over a short range within blocks or over a long range between blocks. In this case, comparisons between theory and data are of limited usefulness, because the effects of averaging across Ss are unknown.

7. CONCLUDING REMARKS

The new data presented here confirm previous findings that filled intervals seem longer than empty ones with the same duration and that the effect is independent of duration. The theorization presented in the previous sections has the following foci: (a) The provision of a schema which allows us to interrelate data from different time-perception tasks. For example, it is shown that, with a particular choice of the encoding function, recognition accuracy and average reproduced interval both depend on encoding variability in a simple manner (Eqs. 3a and 6a). (b) An analysis of the reproduction method when empty intervals are used. It is shown that a reversible encoding model can yield average reproduced intervals that are linear with clock time, and can account for the overestimation of short intervals and the underestimation of long intervals. (c) The provision of an explanation of the filled-duration illusion. It is shown that a chunking model, in which clicks occurring during an interval serve to segment the interval during encoding, can account for certain aspects of the illusion (Theorems 6.1.1 and 6.1.2). In this model, the time-keeping mechanisms responsible for encoding are left unspecified. (d) A consideration of mechanisms that yield certain functions relating average reproduced interval to clock time. It is shown that reversible encoding, counting, and attention models are all consistent with much of the data.

We have been concerned exclusively with "discrete," filled intervals, for which the evidence of the illusion is unequivocal; but Burnside (1971) has shown that underestimation can occur when the interval is filled with mathematical tasks. It may be recalled that the attention model implies that attention to *time-keeping* is *enhanced* by the occurrence of clicks. However, it could be the case that certain types of presented information "capture" the attention mechanism and so *reduce* attention to time-keeping, leading to a reduction in the amount of time encoded. According to this view, the effects of presented information inhere in input processes. Evidence against this view and favoring a storage-size explanation comes from studies in which S's time-keeping is discouraged (Ornstein, 1969), so that the validity of the view, with respect to the majority of tasks, is still an open matter.

APPENDIX 1

A Non-Poissonian Counting Model

Let g(t) be the probability density function (p.d.f.) of the interpulse intervals, and let H(t) be the expected number of pulses in an interval (0,t). Let $g^*(s)$ and $H^*(s)$ be the Laplace transforms of g(t) and H(t), respectively. Then

$$H^*(s) = \frac{g^*(s)}{s[1 - g^*(s)]}$$

(Cox, 1962; p. 46). When

$$g(t) = \frac{1}{2}(\lambda e^{-\lambda t} + \mu e^{-\mu t}),$$

$$g^*(s) = \frac{1}{2}\left[\frac{\lambda}{\lambda + s} + \frac{\mu}{\mu + s}\right],$$

and, after simplification,

$$H^*(s) = \frac{1}{\lambda + \mu}\left\{\frac{2\lambda\mu}{s^2} + \frac{(\lambda - \mu)^2}{\lambda + \mu}\left[\frac{1}{s} - \frac{1}{s + \frac{1}{2}(\lambda + \mu)}\right]\right\}, \quad \text{A.1}$$

which gives, on inversion,

$$H(t) = \frac{1}{\lambda + \mu} \left\{ 2\lambda\mu t + \frac{(\lambda - \mu)^2}{\lambda + \mu} \left[1 - e^{-t(\lambda+\mu)/2} \right] \right\}.$$

So that

$$\rho(t) = c(At + 1 - e^{-Bt})$$

as asserted in Eq. 11.

APPENDIX 2

An Attention-Switching Model

At time 0, S is in the attentive state and stays in this state for a period having p.d.f. $g_a(t)$. Then S switches to an inattentive stage and stays in this state for a period having p.d.f. $g_b(t)$; then S switches to the attentive state, and so on. Let f(t) denote the total length of attentive periods in an interval (o,t). Let

$$R(u,t) = \text{Prob}\,[f(t) \geq u],$$

have a double Laplace transform

$$R^*(w,s) = \int_0^\infty e^{-wu} \int_u^\infty e^{-st} R(u,t)\,dt\,du.$$

Then, as is shown by Thomas (1966),

$$R^*(w,s) = \frac{1 - g_a^*(w + s)}{s(w + s)[1 - g_a^*(w + s)\,g_b^*(s)]}.$$

Let $a(t)$ denote the expected value of f(t). Then

$$a(t) = \int_0^t R(u,t)du,$$

and

$$a^*(s) = \int_0^\infty e^{-st} \int_0^t R(u,t)\,du\,dt$$

$$= \int_0^\infty du \int_u^\infty e^{-st} R(u,t)\,dt$$

$$= R^*(0,s)$$

$$= \frac{1 - g_a^*(s)}{s^2\,[1 - g_a^*(s)g_b^*(s)]}.$$

When $g_a(t) = ae^{-at}$ and $g_b(t) = be^{-bt}$,

$$a^*(s) = \frac{b + s}{s^2\,(a + b + s)},$$

which has the same form as Eq. A.1 and gives, on inversion,

$$a(t) = \frac{bt}{a + b} + \frac{a}{(a + b)^2}\,[1 - e^{-(a+b)t}],$$

as asserted in Example 5.1.3.

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NOTES

1. The data for one S from the 1-sec group were discarded because, on the average, this S's reproductions were 2½ times as great as those of other Ss.

2. In studies of temporal order discrimination (e.g., Sternberg & Knoll, 1973) and perceptual duration (e.g., Efron, 1973), the output from these analyzers is assumed to go to components not shown in the diagram. These tasks are included for the sake of completeness and will not be considered further.

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