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Time, Quantum Mechanics, and Decoherence

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ABSTRACT State-reduction and the notion of "actuality" are compared to "passage" through time and the notion of "the present"; already in classical relativity the latter give rise to difficulties. The solution proposed here is to treat both tense and value-definiteness as relational properties or "facts as relations"; likewise the notions of change and probability. In both cases "essential" characteristics are absent: temporal relations are tenselessly true; probabilistic relations are deterministically true.The basic ideas go back to Everett, although the technical development makes use of the decoherent histories theory of Griffiths, Omnès, and Gell-Mann and Hartle. Alternative interpretations of the decoherent histories framework are also considered.

1 Introduction

In relativistic physics tense is routinely treated in purely relational terms, in such a way as to avoid the concept of "time-flow" altogether. There is no real consensus as to what follows from this, but that there is a difficulty is widely acknowledged:

> Eddington once remarked that time enters into our consciousness through a sort of "back door" and I think it is generally conceded that this so-called psychological time possesses apparent qualities that are absent from the "outside" world of the laboratory. This additional structure consists of an awareness of a *now* or present moment, and an impression that time passes. Modern physics certainly distinguishes past-facing and future-facing directions of time, i.e. temporal asymmetry in the behaviour of physical systems, but it makes no use of either the present (as opposed to a particular moment) or the flow of time. Newton ...described time as passing

("flowing equably" was the expression used), but there is no instrument that can measure its rate of flow (one second per second?) and no physicist that I know who can even make sense of the idea. If our conception of reality is based on our experience of time, therefore, it is seriously at odds with the world whose reality we are concerned with. (Davies, 1981, p.63.)

Consider now quantum mechanics. Here there is a radical (minority) view that value-definiteness is likewise to be understood in purely relational terms, along the lines indicated by Everett, 1957. That is, given a total state-vector of two sub-systems A and B, of the form

$$\Psi = c_1\varphi_1 \otimes \psi_1 + c_2\varphi_2 \otimes \psi_2 \tag{1}$$

with orthonormal vectors $\varphi_i \in H_A$, $\psi_j \in H_B$, no definite vector can be attributed to either A or B. But relative to the state-vector ψ_k of B we can attribute the vector φ_k to A, and in this sense we arrive at product states (the coefficients c_k are irrelevant to these, since they disappear on renormalization). These product states are eigenstates of certain dynamical variables, which are then value-definite in this relational sense (whether they are the "correct" variables is another matter). This is the simplest example of the treatment of value-definiteness as a relational property.

The connection should be quite plain: the difficulties of attribution of tense (or "A-determinations", in McTaggart's terminology), given a space-time description, mirror the difficulties of attributing definite values to observables, given a superposition of eigenstates. It seems that either every event is "present", or that none is, just as every eigenvalue or none is "actual". The method of solution is the same in each case: whilst "event E is now", and "event E' is now" are contradictory, given that E and E' occur at different times, introducing new events W, W' we obtain: "event E is now relative to event W", "event E' is now relative to event W'", and there is no longer a difficulty. Likewise "observable X has value x" and "observable X has value x'" are inconsistent for x and x' distinct, but introducing a new observable Y we may say instead: "observable X has value x relative to value y of Y", and "observable X has value x' relative to value y' of Y", and there is no longer a contradiction.

This sort of difficulty, and the relational strategy used to circumvent it, applies equally to the use of indexicals; the analogy between value- definiteness and spatial determinations ("here" etc.) is just as good. But the parallel with tense is particularly helpful; for one, it makes clear that what is involved in the Everett procedure is poorly made out in terms of the notion of a set-theoretic collective of worlds ("Everett-worlds"). Space-time may be understood as an infinite collection of 3-dimensional worlds, but not in the sense that the total mass or energy is additive.

A second strand to the analogy is that in the case of relativistic space-time the relational account of "the present" and other A-determinations is in a certain sense interest-relative, or of only approximate significance, especially for macroscopic events. As we shall see, the same is true in the case of the Everett relativization procedure, using the decoherence theory to define the preferred basis involved.

Third and most important, the remaining problem of the Everett approach - the interpretation of probability - resembles a cluster of problems still outstanding in the relational interpretation of time. Consider, for example, the more or less instinctive criticisms that have been directed to each: if space-time as a whole is unchanging, then it cannot describe change (if the universal state as a whole is deterministic, then it cannot describe probability); if there is no such thing as "time-flow", then the distinction between past, present and future is unreal (if there is no such thing as state-reduction, then value-definiteness is illusory). Again, recall Laplace's argument that in a deterministic universe time and becoming are effectively eliminated (for more recent variants see e.g. Grünbaum, 1973, p.319-24).

If tenseless relations are adequate to the treatment of tense, then so too are deterministic relations adequate to indeterminism. Temporal facts do not come to be true in time; probabilistic facts are not made true by chance.

In both cases there is the underlying conviction that something "essential" has been omitted; certainly the intuitive notions of "time-flow" and "actualization" no longer enter at the level of foundations. But it need not be claimed that these notions are empty or meaningless: it may be that part of their meaning can be recovered at other levels in the development of the theory, remote from first principles. Whether or not we must allow for anthropocentric factors, not excluding human psychology and elements to the mind-body problem, is an open question. In fact both topics are bedeviled by more or less uncritical appeal to "illusion" and "consciousness"; this is a further strand to the analogy.

I should emphasize that the various approaches to the interpretation of probability so far floated, by those sympathetic to the Everett approach (e.g. Deutsch, 1985; Lockwood, 1989), have made use of the common-sense notion of change even when the authors point out an analogy with the concept of time.[1] Given the recent successes of the decoherence theory *viz à viz* the preferred basis problem (as summarized in e.g. Zurek 1991) there is no question that the interpretation of probability remains the outstanding difficulty. In this context it is fair to say that the analogy with time has been universally ignored, although it is just here that it is most needed.

It will be convenient to introduce a uniform terminology in both cases. I shall call facts of tense or value-definiteness *facts as relations*; the general approach I shall call *relativism*. It will not be possible, in the span of a single article, to systematically review all of the components to the analogy, particularly since we shall make use of the decoherent histories approach (for we must consider value-definiteness as a relation over time as well). This formalism is still quite novel, and the connection with Everett's relativization of state may not be familiar; further, there are those who hold that decoherence theory in and of itself resolves the problem of measurement, without appeal to a relativism of value-definiteness. For these reasons, here most of our business is with quantum mechanics and the analogy with time, rather than the relational treatment of time *per se*. But concerning the latter one point should be made plain. A relational approach to time is obviously possible in the non-relativistic case (Galilean-Cartan space-time); many of the parallels just summarized can still be made out in this case. But here we do not have a metric space, and the symmetries of the theory are fewer. As such, they can be respected in passing to a non-relational approach in terms of "things" which "change". The same absolute structure at work here (absolute simultaneity) appear essential to the Bohm theory, and may turn out to be necessary to the alternative theory of Ghirardi *et al* (the GRW approach). I take it as significant that the relational approach to tense may be *forced* in the case of relativistic space- time; what we are about is an interpretation of quantum mechanics which is not just compatible with relativity, but that shares its spirit too.[2]

Finally, there is an important context to this debate which I shall say nothing about, namely modal logic and possible world semantics. The connection of the latter to DeWitt's framing of Everett's ideas (the "many-worlds" approach) should be quite obvious. It follows that a connection between the relational approach to tense, and to value-definiteness, is likely to extend to a connection between tense and modal metaphysics. And so it does; for a recent review, altogether innocent of quantum mechanics, see Yourgrau, 1991.

2 Relativization of State and Projections.

This section and the next are concerned with some technical background. An important lacuna to early work on Everett's ideas concerned the choice of basis (the "preferred-basis problem"); this is now treated using the decoherence theory. The form we shall use is the decoherent histories approach, which is based on operators rather than states; we must first see how the relativization procedure can be formulated in these terms.

The intuition behind the state-relativization is clear from Eq.(1). In the more general case, where the state is of the form $\Psi = \sum_{ij} c_{ij} \varphi_i \otimes \psi_j$, the relative state of ψ_j in H_A is

$$\psi_{\psi_k} = N_k \sum_i c_{ik} \varphi_i, \quad \frac{1}{N_k^2} = \sum_i \mid c_{ik} \mid^2 \tag{2}$$

(N_k is a normalization constant). Note that failing the particular case where the basis $\{\psi_k\}$ is such that Ψ has biorthogonal form[3] (i.e. $c_{ij} = \delta_{ij} c_{ij}$), the relativization is not symmetric (we shall come back to this shortly).

Now to its analog at the level of operators. A purely qualitative treatment will suffice; we work with projections, the quantum-mechanical analog of the notion of "property" (more precisely, the analog of a characteristic function on phase space). Modulo certain technical complications, we can consider the quantities P as projections onto phase-space cells, where Greek subscripts range over subsets of the spectra of dynamical variables (more precisely: subsets of the Cartesian product of the spectra of a complete set of commuting dynamical variables). These subsets should be disjoint and together sum to give the entire spectra of the operators concerned, corresponding to the orthogonality and completeness properties of a basis set of vectors. In short, we suppose we are given a resolution of the identity, as the analog of a preferred basis, so that the P 's are pairwise disjoint and sum to unity.

To formulate the relativization procedure in terms of projections we first suppose we have as before a decomposition into sub-systems A and B, with associated tensor-product structure, and that in place of the bases $\{\varphi_j\}$, $\{\psi_k\}$, we have resolutions of the identity $\{P_\alpha\}$, $\{P_\beta\}$. We note that the relative state will be eigenstates of some projections, and not others, and in this sense define value-definite properties. It is now obvious how to proceed: beginning with $I \otimes P_\beta$, we project out the vector $(I \otimes P_\beta)\Psi$ (for $P_\beta = P\psi_1$, as in Eq.(1), we then obtain $c_i\varphi_i \otimes \psi_i$). and note that if this is an eigenstate of some projection $P_\alpha \otimes 1$ with eigenvalue 1, then the latter will be "actual" relative to $1 \otimes P_\beta$. This can be summarized by the equation:

$$Tr\left[(P_\alpha \otimes 1)(1 \otimes P_\beta)\rho(1 \otimes P_\beta)\right]/Tr((1 \otimes P_\beta)\rho) = 1 \tag{3}$$

where $\rho = \mid \Psi >< \Psi \mid$ and "Tr" denotes the trace (note once more the lack of symmetry in this expression). The relativization of state takes the form:

$$\rho \to \rho' = (1 \otimes P_\beta)\rho(1 \otimes P_\beta)/Tr((1 \otimes P_\beta)\rho).$$

This may be called the Everett-Lüders rule; recall that the Lüders rule, for conditionalization of the state on a given property P (the result "yes" for a yes-no experiment for P), is

$$\rho \to \rho' = P\rho P/Tr(P\rho).$$

For $P_\beta = P_{\psi_k}$ it is a simple calculation to check that $\rho' = \mid \varphi_{\psi_k} \otimes \psi_k >< \varphi_{\psi_k} \otimes \psi_k \mid$, where φ_{ψ_k} is given by Eq.(2), for the general state $\rho = \mid \Psi >< \Psi \mid$, $\Psi = \sum_{ij} c_{ij}\varphi_i \otimes \psi_j$. Eq.(3) simply says that $P_\alpha \otimes 1$ is actual relative to $1 \otimes P_\beta$ exactly when $Tr((P_\alpha \otimes 1)\rho')$ is unity.

It should also be clear how to proceed in the absence of a tensor-product structure. Given a resolution of the identity $\{P\gamma\}$, we can define more degenerate projections P_δ by integrating over some of the dynamical degrees of freedom (in a field-theory setting, these we can suppose relate to different regions of space). The LHS of Eq.(3) is of the form:

$$\mu_\rho(P_{\delta'}/P_\delta) = Tr(P_{\delta'}P_\delta\rho P_\delta P_{\delta'})/Tr(P_\delta\rho P_\delta).$$

We say that $P_{\delta'}$ is actual relative to P_δ iff $\mu_\rho(P_{\delta'}/P_\delta) = 1$. For the sake of precision we can formalize this as the relation "Def":

$$Def_\rho(\delta'/\delta) \text{ iff } \mu_\rho(P_{\delta'}/P_\delta) = 1.$$

It is easy to see that for commuting projections P_δ, P_δ :

(i) $Def_\rho(\delta'/\delta) \Leftarrow P_\delta \subseteq P_{\delta'}$.
(ii) $Def_\rho(\delta'/\delta)$ & $Def_\rho(\delta^{''}/\delta^{'}) \Rightarrow Def_\rho(\delta^{''}/\delta)$
(iii) $Def_\rho(\overline{\delta}^{'}/\delta) \Leftrightarrow Def_\rho(\overline{\delta}/\delta')$

where $P_{\overline{\delta}} = 1 - P_\delta$ (the quantum negation), and $\sqsubseteq$ is the usual partial ordering of projections in terms of their range; (i) says that Def is reflexive and invariant under coarse-graining (i.e. if $P_{\delta'}$ is value-definite relative to P_δ , then so is any coarse-graining of $P_{\delta'}$); (ii) is transitivity; and the duality condition (iii) is the usual transposition of a conditional on taking negations.[4] These properties of Def hold independent of the state ρ.

We must extend these considerations to the time-dependent case. As before we may work either with states or with projections, but in the first case we use the Schrödinger picture, in the second the Heisenberg picture. For convenience we work with a discrete time variable t_k, denoting Heisenberg picture projections by P_{γ_k} (so that γ_k ranges over subsets of the spectra of dynamical variables at time t_k). For projections P_{γ_j}, P_{γ_i} , at times t_j , t_i ($t_j \geq t_i$), which in general do not commute, we can still define the quantity:

$$\mu_\rho(P_{\gamma_j}/P_{\gamma_i}) = Tr(P_{\gamma_j} P_{\gamma_i} \rho P_{\gamma_i} P_{\gamma_j})/Tr(P_{\gamma_i} \rho P_{\gamma_i}). \quad (4)$$

This is real and positive definite with range [0,1]; for $t_j > t_i$ it is the transition probability for initial state ρ, "state-preparation event" P_{γ_i}, and "state-detection event" P_{γ_j}, using the Lüders rule on each event. In this case we have "probability" as a relation between events (in cases where the interpretation of these quantities as "probability" is in doubt, we shall speak of "relations in norm" instead). But if we go on to define Def as before, (ii) and (iii) no longer hold in general (because of the failure of commutativity). It is also clear that we have as yet no guidance as to how to select the projections $\{P_{\gamma_i}\}$ at each time t_j. The two problems are in fact related.

3 Decoherence Theory

The definition of the "preferred basis" (the class of projections) at each time, is the business of decoherence theory. There are various kinds of decoherence: *dynamical decoherence* (in the terminology of Zurek, 1994), *medium decoherence* (Gell-Mann and Hartle, 1990), and *consistency* (Griffiths, 1984; Omnès, 1988, 1990). The last two explicitly make use of the notion of a history, formalized as a string of time-ordered Heisenberg-picture projections. The first is the familiar notion of decoherence in open systems theory, i.e. where an equilibrium limit is rapidly reached with the reduced density matrix of a particular subsystem diagonal in an "appropriate" representation (appropriate to observed phenomena and quasi-classical models). The other subsystem is typically taken as an "environment" (hence the terminology "environmentally-induced superselction rules"). There are now many models demonstrating dynamical decoherence; given local interactions, with a weak coupling to a large number of light particles (e.g.

photons),[5] we have decoherence on a time scale going as $\hbar^2/n$ in comparison to the thermal relaxation time (n is the number of degrees of freedom). But the deficiencies of this sort of approach, in the absence of a relativized account of value-definiteness, are well-known; see e.g. the recent criticisms of Zurek, 1991, collected with Zurek's 1993 reply, where he makes clear that an interpretation of Everett-type was after all intended.

The single-time relativization of value-definiteness is most at home with the dynamical decoherence, but to understand better the overall picture, and in particular the treatment of probability and the analogy with time, what is needed is a space-time formulation. The best framework so far worked out is the theory of decoherent histories. The connections with dynamical decoherence are not yet completely clear (for some recent results, see e.g. Dowker and Halliwell, 1992; Gell-Mann and Hartle, 1993; Finkelstein, 1993; Diøsi *et al*, 1994), and we can look forward to something better; but the decoherent histories framework will do for our purposes.

The basic idea is quite simple; it is, moreover, independent of any decomposition of the total system into subsystems. In brief, we are to represent the time-development of the total state in terms of a set of mutually-exclusive histories (strings of projections), in such a way that interference effects between disjoint histories are negligible, *just as would be the case were a single history realized through a process of state reduction.*

To see how this works, we once again suppose for each t we have a resolution of the identity $\sum_{\gamma_k} P_{\gamma k} = 1$; time-ordered products of (not necessarily commuting) projections are denoted:

$$C_\gamma = P_{\gamma_k} P_{\gamma_{k-1}} .... P_{\gamma_1}.$$

Were such a sequence of projections subject to sequential measurement, given state preparation event P_{γ_0}, by repeated application of the Lüders rule $\rho \rightarrow P\rho P/Tr(P\rho)$, we would obtain for the probability of a positive outcome for each member of the sequence the expression:

$$\mu_\rho(C_\gamma/P_{\gamma_0}) = Tr(C_\gamma P_{\gamma_0} \rho P_{\gamma_0} C_\gamma^\dagger)/Tr(P_{\gamma_0}\rho). \tag{5}$$

If the theory does not distinguish experimental configurations from others, then Eq.(5) should apply whether or not intermediate events are considered "measurements", hence (since we use only the unitary dynamics) *whether or not there is any intermediary state reduction.* This imposes a powerful constraint; the sum of such quantities over all sequences differing only in the configuration at time t_k, must then be the same as the probability for the strings $P_{\gamma_n}...P_{\gamma_{k+1}}P_{\gamma_k}P_{\gamma_{k-1}}...P_{\gamma_1}$, i.e. where the configuration P_{γ_k} at time t_k is not specified. Of course this will not in general be the case; the probability for the observation of an electron at the screen in the 2-slit experiment is not the sum over probabilities for histories where it is in addition measured as localized at one slit or the other. The necessary and sufficient condition is that the real part of the quantity $Tr(C_\gamma \rho C_{\gamma'}^\dagger)$ approximately vanishes for $\gamma \neq \gamma'$. This is a functional of pairs of histories; it is called the *decoherence functional.* It also has a simple (covariant) expression in terms of path integrals. Sufficient, but not necessary, is the *medium decoherence* condition of Gell-Mann and Hartle, 1990, which requires that both real *and* imaginary parts approximately vanish:

$$Tr(C_\gamma \rho C_{\gamma'}^\dagger) \approx \delta_{\gamma_n \gamma_n'} ... \delta_{\gamma_1 \gamma_1'} Tr(C_\gamma \rho C_\gamma^\dagger). \tag{6}$$

(This automatically holds for commuting sequences.) It then follows, for example, that

$$Tr(P_{\gamma_3} P_{\gamma_1} \rho P_{\gamma_1} P_{\gamma_3}) = Tr(P_{\gamma_3} (\sum_{\gamma_2} P_{\gamma_2}) P_{\gamma_1} \rho P_{\gamma_1} (\sum_{\gamma_2} P_{\gamma_2}) P_{\gamma_3})$$
$$\approx \sum_{\gamma_2} (P_{\gamma_3} P_{\gamma_2} P_{\gamma_1} \rho P_{\gamma_1} P_{\gamma_2} P_{\gamma_3})$$

so that

$$\mu_\rho (P_{\gamma_3} / P_{\gamma_1}) \approx \sum_{\gamma_2} \mu_\rho (P_{\gamma_3} P_{\gamma_2} / P_{\gamma_i}). \tag{7}$$

This certainly requires that relations in the Hilbert-space norm are approximately the same, whether or not state reduction takes place with respect to measurement of the projections concerned, hence whether or not they are measured. Modulo equivocations on the precise condition used, decoherence can be understood as a constraint on what relations in norm are to be interpreted as "probability". Any set of disjoint exhaustive histories satisfying Eq.(6) is called a *decoherent history space*; one that cannot be further fine-grained without loss of decoherence is called a *quasiclassical domain*.[6] Both depend on the universal state and the unitary dynamics.

A bullish assessment then runs as follows: at the very least we have a FAPP theory, given the right quasiclassical domain; but at no point do we mention "the observer" or "measurement". Further, the theory is based on the unitary mechanics, and can evidently be applied to closed systems, without (or so it seems) any need to distinguish one sub-system from any other.[7] That is why the decoherent histories approach has created so much interest in quantum cosmology.

The point deserves due emphasis: given the quasiclassical domain and universal state, the entire empirical content of the theory is fixed once and for all. On the premise of a unique quasiclassical domain, it can be said that *if the problem of measurement consists in the fact that the notion of "measurement" and "the observer" figure explicitly in the fundamental postulates of the quantum mechanics, then there is no longer a problem of measurement.*

This is the selling-point of the decoherent histories approach. It is a claim made out independent of the interpretation of probability. In point of fact there has been little discussion of the latter; the crucial question, of how and in what sense there is an "actualization" of one history to the expense of all others, has scarcely been raised. But the difficulties here should be clear enough; if a single history is "actualized", one projection at a time, then the decoherence condition is being used as a criterion for "objective" state-reduction, contrary to the entire basis of the approach.

We shall consider this in more detail in Section 6. Failing this sort of input into the theory, we have only the branching structure of decoherent histories. In itself, as a network of norms, it is an invariant structure; so long as the smooth space-time symmetries are unitarily implemented in the Hilbert-space, all relations in norm given by the functional μ_ρ are invariant and basis independent, just as all the space-time metrical relations among events are invariant and coordinate independent in relativity theory. *The basic idea of the relational approach is that this is all that is required at the level of the fundamental equations.* What is "actual", just as what is "now", is to be understood as facts as relations. There is nothing more to be put in; neither the "flow" of time, taking us from one "now" to the next, nor the reduction of state, taking us from one "actuality" to another.

But the account of facts as relations does need some further work. The relation Def can be applied to the global projections P_{γ_k} defining a decoherent history space, and applies equally to further coarse-grainings P_{δ_k}. The treatment of equal-time value definiteness can proceed as in the previous section; where the projections are defined in terms of quasi-local algebras (e.g. polynomial algebras in field theory), microcausality automatically implies commutativity for space-like projections. In the unequal-time case we find that medium decoherence is sufficient to ensure conditions (ii) and (iii) are met despite the fact that such projections do not in general commute.[8]. Naturally (failing exact decoherence) the relation Def must be understood as of only approximate significance.

It is at this point that we meet with an ambiguity: should we relativize to a single projection P_{δ_k}, or to a history C_γ preceding P_{δ_k} as well? We can associate different relativized vectors $P_{\delta_k} C_\gamma \Psi$ to P_{δ_k}, one to each distinct history $C_\gamma = P_{\gamma_{k-1}} P_{\gamma_{k-2}} ... P_{\gamma_0}$, noting that these vectors are approximately orthogonal by Eq.(6); the question is whether these should simply be superposed and then normalized, or separately normalized, and used to define a class of history-to-event facts as relations.

This question goes to the heart of the interpretation on offer. There are certain correlations - well and good. The attribution of value- definiteness and probability will depend on the objects used in relativization. As Bell puts it:

> Whereas "measurement" was a dynamical intervention, from somewhere outside, with dynamical consequences, it is clear that "attribution" must be regarded as a purely conceptual intervention. It is made, say, by a theorist rather than an experimenter; he is quite remote in space and time from the action, and simply shifts his attention....it follows that attributing a particular value to some beable cannot change particular values already attributed to some other beables. (Bell, 1987, p.42-3.)

(We can take this over without modification, with projections as "beables", so long as the value-attributions are relativized.) The difficulty is that this "attribution" is also made by the experimenter; what will be *observed* as value-definite? According to relativism, that depends on who or what is "the observer"; the ambiguity arises because we do not know whether this is something historical, a history of projections, or something momentary, a local segment of history. Correspondingly, we have two definitions of value-definiteness; relativization to history:

$$Def_\rho(\delta'_k/\delta_k\gamma) \Leftrightarrow Tr(P_{\delta'_k}P_{\delta_k}C_\gamma\rho C_\gamma^\dagger P_{\delta_k})/Tr(P_{\delta_k}C_\gamma\rho C_\gamma^\dagger) \approx 1 \qquad (8)$$

or, as before, a relativization to single-time configurations:

$$Def_\rho(\delta'_k/\delta_k) \Leftrightarrow Tr(P_{\delta'_k}P_{\delta_k}\rho P_{\delta_k})/Tr(P_{\delta_k}\rho) \approx 1 \qquad (9)$$

There is no contradiction between the two, but only the question as to which is relevant to observation. The most parsimonious hypothesis is that it is the "here and now", to be modeled as a local sequence of configurations, over milliseconds or less. On this basis we should use Eq.(9). The alternative, to relativize to global histories, has more in keeping with a realist approach to state reduction: doing this we obtain vectors $C_\gamma\Psi/\|C_\gamma\Psi\|$, identical to what would have been obtained were state-reduction an objective physical process. This is the version of relativism that will be attractive to someone who wants to abandon it.

On the more radical view the only notion of history available is that given by past-directed relations in the Hilbert-space norm, conditionalized on the present. For $C_\gamma = P_{\gamma_{k-1}}P_{\gamma_{k-2}}...P_{\gamma_0}$

$$\mu_\rho(C_\gamma/P_{\gamma_k}) = Tr(P_{\delta_k}C_\gamma\rho C_\gamma^\dagger)/Tr(P_{\gamma_k}\rho) \qquad (10)$$

(Omnès, 1990; Gell-Mann and Hartle, 1990; these quantities are only correctly normalized given decoherence, in contrast to the forward-directed relations of Eq.(4).) These may well be concentrated on a class of very similar histories, but given only P_{δ_k} and the universal state and history space, there is nothing else in the mathematics to say which is "the real" history. But neither can there be any other way to decide the matter; as a matter of general principle, we can only have knowledge of the past based on present states of affairs (including present memories and records). That is, we can only proceed from some particular P_{δ_k}, even if we are given the universal state and decoherent history space (i.e. in the context of a cosmological model). In practice, as in retrodiction to laboratory events, we first use classical physics to retrodict from δ_k; Eq.(10) adds corrections to this. So if we were to try to estimate the probability for $P_{\delta'_k}$ given ρ and P_{δ_k}, using the history-dependent expression Eq.(8), we would have to treat the latter as an ensemble, weighted by the quantities Eq.(10); the result (using medium decoherence) is exactly what is given by Eq.(9).[9]

It follows from this that the question before us is purely metaphysical. No experiment can bear on the matter; by definition an experiment is a controlled process in which such correlations (records) are reliably produced, so just insofar as we consider processes involving the reliable production of records, to that extent the present brings with it the past. It would seem that the issue is primarily philosophical. In point of fact, the links here with philosophical debates over the reality of the past are so close that there is no point in pursuing them independent of that literature (see e.g. Dummett, 1969; Wright, 1993, Ch.5). For the moment let me only indicate a new question that is raised in quantum mechanics: for medium decoherence is not only used in proving the equivalence just established, but it can in turn be *derived* from the assumption that there exist records in the first place (Halliwell, 1994, p.59-63). Medium decoherence can also be understood as a stability condition on relations in the Hilbert space norm; the important question (to which we shall return in Section 6) then arises as to how and in what sense such conditions can be taken as explanatory; one sense, hinted at by a number of authors (Gell-Mann and Hartle, 1990, Zurek, 1994), amounts to a constraint on the possibility ofcomplex organization, in effect a generalized evolutionary constraint (Saunders 1993a, 1993b).

There is a related matter which is equally pressing. It is clear that in view of the fact that Def is not symmetric, value definite events (facts as relations) cannot be partitioned into equivalence classes, given medium decoherence alone. In fact this would be quite unphysical in the general case: past-directed relations in norm *should* differ from their future-directed counterparts. But space-like relations are another matter; here symmetry, at least in an approximate sense, is desirable, failing which the notion of an intersubjective value-definite world is also in question. On this two things can be said; first, we see something similar in connection with tense: the relation "spacelike" is symmetric but non-transitive, so "the now" (if defined in this way) is not an equivalence relation. Or using Einstein synchrony, we have a relation which is neither transitive nor symmetric, although in special cases, e.g. given a class of parallel world-lines, we obtain both. Yet despite this, there is no difficulty in relativizing to a family of spacelike hypersurfaces, no matter that it may not be defined in terms of a synchrony procedure or light-cone structure for a family of observers. Equally, there is no difficulty in relativizing to a global projection ("our classical world").[10] But second, it is clear that this issue bears directly on the relation of medium to dynamical decoherence. The basis which diagonalizes the reduced density matrix for a given subsystem is the same as that obtained by appeal to the biorthogonal form for the composite system. In this case, as we have seen, the relativization is symmetric. Obviously the connection between the two kinds of decoherence is in need of clarification; there is certainly reason to think that Def will turn out to approximately symmetrical for the special sorts of subsystems that might count as an epistemic community. Again, the same is true in relativity (compare Stein 1991).

4 Relativism and the Copenhagen Interpretation

Despite the ontological extravagance - properly understood[11] - of the relational strategy, it is in a certain sense quite austere. That is particularly true of the radical version just sketched. Nothing is added to quantum mechanics, but only a certain methodology: we are to determine a stable set of correlations as described by the state. What appear to be unconditional facts are to be understood as tacitly presupposing a context ("facts for ....." or "....... relative to ...."), pointed to by the use of token-reflexives or demonstratives ("I", "here", "now" etc.). When this is made explicit we have only facts as relations. Where the context includes a human being (e.g. in perception), then relative to that person, at that time, facts about tense and value-definiteness are perfectly objective; there is no appeal to "consciousness" or "illusion",[12] and we can perfectly well work with automated experiments, so long as they figure in the same decoherent history space,[13] or with arbitrary quasiclassical phenomena (the facts concern what is actual-for, and not only what is known to be actual-for). In the case of global projections the facts as relations are correlations between microscopic and "classical world". There is no real difference between using the Lüders rule here, and using it to model the state-preparation or detection in a simple experiment; in both cases enormous idealizations are required.[14]

Clearly there are a number of distinctions that must be carefully handled; between the local and global, between ontology and epistemology, and between relativization and the more familiar notion of a dependence of what is observed on the mode of observation. It is illuminating to study how these distinctions have been handled in the more influential (and familiar) interpretations of quantum mechanics, in particular the Copenhagen interpretation. Roughly speaking, Bohr was concerned with the second in each case; in particular, it is doubtful whether his "relational" concept of state (Jammer, 1972, p.197-211; Murdoch, 1986, p.137-9) has to do with relativism in the sense used here, so much as with the notion of a choice of "complementary" descriptions. His response to the EPR analysis, which threw into sharp relief the shortcomings of a "disturbance" interpretation of measurement, was to limit talk of "elements of reality" to what is specified given the entire experimental context (even when these "elements" appear to refer to systems spacelike to the apparatus). Exactly the same is true according to relativism (relativization in the case of value-definiteness is non-local in exactly the same sense as it is in the case of tense; in neither case as a physical process). But Bohr did not only consider the physical configuration of the apparatus, but also the type of measurement (position, momentum etc.), and hence - as understood by Bohr - the preferred basis to be used. Excepting the latter, there are numerous connections between Copenhagen philosophy and relativism. Bohr's views may be summarized (i) "The actual" (an "element of reality") is only defined relative to an experimental context. (ii) The "classical description" (set of concepts etc.) has a quasi- Kantian a priority, both reflecting “our forms of perception” (Bohr, 1934 p.1,19,118, 196, “there is no use in discussing what could be done if we were other beings than we are”, Heisenberg, 1959, p.55), and also requiring that (iii) “the observer” is invariably coupled to a larger world (“the classical world”). (iv) The use of classical concepts implies an incompleteness to the description of the observer, just because it must be considered a part of a larger system (in this sense observation necessarily is treated in open quantum systems theory).

The last point deserves clarification, for it is another and quite distinct formulation of the notion of complementarity: if measurement apparatus + object system are considered in isolation from the external world, then the apparatus does not function as “the observer” (and correspondingly the description is purely quantum mechanical). In this case we have a “causal” description, but at the cost that there is no “space-time coordination” (Bohr, 1934 p.94, 98, 114, 1935, p.146-7); if now the joint system is subject to measurement, it must be coupled to an open system and the description is no longer “causal” (or no longer precisely “defined”). There is therefore an opposition or limitation to simultaneously upholding “definition” and “observability” (cf. Bohr, 1934, p.54, 87; Heisenberg, 1959, p.52-3). It is also significant that this notion of complementarity (what I shall call the principle of complementarity), in contrast to ideas centering on the wave-particle duality, or to descriptions associated with non-commuting observables, has a parallel in space-time theory. The analog of the classical concepts is tense; the analog of quantum mechanics is a “block universe”. The latter is “causal” (or better, “well-defined”), in that it is uniquely specified (with the dynamics understood in the sense of constraints), but since the demands of “observability” require the selection of a particular “now”, and this is not determined by the dynamical constraints, we may no longer deal with the block-universe alone (“the now” is the analog of Bohr’s “classical world”). The coupling of “the now” to the block-universe is not “causal” (and it may not be included in the block-universe and remain “the now”).

It is again this notion of complementarity which motivates (v) Copenhagen prohibits the application of quantum mechanics to the world as a whole, including “the observer” (this strengthens (iv)); at the same time, Bohr insists: (vi) “the observer” has no anthropomorphic content, and in particular has nothing to do with “consciousness” (this sits uneasily with (ii) and the principle of complementarity as used to underpin (v)). Turning now to the relational approach, which explicitly violates (v), the Bohr-Heisenberg notion of “actuality” goes over to: (i)′ relativization to projections. To this we may add (iii)′ we are particularly concerned with global projections (this on the grounds of intersubjectivity). Here the “necessity” of classical concepts amounts to: (ii)′ restriction to a decoherent history space.[15] Further: (iv)′ non-trivial (medium) decoherence requires some degeneracy in the projections involved (the principle of complementarity is incorporated as an epistemic constraint according to the basic concept of relativization, and no longer implies (v)). Finally (vi)’ decoherence is purely a matter of the objective physics, and has nothing to do with “mentality”. This is likewise tempered with the recognition that if there exists more than one (non-trivial) quasiclassical domain, anthropocentric criteria may be invoked (see Section 6).

5 Time and Identity

What has no parallel in relativism is Bohr's notion of "complementary descriptions" and the connection between this and "the observer" (in the sense of choice of apparatus). Insofar as this was understood to determine preferred basis, there is no longer any common ground.[16] It was exactly in this context (and not in terms of the principle of complementarity) that Bohr (1934, p.97-8, 1935, p.150, 1949, p.224) made out an analogy with relativity; what is more the parallel was made out with respect to the choice of inertial frame. In common with most physicists of his generation, Bohr never considered the problem of transience, in McTaggart's sense. The mere use of Minkowski diagrams did not herald the metaphysics of the "block universe", no more than did Galileo's use of space-time diagrams, and the question as to whether or not relativity rules out an absolute account of tense was not even posed. In itself the relativity of simultaneity is a red-herring; it may be that it forces a relational approach to tense, but it does not capture its significance.

In particular, Bohr did not acknowledge the parallel between the principle of complementarity and the problem of tense; neither did he remark on the obvious alternative to Copenhagen suggested by the relational approach in the framework of space-time theory. In the hands of Minkowski, the space-time concept provides an integration of a plurality into a single, unified, totality; precisely what is explicitly denied by Bohr: "evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as *complementary*" (Bohr, 1949, p.210). And where Bohr did acknowledge the invariant structure of Minkowski space (Bohr, 1934, p.97), he did not consider that the price is a radical change in ontology, and that a similar strategy may apply to quantum mechanics as well. In this sense Everett stands to Bohr much as Minkowski stands to Einstein as of 1905.

Consider now the way in which questions of mentality have arisen in the two contexts. In both the philosophy of time and quantum mechanics there is the repeated temptation to relocate something supposedly lost to the world of physics in the mind instead. The tradition is of course much older than that; it is at bottom the same strategy that was used in the face of the primary- secondary quality distinction, what stands at the beginning of modern philosophy and physics alike. In quantum mechanics the strategy was clearly intimated by Schrödinger (the relative state "is born anew, is reconstituted, is separated out from the entangled knowledge that one has, through an act of perception"; Schrödinger, 1935, p.162), but it was most sharply posed in the analysis of London and Bauer of 1939. The von Neumann theory emphasized the role of subjective experience in quantum mechanics, but this was to be checked by the constraint of the "psychophysical parallelism" (so that physics need only deal with the physical correlates to subjective experience). London and Bauer provided the first accessible account of von Neumann's analysis, but the discipline was easily lost:

> The observer has a completely different impression. For him it is only the object x and the apparatus y that belong to the external world, to what he calls "objectivity". By contrast he has with himself relations of a very special character. He possesses a characteristic and quite familiar faculty from which we can call the "faculty of introspection". He can keep track from moment to moment of his own state. By virtue of this "immanent knowledge" he attributes to himself the right to create his own objectivity - that is, to cut the chain of statistical correlations summarized in Ψ by declaring "I am in the state φ_1" ... . (London and Bauer, 1939, p.252.)

How may the observer "keep track from moment to moment of his own state"? What is the psychophysical correlate of this[17]? As for their subsequent remark:

> Thus it is not a mysterious interaction between the apparatus and the object that produces a new φ for the system during the measurement. It is only the consciousness of an "I" who can separate himself from the former function Ψ and, by virtue of his observation, sets up a new objectivity in attributing to the object henceforward a new function $\varphi = \varphi_1$. (Ibid p.252.))

this can be understood in terms of relativization, but only insofar as the new product state $\varphi_1 \otimes \psi_1$ is only one term of the total superposition. If the result of "setting up a new framework of objectivity" is a change in the total state, then this "setting-up" is the cause of state-reduction in a non-relational sense, and again the psychophysical parallelism is abandoned. The appeal to mentality in the case of time is even more familiar. Weyl's picture of consciousness "crawling up the life-line of the body" is an example; it is a mentalistic version of the metaphor of the river of time (that is, paddling up the creek instead). With this Weyl hoped to reconcile the notions of "becoming" and identity over time with relativity, but at the cost of introducing a mentalese meta-time, with the explicit violation of the psychophysical parallelism (all the waking moments on the life-line are conscious). Grünbaum, to take one of the foremost champions of a mind-dependence thesis, insists not only that the notion of "becoming" or transience is mind-dependent (and with it identity over time), but emphatically declares:

> My characterization of *present* happening or occurring *now* is intended to deny that belonging to the present is a physical attribute of a physical event E which is *independent* of any *judgmental awareness* of the occurrence of either E itself or of another event simultaneous with it. ...This formulation...serves to articulate the mind-dependence of nowness, not to claim erroneously that nowness has been eliminated by explicit definition in favor of tenseless temporal attributes or relations. (Grünbaum, 1971, p.209.)

It may be Grünbaum intends throughout that the terms "now", "present" etc. refer to whatever is required over and above facts as relations ("Thus our question is: what over and above its otherwise tenseless occurrence at a certain clock time t, in fact at a time t characterizes a physical event as now or as belonging to the present?", ibid p.206), but it is not quite clear whether he recognizes "occurrence at a clock time t" as a relation; nor, consequently, what work there is to do for the notion of a judgmental awareness over and above the needs of epistemology. The difficulty in interpreting Grnbaum is brought to the fore by his explicit appeal to color-vision as an analogy for the notion of tense; for whatever the adjectival use of color words, there are non-mentalistic characterizations of radiation fields and surfaces which do the same job.

Shimony's recent criticism is a case in point; if "the now" is understood in a non-relational sense, and is modeled instead on the notion of perception, then what is it that is perceived? It would seem that either every event is "now", or there is no such thing, not even as an "illusion" . Shimony's objection is irrelevant to a relational account of tense, for it is granted that every event is "now" relative to itself (just as every event is "here" relative to itself). It likewise follows that every projection is value-definite relative to itself (because Def is reflexive), or that every Everett-world is actual relative to itself; but not that every event is "now", as Shimony interprets Grünbaum,[19] or that every Everett-world is "actual", as DeWitt interprets Everett.

The crucial problem, as understood here, concerns identity over time. We have the criterion of structural similarity of events (and in the case of persons, psychological continuity, even given a bifurcation of histories[20]); how this may fit with the concept of probability is the key question.

My claim is that there is no sense to a relational account of probability failing a relativism of tense. I have stressed that the interpretation of probability, in the absence of state-reduction (or the explicit introduction of a stochastic process), parallels the interpretation of time in the absence of "time-flow"; in both cases we have only formal criteria of adequacy. But if we appeal to a relativism of value-definiteness in the framework of everyday notions of change, there is an insuperable difficulty. Consider:

> There does not seem to be any reasonable motivation (other than traditionalism) for introducing concepts like particles, quantum jumps, superselection rules, or classical properties on a fundamental level. It also appears unfortunate, therefore, that the now very popular technical concept of path integrals may suggest a fundamental role of paths or "histories" of classical states, although their required superposition is nothing but another representation of the Schrödinger wave function....Quasiclassical histories emerge instead from the global wave function as traveling and thereby smoothly branching narrow wave packets (not as time sequences of "events") only because of the continuous action of decoherence (which leads to increasing complexity, symmetry-breaking, and fine-graining into dynamically independent wave packets) (Zeh, 1993, p.191.)

Zeh is engaged in a polemic against those who see in the decoherent histories approach a one-history solution of the problem of measurement, with no appeal to Everett's ideas. In fact he endorses a radical version of relativism, based on the dynamical decoherence theory, save in the crucial respect that he appeals to the common-sense notion of change. But now he is not about to say anything about probability: given that we have a "dynamically independent wave packet", let us suppose the packet divides in two, with unequal amplitudes. This corresponds to a bifurcating history with unequal relations in norm. In what sense is ".....'' more likely to go one way or the other, and what is "....."? Here a traditional intuition is brought into play which makes the notion of probability unintelligible: what of the difference in numbers, in the Hilbert-space norm, if one thing becomes two willy-nilly?

I do not claim to eliminate one problem (probability) by compounding it with another (tense); I do claim an intuitive (but "opaque") concept on the one hand ("passage through time"), eliminating a relational approach to tense, makes the corresponding approach to value-definiteness incoherent.

6 Alternatives to Relativism

The vast bulk of the literature on the decoherent histories approach says nothing of value-definiteness or probability as relations. We must consider with some care the alternatives. An immediate difficulty with the decoherent histories approach is the question: How is a quasiclassical domain to be selected? There are certainly an enormous number of "trivial" quasiclassical domains (Zurek, 1994); to hope that there may yet be some kind of proof of uniqueness, for the "right kind" of decoherence, would still require some additional physical principle, for what is the "right kind" of decoherence?

Might we look on a particular "choice" of quasiclassical domain as analogous to a particular solution or boundary condition to the field equations of general relativity? This sort of proposal has not been fleshed out in the literature, and it is hard to see how it can be given substance. But let us suppose that a quasiclassical domain may in some sense be understood as a fundamental object of the theory, along with the universal state, the dynamical variables, and the equations of motion; that it need not be "explained", no more than the latter. Even then, it must be shown that quasiclassical domains exist. The notion of a "limit" decoherent history space must be mathematically well-defined, and subject to classification. Otherwise there are only shifting standards of decoherence, FAPP.

In contrast, there is no problem with only an approximate notion of decoherence given the radical relativism of Sec.3. The representation of the physics, as a decoherent history space, is only a way of specifying a structure to the correlations present in the universal state. This structure may be of interest for a number of reasons, and it is certainly objective, but it is perhaps one among others (or it is an interest-relative affair, similar to the definition of a phenomenological model or effective theory in particle physics). In particular, it may be of interest to ourselves, and thence to the epistemology of the theory, depending on the way in which we, *qua* biological organisms, or "information-gathering and utilizing systems" (IGUS), in the terminology of Gell-Mann and Hartle, 1990, happened to evolve:

> If there is essentially only one quasiclassical domain, then naturally the IGUS utilizes further coarse grainings of it. If there are many essentially inequivalent quasiclassical domains, then we could adopt a subjectivist point of view, as in some traditional discussions of quantum mechanics, and say that the IGUS "chooses" its coarse graining of histories and, therefore, "chooses" a particular quasiclassical domain, or subset of such domains, for further coarse graining. It would be better, however, to say that the IGUS evolves to exploit a particular quasiclassical domain or set of such domains. Then IGUSes, including human beings, occupy no special place and play no preferred role in the laws of physics. They merely utilize the probabilities presented by quantum mechanics in the context of a quasiclassical domain. (Gell-Mann and Hartle, 1990, p.454.)

With this the Kantian element to Copenhagen philosophy is appropriated with a vengeance: it is *the functional organization of the organism*, no less than the measuring instrument, which exploits one decohering regime to the universal state, rather than another. But we must add that this "genetic" approach to the definition of a decoherent history space requires a notion of evolution or "emergence" in a regime which is not, or not yet, a decoherent history space. In this sense not only do we deal with the question of which particular decoherent history space is to be used, but we also must also explain why medium decoherence should be relevant to "life", to be understood in an abstract sense of functional organization and complexity theory (cf. the remarks at the close of Section 3). Zurek, 1994, has also hinted at this sort of framework, but in the framework of dynamical decoherence.

The alternative approach to the "choice" of a decoherent history space, as in some sense a "given", has more in common with the view that probability (as opposed to the Hilbert-space norm) is fundamental. An example is the consistent histories condition of Griffiths, 1984 and Omnès, 1988, where the stress is on an *a priori* notion of probability, merging with the appeal to "logical consistency" or "the validity of classical logic", (Omnès, 1990, 1992), similar to the more rationalist strands to Bohr's thinking. Remarks which hint at this can also be found in Gell-Mann and Hartle, 1990, despite the appeal to evolutionary criteria; and likewise in Zurek, 1991.

On the genetic relativism just sketched, the Hilbert-space norm is taken as something more fundamental than "probability", which may have only a phenomenological or anthropocentric significance. In contrast, on the absolutist approach, it is in some sense "essential" to the theory that relations in norm obey a classical probability calculus (forcing decoherence and, it is hoped, a unique quasiclassical domain). One might similarly rule out certain solutions to general relativity (e.g. Gödelian space-time, which admits closed time-like curves), on the grounds that an "essential" feature of time has been lost (that this *may not count* as a space-time model).[21] Alternatively, one can explain how it is that we do not see too pathological a space-time geometry on evolutionary grounds, in the sense of a relatively mild application of the weak anthropic principle.

It should be clear that if relativism is to be abandoned, there is no sense to the appeal to evolutionary criteria; and conversely, if there is a role for such principles in the determination of a decoherent history space, then the interpretation of probability must respect the existence of other inequivalent regimes of decoherence. In particular, the interpretation had better not introduce a notion of privilege for one domain over another, in anything more than an interest-relative sense. Again there is an analogy in the case of time: if we are to provide an interpretation of the "flow" of time, consistent with a relational account of tense, then it had better not lead to any "absolute" significance of one space-time foliation over another; and conversely, if we do have the latter, then there is no sense to the appeal to anthropocentric factors.

The worst of both worlds is to insist on a plurality of histories, as well as a non-anthropocentric criterion for the definition of quasiclassical domain. This applies in particular to the view that all histories are "simultaneously actualized". The idea is self-defeating, for if all histories are "actualized" it would seem that probabilities can have nothing to with it; at the very least the relations in norm should all be the same.

This discouraging conclusion was reached by Graham, 1973, in the very process of formulating the quantum mechanical Bernouilli theorem (of which more below); it arose in the context of DeWitt's interpretation of the universal state as an infinite ensemble of universes, subject to spontaneous "splitting", of which ours is only one. Likewise one could suppose that each history of a quasiclassical domain is a "universe-history" (presumably terminating in a "present"[22]), and that bifurcations in sequences of projections should be understood as a "splitting" of universe-histories. The approach amounts to an abandonment of the relational approach to both value-definiteness and tense; the intuition which underwrites the problem of probability (this is *how* we arrive at the problem of probability) is just what is abrogated by a relational approach to time.

With the first formulation of the decoherent histories approach (Griffiths, 1984), the DeWitt picture was emphatically rejected. But when Griffiths reluctantly considers that "a certain "splitting" (i.e. bifurcations of strings of projections) is also to be met in his own approach, he claims:

> This phenomenon is interpreted either by saying that a single system starting in the initial macrostate will at a later time be in one of the macrostates which has significant probability (but the theorist, owing to his ignorance, cannot say which), or that in an ensemble of systems corresponding to the original probability distribution, some will later be found in one macrostate and some in another. In nei-

ther case does one think of an individual system as somehow "split" between different macrostates. We grant that quantum probabilities do not behave in all respects like their classical counterparts. But given the consistency condition selects families of histories whose probabilities satisfy the classical rules (in a mathematical sense), it seems most natural to interpret the "splitting" which takes place inside a consistent family - this seems to be the situation which Everett has in mind - in terms of the classical analogy, or at least there seems to be no reason *not* to do so. (Griffiths, 1984, p.ý64-5.)

The argument is uncertain. Griffiths appeals either to ignorance, hence presumably to a hidden-variable theory, or to the classical model of an ensemble, which again (traditionally) was formulated in the context of an underlying microscopic physics. Griffiths may have it in mind that the microscopic physics is described by the state (with the projections playing the role of characteristic functions of thermodynamic variables). If so there is no analog of the argument from ergodicity. Clearly ignorance of the state is not an option; surely no combinatorial argument applies; what is the interpretation of probability in terms of an "ensemble"?

To see the difficulty consider an ignorance interpretation formulated in terms of an ensemble as an infinite comparison class of histories (cf. Bell, 1987, p.133). One of these is "actual" (but we do not know which), so that the probability that it has such-and-such properties equals the measure of the subset of histories of the comparison class with those properties. The problem with this is that failing a dynamics of "actualization", mysteriously, the entire history of the universe is "selected" by quantum chance, either terminating in "the present" (with respect to which choice of hypersurface?) or extending to all times. When is this history selected? If it is selected at each instant, or according to the decoherence timescale, and this is a fundamental physical process, then surely the associated conditionalization

$$C_\gamma \Psi / \|C_\gamma \Psi\| \rightarrow P_{\gamma_{n+1}} C_\gamma \Psi / \|P_{\gamma_{n+1}} C_\gamma \Psi\| \qquad (11)$$

is also an objective physical process (state-reduction). Hartle, 1993, indeed appears to accept that state-reduction must be taken as part of the basic physics; but he has not made clear the sense, if any, that this can be maintained consistent with the unitary evolution, or how to avoid the slide to a variant of the GRW proposal.

It would be an unrewarding exercise to consider the literature on decoherent histories further; a discussion of the interpretation of probability is there not to be found. But the problem is elsewhere considered to be extremely serious, as is clear from the lengths that some have been prepared to go to find a solution. There is an analog of the difficulty in the modal interpretations of Van Fraassen, Kochen, Dieks, and Healey, in the purely "mentalistic" approaches of Albert and Loewer, 1988; Squires, 1990; and Lockwood, 1989, and (as we have seen) in the DeWitt interpretation, i.e. in every other interpretation where we have only the unitary mechanics. A common solution has been offered by Deutsch, 1985: an ensemble is to be reified as an uncountable infinity of identical systems for each of the plurality of projections (or Everett worlds) at a given time, already represented in the universal state. This class is subject to a stochastic partitioning, with probabilities exactly matching the corresponding relations in norm.

We have met something similar before (the use of a "meta-time"; one second per second). The hypothesis presupposes a notion of identity over time, so that it makes sense to consider the probability in terms of a chance "passage" from one projection to the next. The infinite multiplicity of the entities involved - which are clearly being treated as "things" - is required in order that every projection at each time is "occupied" (that has something "pass to it"), and so that there may yet be different chances for arriving at one rather than another. What are these things? In the case of Deutsch, they are entire spatial universes (and the projections, as it were, descriptions to be satisfied). The criterion of identity over time is tacit, just because there is no analysis of tense (Deutsch explicitly declares that his model only applies in the non-relativistic case). It has nothing to do with the physics, because all the elements in each partition are physically identical. For Albert, Loewer, and Lockwood these things are "minds" or "perspectives", likewise endowed with a transcendental individuality.[23]

If we are to abandon the relational approach, it is much better to suppose we have a single 3-dimensional universe, passing from one description (projection) to the next, in accordance with the relations in norm. Reviewing the difficulties just discussed, the best that can be done on these lines seems to be something like this: decoherence is indeed to provide a criterion for "actualization", but there is no state-reduction to go with it. The vectors figuring in the process Eq.(11) are only mathematical artifacts. The true state is always the same, and the probabilities are always given by Eq.(5), not by the RHS of Eq.(7) (which only approximately agrees with (5)). This is to give up the eigenvalue-eigenvector link, in the manner of Dieks, but unlike that theory (or any modal interpretation using completely fine-grained projections, e.g. "value-states"), we only have value-definiteness for the coarse-grained projections; we cannot relativize to these, to get the microscopic ontology, because that would be to take seriously the reduced states and the reduction Eq.(11). It is by calling these artifacts that we do not have to account for state-reduction. The price is that although there is a universal state, there is no microscopic reality, and there is a promissory note to be made good: the classification of quasiclassical domains, and the selection of a single one of these on the basis of some principle as yet unknown.

And there is the further difficulty; need this "actualization" be covariant? As a fundamental physical process, it would seem that it should, and that we should make use of the GRW notion of "stochastic covariance",[24] with all the difficulties that follow in its wake; that in consequence, relativistic quantum field theory must be substantially reworked, as Ghirardi *et al* clearly recognize. How is it that this problem is so severe in the GRW context, if it can be ignored in its entirety in the decoherent histories framework? It could be replied that covariance is required only of the universal state, that there is no need for the "actualization" to respect the fundamental space-time symmetries; but increasingly what is the "real" physics (governing what becomes "actual") has little in common with the physical principles otherwise deemed fundamental. The really important thing, on a non-relational approach, the "actualization", on the contrary arises when the exact values of enormously small numbers, as determined by a quasiclassical domain, pass certain non-zero thresholds, varying from one phenomenon to the next.

The more "real" one history over the others, the more "real" the stochastic process by which it unfolds; the more reason to suppose that the fundamental physics should concern this stochastic process. Conversely, if the everyday notion of reality is a matter of facts as relations, and relations among these in the Hilbert space norm; if further what is the fundamental object of physics is the unitary system in which they (all of them) are embedded; then it is to this that the Hilbert space symmetries attach. Systems of correlations within this can be better or worse specified, in terms of effective equations.

In the analogous case of time, the more "real" one hypersurface over the others, the more "real" the flow of time; the more reason that the fundamental equations should make use of this global time parameter. On a relational reading, what previously appeared fundamental is seen rather as parochial or interest-relative, a matter for an analysis FAPP, but quite possibly involving a great deal of complexity, whereas before there was a supposedly "simple" fundamental concept.

7 Prognosis and Summary

To summarize, the problem of measurement appears to parallel the problem of tense in some if not all of its most significant features. But the latter problem has been widely disregarded by physicists and philosophers alike, indeed, a relational approach is on the whole the orthodoxy. If we take a similar approach in quantum mechanics, the intuitive objections that are raised are exactly those which are already undercut in the treatment of tense. The two are partners in guilt; if the one is satisfactory, then so is the other.

The accounts run in tandem as follows. A relational approach requires:
(1) A relativization of "the actual", as a transitive asymmetric relationship between one fragment ("target") of the universal state and another (the Everett relative state), in parallel to the relativization of "the now" as e.g. an intransitive symmetric relationship ("spacelike") between one fragment of space-time and another (events).
(2) A specification of the target of relativization, if what results as value-definite is to be quasiclassical (the "preferred-basis" problem).
(3) An interpretation of time-like relations in the Hilbert space norm, between targeted projections and their relative states, as "probability", in parallel to the interpretation of time-like metrical relations between events as "time".

A relational approach to tense in the classical context brings with it supplements to (1) and (3):

(1T) Ontological excess: we pass from one 3-dimensional world to a (potentially infinite) collection of worlds.
(3T) Identity over time is to be made out at the level of structural relations between events, supplemented (if necessary) by psychological or mentalistic considerations.

If there is substance to the appeal to psychology (so that something "essential" to the concept of time is to be reduced to the mental), or if certain pathological space-times or regions of space-time (hence temporal structures) are to be ruled out on anthropocentric grounds (e.g. that there exist no closed time-like curves), then there is a supplement (2T) of (2) as well: the target of relativization had better be a "judgmental awareness", as Grnbaum has it, or some system which otherwise precludes the pathology at issue.

If we now consider quantum mechanics, we find:
(1Q) Ontological excess: we pass from one 3-dimensional world to a (potentially infinite) collection of worlds.
(3Q) Identity over time is to be made out at the level of structural relations between projections, supplemented (if necessary) by psychological or mentalistic considerations.

The difficulty to do with identity over time arises already at the level of bifurcation of observer-histories, regardless of the relational approach to tense. But equally, it arises in relativity, regardless of the relational approach to value-definiteness. In both cases, formal criteria of adequacy are all that figure explicitly in the physics. In the case of time, we have the space-time metric, including the light-cone structure, and "special" constraints, such as the non-existence of closed time-like lines. In the case of probability, we have the Hilbert-space metric and Mackey axioms, including quantum mechanical analogs of the Bernouilli theorems (Finkelstein, 1963; Hartle, 1968; Ochs, 1977; Fahri et al, 1989), and "special" constraints, in particular medium decoherence.

I have argued that the relational interpretation of probability has been made much more difficult by the neglect of the problem of tense, and the meaning of identity over time. As far as goes physics, we take our cue from Everett: having given a derivation of sorts of the Hilbert-space norm as a "natural" candidate for a notion of "measure" (this was made redundant by Gleason's theorem), Everett points out

> The situation is fully analogous to that of classical statistical mechanics, where one puts a measure on trajectories of systems in the phase space by placing a measure on the phase space itself, and then making assertions (such as ergodicity, quasi-ergodicity, etc.) which hold for "almost all" trajectories. This notion of "almost all" depends here also upon the choice of measure, which is in this case taken to be the Lebesgue measure on the phase space. One could contradict the statements of classical statistical mechanics by choosing a measure for which only the exceptional trajectories had nonzero measure. Nevertheless, the choice of Lebesgue measure on the phase space can be justified by the fact that it is the only choice

> for which the "conservation of probability" holds, (Liouville's theorem) and hence the only choice which makes possible any reasonable statistical deductions at all. (Everett, 1957, p.321.)

Everett's approach to probability has been repeatedly condemned as question-begging or otherwise inadequate (see e.g. Graham, 1973, p.236; Kent, 1990, Maudlin, 1994, p.5), but I think we do better to recognize that there can be no such thing as a "proof" that the Hilbert-space norm can (in some circumstances) be interpreted as probability, no more than we can "prove" that the Minkowski space metric can be interpreted in terms of space and time. For what is desired here? It seems nothing other than this, that the theory in question can be reformulated in terms of some other - where this other theory sets the standard of intelligibility - but that this has nothing to do with the question of empirical adequacy. For if it is empirical adequacy that is at stake, then let the objection be posed as such.

I equate empirical adequacy with purely formal criteria of adequacy, and I suppose that some of them have just been advanced by Everett. Any other desiderata will ultimately amount to an a priori or philosophical theory of probability. As such it must also address the concepts of change, identity over time, and (perhaps) issues in psychology and the philosophy of mind. Exactly the same is true of space-time theory in Minkowski's sense.

It is significant that all of these concepts are also in question due to difficulties arising in the framework of quantum theory and general covariance (the so-called "problem of time"[2]); my point is not to trivialize the difficulties, but to indicate their depth and importance; whether to physics, philosophy, or (as seems most likely) to both. Here our aim is much more modest; the first issue is the comparison with tense in Minkowski space.

I see no reason to suppose that the relational interpretation of probability is in any worse shape than that of time. If the relational approach is held adequate in this field, the appeal to intuition in the context of probability is effectively neutralized (in whatever sense it is neutralized in the context of tense); conversely, if a non-relationalist account of tense is reconcilable with the symmetries of Minkowski space, then we will have learnt better how to deal with the Hilbert-space symmetries in the context of probability.[25]

I shall close with a final comment on the basic analogy. Consider once again the question of "ontological excess" involved in the relational approach, the sense that we must admit something more to the world than the now (tense) and the actual (probability). In both cases I hold we do not have an adequate grasp of the collective ("every instant", "every possibility") at issue, as both DeWitt's picture and the objection to it (based on energy conservation) make clear.[11] Related to this, however, there is the more tractable question of epistemic access; whatever the ontological status of the entities in question (space-time, the universal state), there is surely a contrast at the level of epistemology; the universal state is almost entirely, and as a matter of principle, inaccessible to any direct observation. Surely the same is not true of the large-scale structure of space-time; surely, but for this, Everett's proposal would have met with a very different reception.

To respond to the substance of the objection, the key question concerns indirect test. In one sense the response is positive, for any experimental violation of quantum mechanics will count against relativism. But this is a weak basis for the differentiation of a relational interpretation from others. It is likely to do for the GRW proposal, with its new fundamental constants of nature; and it may yet distinguish between relativism and the Bohm theory in the relativistic regime (if and when a non-trivial relativistic Bohmian mechanics is constructed); but it is hard to see how relativism can be distinguished from the Copenhagen formulation of quantum mechanics on this basis. The most that can be done is to develop applications that are simply outside the scope of the Copenhagen interpretation. In view of the vagueness of the latter, concerning the question of what is to count as an "observation", it would seem the question will only take on a concrete form when we have testable predictions in quantum cosmology.

But the more immediate question for our purposes concerns the parallel with tense; for how is it, if tense and value-definiteness are so intimately linked (as I have claimed), that the epistemological difficulties are so severe in the one case, and not in the other?

Consider the following objection: there is an important disanalogy in the reification of past and future times, as compared to the reification of alternative possibilities. For whereas any particular hypersurface will do for the complete specification of Cauchy data for a (classical) space-time model (therefore any specification of "the now"), so that from this the "block representation" can be constructed, that is not the case for a relational approach to value-definiteness. Given the relativization of the universal state to a global configuration, one cannot recover its relativization to others at that time. In classical theory the future will be encountered in the course of time, and even if the past may not it once was; but in quantum mechanics other present possibilities not actual (for us) never were and never will be actual (for us); correspondingly, they do not figure in the equations for (our) relativized state.

As it stands the objection begs the question; according to the relational approach, in the classical case it is not "we" who shall witness future events, for this smuggles in the notion of identity over time. What is true is that they can be predicted (and observations predicted) in the relationalist sense. But now the same can be said of future possibilities in quantum mechanics; we say, correctly, that such-and-such an event has such- and-such probability, meaning in relation to some other (typically what is "now actual"). The only difference is that we cannot straightforwardly obtain data relating projections one of which represents an alternative to what is "now actual".

This does not mean that there are not indirect means of obtaining such descriptions; insofar as we can retrodict to some past global projection, we can also obtain information on the universal state subject to different histories bifurcating at that projection. That is exactly what we do, crudely, when we consider what would have happened had such-and-such happened. The situation is indeed very similar to the epistemology of space- time theory taking into account the restriction to place; we describe what is now space-like only by retrodicting to previous times and then predicting from the latter events. (Indeed, the same applies to descriptions of future events.)

The great difference between the two cases rests exactly on the notion of identity; that we are prepared to count all past and future observers as part of our epistemic community, or at least those not too remote in time, whereas we are not prepared to count all possible observers in this way, not even those not too improbable in terms of the Hilbert-space norm. But if we give up the notion of personal identity it is not so obvious that we should not count them so; after all, by the Bernouilli theorems those with records of relative frequencies of experimental outcomes not approximately equal to the quantum mechanical probability will have vanishingly small probability (in the relational sense).

But of course we cannot, in any simple way, give up the notion of personal identity. At the very least we must put in place the ersatz criterion of structural continuity over time. Involved in this is also the notion of signaling or information transfer; what is wrong with an epistemic community of mutually exclusive observers is that they cannot communicate with each other. This point needs to be understood in the more general context of a restriction to place, for it is also true that observers at space-like regions cannot signal to each other; they can however signal to a third region of space-time, time-like to both. It is the analog to this which is ruled out by the medium decoherence condition, at least in the case of deterministic signals (i.e. where the relations in norm between records at different times are approximately one).[26]

Evidently further pursuit of this question will require a much more systematic discussion of the criteria that motivate medium decoherence in the first place; it is clear that on any evolutionary approach to the specification of a decoherent history space, constraints on what is to count as an information processing system are also constraints on what can reasonably be understood as an "epistemic community". In other words the objection must be ceded, but the epistemological contrast at issue is actually built into the theory *ab initio*, as constraints on information transfer and stability; if we are to live in Plato's cave, at least we can understand how it is that we are confined there.

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Endnotes

1. Deutsch, 1986, p.244; Lockwood, 1989, p.283. The analogy has also been hinted at by Putnam, 1981, p.410; Geroch, 1984, p.624-7; Stein, 1984, p.644-6; and Maxwell, 1985 p.27-28, but only in passing. Davis, 1977, argues for a "perspectivalist" approach to quantum mechanics, and compares the situation to the choice of an inertial frame in space-time theory; but the latter goes back to early writings of Bohr, and insofar as perspectivalism denies the existence of an objective totality (compare Healey, 1984, p.601), this is not our thesis here.
2. General covariance (or "reparameterization invariance") is another matter; nothing in what follows speaks to the so-called "problem of time" in quantum

gravity (see e.g. Kuchar, 1992), although among quantum cosmologists some variant of Everett's approach is quite common. There is also reason to think the decoherent histories approach offers new leverage on the problem; see e.g. Isham and Linden, 1994.

3. This has given rise to a general approach to the definition of the preferred basis; I refer to Dieks, 1989, and references therein. In certain cases this basis yields decoherence, in others not (see Gell-Mann and Hartle, 1983, p.3356 and references).

4. The proofs involves simple manipulations of the trace, and identities of the form $I = Tr((P_{\delta'} + P_{\overline{\delta}'})P_\delta \rho P_\delta (P_{\delta'} + P_{\overline{\delta}'}))/Tr(P_\delta \rho)$.

5. For a simple model which brings out the essential feature, see DeWitt, 1993. Decoherence of e.g. a small particle of dust, coupled only to the microwave background, is already quite fast, of the order of nanoseconds (Joos and Zeh, 1985).

6. After Gell-Mann and Hartle, 1990. The set of decoherent history spaces is partially ordered under coarse-graining. Aside from the trivial cases (where the histories essentially track the universal state), there is always a lower bound (but it does not follow that this poset is a lattice, hence that quasiclassical domains exist).

7. Global criteria of decoherence, which do not require a system/sub-system distinction, are also familiar from thermodynamic approaches to the measurement problem (ranging from e.g. the DPL theory of Daneri et al, 1962, to Hepp, 1972, and variants on this). This program makes clear the limitations of decoherence theory; there is always a residual coherence, given that the dynamics is implemented as a smooth algebraic automorphism, unless the class of observables considered is "suitably" restricted. The specification of a quasiclassical domain amounts to such a choice, but even then coherence is not entirely eliminated save in the "trivial" cases remarked on above.

8. Saunders, 1994; for this Def must be defined using a time-ordering operator, taking into account past-directed transition probabilities as well (see below). We then obtain a coordinate and basis independent relation which is transitive, even in the case of zig-zag histories.

9. Consistency conditions of the following form can be easily checked: if $P_{\delta'_k}$ is value-definite relative to P_{δ_k}, for which a history C_γ is also value-definite, then C_γ is also value-definite relative to $P_{\delta'_k} P_{\delta_k}$.

10. For further discussion, see my 1993a; reintroducing a tensor-product structure, the natural consistency condition, that relations in norm for a "1-observer" history $1 \otimes P_\beta$ equal the sum over α of norms for compatible finer-grained histories $P_\alpha \otimes P_\beta$, where α specifies the environment (and perhaps other observers as well), follows trivially from medium decoherence

11. The "plurality" is the universal state, and physical operators are linear, not additive, over its components. The sum of orthogonal projections corresponds to the quantum disjunction; for a complete set we obtain simply the identity, projecting out the universal state.

12. The situation may be different with respect to identity over time and the sense of time's "passage".

13. Using (ii) of Section 2, Def(atom/probe) & Def(probe/apparatus) $\Longrightarrow$ Def(atom/apparatus), so long as the projections decohere, where it is implicit that Def(apparatus/us).
14. It is because models of the measurement process are enormously idealized that the projection postulate (or Lüders rule) is called into question (e.g. in "demolition" experiments). Were the full Hamiltonian used, the demolition would figure explicitly in the equations, and relativization (using the projection postulate) would faithfully describe the fragments, if any, that remain. The artifact is not the relativization, but the idealization.
15. This can also be given an evolutionary gloss, cf. the remarks of Section 3.
16. Non-trivial decoherence typically requires center-of-mass variables, or more generally, quasi-hydrodynamic variables involving local integrals over conserved quantities (Gell-Mann and Hartle, 1993); whatever the options here, the key point is that it has nothing to do with any choice of the experimenter (or design of the apparatus or whatever). If any selection is involved, it is at level of evolutionary desiderata (Section 6); the preferred-basis is determined from the "bottom up", not in the "top-down" sense of Bohr.
17. Everett's notion of "memory-configurations" does not of course provide a criterion of individuation over time, and applies in the case of branching as well.
18. If there is a perception of "nowness", even understood as an "illusion", there must still be something which is "now" (Shimony, 1993, p.277-8).
19. The connections with Kant's treatment of "existence" as a predicate is strongly endorsed by Shimony, 1993, p.274, although I do not think the implication is quite what he thinks. See further Zelicovici, 1987, p.189; there is also a link with Albert's attribution of "belief" to entanglements of states of "belief" (Albert and Loewer, 1988; Albert, 1992, p.126-7)), i.e. this is the same as attributing "now" to every event in space-time, hence to space-time as a whole (the analog of the entanglement).
20. An idea of the implications of this move can be gained from the impact of Parfit's arguments (Parfit, 1984) on the basis of "split-brain" scenarios. If personal identity amounts to psychological continuity over time, then why should we care for our own future, rather than that of someone else similar? In a well-known criticism of Everett, Bell, 1987, p.136, makes essentially the same point, ignoring the same dilemma as it occurs in a relational account of time, or in any philosophy of personal identity which tries to accommodate the split-brain scenario.
21. For Gödel's (unfavorable) comments on this strategy see his 1949, p.562. In the case of quantum mechanics, compare Zeh: "it would be insufficient simply to *require* decoherence just because it leads to classically plausible consequences (such as 'consistent histories')" (Zeh, 1993, p.191).
22. This is what is wanted on an essentialist (non-relational) view of time; pictures like this have often been urged in the literature (see e.g. McCall, 1974; Belnap, 1992).
23. This is explicit in Albert and Loewer but implicit in Lockwood, 1989. Recently Lockwood has proposed to renounce this notion of identity (Lockwood, 1992, p.379-80)); as should be evident from the text, I challenge that there is

then anything left for the stochastic process to do, thence neither the infinite multiplicity of "perspectives".

24. This remains an outstanding difficulty of the GRW approach; for alternative attempts to Pearle, 1990, see Rimini, 1992, and Dowrick, 1993. Which if any can be followed through remains an open question.

25. The problem has been broached by Putnam, 1967, and Maxwell, 1985, both of whom have been strongly criticized by Stein, 1968, 1991. The controversy continues in the pages of *Philosophy of Science*, but has yet to show focus. Stein does not recognize, and Maxwell does not emphasize, that the criterion for what is past may not be anthropocentric, on a non-relational account of tense.

26. The analysis of probabilistic signalling is at an early stage, and may yet yield surprises; see Royer, 1994, for a possible example.

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