# tIME-SERIES FORECASTING TECHNIQUES FOR SCHEDULING OF MULTIPROCESSOR COMPUTER JOBS 

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# TIME-SERIES FORECASTING TECHNIQUES FOR SCHEDULING OF MULTIPROCESSOR COMPUTER JOBS 

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## SUMMARY

In executive-request scheduling for increased throughput in a multiprocessor computer system, choice of a method of forecasting execution times is complicated by the high cost of tracing actual program tasks, by the difficulty of defining and obtaining a truly representative sample of jobs processed by a computer center, by the lack of theory for selecting appropriate forecasting methods for these series that have a special structure reflecting computer programming practices, and finally by uncertainty as to the cost/accuracy tradeoff in using the forecasts in a scheduling algorithm.

Previously, a 'level-reset' forecasting method developed by Young had been found by Raynor to be more accurate and less costly than standard forecasting methods, when the forecasts were used in Raynor's specific scheduling algorithm applied to a very limited sample of real program tasks. The present work extends Raynor's empirical sample, establishes a theoretical basis for forecasting (based on assumptions concerning piecewise constant time series and empirical verification of piecewise constant structure), derives extensions of level-reset forecasting, and empirically compares levelreset forecasting and extensions to alternative forecasting methods. An improved criterion for evaluating forecast errors is derived and applied. A less costly and perhaps more accurate version of Raynor's level-reset forecasting is developed and is recommended as the method of choice for scheduling of multiprocessors.

## CHAPTER I

FORECASTING FOR MULTIPROCESSOR SCHEDULING

Today's computer industry stands at the threshold of a new and exciting generation of electronic computer systems, the multiprocessor computer. In the thirty years preceding 1974, the industry has proceeded from the vacuun tube, through the transistor, to the modern-day central processing units (CPUs) composed of modules of printed circuitry. The result has been a significant reduction in the size of conputer systems, as well as an increase in both efficiency and reliability of such systems. The next logical step is to unite many of these modern CPUs into a complex system linked together by both hardware (physical equipment) and software (supervisory programs, data banks, etc.).

Such a system would have several inherent assets. First, there would be a consolidation of the large data files (subroutines, special libraries, etc.) that would otherwise have been duplicated in the separate system concept. Along with the multiplicity of the CPUs would be the replication of the many peripheral devices associated with a computer system. Such replication (which is being considered on a large scale [8][14][16][37]) would make it worthwhile to maintain an inventory of repair parts and probably an in-house repairman at the facility. This should conceivably reduce the down time on those devices, enhancing the efficiency of the entire computer system. Although $M$ processors cannot do $M$ times as much work as one processor,
cost savings stem from the fact that far less than M times as much peripheral equipment is necessary. The savings are amplified by the fact that the cost of processors has decreased much faster than the cost of peripheral equipment [2].

Efficient design of a multiprocessor system presents challenging difficulties. The most significant is the need to assemble the system in such a way that all components are efficiently utilized. In other words, the jobs to be processed by the system must somehow be scheduled into each processor in such a way that the processors do not interfere with each other's operation. Madnick [23] showed that such interference, called multiprocessor lockout, is indeed a significant factor to be dealt with. For example, with no scheduling algorithm to reduce lockout, it was demonstrated under real operating loads that if there were 15 processors in the system, an average of one would be idle. The reason for this idleness is that the supervisor is busy assigning a job to another processor. The supervisor can schedule only one processor at a time. Any other processor needing the supervisor is put in a queue until the supervisor becomes available. An increase to 40 processors results in 19 idle processors, while 41 processors results in 20 idle. In other words, the $41 s t$ processor has zero marginal effectiveness! (See Figure 1.) Thus, before systems beyond the research level are produced, a scheduling algorithm must be developed to minimize mutual interference among the processors. The first steps have already been taken in this area. Most recenty Pass [28] and Raynor [29] at Georgia Tech have pursued this matter and offer excellent references for the most up-to-date Iiterature such


Figure 1. Idle Processors
sxosseวotd (子noxpot) erpi fo xequnn peqpedxg
as that of Lampson [19] and Sherman, Baskett, and Browne [32]. They also provide valuable initial results from which to continue development and refinement of the needed scheduling algorithms.

One of the necessary assumptions for the algorithm development is the assumption of being able to forecast the times between input and output ( $1 / 0$ ) interrputs. These interrupts characterize the jobs generated by the system's workload. We will use the symbol ER (for executive request) interchangeably with $I / 0$ interrupts, following the terminology employed by the staff of the Georgia Tech computer center. It is not necessary for a program being computed by the system to be completed from start to finish. Instead the program is done in segments (jobs) which are separated by I/O interrupts. Forecasting accuracy was demonstrated to have a definite effect on the amount of work that can be processed through a multiprocessor system. Table l shows such effect when using the Raynor algorithm for scheduling in a multiprocessor environment [29].

## Objective of the Research

Forecasting of the times between successive I/O interrupts is the subject of this research. Certain preliminary results obtained by Pass and Raynor will serve as the starting point for our research efforts. These preliminary results will be discussed in the following chapter as part of the survey of forecasting techniques.

It is the objective of this research to determine to what extent and precision it is possible to forecast times between successive I/O interrupts generated by actual computer programs. It is not enough to say we can forecast, we must know whether or not our

Table 1. Forecasting Errors Effect

| Standard <br> Deviation of <br> Error Distribution* | Average <br> Throughput | Percent Increase <br> in Throughput |
| :---: | :---: | :---: |
| 0 | 6.78 | 10.04 |
| $5 \%$ | 6.73 | 9.24 |
| $10 \%$ | 6.66 | 8.10 |
| $15 \%$ | 6.57 | 6.64 |
| $20 \%$ | 6.57 | 6.64 |
| $35 \%$ | 6.53 | 5.99 |
| $50 \%$ | 6.48 | 5.18 |

*As a percentage of the true value.
forecasts are acceptably accurate and if so at what cost the forecasts themselves use computer time). Forecasts must be timely as well as accurate and efficient; for example, it is useless to forecast if the times between interrupts are smaller than the time it takes to forecast. In such a case the answer would arrive too 1 ate to be of any value.

Summary of the Chapters
Chapter II will present a survey of the literature as to the types of forecasting techniques currently employed today with emphasis on some of the results of Pass and Raynor. Chapter III will explain the specific techniques of forecasting that were examined. Also included will be a section on how the actual time series were generated, for the question of what kind of series best represents actual workloads at an operating computer center remains unresolved. Chapters IV and $V$ will present the results and conclusions of the research and suggestions for further research.

## CHAPTER II

## SURVEY OF THE PREVIOUS RELATED WORK

Many examples of forecasting systems are found in the literature. Most of the current literature is concerned primarily with forecasting systems that have evolved from the basic writings of Brown [9] on moving-average and exponential smoothing techniques and Box and Jenkins [7] on linear filtering. Many efforts have been made to extend these techniques for more powerful use in specific applications in industry and business [5][15][18][31].

## Need for Self-Adaptive Systems

In the context of the technical literature in forecasting, to forecast means to assign estimates of future values--forecasts--of a randon variable whose values are assumed to constitute a non-stationary stochastic process. Forecasting systems vary as to what information is formally taken into account and as to the assumed structure of the stochastic process, but many forecasting techniques may be viewed as including a smoothing constant, $0 \leq \alpha \leq 1$, or its equivalent.

The choice of smoothing constant chosen is extremely important since regardless of the model chosen, the ability to detect changes in the time series depends on the value of $\alpha$. If the constant is large, say close to one, more weight will be placed on the more recent observations. When it is close to zero, it will give more weight to the historical data. Exponential smoothing also requires an initial value
of the smoothing statistics to start the smoothing process. Much of the iiterature concems development of an adaptive technique, a system to adapt to changes in the time series and to correct for an improperly chosen initial smoothing constant. Wichern [36] at the University of Wisconsin showed that even when the proper model is used for a given time series, if an improper value of $\alpha$ is chosen, the variance of the forecast errors will be significantly underestimated. The result is not only to fail to minimize the variance of the forecast errors, but also to fail to get an accurate estimation of the actual variance.

## Review of Some Self-Adaptive Systems

Let us now examine some systems that have been developed to try to deal with this problem of smoothing parameters. Such systems are called "self-adaptive" in that they examine themselves and make the appropriate change in the smoothing constant when the system appears not to forecast the monitored time series adequately. This often occurs when there is a large change in the underlying stochastic process. If the forecasting parameters were fixed it might take an unacceptably long time for the system to readjust itself. Box [5][6], in his articles on evolutionary operations (EVOP) proposed a method of using a factorial experimental design such as that used in response surface analysis to determine when and how to modify the independent variables of an experiment or process to obtain a desired change in the dependent variable. Such a method consists of setting up the design in such a way that the effect of changing each variable can be determined and action taken according to established rules.

Roberts and Reed [30] developed a self-adaptive forecasting technique (SAFT) which combines exponential smoothing with a response surface analysis technique to test the forecast accuracy of various smoothing parameters in a forecasting model. The technique is a specific application of Box's evolutionary operations technique.

Chow [12] proposed a technique of establishing a high, normal, and low value of the smoothing constant to be utilized in the exponential smoothing technique. The constants are initially chosen arbitrarily, but are modified as the time series progresses. Whenever, on the basis of an error criterion, one of the "outer" forecasts turns out better than the normal forecast, the next period's forecast is made based on the new "best" value. At the same time new high and low values are introduced around the reset normal value. This is in reality a one-parameter version of the evolutionary operation design of Roberts and Reed.

Montgomery [25] has also used an evolutionary operation scheme for an adaptive forecasting system. However, he proposed the use of an orthogonal, first order experimental design called the simplex. His procedure involves the changing of the exponential smoothing parameters each pexiod by the sequential application of the simplex design. A new simplex is determined each period by deleting the worst parameter combination (that which gives the worst forecast error) and creating a new point according to fixed relationships. These relationships generally create a point geometrically opposite of the deleted point. An example in two-space is shown in Figure 2.


Figure 2. Montgomery's Simplex Design for Forecasting

Brown [9] proposed the use of either the tracking signal or the mean absolute deviation (used as an approximation of standard deviation) of the forecast errors as the criterion for monitoring the forecasting technique to determine when it goes out of control. The tracking signal is the sum of recent forecast errors, which, if the system is under control, should oscillate around a mean value of zero. If the signal significantly moves away from zero, the system is to be considered as out of control and corrections to the parameters are made.

Burgess [11] proposes an automatic adaptive system using the tracking signal as the out-of-control indicator. The smoothing parameter is defined as $\alpha=1 /(1+M)$ where $M$ is the number of time periods to the midpoint of an exponentially smoothed moving average. For each period that the system is in control, $M$ is incremented by 1 up to a value of $M=20$ (which corresponds to $\alpha$ of approximately .05). This heavily weights historical data when the system is in control.

When the system goes out of control, a constant value is subtracted from the current value of $M$. This effectively increases the value of $\alpha$, putting more weight on the most recent information.

Trigg and Leach [35] proposed a method of equating the smoothing constant to the modulus of the tracking signal.

Pass [28] used a modification of double exponential smoothing which used a relative error ( $e_{t-1}$ ) and a threshold value ( $\tau$ ) as the means of determining when the system is out of control.

$$
\begin{equation*}
e_{t-1}=\frac{\hat{x}_{t-1}-x_{t-1}}{t_{t-1}} \tag{1}
\end{equation*}
$$

where $\hat{x}_{t-1}$ is the forecast of the actual observation $x_{t-1}$. If $e_{t-1}$ is greater than $\tau$ and the sign of $e_{t-1}$ is the same as the sign of $e_{t-2}$, it is assumed that the system was not responsive enough; $\alpha$ is changed by a small fixed increment according to appropriate rules.

Raynor [29] used a similar measure of error, but did not use it as a means of updating $\alpha$. Instead, when it was determined that the system was out of control, the smoothed value used for the next forecast is reset to the value of the most recent observation. This is an example of the level-reset class of methods to be discussed in Chapter III. In equations we would write:

$$
\begin{aligned}
\hat{x}_{t} & =\alpha x_{t-1}+(1-\alpha) \hat{x}_{t-1} \\
& \text { when } \frac{\left|x_{t-1}-\hat{x}_{t-1}\right|}{x_{t-1}}<\tau \\
& =x_{t-1}
\end{aligned}
$$

We are in effect setting $\alpha$ equal to one when out of control and equal to a predetermined value when in control.

## Results of Raynor's Research

Results of comparison among Raynor's, Pass', current-observation forecasting (Raynor's with $\tau=0$ ), and double moving average techniques indicated that Raynor's method surpassed the others in forecasting the times between ERs. Table 2 is from Raynor's work.

Table 2. Forecast Technique Comparison

| Forecasting <br> Technique | Average Percent of Forecasts <br> within $+15 \%$ of the Observation |
| :--- | :--- |
| Double Moving Average | 43.0 |
| Pass' Method | 44.5 |
| Raynor's Method | 74.4 |
| Current Observation <br> $\left(\hat{x}_{t}=x_{t-1}\right)$ | 62.5 |

This result is not unrealistic. It is not surprising that the $\tau=0$ version of single exponential smoothing, which is merely currentobservation forecasting, did well. Computers are built to handle repetitious data. The routines that accomplish this digestion contain loops which tend to cause times between ERs to form an approximately constant series with jumps from one level to another as we proceed from one loop to another. Raynor's results suggest our research should include methods of adapting a constant forecasting scheme that resets data to the new level when the process is out of control.

With this method we hope to reduce the time it takes for our forecasting system to reset to the new level and thus increase forecasting accuracy.

We will, therefore, concentrate on a constant model and utilize techniques to determine when to reset to a new level. Methods for adapting both single exponential smoothing and moving average will be tested. Moving average will be discussed more fully in the next chapter.

## CHAPTER III

## DESCRIPTION OF THE RESEARCH

Raynor's work [29] showed that there exists at least one scheduling algorithm, using forecasts of times between successive I/0 requests, that is capable of significantly increasing throughput in a multiprocessor computer system. For his scheduling algorithm, which considered CPU time in rather coarse blocks of $200 \mu-\mathrm{sec}$, several forecasting methods were found to perform adequately. He reported a version of "level-reset" forecasting as both lowest-cost and highestbenefit for the programs he ran and the scheduling algorithm he used, but two important considerations were beyond the scope of his study. First, Raynor did not make a systematic study, either theoretical or empirical, of appropriate forecasting methods, and second, his sample of programs was so small as to leave in doubt whether they were typical of prograns submitted to a computer center.

The present research attempts to make a systematic study of available forecasting methods for times between successive $I / O$ requests. It was hoped the results would (1) either provide a better forecasting method or verify Raynor's selection, and (2) provide additional samples of typical I/O-request time series. This work should be useful for scheduling by any method (Raynor evaluated forecasting methods only as applied to his own scheduling algorithm).

The research consisted of three parts: (1) data generation from typical programs submitted to the Georgia Tech computer center,
(2) theoretical work to derive appropriate forecasting techniques, and
(3) evaluation of the forecasting methods.

## Data Generation

All the electronic calculations for this research were carried out on the Univac 1108 computer. Within the Univac System Library, there exists a program trace routine called SNOOPY. SNOOPY provides an account of every instruction executed and its effect. Univac affiliated programming personnel are familiar with this trace routine and are capable of modifying the routine's output in several ways.

Figure 3 below is representative of the type of information that may be generated as output by SNOOPY. The first line of output indicates that a command from the program called TEST1 is beginning to


Figure 3. SNOOPY Output
be processed (traced) by SNOOPY. The line could be an equation, logic statement, or any other FORTRAN instruction. The second type of output line is one that represents a breakdown of the first line into computational jobs such as addition or subtraction. For example, the equation $Y=X * * 2+2 * W * X+W * * 2$ would be broken down into six jobs of exponentiation, addition, and multiplication. This type of output is expressed as the second underlined line in Figure 3. Under each of the two previously mentioned outputs are found a third type (numbered 3) which indicates every individual step the computer goes through to solve the problem it is given. Output that would normally result from the program being traced is separated from the SNOOPY output by a dashed line (------) above and below. By examining the type-one or type-three lines, the researcher can determine how far SNOOPY has progressed through the traced program. The final line in the figure is representative of that output generated when an ER is initiated by the computer.

All of the output mentioned can be turned off by program modification of SNOOPY. This can be done by sending the information to a subroutine to be analyzed rather than to memory to be printed in the output, or by simply flagging the output so that it is not routed to any location. In the present research, a subroutine was written to examine each line as it was sent to determine the time it took to execute each instruction. The times are determined according to specific rules found in the Exec 8 Handbook distributed by Univac. A running total of time is maintained until an ER line is sent. The tine on hand is then printed and the running total reset to zero to begin the process again until the next ER. This continues until the
program being traced has completed its run or the maximum allowable computation time on the computer has been reached.

The exact method of setting up a program for the use of SNOOPY is found in Appendix 1A. A copy of the subroutine used is found in Appendix 1 B . A copy of $S N O O P Y$ is too lengthy to be contained herein, but is contained in the Univac Executive 8 Library.

The system of routines and subroutines offers an excellent means of obtaining accurate times between $I / O$ interrupts. However, the necessity of screening a line for many possible values and the movement of logic into and out of many subroutines utilizes large quantities of CPU time. As a result, one must have access to large amounts of CPU time for at the maximum run time all computations cease whether or not the process is completed. Thus one must be careful to insure enough run time is used to complete at least one full cycle of the program as a minimum and to insure that an adequate number of times are generated. This generation of an adequate number of times is important for the proper analysis of any forecasting technique that is proposed. In general, one should attempt to get a minimun of 100 times in the series. With less data, it would be presumptuous to speak of analyzing its structure as a non-stationary stochastic process.

## Piecewise Constant Time Series

Multiprocessor computer systems are designed for flexible simultaneous handling of many computing jobs submitted by many users, such as is the situation at large university computing centers. Experience shows that the available job mix is generally dominated
by tasks from "large" programs full of repetitive "number crunching" [22].

Large prograns exhibit a strongly repetitive structure consisting of loops, in each of which an identical set of instructions is executed many times. The most commonly encountered loop structure contains one executive request in each execution of the loop (for example, one READ statement or one WRITE statement), and uses approximately a constant time for the execution between successive requests. This motivates the piecewise constant structure of the series of execution times expected in processing a program.

Variations among the successive execution times in a single loop are generally of two distinctive kinds. There are small highlyautocorrelated fluctuations caused by very small variations in the time required for each arithmetical, logical or transferral operation. These variations are dwarfed by program logic variations within a loop, which are also usually highly autocorrelated and which can range from less than $1.0 \mu-s e c$ to any amount whatsoever. Conditional control transfers (IF statements) are the most commonly encountered program logic variations found within a loop. The computation time between two executive requests varies anywhere from less than $1 \mu-\sec$ up to about $10,000 \mu-\mathrm{sec}$, but the variability cannot be shown to increase significantly with computation time. This independence of variability and level has convenient implications in choosing forecast parameters. Its cause is apparently that the main difference between a longer interval between $I / O$ statements and a shorter interval is that the longer interval is packed with more number crunching of almost zero
variance. In other words, this phenomenon is apparently an artifact of programing practice.

The following arguments are adapted from Young [39].
Let us postulate a piecewise constant time series, in which each observation $x_{t}$ is either (Event A) a further observation from the current constant process whose mean is $\mu_{o}$ or (Event A') the first observation from a new constant process whose mean is $\mu_{1}$. We assume that the standard deviation of $x_{t}$ under Event $A$, denoted $\sigma_{A}$, is far smaller than $\left|\mu_{1}-\mu_{0}\right|$, i.e., that the variation of observations in any one single constant process is far smaller than the variation of observations from two different processes.

In forecasting a piecewise constant series there are obviously two separate kinds of error: ordinary forecast errors (A-errors) within a single process and much larger process-change errors (A'errors) incurred when the process changes levels from $\mu_{0}$ to $\mu_{1}$. From our assumption $\sigma_{A} \ll\left|\mu_{1}-\mu_{0}\right|$, we see that avoidance of $A^{\prime}-$ errors is paramount, and hence that standard methods such as exponential smoothing, moving average and linear filtering will incur large errors. In fact, exponential smoothing forecasts with smoothing constant $\alpha$ will incur a total $A^{\prime}$-error approaching $\left|\mu_{1}-\mu_{0}\right|(1-\alpha) / \alpha$ in the first few forecasts after a change in level from $\mu_{0}$ to $\mu_{1}$, and moving average forecasts of length N will incur a total $\mathrm{A}^{\prime}$-error approaching $\left|\mu_{1}-\mu_{0}\right|(N+1) / 2$. This is easily seen by referring to Figure 4, where denotes an observation with the smaller A-error suppressed and $\square$ denotes a forecast calculated one period earlier:



Exponential smoothing with $\alpha=.6$

Moving average
with $N=3$

Figure 4. A'-errors in Forecasting a Piecewise Constant Time Series by Exponential Smoothing and Moving Average

To reduce the large $A^{\prime}$-error in forecasting a piecewise constant time series to its theoretical minimum of $\left|\mu_{1}-\mu_{0}\right|$, which corresponds to imnediate recovery, we can set $\alpha=1$ in exponential smoothing or set $\mathrm{N}=1$ in moving average forecasting, in either case obtaining the simple forecasting method $\hat{x}_{t}=x_{t-1}$, i.e., the forecast calculated for time $t$ equals the observation obtained at time $t-1$. Raynor [Ref. 29, page 112] found this method to outperform all others for multiprocessor scheduling except the level-reset method to be described below.

A natural extension, after reducing $A^{\prime}$-error to its theoretical minimum, would be to attempt to reduce A-error without sacrificing the feature of immediate recovery from a process level change. From our assumption $\sigma_{A} \ll\left|\mu_{1}-\mu_{0}\right|$, we can almost always distinguish whether an observation $x_{t}$ signals Event $A$ or Event $A^{\prime}$; when $\left|x_{t}-\hat{\mu}_{o}\right|$
is small enough to be comparable to $\sigma_{A}$, Event $A$ is likely, otherwise Event $A^{\prime}$. (Here $\hat{\mu}_{0}$ represents the current estimate of the process level.) If Event $A^{\prime}$ is indicated, the next forecast should certainly be $x_{t}$, which is the best and only estimate available for the new Ievel $\mu_{1}$; on the other hand, if Event $A$ is indicated, we are free to forecast by any appropriate method that assumes continuation of a constant process. Thus a promising class of forecasting methods for piecewise constant series includes all those constant-model methods that reset the level of the forecast when an outlying observation is received. Members of this class $c$ an be called level-reset methods. Level-Reset Forecasting

Level-reset forecasting differs from the variety of useful methods that dynamically adjust the smoothing constant. The latter methods apply especially well to highly autocorrelated series that exhibit changes in variability, and they focus mainly on reacting to changes in the relative sizes of permanent and temporary errors. By contrast, level-reset forecasting is specifically intended for piecewise constant time series, in which permanent errors are far larger than temporary errors. Application of both methods to a piecewise constant series is shown in Figure 5. On the left, the levelreset method forecasts the new level after a large change. On the right, following Brown [Ref. 9, page 296, and proprietary IBM forecasting software], $\alpha$ is reduced after two successive outliers, accelerating the recovery. Of course, the simple forecast $\hat{x}_{t}=x_{t-1}$ is a special case of both methods.

The level-reset forecasting method is as follows:


Figure 5. A'-errors in Forecasting a Piecewise Constant Time Series by Level-reset and by Dynamic Adjustment of the Smoothing Constant

$$
\hat{x}_{t}=\left\{\begin{array}{lll}
\alpha x_{t-1}+(1-\alpha) \hat{x}_{t-1}  \tag{1}\\
x_{t-1} & \text { if } & g\left(x_{t-1}, \hat{x}_{t-1}\right)<T \\
\text { otherwise. }
\end{array}\right.
$$

Level-reset forecasting has two parameters: $\alpha$ is the usual smoothing constant used when the process is judged not to have changed levels, and $T$ is a "gate" or maximum error function that represents the highest value of the current forecast error function $g\left(x_{t-1}, \hat{x}_{t-1}\right)$ that is considered not to signal a level change. In the definitions to follow, $g$ is an increasing function of forecast error, and is also normalized so that $T=0$ means "always reset" $\left(\hat{x}_{t}=x_{t-1}\right)$, and $T=\infty$ means "never reset" (exponential smoothing).

There are three forms of the forecast error function $g\left(x_{t-1}, \hat{x}_{t-1}\right)$ of special interest. Raynor [29] and Pass [28] have
used a relative error (or percentage error if expressed in percentage), so that $g\left(x_{t-1}, \hat{x}_{t-1}\right)<T$ in Equation 1 becomes specifically

$$
\begin{equation*}
\frac{\left|x_{t-1}-\hat{x}_{t-1}\right|}{x_{t-1}}<T \tag{la}
\end{equation*}
$$

Relative error is meaningful in the context of using the forecasts for scheduling, but its use introduces a bias that makes the parameter T difficult to choose; as a matter of empirical fact, large relative errors are rare when $x_{t}$ is large and common when $x_{t}$ is small, so that a given value of the gate $T$ camot be satisfactorily related to the probability that an error signals a change in level.

From a probabilistic point of view it would seem more logical to use the relative squared error:

$$
\begin{equation*}
\frac{\left(t_{t-1}-x_{t-1}\right)^{2}}{x_{t-1}}<T \tag{lb}
\end{equation*}
$$

The relative squared error criterion can be justified by assuming the execution time to be a sum of independent execution times. However, computer programing practices seem to favor loops that contain only one or two highly variable statements (such as conditional control transfers), with the remainder being made up of number-crunching statements with very low variance. Thus in actual practice a long loop actually has about the same execution-time variability as a short one, leading to the most truly appropriate error function for forecasting execution times:

$$
\begin{equation*}
\left|x_{t-1}-\hat{x}_{t-1}\right|<T \tag{1c}
\end{equation*}
$$

The experimental work in the present study uses level-reset forecasting with two error functions: that of Inequality la for comparison with previous work, and the more appropriate one of Inequality lc. (The error function of Inequality lb would be applicable for piecewise constant time series in more general contexts, but it is not useful here.)

Evaluation of Forecast Errors
In earlier work [Ref. 28, Ref. 29] forecasts were evaluated directly in terms of the increase in work throughput that was achieved by scheduling based on the forecasts. From Raynor's empirical results given in Table 1, Chapter I, perfect forecasting gave a 10 per cent increase in throughput, "ballpark" forecasting ( 68 per cent of the forecasts falling between half and twice the true execution time) gave a 5 per cent increase in throughput, and of course completely random forecasting would have given no increase in throughput. Such results suggest that the usual evaluation of forecasts on the basis of variance of forecast error is quite inappropriate in this application context. The paradox of variance versus usefulness is illustrated repeatedly in the six actual time series studied herein. The variance depends most strongly on the largest errors whereas the usefulness depends most strongly on the smallest errors.

Figure 6 shows a time series (with A-errors suppressed) illustrating a type-1 pathology which is the commonly occurring case of a piecewise constant time series interrupted by one outlier. The observations (O) are forecast by level-reset ( $\square$ ) and exponential smoothing ( $\Delta$ ); parameters of the level-reset forecast are $0<\alpha<1$,
$0<T<\mu_{1}-\mu_{0},\left|x_{t-1}-\hat{\mathrm{x}}_{\mathrm{t}-1}\right|<\mathrm{T}$; the exponential smoothing constant is $\alpha=.5$; and with the chosen parameters Raynor's empirical results would predict roughly an 8 per cent increase in throughput by either method.


Figure 6. A'-errors in Forecasting a Piecewise Constant Time Series with a Type-l Pathology, Using Level-reset and Exponential Smoothing

Directly from Figure 6 we can calculate the variance of forecast errors, which for the six observations shown is $\left(0+0+1^{2}\right.$ $\left.+1^{2}+0+0\right) / 6=2 / 6$ with level-reset forecasting and $(1+.25$ $+.0625+.015625)=1.33 / 6$ with exponential smoothing. If we compare mean absolute deviations, we get $2 / 6$ for level-reset forecasting and $1.875 / 6$ for exponential smoothing. Since the forecasts were chosen specifically as those yielding approximately equal usefulness, we can conclude that unfortunately neither variance nor mean absolute deviation gives an appropriate measure of forecast usefulness.

Raynor [Ref. 29, page 112] used the average percentage of forecasts lying between 85 per cent and 115 per cent of the true value as his measure of forecast performance. This criterion was apparently selected over variance, over mean absolute deviation, and over other functions of relative error for its ability to rank the tested forecasting methods in the same order as the throughput increases obtained by their use in scheduling. It is uncertain whether this criterion would be appropriate when used in conjunction with scheduling algorithms other than Raynor's. Certainly the bias of relative error, as discussed earlier, suggests that a criterion based on some absolute rather than relative error would be more appropriate. For discrete scheduling in blocks of $W \mu-s e c, ~ a ~ c r i t e r i o n ~ t h a t ~ s u g g e s t s ~ i t s e l f ~ i s ~$ the percentage of forecasts with error less than $W \mu$-sec. Under Raynor's scheduling algorithm, this criterion at $W=200 \mu-s e c$ gives the approximate percentage of essentially perfect forecasts--those where the actual execution time falls within one $200-\mu$-sec block the forecast.

Generally, errors in smaller ranges (see Table 1) should be weighted more heavily in ranking forecast methods than errors in larger ranges. The question of exactly what weights to give to errors in various ranges can be sidestepped, as the actual results reported in the next chapter fortunately rank various methods in the same order for all values of $W$ small enough to provide significant improvements in scheduling (although variance, with its overwhelmingly large weighting of the Iargest errors, gives rankings that differ).

## Description of the Adaptive Systems Tested

The methods tested were based, as mentioned previously, on an adaptive system that resets the past data to the new level (level-reset) of the constant model. Both moving average and single exponential smoothing techniques were modified to do this. Each of the techniques tested under each of the two main categories differ from the other only in the rules by which we determine whether or not to reset to the new level.

## Standard Constant Model Techniques

As a reference point we begin by using a single exponential smoothing technique in which the value of the smoothing constant $\alpha$ is examined at six levels. We use exponential smoothing since we know that the expected value of the smoothed value is equal to the expected value of the coefficient of a constant model (see below). In single exponential smoothing we express the next forecast by

$$
\begin{equation*}
S_{t}(x)=a x_{t}=(1-\alpha) S_{t-1}(x) \tag{2}
\end{equation*}
$$

where $\alpha=$ the smoothing constant

$$
\begin{aligned}
S_{t}(x) & =\text { the smoothed value of } x \text { at time } t \\
x_{t} & =\text { the observation of } x \text { at time } t
\end{aligned}
$$

In general form we have

$$
\begin{align*}
s_{t}(x) & =\alpha x_{t}+(1-\alpha)\left[\alpha x_{t-1}+(1-\alpha) S_{t-2}(x)\right] \\
& =\alpha x_{t}+\alpha(1-\alpha) x_{t-1}+(1-\alpha)^{2}\left[\alpha x_{t-2}+(1-\alpha) s_{t-3}(x)\right]  \tag{3}\\
& =\alpha x_{t}+\alpha(1-\alpha) x_{t-1}+\alpha(1-\alpha)^{2} x_{t-2}+\ldots+\alpha(1-\alpha)^{n} x_{t-n}+\ldots+(1-\alpha)^{t_{x}} x_{0}
\end{align*}
$$

$$
\begin{equation*}
S_{t}(x)=\alpha \sum_{k=0}^{t-1}(1-\alpha)^{k} x_{t-k}+(1-\alpha)^{t} x_{0} \tag{4}
\end{equation*}
$$

That is, $S_{t}(x)$ is a linear combination of all past observations. The expected value of $S(x)$ is shown below.

$$
\begin{align*}
E[S(x)] & =\sum_{k=0}^{\infty} \beta^{k} E\left[x_{t-k}\right]  \tag{5}\\
& =E[x] \alpha \sum_{k=0}^{\infty} \beta^{k}=\frac{\alpha}{1-\beta} E[x]=E[x] \tag{6}
\end{align*}
$$

since $1-\beta=\alpha$.
Since the expectation of the smoothed value is equal to the expectation of the data, we have a method of estimating a value of our constant model.

A moving average of length $N$ is similar to exponential smoothing. In this case rather than weighting the past observations geometrically, the $N$ most recent observations are given a weight of $1 / \mathrm{N}$ and the remaining observations a weight of zero. The moving average is computed as follows:

$$
\begin{equation*}
M_{t}=M_{t-1}+\frac{x_{t}-x_{t-N}}{N} \tag{7}
\end{equation*}
$$

where $M_{t}$ is the current moving average
$M_{t-1}$ is the previous moving average
$x_{t}$ is the current observation
$x_{t-N}$ is the observation $N$ periods ago

## Level-Reset Techniques

Two modifications of single exponential smoothing were developed to determine when the system goes out of control. The first method is that developed by Young (Raynor's best method) which consists of resetting to the new level when the latest observation is outside some specified percentage limit. We express this modification as

$$
\hat{x}_{t}= \begin{cases}\alpha x_{t-1}+(1-\alpha) \hat{x}_{t-1} & \text { if } \frac{\left|x_{t-1}-\hat{x}_{t-1}\right|}{x_{t-1}}<\tau  \tag{8}\\ x_{t-1} & \text { otherwise }\end{cases}
$$

This is the same method derived earlier herein from theoretical considerations assuming a piecewise constant time series, and given in Equation (1) and Inequality (1a). When the system is out of control we wish to reset to the new level and then continue smoothing at some fixed value of $\alpha$ until the system goes out of control again. Table 3 demonstrates this technique with $\tau=.5$ and $\alpha=.1$.

Table 3. Example of SAES Method ( $\tau=.5, \alpha=.1$ )

| $t$ | $x_{t}$ | $\hat{x}_{t-1}$ | UL <br> (upper <br> limit) | LL <br> (lower <br> limit) | In <br> Control? |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\ldots$ | 110.0 | 100.0 | 150.0 | 50.0 | yes |
| 46 | 110.0 | 101.0 | 151.5 | 50.5 | yes |
| 47 | 50.0 | 101.9 | 152.85 | 50.95 | no |
| 48 | 52.0 | 50.0 | 75.0 | 25.0 | yes |

Graphically we would have


Figure 7. Graphical Representation of Table 3.

The second modification is similar to the first except that rather than setting $\left|x_{t}-\hat{x}_{t-1}\right| / x_{t-1}<\tau$ we set the criterion as $\left|x_{t}-\hat{x}_{t-1}\right|<\Delta$ where $\Delta$ is some fixed constant. That is, rather than changing the width of the acceptance region according to the time level, we will keep the region a fixed width at all levels.

Two rules were used to set the acceptance region for the two moving average level-reset methods. First a percentage rule similar to SAES was used. The moving average was computed as follows:

$$
M_{t}= \begin{cases}\sum_{k=0}^{N-1} x_{t-k} \\ N & \text { if } \\ \frac{\left|x_{t-1}-\hat{x}_{t-1}\right|}{x_{t-1}}<\tau \\ x_{t-1} & \text { otherwise }\end{cases}
$$

Calculations would proceed as in Table 4.

Table 4. Example of SAMA Method ( $\tau=.1$ )

| t | $\mathrm{x}_{\mathrm{t}}$ | Total | $\hat{x}_{\mathrm{t}}$ | UL | LL | In <br> Control? | $\mathrm{N}_{\text {old }}$ | $\mathrm{N}_{\text {new }}$ |
| :--- | ---: | ---: | ---: | :--- | :--- | :--- | :--- | ---: |
| 46 | $\ldots$ | 1000 | 100 | $\ldots$ | $\ldots$ | yes | 9 | 10 |
| 47 | 106 | 1106 | 105.45 | 110.0 | 90.0 | yes | 10 | 11 |
| 48 | 90 | 90 | 90 | 115.9 | 94.9 | no | 11 | 1 |

The second level-reset moving average consists of the rule in which the acceptance region is of a fixed width no matter at what level the time series is located. The only difference between this method and the second modification for exponential smoothing is the substitution of moving average in place of exponential smoothing. Thus the six methods used to forecast the real time series were:

1. Single Exponential Smoothing (ES)
2. Single Moving Average (MA)
3. Self-Adaptive Exponential Smoothing (SAES ( $\tau$ ))
4. Self-Adaptive Moving Average (SAMA ( $\tau$ ) )
5. Self-Adaptive Exponential Smoothing (SAES ( $\Delta$ ) )
6. Self-Adaptive Moving Average (SAMA ( $\Delta$ ))

Description of the Time Series Used
The question of what kind of series best represents the actual workloads at an operating computer center remains unanswered. No one computer program or set of programs has been developed that is representative of the majority of programs processed at a computer center. Thus the time series were generated from a random sampling of programs in an attempt to reduce bias of the results of the research.

Unfortunately, due to computer time limitations, we were somewhat restricted in that the programs chosen had to be of fairly short execution time themselves (that is, when not being traced). Also, due to the number of observations (I/O times) needed, the programs had to generate considerable input and output in a short run time.

However, within these restrictions, it is felt that a representative sample was achieved of the types of programs processed at the Georgia Tech computer center. No two programs were written by the same person, thus eliminating the possible bias of results due to one person's programning technique. Also, the six prograns used were accumulated from five different schools (academic departments) at Georgia Tech. This should help elininate duplication of possible types of problems that might be processed by the computer center.

## Time Series 1 (COBOL)

Time series 1 (TS-1) was generated by a COBOL program of the types employed by students in the School of industrial Management at Georgia Tech. This type of program is similar to those used by the business world and would be commonly used at a central computer facility used by many businesses. Figure 8 is a graph of this time series.

## Time Series 2 (DIFFER)

The second time series (TS-2) was generated from a program written by a mathematics student. This program was used to examine two methods for approximating a differential equation. This program used a FORTRAN FUNCTION which is similar to a FORTRAN subroutine in
its use. The graph of this time series is Figure 9.

Time Series 3 (METHANE)
A chemistry program, comparing several techniques for determining the pressure of methane gas at several temperatures, was used to generate the third time series (TS-3). This program read no input and contained one basic DO LOOP for incrementing the temperature. Figure 10 depicts this series of times.

## Time Series 4 (OUT-OF-KILTER)

Time series 4 (TS-4) was generated from the OUT-OF-KILTER algorithm program from the School of Industrial and Systems Engineering program library. This program is representative of the linear programming problems found. The program reads in all its data, has several DO LOOPS (some within the loop of other DO LOOPS) and prints all of its output at one time at the end of the program versus at each iteration calculated by the program. Figure 11 is a plot of the times from this series.

## Time Series 5 (SIM)

A FORTRAN simulation was the program used to generate the fifth series (TS-5). It is representative of programs written by students in the Information and Computer Science Department at Georgia Tech. This program specifically describes the operation of a computer system designed by the programer. This program differs from programs one and two in that it contains several FORTRAN subroutines. Time series five is depicted in Figure 12.

## Time Series 6 (NLS)

The sixth time series (TS-6) was generated from a program that conducted a simple coordinate search of a non-linear programming problem in industrial engineering. This is a simple, repetitious program that reads in the initial data and proceeds to calculate until specific criteria are met. Each calculation is printed as the program progresses. It contains no standard DO LOOP, but does repetitious operations due to IF statements that recycle when specified criteria are not met. Another feature of this program is the additional END $=$ $\qquad$ statenent within the READ command that abruptly terminates the program if there is no more input data. This again is another instance where a DO LOOP was not used but the program cycles are similar to those in a DO LOOP. Figure 13 is a graph of the time series.

Where time series (TS-1 and TS-5) were available from earlier work by Raynor [28, page 104], they were given in units truncated down to the next lower $200 \mu-s e c$. These were randomized by replacing each observation $x_{t}$ by $\left(x_{t}+R\right) 200$, where $R$ is a pseudo-random variate from a uniformly distributed population on the interval ( 0,1 ). This allowed approximate calculation of forecast errors within the range of $200 \mu$-sec. Of course, all results depending on errors in this range were checked for consistency with errors in larger ranges, because the randomization could introduce a bias in the smaller range. Appendix 3 contains listings of the times for each of the six time series.

Visual examination of each of the time series provides us with two useful conclusions. First, time series have specific structure that can be exploited in forecasting. Basically, all the programs
displayed varying degrees of the piecewise constant structure mentioned previously. It was possible to relate the individual time series observations to programing statements in all time series. From doing this, one obvious conclusion was that type-l pathologies (one outlier within a series) could often be avoided by improved programming practice. The large errors at the beginning of the OUT-OF-KILTER program were a result of unnecessary line skipping between lines of output as were the large deviations in the non-linear search program. Corrections to programs such as these would remove those small line skip interrupts, which add nothing in the way of useful information to the programmer and cause the program to compute longer because of (1) the additional commands necessary for output of a blank line, and (2) the need to reschedule even this small task since it is an I/O-interrupt which breaks the program into even smaller jobs. The second conclusion is that variance of times is not related to the times themselves (that is, their level). There is no noticeable significant increase in variance of the times with an increase in time level. The programming practices mentioned on pages 17-18 explain this phenomenon. The concept of relative error is not really meaningful. In fact, as was demonstrated, unnecessary forecast errors are encountered when the level is very low or very high, since the acceptance region is too narrow or too wide, respectively.







## CHAPTER IV

## RESULTS AND CONCLUSIONS

The forecasting techniques described in Chapter III were applied to the six series TS-1 to TS-6. A search for optimal parameters in each forecasting technique was made to identify the best version of each technique when applied to each series separately and when applied to the combined series. The criterion for "best" was the number of forecast errors within $+W-\sec$, with $W=200$ showing the most discrimination among various parameters and methods--a fortunate coincidence, since this is the smallest $W$ allowed by the data (recall that numbers of errors in the smallest range are most important in determining actual throughput increases achieved by scheduling based on the forecasts). Among the techniques found to be relatively accurate, the parameter choices using larger values of $W$ are identical (as will be shown in Tables 8 through 13 below). The searches for optimal parameters were limited to the following parameter values: $\alpha$ from .l to 1 in increments of $.1, \mathrm{~N}$ from 1 to 9 in increments of 1 , $\tau$ from . 1 to . 9 in increments of .1 , and $\Delta$ from 200 to 1200 in increments of 200 and also at 250,300 , and 350 for those series (TS-1 and TS-5) where the original data had been truncated to the next lower $200 \mu-\mathrm{sec}$.

## Best Forecasting Parameters

Table 5 summarizes the forecasting results using the best parameters for each forecasting technique when applied to each

Table 5. Performance of All Tested Forecasting Methods on Each Series, Using Parameters Found Best for Each Series Separately

| Forecasting Technique | $\begin{gathered} \text { TS -1 } \\ \text { (COBOL) } \\ (298 \\ \text { errors) } \end{gathered}$ | $\begin{gathered} \text { TS-2 } \\ \text { (DIFFER) } \\ \text { (107 } \\ \text { errors) } \end{gathered}$ | $\begin{gathered} \text { TS-3 } \\ \text { (METHANE) } \\ \text { (122 } \\ \text { errors) } \end{gathered}$ | $\begin{aligned} & \text { TS-4 } \\ & \text { (OOK) } \\ & \text { (150 } \\ & \text { errors) } \end{aligned}$ | $\begin{gathered} \text { TS-5 } \\ \text { (SIM) } \\ \text { (358 } \\ \text { errors) } \end{gathered}$ | $\begin{aligned} & \text { TS -6 } \\ & \text { (NLS) } \\ & \text { (298 } \\ & \text { errors) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of forecast errors within $+200 \mu-\mathrm{sec}$ of observation |  |  |  |  |  |
| ES |  |  |  |  |  |  |
| Exponential | 60 | 90 | 120 | 59 | 212 | 247 |
| Smoothing | $\alpha=1.0$ | $\alpha=1.0$ | $\alpha=1.0$ | $\alpha=1.0$ | $\alpha=1.0$ | $\alpha=1.0$ |
| MA | 60 | 90 | 120 | 59 | 212 | 247 |
| Moving Average | e $\mathrm{N}=1$ | $\mathrm{N}=1$ | $\mathrm{N}=1$ | $N=1$ | $\mathrm{N}=1$ | $\mathrm{N}=1$ |
| SAMA ( $\tau$ ) |  |  |  |  |  |  |
| Self-Adaptive | 65 | 94 | 120 | 57 | 241 | 247 |
| Moving Average | e $\tau=.6$ | $\tau=.5-.9$ | any $\tau$ | $\tau=.1-.8$ | $8 \tau=.9$ | $\tau=.1-.6$ |
| SAES ( $\tau$ ) |  |  |  |  |  |  |
| Self-Adaptive | 68 | 94 | 120 | 59 | 224 | 247 |
| Exponential | $\alpha=.1$ | $\alpha=.9$ | $\alpha=.1$ | $\alpha=.1$ | $\alpha=.9$ | $\alpha=.1$ |
| Smoothing | $\tau=.5$ | $\tau=.5-.9$ | $\tau=.5$ | $\tau=.5$ | $\tau=.9$ | $\tau=.5-.9$ |
| SAMA ( $\triangle$ ) |  |  |  |  |  |  |
| Self-Adaptive | 69 | 94 | 120 | 60 | 274 | 248 |
| Moving Average | e $\Delta=800$ | $\Delta=800$ | $\Delta=600-1000$ | $\triangle=200-800$ | $0 \triangle=800 \triangle$ | $\triangle=600-800$ |
| SAES ( $\triangle$ ) |  |  |  |  |  |  |
| Self-Adaptive | 71 | 95 | 120 | 59 | 274 | 248 |
| Exponential | $\alpha=.1$ | $\alpha=.1$ | $\alpha=.1$ | $\alpha=.1$ | $\alpha=.1$ | $\alpha=.1$ |
| Smoothing | $\Delta=600-1000$ | $\Delta=1200$ | $\triangle=800$ | $\Delta=200 \Delta$ | $\Delta=800-1200$ | $\begin{array}{r} 10 \quad \Delta=200- \\ 800 \end{array}$ |

series separately.
The best version of ES (exponential smoothing) and of MA (moving average) is the special case of current-observation forecasting ( $\alpha=1$ in $E S$ and $N=1$ in MA). This is true for every series and hence also true for the combined series.

The best version of SAMA( $\tau$ ) (self-adaptive moving average with level-reset criterion based on relative error) is that with $\tau=.6$ for each series except TS-5, for which $\tau=.9$ is best.

The best version of $\operatorname{SAES}(\tau)$ (self-adaptive exponential smoothing with level-reset criterion based on relative error) is that with $\alpha=.1$ and $\tau=.5$ for four of the series, and that with $\alpha=.9$ and $\tau=.9$ for TS-2 and TS-5.

The best version of SAMA( $\Delta$ ) (self-adaptive moving average with level-reset criterion based on absolute error) is that where the level is reset after an error exceeding $\Delta=800 \mu$-sec.

The best version of SAES ( $\Delta$ ) (self-adaptive exponential smoothing with level-reset criterion based on absolute error) is that with $\alpha=.1$ for every series, but the best value of $\Delta$ varies slightly from series to series. For TS-2 and for TS-4, resetting the level upon encountering errors exceeding 1200 and $200 \mu-s e c$, respectively, gives slightly better forecasting (one extra forecast error within $W=200$ $\mu-$ sec in each case) than resetting using $\Delta=800 \mu-s e c$. For the remaining four series, $\Delta=800 \mu-\mathrm{sec}$ was best.

Appendix 2 contains histograms of the best versions of each technique for each time series. The time series and technique (with its parameters) are listed on each histogram. The vertical axis
numbered from -4 to +4 indicates the number of standard deviations each group is from the mean of the forecast errors.

Table 6 summaxizes the forecasting results using the best parameters for each forecasting technique when applied to the combined series. For every technique, the set of parameters that is best for the majority of the individual series is also best for the combined series.

We conclude that the empirical evidence indicates that unmodified exponential smoothing and moving average techniques are not appropriate (except in their trivial versions that collapse to currentobservation forecasting), that $\alpha=.1$ is an appropriate smoothing constant wj.thin each piece of a piecewise constant series and that $\Delta=800 \mu-$ sec is an appropriate forecast error beyond which to assume a change in level.

## Best Forecasting Techniques

Choice of forecasting techniques depends both on accuracy and cost. Table 7 gives accuracy information summarized from Table 6 for each forecasting technique and also gives the cost of a single forecast by each technique in terms of the actual UNIVAC 1108 computation time required (as measured by SNOOPY). The same information is presented graphically in Figure 14.

We conclude that two techniques, current-observation and $\operatorname{SAES}(\Delta)$, are dominant over the other techniques in terms of being significantly more accurate or less costly or both. The choice between current-observation forecasting and SAES( $\Delta$ ) forecasting would depend on the scheduling algorithm being used, because of doubt as to

Table 6. Performance of All Tested Forecasting Methods on Each Series, Using Parameters Found Best for the Combined Series

| Forecasting Technique $\xi_{G}$ Parameters | $\begin{gathered} \text { TS-1 } \\ \text { (COBOL) } \\ \text { (298 } \\ \text { errors) } \end{gathered}$ | $\begin{gathered} \text { TS -2 } \\ \text { (DIFFER) } \\ \text { (107 } \\ \text { errors) } \end{gathered}$ | ```TS-3 (METHANE) (122 errors)``` | $\begin{aligned} & \text { TS-4 } \\ & \text { (00K) } \\ & \text { (150 } \\ & \text { errors) } \end{aligned}$ | ```TS-5 (SIM) (358 errors)``` | $\begin{aligned} & \text { TS-6 } \\ & \text { (NLS) } \\ & \text { (298 } \\ & \text { errors) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of forecast errors within $\pm 200 \mu-$ sec of observation |  |  |  |  |  |
| ES |  |  |  |  |  |  |
| Exponential |  |  |  |  |  |  |
| Smoothing, $\alpha=1$ | 60 | 90 | 120 | 59 | 212 | 247 |
| MA |  |  |  |  |  |  |
| Moving |  |  |  |  |  |  |
| Average, |  |  |  |  |  |  |
|  | 60 | 90 | 120 | 59 | 212 | 247 |
| SAMA ( $\tau$ ) |  |  |  |  |  |  |
| Self-Adaptive |  |  |  |  |  |  |
| Moving Average, $\tau=.6$ | 65 | 94 | 120 | 47 | 218 | 247 |
| SAES ( $\tau$ ) |  |  |  |  |  |  |
| Self-Adaptive |  |  |  |  |  |  |
| Exponential |  |  |  |  |  |  |
| Smoothing, |  |  |  |  |  |  |
| $\alpha=.1 \quad \tau=.5$ | 68 | 94 | 120 | 59 | 196 | 247 |
| SAMA( $\triangle$ ) |  |  |  |  |  |  |
| Self-Adaptive |  |  |  |  |  |  |
| Moving Average, $\Delta=800^{\circ} \mathrm{H}-\mathrm{sec}$ | 69 | 94 | 120 | 60 | 274 | 248 |
| SAES ( $\triangle$ ) |  |  |  |  |  |  |
| Self-Adaptive |  |  |  |  |  |  |
| Exponential |  |  |  |  |  |  |
| Smoothing, $\alpha=.1, \Delta=800 \mu-\sec$ | 71 | 94 | 120 | 58 | 274 | 248 |

Table 7. Forecasting Results for Combined Serics TS-1 through TS-6

| Forecasting Technique | Parameters <br> Found Best <br> for Com- <br> bined Series | Errors Within $+200 \mu-\mathrm{sec} /$ No. of Errors | Percentage Within $+200 \mu-\mathrm{sec}$ | Computation <br> Time Above <br> Minimum <br> Possible, $\mu-s e c$ |
| :---: | :---: | :---: | :---: | :---: |
| ES | $\alpha=1$ |  |  | 0.00 |
| Exponential | (Current | 788/1339 | 58.8 | (Would be 10.25 |
| Smoothing | Observation) |  |  | for $\alpha<1$ ) |
| MA | $\mathrm{N}=1$ |  |  | 0.00 |
| Moving | (Current | 788/1339 | 58.8 | (Would be 16.25 |
| Average | Observation) |  |  | for $N>1)$ |
| SAMA( $\tau$ ) |  |  |  |  |
| Self-Adaptive | $\tau=.6$ | 801/1339 | 59.8 | 38.75 |
| Moving Average |  |  |  |  |
| SAES ( $\tau$ ) |  |  |  |  |
| Self-Adaptive | $\alpha=.1$ | 784/1339 | 58.6 | 25.00 |
| Exponential | $\tau=.5$ | 784/1339 | 58.6 |  |
| Smoothing |  |  |  |  |
| SAMA( $\triangle$ ) |  |  |  |  |
| Self-Adaptive | $\Delta=800$ | 865/1339 | 64.6 | 33.50 |
| Moving Average $\quad \mu$-se |  |  |  |  |
| SAES ( $\triangle$ ) |  |  |  |  |
| Self-Adaptive | $\alpha=.1$ |  |  |  |
| Exponential | $\Delta=800$ |  |  |  |
| Smoothing | $\mu-\mathrm{sec}$ | 865/1333 | 65.00 | 18.75 |

ACCURACY,
Percentage of Forecasts
Within $+200 \mu-\mathrm{sec}$

Figure 14. Dominance Graph of Forecasting Methods (Data from Table 7).
the relative contribution (to reducing supervisor queuing) of better scheduling versus reduced supervisor computation time. SAES ( $\triangle$ ) gave forecast errors within $\pm 200 \mu-\sec$ in 65 per cent of all forecasts, and current-observation forecasting in 58.8 per cent. In testing the null hypothesis that the two methods are equally accurate against the hypothesis that SAES ( $\Delta$ ) is more accurate, the advantage of SAES ( $\Delta$ ) over current-observation forecasting is statistically significant at the .001 level. The accuracy advantage of $\operatorname{SAES}(\Delta)$ over $\operatorname{SAMA}(\Delta)$ is not significant, but the cost difference is substantial. The accuracy advantage of SAES ( $\Delta$ ) over SAES ( $\tau$ ) (which is the method found best by Raynor of those tested by him) is significant at the . 001 level, and the cost difference is also substantial.

We find SAES ( $\tau$ ) and current-observation forecasting to be equally accurate when applied to the six time series. This does not corroborate Raynor's finding that SAES( $\tau$ ) was slightly but significantly more accurate than current-observation forecasting. However, Raynor's conclusion was based on the series TS-1 and TS-5 only, and as discussed earlier, his accuracy measure was biased.

The forecasting results for each series using $\operatorname{SAES}(\tau)$ and current-observation forecasting are given in Tables 8 through 13. Since these two techniques are the best found by this research, we present these tables to demonstrate the differences between the two techniques for each error range examined. We can compare forecasting accuracies using the best parameters for each individual series with those using the best parameters for the combined series. Note that SAES( $\Delta$ ) forecasting was significantly more accurate than the second-best

Table 8. Forecasting Results for Series TS-1 (COBOL), Based on 298 Forecast Errors, Using SAES ( $\triangle$ ) and Current-Observation Forecasting


Table 9. Forecasting Results for Series TS-2 (DIFFER), Based on 107 Forecast Errors, Using SAES ( $\Delta$ ) and Current-Observation Forecasting

|  | No. of forecast errors less than W $\mu-\mathrm{sec}$ |  |  |  |  |  | $\begin{gathered} \text { Error } \sigma, \\ \mu-\text { sec } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | W=200 | $W=400$ | $W=600$ | $W=800$ | $W=1000$ | W=1200 |  |
| SAES ( $\triangle$ ) |  |  |  |  |  |  |  |
| Best level-reset parameters for TS-2: $\alpha=.1$, |  |  |  |  |  |  |  |
| $\Delta=1200$ | 95 | 95 | 95 | 100 | 100 | 101 | 934.8 |
| Best level-reset parameters for combined series: $\alpha=.1, \Delta=800$ | 94 | 94 | 94 | 99 | 99 | 101 | 943.3 |
| Current Obser- <br> vation (ES $\alpha=1$ ) <br> (MA N=1) | 90 | 90 | 92 | 99 | 99 | 101 | 965.2 |

Table 10. Forecasting Results for Series TS-3 (METHANE), Based on 122 Forecast Errors, Using SAES ( $\Delta$ ) and CurrentObservation Forecasting

|  | No. of forecast errors less than W $\mu$-sec |  |  |  |  |  | $\begin{gathered} \text { Error } \sigma, \\ \mu-\sec \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $W=200$ | $W=400$ | $W=600$ | W=800 | W=1000 | $W=1200$ |  |
| SAES ( $\triangle$ ) |  |  |  |  |  |  |  |
| Best level-reset parameters for |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { TS - } 3: \quad \alpha=.1, \\ & \Delta=800 \end{aligned}$ | 120 | 120 | 120 | 120 | 120 | 121 | 282.8 |
| Best level-reset parameters for combined series: $\alpha=.1, \Delta=800$ | 120 | 120 | 120 | 120 | 120 | 121 | 282.8 |
| Current Observation (ES $\alpha=1$ ) (MA $N=1$ ) | 59 | 62 | 65 | 65 | 66 | 67 | 282.8 |

Table 11. Forecasting Results for Series TS-4 (OUi'-OF-KILTER), Based on 150 Forecast Errors, Using SAES ( $\Delta$ ) and Current-Observation Forecasting


Table 12. Forecasting Results for Series TS-5 (SIM), Based on 358 Forecast Errors, Using SAES ( $\Delta$ ) and Current-Observation Forecasting


Table 13. Forecasting Results for Series TS-6 (NLS), Based on 293 Forecast Errors, Using SAES ( $\Delta$ ) and Current-Observation Forecasting

|  | No. of | forec | t erro | s less | than W | - sec | $\begin{gathered} \text { Error } \sigma, \\ \text { p-sec } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $W=200$ | $W=400$ | $W=600$ | $W=800$ | $W=1000$ | $W=1200$ |  |
| SAES ( $\triangle$ ) |  |  |  |  |  |  |  |
| Best level-reset parameters for TS-6: $\alpha=.1$, |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { TS-6: } \quad \alpha=.1, \\ & \Delta=800 \end{aligned}$ | 248 | 248 | 248 | 248 | 250 | 258 | 863.0 |
| Best level-reset parameters for combined series: $\alpha=.1, \Delta=800$ | 248 | 248 | 248 | 248 | 250 | 258 | 863.0 |
| Current Observation (ES $\alpha=1$ ) <br> (MA $N=1$ ) | 247 | 248 | 248 | 248 | 250 | 257 | 851.9 |

method of current-observation forecasting in individual series TS-1, TS-2, and TS-5 according to the $W$ criterion. The variance of forecast errors failed to indicate this except in the case of TS-2, and in the case of TS -6 the variance falsely indicates a reverse-order accuracy ranking. Also note that in every case, including the two series with truncated data (TS-1 and TS-5), the results using $W=200$ are corroborated by similar results using higher values of $W$.

## Recapitulation of Results

The purpose of this research was to develop an improved technique for forecasting execution times between $I / 0$ interrupts, so that throughput of a multiprocessor computer system could be increased by using the forecasts in a scheduling algorithm to reduce queueing of processors attempting to obtain jobs. Previous work by Pass and Raynor had developed a method that gives essentially perfect forecasts for 59 per cent of all jobs, giving an assumed 6.6 per cent increase in throughput. The present work has developed a method that gives essentially perfect forecasts for 65 per cent of all jobs, and furthermore uses only three-fourths as much computation time as previous methods. Reasoning from Raynor's results, the improvement of our method over Raynor's should boost the throughput increase to 7.0 per cent or higher. The forecasting method, $\operatorname{SAES}(\Delta)$, is

$$
\begin{aligned}
\hat{\mathrm{x}}_{\mathrm{t}} & =.1 \mathrm{x}_{\mathrm{t}-1}+.9 \hat{\mathrm{x}}_{\mathrm{t}-1} \text { when }\left|\mathrm{x}_{\mathrm{t}-1}-\hat{\mathrm{x}}_{\mathrm{t}-1}\right|<800 \mu-\mathrm{sec} \\
& =\mathrm{x}_{\mathrm{t}-1} \quad \text { otherwise }
\end{aligned}
$$

Our results, based on Raynor's 656 observations from two computer
programs plus 683 additional observations from four additional programs of widely varying types, corroborate and strengthen previous suggestions that scheduling based on forecasts can significantly increase the throughput of future multiprocessor computer systems.

## CHAPTER V

## RECOMMENDATIONS FOR FURTHER RESEARCH

Six areas of further research could continue the work done for this thesis. The first two deal with the generation of the real time series. The next two pertain to the actual utilization of the results and conclusions of this thesis. The fifth area considers forecasting before a program is run in the computer. Finally, further extensions of forecasting methods could be investigated.

First, it is quite apparent that a more efficient method of tracing the prograns to generate the time series is needed. Simply too much time and effort are expended in generation of these times. This is not only important for our purposes, but also such research might provide the software that will be needed when multiprocessor systens actually are put into operation in more than just a research configuration.

The second area is that area which at the start of this research was ambiguous and remains so, that is, the search for a program or set of programs that is representative of those habitually processed at a computer center. The more programs that are analyzed, the broader the basis for the results and conclusions enumerated by the researcher.

This thesis dealt with the work of Raynor and his specific scheduling algorithm. Further research is needed to utilize the proposed forecasting techniques in other scheduling algorithms since
it is the scheduling algorithm that establishes the accuracy desired from the forecasts. In one algorithm, it may be that a more costly forecasting technique is needed in order to obtain the desired accuracy, whereas in another algorithm not designed to use such great accuracy, a less costly technique might be more satisfactory.

The fourth area for further research is the actual application of the forecasting techniques proposed. That is, the best technique should be put into the computer system, and its performance measured. Since these techniques were developed with Raynor's work in mind, the logical use would be to apply Raynor's scheduling algorithm to a multiprocessor system with the best technique as the forecasting routine. The fifth area for further research was beyond the scope of this thesis. It appears possible that when a program is compiled by the computer, that the computer could at that time tag each computer job with a guessed time to next $1 / 0$-interrupt based on the FORTRAN statements between requests for input or output.

As the sixth area for further research, there are at least two classes of time-series forecasting methods that show some promise but have not been fully investigated.

One of these classes includes methods that dynamically readjust the criterion for deciding whether or not a time series has changed levels. Preliminary examination was made into a level-reset technique that used $\left|x_{t-1}-\hat{x}_{t-1}\right|<k \hat{o}$ as a reset criterion, where $\hat{\theta}$ was an estimate of the standard deviation of forecast error and $k$ is a constant, say 2.0. It is not yet clear whether $\hat{\circ}$ should be reset when the level is reset.

Another class of methods would exploit the repetitive structure of loops explicitly. When an observation or series is encountered that closely matches an earlier observation or series, then the forecast would assume continuation of the previous pattern.

APPENDIX 1A

## APPENDIX 1A

SET-UP OF THE PROGRAM FOR
A SNOOPY TRACE

This appendix is presented under the assumption that the roader has a basic knowledge of FORTRAN programming and Univac 1108 control techniques.

Before a trace can be run, a file (we will call it FILE) raust be catalogued containing the following elements.

Element Where located

1. RELOCATABLE TRA\$ER. . . . . . . . . . . . EXEC 8 LIBRARY
2. RELOCATABLE SNOOPY. . . . . . . . . . . . EXEC 8 LibRARY
3. RELOCATABLE PROGRAM TO BE TRACED . . . . . . . . PROGRAMAER
4. RELOCATABLE SUBROUTINE TO PRODUCE TIMES. . . . . . PROGRAMAER
5. RELOCATABLE DUMMY ELEMENT . . . . . . . . . . .SEE BELOW

The relocatable DUMMY element is produced through a mapping command as below.

@MAP,R ,FILE.DUMMY<br>IN FILE.TRA\$ER<br>IN FILE.SNOOPY<br>IN FILE.SUBROUTINE<br>DEF TRON<br>LIB SYS\$*RLIB\$.<br>END

Then the executable absolute of the progran is produced by mapping

```
@Map,N ,FILE.PROGRAM
IN FILL.PROGRAM
IN FILE.DUMMY
END
```

Once the absolute has been produced, the progran can be executed from either batch (cards) or demand. For short tests demands can be used, but for the actual runs batch is necessary due to the large number of pages of output generated. Figures 15 and 16 depict the commands and the check set up for batch.

```
@RUN CARD
@PWRD CARD
aCOL 9 (if used 029 key punch)
@ASG,A FILE.
@XQT FILE.PROGRAM
    [ DATA [ IARS, IF ANY }]\mathrm{ or @ADD DATAFILE.
    EEOF
        @FIN
```

                                    DATAFILE is a file with
                                    your data previously entered
    Figure 15. Batch Deck for SNOOPY

```
>> RESPONSES TO GET ON
> XCTS (must be in EXEC MODE)
> EASG,A FILE.
> QXQT FILE.PROGRAM
>LIB A
> GO
>DATA AS REQUESTED
BY COMPUTER FOR YOUR
PROGRAM (TERMINAL WILL PRINT > sign
AND WAIT FOR YOUR DATA)
> @EOF
> @FIN
```

Figure 16. Demand Commands for SNOOPY

Note: DO NOT @eCQUE since you need to know when computer is requesting information from you.

Wue to slowness of demand terminal output, you probably will not be able to let program run more than a short time. Use of the demand should be limited to execution of the program to see that everything is in working order. Once you can establish that fact, terminate the run with normal control procedures.

## Subroutine for Use with SNOOPY






$\qquad$
-
$\qquad$

APPENDIX 2
















APPENDIX 3

ACIUAL TIMES FOR TS-I (COBOL) UNITS: pi-sec (Randomized from data originally grouped into $200 \mu$-sec blocks).
Read Down Each Colum

| $8+9.311$ | 904.512 | 50.59 .354 |  |
| :---: | :---: | :---: | :---: |
| 25647.931 |  | 701.314 | 30, 3030 |
| 4295.661 | 43'to3is | 283, 483 | 5101.213 |
| 495.104 | 2.047 .210 | 17214.462 | 493.735 |
| 539.600 | 5 62, 329 | S097,750 | 735:95 |
| 2535.560 | 4020.03 | 724.349 | 495.3n3 |
| 25380,978 | \%37,096 | 303.257 | 19070,160 |
| 3775, 167 | 2930445 | 19531.475 | 5 5 44.23 |
| 4918.945 | $3: 1509.103$ | 3509.003 | 410.546 |
| 1,12, 0.54 | 5659.071 | $5135 \cdot 3+1$ | 612.35 |
| 554.452 446.170 | 2872.349 | 711.988 | $555 \cdot 370$ |
| 45450170 10709 | 3235.936 | 237.445 | 25457.597 |
| 19789.93 3577035 | 3044.934 | 17112.212 | 4689.334 |
| 3577.335 | 402.041 | 5335.289 | 480, 280 |
| 52,183 | 603.558 | 509.576 | 520.535 |
| 7950.440 | 554.354 | 232.372 | 3in-102 |
| 2640407 20.603 | 20069.521 | 19504.378 | 22;40.273 |
| 20731.933 | 4123.957 | 3325.594 | 9331,33 |
| 3759.370 | 327.723 | 3495.320 | 919,6\% |
| 3753.066 | 430.789 | 3016.255 |  |
| 400,672 675.33 | 303.165 | 483.500 |  |
| 875.343 $2006 \% 303$ | $20434 \cdot 830$ 3655.845 | 307.0 .95 15271.920 | absentob |
| 3992, 714 | ?285.15 | 4799.035 | 431.383 |
| 3925.694 | 4085.765 | 555.579 |  |
| 607.748 | 334, 090 | 271.373 | 30,9060 |
| 459.13 | 532.924 | 21035.477 | 747.174 |
| 210.7 BC | 230.464 | 4150.891 | 231.057 |
| 22749.711 | 27377,32? | 4814.015 | 2806? 550 |
| 3220.066 | 3423.486 | 427.643 | 404:533 |
| 4057.570 | 419.959 | 389.991 |  |
| 435.543 | 163.742 | 301,544 | 3n49,2088 |
| 562.307 | 17457.835 | 25752.505 | 3 n 77.1 ? 1 |
| 237.340 | 5301.233 | 3772.879 | 3 354,203 |
| $255 \cdot 183$ | 493.951 | 4132.461 | 530.715 |
| 22340.335 | $235 \cdot 973$ | 441.353 | 555.475 |
| $55 \% .798$ | 22327.305 | 499.555 | 291-548 |
| 433.570 | 3167.947 | 25107.0.6 | 24655.929 |
| 261.532 | 3757.399 | 3463.858 | 5179.520 |
| 25034.044 | 4097.150 | 3770.019 | 452.21 |
| 4555.746 | $5385 \cdot 732$ | 4225.450 | 674, $9 \times 1$ |
| 425.757 | 423.613 | 430.211 | 873031 465062 |
| 447.979 | 210.304 | 3R4.271 | $2356 \%+42$ |
| 216.710 | 24147.304 | 20207,641 | -3837.722 |
| 13733.630 | 2933.1:5 | 4140.322 | 3657,272 |
| 5003.901 | 3853.235 | 542,312 | 4626.131 |
| 4221.451 | 3652.565 | 493.611 | 54.4 .300 |
| 5389.336 | 3136.415 | 394821 | 211:7,9 |

ACTUAL TIMES FOR TS-1 (COBOL) UNITS: k-sec
(kandomized from data originally grouped into 200 小-sec blocks)
Read Down Each Column

| 549,311 | 904.512 |
| :---: | :---: |
| 23407.95 | 770.001 |
| $4835 \cdot 61$ | 4634-301 |
| -105.10. | $\because 273010$ |
| - 639.650 | $3452,3,2$ |
| 335.010 | $4925.05 \%$ |
| 25380,578 | 757.096 |
| 3775,157 | 297.445 |
| $4018.9+5$ | 511090.103 |
| 1412,046 | З'5?,071 |
| 654.952 | ? $270.3+9$ |
| 4, 56.170 | 3.35 .935 |
| 19707,190 | 3944.834 |
| 3577.355 | 492.051 |
| 51.7 .1 .33 | 603.553 |
| $70=8140$ | 3040384 |
| 2450.407 | 20659.521 |
| 2073.233 | 4123.957 |
| 3763.670 | 327.723 |
| 3955066 | 480.789 |
| 490.472 | 303, 165 |
| 875.343 | 20954.350 |
| 20603,503 | 3635.945 |
| 3902,943 | 2935.151 |
| $3929+594$ | 4095.765 |
| 6979749 | 334, 340 |
| 469.155 | 5.52 .924 |
| 219.718 | 330.468 |
| 22749.771 | 27.777 .322 |
| 3229.066 | 3423.486 |
| 40570570 | 1.18 .959 |
| 435,533 | 863.742 |
| 562.307 | 17657.335 |
| 239.340 | 5301.238 |
| 255.193 | 493.951 |
| 22140.335 | 235.773 |
| 554.798 | $22.27 \cdot 305$ |
| 433.570 | 3167.947 |
| 261.032 | 3757.399 |
| 25034.044 | 4997.160 |
| 4555.746 | 5385.732 |
| 426.757 | 423.613 |
| 447.079 | 210.954 |
| 215.710 | 24147.304 |
| 13735.630 | 2\&33,1:5 |
| 5003.901 | 3853.235 |
| 4221.65 | 3652.565 |
| 5.3.4.334 | $3735: 65$ |

5159.454 701,314 293.493

17"14.4.22
3094,750 724.349 $303 \cdot 257$
$19531 \cdot 475$
3509.003
$5135 \cdot 8+1$ 711.968 237.445
$17112.21 ?$
5335.289
509.576
232.372
19504.378
3325.594
3490.320
3016.255
$413: 500$
$314+0.95$
15271. 220
4939.095
655.579
271.373
21036.477
4150.891
4814.015 427,648 389.991 301,644
2576?.506
3772.379
4132.461 441.353 499.555
25107.066 3453.958
3770.019
4325.460 430.211 3A4.?.71
20707,641
4140.322

542,312 493.611 $34.2+1$
ab:44, ! 40
$3 \operatorname{Bin}_{3}-30$ ?
$5191: 913$ $4.19 .7: 30$ $73=95$ 535.545

19079,160
$5 \mathrm{~S} / 4,243$ 413,646 42 3 3 5 3 555.370

25457:547
4699.334 480,240 620.935 $3+0.112$
2293073
4351,63 471:399
 $5 \% .57$
a.6.5: 205
$4.14,735$
$3,111 \cdot 102$
3465 - 10 $\therefore 3 ; 260$ 749.774 231,057
26964,550
4045.533
4170.105
3049.253
3677.121
 $530 \cdot 71 \%$ 553,475 2.91:548
24655.929
5179.520 452.521 6716.931 445052
$23569+42$
3 P5\%.722
3657,272
4he? 131 54.4 .300 211.779

|  |
| :---: |
| 15423.568 306.657 |
| 3710.075 |
| 0374.793 |
| 784.321 |
| 592.159 |
| 2206!-307 |
| 33\%5.754 |
| 3339.036 |
| 6599.6i9 |
| 505, 95 |
| 267.631 |
| $2.75 \cdot 193$ |
| 2.34.0.14 |
| 27541.139 |
| 3597.575 |
| 51103,320 |
| 539.376 |
| 652,741 |
| 515.415 |
| 519.400 |
| 1371.596 |
| 373.298 |
| 16124.212 |
| 3524.435 |
| 373.969 |
| 472.313 |


| 2763n.1.45 |
| :---: |
| 4637.323 |
| 667.159 |
| 794.606 |
| $371.36 ?$ |
| 25497.423 |
| 5472.304 |
| 5397,490 |
| .371,4:36 |
| 594.792 |
| 467.405 |
| 23597,330 |
| 4050.505 |
| 401.110 |
| . 650.964 |
| 370.129 |
| 24533,601 |
| 4056.365 |
| 5423.478 |
| 539.381 |
| 305.594 |
| 31F,20.517 |
| 3384.349 |
| 4293.592 |
| 5161.544 |
| 373.806 |
| 535.377 |

246.36
31616.449
5715.951
4374.761

592, 392
大4, 312
247,052
25403.102
5009.422

5103,151
567.410

727499
222.938

22574,916
$7 n 75,125$
225,003
$325 \cdot 491$
375.539
22174.006
$3121: 363$
3219.961
3.55 .327
5251.024
205.050
$700 \cdot 3+7$
$343 \div 773$
25135,908

ACTUAL TIMES FOR TS-2 (DIFFER)
Units: $\mu-s e c$
Read Down Each Column

| 55.125 | 6038.125 | 0286.875 |
| :---: | :---: | :---: |
| 97.0100 | 6043.500 | 6311.375 |
| 2168.375 | 6019.000 | 6299.125 |
| 1427.625 | 6745.250 | 0299.125 |
| 2168.375 | 6160.375 | 6317.500 |
| 1569.125 | 6191.000 | 6311.370 |
| 4003.375 | 6178.750 | 6299.125 |
| 70.125 | 6172.025 | b305.250 |
| 1105.125 | 0191.000 | 5317.500 |
| 70.125 | 6179.750 | 6293.000 |
| 70.125 | 6.160.375 | 6311.375 |
| 70.125 | 6191.750 | 6.305 .250 |
| 3699.250 | 6179.500 | 6305.250 |
| 70.125 | 6185.625 | 6311.375 |
| 87.375 | 6.773 .375 | 6293.000 |
| $6<16.500$ | 6179.500 | 6318.250 |
| 0660.125 | 6185.625 | 0299.875 |
| 0454.000 | 0173.375 | 7001.625 |
| 6050.125 | 6179.500 | 6399.750 |
| 9072.370 | 5173.375 | 6405.875 |
| 6054.750 | 6179.50 u | 6387.500 |
| 6034.250 | 6179.500 | 6405.875 |
| 6034.250 | 6173.375 | 6393.625 |
| 6046.500 | 6173.375 | 6412.000 |
| 6060.125 | 0185.625 | 6393.025 |
| 6039.125 | 6197.875 | 6400.500 |
| 6032.000 | 6190.250 | 6406.625 |
| 0032.000 | 6174.125 | 6394.375 |
| 0425.875 | 6871.000 | 6394.375 |
| 6038.125 | 6504.500 | 6394.375 |
| 6038.125 | 0236.125 | 6418.075 |
| 5032.004 | 6298.375 | 6582.125 |
| 6025.875 | 6492.250 | 0406.026 |
| 6032.000 | 6304.500 | 6.382 .125 |
| 6433.125 | 0304.500 | 6418.1375 |
| 6U56.500 | 6304.500 | -394.375 |
|  |  | 6406.625 |

--

ACTUAL TIMES FOR TS-3 (METHANE)
Units: $\mu-\sec$

Read Down Each CoIumn

| 63.375 | 0052.875 | 5993.750 | 6111.375 |
| :---: | :---: | :---: | :---: |
| 2145.125 | 6040.625 | 6012.125 | 6093.004 |
| 3299.250 | 6052.875 | 5993.750 | 6122.000 |
| 6215.000 | 6040.625 | 6012.125 | 6122.0100 |
| 6197.500 | 0452.875 | 6012.125 | 6134.250 |
| 6197.000 | 5993.375 | 5999.875 | 6115.375 |
| 6603.625 | 6024.375 | 5993.750 | 6115.870 |
| 6222.000 | 5993.750 | $5 \rightarrow 93.750$ | 6144.875 |
| 6209.750 | 5499.875 | 6012.125 | 6185.125 |
| 6222.100 | -993.750 | 6005.000 | 6202.375 |
| 0203.625 | 6005.000 | 5999.375 | 6190.625 |
| 6203.623 | 5499.875 | 6006.000 | 6233.500 |
| 6175.125 | 6430.500 | 6006.000 | 6202.875 |
| 0193.290 | 5999.875 | GU12.125 | 6209.000 |
| -128.250 | $6 \cup 13.250$ | 6006.1000 | 6196.750 |
| 6109.837 | 6012.125 | 6006.000 | 6202.675 |
| 6122.125 | 0013.250 | 5937.625 | 6215.125 |
| 6122.125 | 6005.000 | 6012.125 | 0227.375 |
| 6128.250 | $\bigcirc 999.875$ | 6035.000 | 6202.075 |
| 5128.250 | 5991.500 | 6035.000 | $6165 \cdot 000$ |
| 6ub1.375 | 5999.875 | _ ¢U2R.875 | 6202.375 |
| $0 \cup 87.500$ | 5999.875 | 6035.000 | 6209.000 |
| 6059.125 | 5997.625 | 6035.000 | 6209.000 |
| 6U87.500 | 5999.875 | 5984.875 | 5215.125 |
| -u87.500 | 6024.375 | 6070.125 | 0215.125 |
| 6093.025 | 5993.750 | 5054.000 | 6196.750 |
| $0 \cup 81.375$ | 6012.125 | 5051.750 | 6215.125 |
| 20837.500 | GU12.125 | 6064.000 | 6215.125 |
| 6051. 375 | 6012.125 | 5070.125 | 6190.625 |
| 0487.500 | 5943.625 | 0051.750 | 6196.750 |
| 6046.750 | 6006.000 | ¢U51.750 | 6227.375 |

TS-4 (OUT-OF-KILTER)
Units: $\mu$-sec

Read Down Each Column

| 55.125 | 2642.875 |
| ---: | ---: |
| 1293.625 | 155.500 |
| 1369.625 | 2646.875 |
| 1410.875 | 156.500 |
| 479.875 | 2642.875 |
| 92.000 | 156.500 |
| 97.000 | 2656.000 |
| 2827.250 | 156.500 |
| 1221.125 | 2660.750 |
| 954.625 | 156.500 |
| 590.875 | 1607.750 |
| 156.500 | 156.500 |
| 2590.000 | 2655.500 |
| 156.500 | 156.500 |
| 2594.750 | 2669.250 |
| 156.500 | 156.500 |
| 2603.250 | 2673.000 |
| 156.500 | 156.500 |
| 2608.000 | 2668.375 |
| 156.500 | 156.500 |
| 2608.000 | 2658.375 |
| 156.500 | 156.500 |
| 2611.750 | 2681.500 |
| 156.500 | 155.500 |
| 2616.500 | 2690.000 |
| 156.500 | 156.500 |
| 2616.500 | 2685.375 |
| 156.500 | 156.500 |
| 2520.250 | 2694.750 |
| 156.500 | 156.500 |
| 2634.375 | 2634.375 |
| 156.500 | 156.500 |
| 2634.375 | 2704.125 |
| 156.500 | 256.500 |
| 2638.125 | 2704.125 |
| 156.500 | 156.500 |
| 2642.875 | 2694.750 |
| 156.500 | 156.500 |


| 2685.375 | $306 \% .125$ |
| :---: | :---: |
| 155.509 | 3075.500 |
| 2694.750 | 3079.500 |
| 156.500 | 3053.125 |
| 2685.375 | 3079.500 |
| 156.500 | 3079.500 |
| 2694.750 | 3095.875 |
| 156.500 | 3095.875 |
| 2594.750 | 3079.900 |
| 156.500 | 3063.125 |
| 2694.750 | 3079.500 |
| 156.500 | 3063.126 |
| 2094.750 | 3079.500 |
| 155.500 | 3079.500 |
| 31776.175 | 3079.500 |
| 3047.875 | 3079.500 |
| 3045.250 | 3081.000 |
| 3046.750 | 197.875 |
| 3046.750 | 70.125 |
| 3046.750 | 526.250 |
| 3048.250 | 1051.125 |
| 3048.250 | 1144.750 |
| 3045.750 | 1137.750 |
| 3046.750 | 1144.000 |
| 3063.125 | 1144.000 |
| 3063.125 | 1136.250 |
| 3063.125 | 1135.250 |
| 3063.125 | 1144.000 |
| 3064.625 | 1160.375 |
| 3063.125 | 1144.000 |
| 3063. 125 | 1144.000 |
| 3079.500 | 1154.125 |
| 3079.500 | 1152.625 |
| 3081.000 | 76.750 |
| 3063.125 | . 750 |
| 3079.500 | 6.375 |
| 3079.500 | 150.000 |
| 3063.125 | 13.875 |

ACTUAL TIMES FOR TS-5(SIM) Units: $\mu-\mathrm{sec}$ (Randomized from data originally grouped into $200 \mu-\mathrm{sec}$ blocks)
Read Down Each Colunn

| 749.311 | -14.51d | 69.454 | 44.140 | 29.623 |
| :---: | :---: | :---: | :---: | :---: |
| 2470.95 | \%o.0t1 | 301.314 | $43 \cdot 35$ | 594, 6,7 |
| 95.51 | $33+301$ | 83.4.3 3 | 391.903 | 111.075 |
| 35, 101 | 14:3.910 | 214.462 | 239.755 | 174,794 |
| 33.560 | 762.329 | 94.720 | 136.215 | 133.321 |
| 135.210 | 125.058 | $32+3.39$ | 33.3-353 | 352,109 |
| 139.579 | $\because 37.035$ | $103.2,7$ | 279.61 | 64, 807 |
| 175.157 | 293.445 | 31.475 | 174.2+3 | 325,754 |
| 119.945 | 9,143 | 8.97 .9103 | 219.545 | 133.031. |
| 12.0+4 | 263.071 | 335.341 | 12.353 | 94.603 |
| 254.352 | 278.369 | 311.963 | 355.370 | $8 \cdot+95$ |
| 2450.170 | 36.935 | 37,445 | 47.697 | 67.591 |
| 137.193 | 14.4 .334 | 112.212 | 289, 334 | 275.198 |
| 77.535 | 202.041 | 336.259 | 280.200 | 254,014 |
| 117:183 | 209.503 | 307.676 | 20.536 | 14.1.15 |
| 10:5.140 | 364.384 | 232.372 | 310.102 | 397.575 |
| 44.407 | 209,521 | 104.379 | 143.973 | 203.321 |
| 331.043 | 323.967 | 125.6.94 | 337, 3.3 | 530,376 |
| 309,870 | 397.723 | 95.329 | 274.659 | 262.741 |
| 355.006 | 6n. 769 | 16.205 | 161,464 | $316 \cdot 815$ |
| 297.572 | 133.155 | 235.500 | 397.579 | 310.400 |
| 875.5083 | 234.800 | 107.00, | 183.003 | 71. 514 |
| 719.215 492,943 | 035.845 | 271.320 | 317.7.38 | 383.243 |
| 92, 0.56 | 3060250 | 197.095 | 1.762 | 224212 |
| 325,594 7.748 | 285,765 | 255.579 | 235.135 | 224.436 |
| 337.113 | 134.690 | 71.37 .3 | 217.300 | 373.959 |
| 217.768 | 132924 | 35.477 | 14.3074 | 72,513 |
| 347.77? | 239.468 | 150.891 | 31.057 | 320.966 |
| 229.066 | 177.322 | ?14.515 | 61.050 | 115.428 |
| 257.670 | 293.486 213.759 | 27.548 389.391 | 241.553 | 65.201 |
| 2.35*583 | 363,74? | 301, 64.4 | 370.766 | 501. 20.576 |
| 362.307 | 57.8.35 | 352.507 | 277.121 | 1.378. |
| $239+3+9$ | 301.238 | 872.379 | 254.23 | 845.359 |
| 265.133 | 293.701 | 13?.461 | 180.715 | 239.711 |
| 240.355 | 235.973 | 41.303 | 255.476 | 31.342 |
| 364.793 | 127.305 | 299.555 | 21.5.9 | 273.293 |
| 239.370 | 167.947 | 107.067 | 55.397 | 214.534 |
| 261.532 | 357.899 | 253.883 | 179.020 | 105.075 |
| $234 \cdot 9+4$ | 97.160 | 170.019 | 252.021 | 144.955 |
| 355.746 | 395.732 | 225,460 | 274.931 | 134.145 |
| 226.757 | 23,513 | 30.212 | 246.502 | 272.535 |
| 247.079 | 210.8144 | $33^{3}+271$ | 367.4!2 | 160.435 |
| 215.710 | 347.304 | 297.642 | 237.722 | $197 \cdot 3+5$ |
| 135.590 | 33.115 | 140.322 | 57.272 | 303.9:54 |
| 20.5.941 | 63.235 | 342,312 | 225.151 | 310.593 |
| 221.4.01 | 52.685 | 93.514 | 344.300 | 4.732 |
| 193.331 | 355:405 | 394.221 | 211.779 | 230.041 |

TS-5 (Continued)

| 20.759 | 223851.641 | 335079.152 | 175.570 |
| :---: | :---: | :---: | :---: |
| 355.703 | 378773.501 | 353970.703 | 355.419 |
| 237.906 | 319335.375 | 313511.570 | 65.577 |
| $\therefore 29.373$ | 317045.254 | 329001.746 | 235.0446 |
| 50.471 | 423705.445 | 356341.230 | :54.024 |
| 830.679 | $145115.9+9$ | 279031.023 | 122.212 |
| 2519.196 | 424274.754 | 296069.129 | 20.310 |
| 199.023 | 2165.82. 379 | 375455.543 | 30\%.017 |
| 267.1:9 | $456: 40 \cdot 305$ | 180192.270 | 72300.35 |
| 194.615 | 217847.051 | 439079,301 | 239.3o? |
| 71.362 | 393003.098 | 2251.13.543 | 2.209 |
| 297.423 | 259108.459 | 2099.301 | 266.245 |
| 272.314 | 400563.125 | 232.261 | 60. 20 ? |
| 197.400 | 194167.109 | 115.536 | 242.7.3 |
| 5471.585 | $434520 \cdot 395$ | 149.119 | 3548044 |
| 294.791 | 402622.995 | 330.011 | 25.3.30 |
| 57.406 | 213574.904 | 261.213 | 226.326 |
| 557089.320 | 410075.121 | 341.725 | 385.131 |
| 205250.664 | 249225.050 | 371.547 | 295.246 |
| $350091-109$ | 373125.458 | 344550.676 | 153.671 |
| 325950.961 | 31.775 .537 | 274279.113 | 361.406 |
| 296970.125 | 315776.090 | 333955.867 | 319.450 |
| 359639,596 | 395321.959 | 284583.926 | 2.24.805 |
| 512755.383 | 305.19.937 | 302160.297 | 80.469 |
| 323:23, 477 | 312665.324 | 115485.781 | 285.44, |
| 369939.979 | a91461.020 | 160.972 | 239.725 |
| 218405.534 | 303205.027 | 185.272 | 143.320 |
| 363:320.513 | 307300.344 | 153.842 | 195.223 |
| 209384.947 | 317543.759 | 31.801 | 398.436 |
| 441093.590 | 283035.906 | 354.031 | 149.369 |

ACTUAL TIMES FOR TS - 6 (NLS)
Units: $\mu-\mathrm{sec}$
Read Down Each Column

| 55.125 | 343.375 | 1715.000 | 278.625 |
| :---: | :---: | :---: | :---: |
| 97.000 | 2751.500 | 459.625 | 2704, 575 |
| 999.500 | 2773.125 | 3'43.375 | 1043.000 |
| 1824.875 | 2.772 .000 | 2774.000 | 259.6\% |
| 257.250 | 2794.500 | 2794.500 | 34.3 .375 |
| 343.375 | 2783.375 | 2794.500 | 2767.675 |
| 2.54,750 | 2773.875 | 2794.500 | 2795.250 |
| 2391.375 | 2795.250 | 2794.500 | 277.000 |
| 2401.625 | 2753.375 | 2709.125 | 343.375 |
| 2407.750 | 2794.500 | 2795.250 | $2 \% 74.625$ |
| 2413.000 | 2789.125 | 2789.125 | 2794.375 |
| 2405,500 | 2799.125 | 2789.375 | 1581.000 |
| 2403.500 | 2793.375 | 2794.500 | 255.875 |
| 2401.625 | 2783.375 | 2195.250 | 92.125 |
| 2424.125 | 2789.125 | 2001.375 | 92.125 |
| 2403.500 | 2795.250 | 2789.125 | 92.125 |
| 2402.375 | 2739.125 | 277.000 | 92.125 |
| 2401.625 | 2794.500 | 343.375 | 343.375 |
| 2402.375 | 2794.500 | 2793.000 | 2053.750 |
| 2424.525 | 2794.500 | 2787.500 | 155.000 |
| 2601.625 | 2795,250 | 2787.500 | 3403.7517 |
| 2402,375 | 277,000 | 2793.625 | 257.250 |
| 277.000 | 343.375 | 2793.625 | 34.3 .375 |
| 343.375 | 2774.625 | 2794.375 | 2757.675 |
| 2719.375 | 2749.750 | 2793.250 | 2788.375 |
| 2738.375 | 2787.500 | 2797.500 | 2780.375 |
| 2760.875 | 2793.625 | 2789.250 | 278日.375 |
| 2754.750 | 2793.625 | 2794.375 | 2794,500 |
| 2771.125 | 2787.500 | 2788,250 | 2799.125 |
| 2771,125 | 2788.250 | 2787.500 | 2772.000 |
| 2761.625 | 2794.375 | 2703.250 | 2789.125 |
| 2729,000 | 2794.375 | 1715,000 | 2800.625 |
| 2778,000 | 2793.625 | 259.625 | 2799.125 |
| 2771.875 | 2793.625 | 343.375 | 2772.000 |
| 2779.000 | 2789.250 | 2730.125 | 2739.125 |
| 2777.250 | 2789.250 | 2794.500 | 2759.125 |
| 2777.250 | 2793.625 | 2794.500 | 2759.125 |
| 2775.000 | 2793.625 | 2789.375 | 2730,375 |
| 2727.250 | 2789.250 | 2789.125 | 2794.500 |
| 2778.000 | 2787.500 | 2739.125 | 2739.125 |
| 2777.250 | 2789.250 | 2795.250 | 2789.125 |
| 2771.875 | 2793.675 | 2795.250 | 2794.500 |
| 2777.250 | 2793,250 | 2789.375 | 2794.500 |
| 2771.175 | 2783.250 | 2788.375 | 2795.250 |
| 2777.250 | 2794.375 | 2795.250 | 2799.125 |
| 2771.875 | 2737.500 | 277.000 | 2783.375 |
| 1731.375 | 2793.525 | 343.375 | 2793.375 |
| 259.625 | 2794.375 | 2744.625 | $\therefore 799.125$ |

TS-6 (Continued)

| 277.000 | 2709.125 | 2789.125 |
| :---: | :---: | :---: |
| 343.375 | 2795,250 | 2600.625 |
| 2769.500 | 277.000 | 2799.125 |
| 2797,500 | 343.375 | 2769.125 |
| 2797.500 | 2774.625 | 277.000 |
| 2793.625 | 2793.625 | 343.375 |
| 2793.625 | 2737.500 | 2763.500 |
| 2793.525 | 2793.625 | 2737.560 |
| 2800.500 | 2773.625 | 2737.500 |
| 2794.375 | 2789.250 | 2793.525 |
| 2794.375 | 2708.250 | 2799.250 |
| 2787.500 | 2787.500 | 2794.375 |
| 2787.500 | 2794.375 | 2793.625 |
| 9794.375 | 2794.375 | 2787.500 |
| 2769.250 | 2793.625 | 2783.250 |
| 2794.375 | 2799.750 | 2794,375 |
| 2787.500 | 2739.250 | 2783.250 |
| 2793.625 | 2733.250 | 2787.500 |
| 2738.250 | 2737.500 | 2787,500 |
| 2794.375 | 2787.500 | 2788.250 |
| 2793.625 | 2794.375 | 2738.250 |
| 2793.625 | 2783.250 | 1715.000 |
| 2799.250 | 1745.000 | 259.625 |
| 2798.250 | 259.623 | 343,375 |
| 2737.250 | 343.375 | 2757.875 |
| 2794.275 | 2774.000 | 2794.509 |
| 1731.375 | 2183.375 | 2794.500 |
| 259.625 | 2794.500 | 2789.125 |
| 343.375 | 2783.375 | 2795.250 |
| 2767,575 | 2799.125 | 2794.500 |
| 2733.375 | 2709.125 | 2794.500 |
| 2788.375 | 2794.500 | 2795.250 |
| 2788.375 | 2795.250 | 2739.125 |
| 8394.500 | 2795.250 | 277,000 |
| 2795.250 | 2789,375 | 343.375 |
| 2795.250 | 2794.500 | 2763.500 |

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