Time-varying asset volatility and the credit spread puzzle *

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Abstract

Structural credit risk models have faced difficulties in matching observed market credit spreads while simultaneously matching default rates, recoveries, leverage and risk premia - a shortcoming that has become known as the credit spread puzzle. We ask whether stochastic asset volatility, as an extension to this model class, has the ability to help resolve this puzzle. We identify that although there are three ways in which uncertainty about asset risk can influence spreads (asset risk volatility itself, dependence between the levels of risk and asset value and finally volatility risk premia), in a calibration setting only the volatility risk premium channel is economically significant. We show that this feature of a stochastic asset risk model allows it to match historical spreads and equity volatility as well. We also provide estimates of the required variance risk premia.

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1 Introduction

Structural credit risk models have met with significant resistance in academic research. First, attempts to empirically implement models on individual corporate bond prices have not been successful.[1] Second, subsequent efforts to calibrate models to observable moments including historical default rates uncovered what has become known as the credit spread puzzle - the models are unable to match average credit spreads levels[2]. Finally, econometric specification tests further document the difficulties that existing models encounter in explaining the dynamics of credit spreads and equity volatilities[3]

In this paper, we develop a structural model with time-varying asset volatility in order to address both the levels and dynamics of credit spreads. Our first contribution is to show that the presence of a variance risk premium resolves the credit spread puzzle in terms of levels. Second, we show that the modelling of stochastic asset volatility allows the model to explain time series of equity volatilities while doing a better job at fitting time series of credit spreads at the individual firm level. Finally, we provide estimates of the size of variance risk premia required to explain credit spread levels and benchmark these to existing empirical evidence.

The credit spread puzzle is defined as the inability of structural models, when calibrated to default probabilities, loss rates and Sharpe ratios, to predict spread levels across rating categories consistent with historical market spreads. Huang and Huang (2003), hereafter abbreviated HH, perform this calibration analysis for a broad and representative selection of models and find that, as an example, the latter never predict spreads in excess of a third of the observed levels for 4- and 10-year debt issued by A-rated firms. The performance is typically worse for more highly rated firms and somewhat better for low grade firms. [4]

Huang and Zhou (2008) test a broad set of structural models by designing a GMM-based specification test that confronts the models with panels of CDS term structures and equity volatilities. In addition to ranking the models by rejection frequency, their paper provides insights into the specific shortcomings of the models. One important weakness that emerges from their study is the models' inability to fit the dynamics of CDS prices *and* equity volatilities. In particular, the models find

[1]See Jones Mason and Rosenfeld (1985) and Eom Helwege and Huang (2003).

[2]See Huang and Huang (2003)

[3]See Huang and Zhou (2008).

[4]Chen Collin-Dufresne and Goldstein (2009) argue that if one accounts for time variation in Sharpe ratios over the business cycle, then spreads are more closely aligned with historical averages. Other papers have followed and reinforced the point that macroeconomic conditions can help explain spread levels.

it difficult to generate time variation in the equity volatility of the same magnitude as is actually observed, suggesting that an extension to allow for stochastic asset volatility is desirable.

In addition to these two important findings, recent empirical work on default swap spreads provides evidence suggestive of an important role for stochastic and priced asset volatility in credit risk modelling. Zhang Zhou and Zhu (2005) perform an empirical study of the influence of volatility and jumps on default swap prices. Although we abstract from jumps in this paper, their results point to the importance of modelling time-varying volatility.[5] Further evidence is provided in Wang Zhou and Zhou (2010) who show that in addition to volatility being important for the price of default protection, the variance risk premium is a key determinant of firm-level credit spreads. Both these studies provide evidence indicating that a structural credit model with time-varying and priced asset risk may be better poised to explain spreads than its constant volatility predecessors.

In addition to this recent work on credit markets, there is a significant body of literature documenting time variation in equity volatilities. Given this evidence, financial leverage would have to be the sole source of variation in stock return volatility in order for asset volatility to be constant, as it is assumed to be in the majority of structural credit risk models. In fact, recent empirical work by Choi and Richardsson (2009) clearly documents time variability in asset risk as well as a degree of asymmetry at the asset level, which complements the leverage effect generated even in a constant volatility model – equity volatility may increase when stock prices decline mechanically because leverage increases as asset values drop or because asset volatility increases as asset values drop, or both.

Overall, stochastic asset volatility would appear to be a compelling extension to a class of models that has been around for more than thirty-five years. However, likely for technical reasons, it is one that has not garnered much attention.[6] We present, in closed form, solutions to debt and equity prices in stochastic asset volatility model framework where default is triggered by a default boundary, as in Black & Cox (1976), Longstaff & Schwartz (1995) and Collin-Dufresne Goldstein (2001). In doing so, we are, to the best of our knowledge, the first to solve the first passage time problem of stochastic volatility dynamics to a fixed boundary.

[5]The authors include both intra-day realized volatility and historical volatility as measures of short term and long-term volatility, consistent with the notion that equity volatility varies both because of changes in leverage and because of changes in asset volatility. Their results suggest that disentangling the two sources of variation is important for explaining default swap prices.

[6]Huang (2005) describes the analytics of such a model, which in its simples form can be thought of as a Heston (1993) model augmented with jumps. In contrast to our model, this framework does not allow for a default threshold permitting default at any time; default occurs only at the maturity of the (zero-coupon) debt.

We first consider the comparative statics of our benchmark model, which can be thought of as an extension of the Black & Cox (1976) model. This permits us to study the channels through which stochastic asset volatility influences bond yields. The three important determinants of spreads are the volatility of volatility itself, the asymmetry of volatility, and the presence of a volatility risk premium. That the volatility itself is made an idiosyncratic risk source, uncorrelated with the level of a firm's asset value, does impact credit spreads in a model with intermediate default. While it does so in a modest way for longer-term credit spreads, it stands to make a significant impact on spreads for up to ten years to maturity. The same is true for the correlation between shocks to asset values and variances. A modest "leverage effect" at the unlevered firm value level leads to higher spreads with a pattern similar to that of the volatility of asset risk: a non-trivial increase in long-term spreads, but an important increase in relative spreads for maturities up to ten years. Finally, the dominant effect stems from the market price of asset volatility risk, which can increase spreads by an order of magnitude.

Of course, comparative statics are limited in that they do not reflect the constraints faced when taking a model to the data. To address this, we rely on the Huang and Huang (2003) calibration setting as a benchmark. This involves requiring the model to simultaneously fit four moment conditions: the historical probability of default, recovery rates, equity risk premia and leverage. We first confirm that, in the absence of stochastic asset risk, our model replicates the credit spread puzzle - that is, it is unable to generate credit spreads in line with historical averages. However, we find that for reasonable parameter values governing volatility dynamics and risk premia, our model resolves or significantly mitigates the underestimation that forms the basis for the credit spread puzzle. In other words, our framework is not subject to an inherent inability to explain historical credit spread levels while matched to the moments used in HH, where the puzzle was first documented.

One potential concern with this result is that several parameters remain free in our exercise.[7] To address this, we conduct an analysis to identify which of the three channels have the ability to significantly impact credit spreads. We find that the only means by which a stochastic asset risk model can influence spreads, given the four chosen moments, is through the volatility risk premium. The other two channels - volatility of asset risk and an asset level "leverage effect" - are counteracted by the matching of empirical moments. Hence, it is the size of the volatility risk premium parameter that determines how reasonable our spread estimates are. Since we do not have rating-level data on the volatility risk parameters to use as inputs, we ask instead what level of volatility risk-adjustment is necessary, within each rating group, to match not only the previous four moments, but also the historical spread level. We express our risk adjustment as a ratio that

[7]Note that this issue is present also in the Huang and Huang (2003) study.

can intuitively be thought of as analogous to the ratio between option-implied and historical equity volatilities. We find that risk-adjusted 3-month volatilities need to be between 20% and 60% higher than their physical measure counterparts to match credit spreads and the other four moments.

In addition, we find an interesting pattern across credit rating groups. Greater proportional risk premia are necessary for higher grade firms. This is consistent with recent findings by Coval Jurek and Stafford (2008) in structured credit markets. They find that although default risk is less important in an absolute sense for senior CDO tranches, systematic risk is extremely important as a proportion of total spreads for these tranches.

Like Huang and Huang (2003) , we find that the implied levels of asset volatilities are higher than historical estimates in the literature and are, in fact, more in line with levered equity volatility levels. To address this, we match our model to six moments: the four Huang and Huang moments, the spread level and the equity volatility. We find that our model with priced systematic volatility risk is able to match these moments quite easily. Although this model implies higher levels of lung run means for the risk-adjusted variances, the quantitative impact on risk-adjustment for the 3 month horizon is limited.

Since the premia may be biased by the presence of non-default components, we carry out one final calibration exercise. It has long been recognized that bond market illiquidity may be an important determinant of spreads. Given the magnitude of the credit spread puzzle documented in previous work, it is unlikely that illiquidity by itself would resolve the puzzle. However, it may well be the case that placing some of the burden of explaining total spread levels on liquidity will generate more accurate implied volatility risk premium levels. When using the level of a AAA short term spread index as a proxy for the level of illiquidity compensation, we find that required risk-adjustments as measured by the ratios of 3 month risk-adjusted to physical volatilities are reduced, in particular for higher grade firms, to a maximum of about 40%.

We then refocus our analysis on the ability of our model to fit the time-series of default swap spreads. We estimate our model firm by firm using GMM, relying on moment conditions matching default swap spreads across five maturities and realized equity volatility. For our sample of 49 firms, we document risk-adjusted mean reversion, a correlation between asset value and volatility shocks of -0.58, and asset risk volatility of 37% on average. The fit for CDS spreads is improved significantly as compared the constant volatility model. In addition, the average pricing errors are overall smaller than in four of the five models studied in Huang and Zhou (2008). While the constant volatility model generates a spread underestimation of 65%, stochastic asset risk reduces this to an underestimation of 10%. Absolute percentage pricing errors are smaller for 34 out of

35 rating / model combinations which they study and are approximately halved in comparison. The model with stochastic volatility could be rejected only for 3 firms out of 49 whereas for a constrained constant volatility version, 48 out of 49 firms lead to a rejection of the model.

This paper is organized as follows: Section 2 describes the model and explains how we derive closed-form solutions for a stochastic volatility credit risk model with fixed default boundary. Section 3 covers the comparative statics, while Section 4 discusses the various calibration exercises. Section 5 reports on our time-series specification tests, and finally Section 6 concludes.

2 The model

We model the firm's unlevered asset value X as the primitive variable. Asset value dynamics can be described by the following two SDEs

$$\frac{dX_t}{X_t} = (\mu - \delta)dt + \sqrt{V_t}dW_1, \tag{1}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_2, \tag{2}$$

where δ is the firm's payout ratio and $E(dW_1 dW_2) = \rho dt$. Under the risk-neutral measure Q, X_t follows

$$\frac{dX_t}{X_t} = (r - \delta)dt + \sqrt{V_t}dW_1^Q, \tag{3}$$

$$dV_t = \kappa^*(\theta^* - V_t)dt + \sigma\sqrt{V_t}dW_2^Q, \tag{4}$$

with $\kappa^* = \kappa + \lambda_V$ and $\theta^* = \theta\kappa/\kappa^*$, where λ_V is the volatility risk premium.[8] As asset variance V_t follows Cox-Ingersoll-Ross (CIR) dynamics, the expected asset variance at t under the objective probability measure is, conditional on an initial variance V_0, given by

$$E(V_t) = V_0 e^{-\kappa t} + \theta\left(1 - e^{-\kappa t}\right). \tag{5}$$

Under the risk-neutral probability measure, it can be written

$$E^Q(V_t) = V_0 e^{-\kappa^* t} + \theta^*\left(1 - e^{-\kappa^* t}\right). \tag{6}$$

In what follows, we provide the solutions for the firm's equity value and equity volatility. To solve for the firm's equity value, we assume that the firm issues consol bonds.

[8]Note we later assume that the market price of volatility risk, $\lambda_V V$, is proportional to the volatility of asset variance, $\lambda_V = k\sigma$, where k is a constant.

Then the equity value can be written as the difference between the levered firm value (F) and the debt value (D), i.e., $E(X) = F(X) - D(X)$. The firm's levered asset value is given by

$$F(X) = X + \frac{\eta c}{r}(1 - p_D) - \alpha X_D p_D, \tag{7}$$

where X, η, c, α, X_D and p_D denote the initial unlevered asset value, the tax rate, the coupon rate, the liquidation cost, the default boundary and the present value of \$1 at default respectively. In equation (**??**), the first term is the unlevered asset value, the second term is the tax benefit and the third term is the bankruptcy cost. The debt value is the present value of the coupon payments before default and recovered firm value at default, which is given by

$$D(X) = \frac{c}{r} + \left[(1 - \alpha)X_D - \frac{c}{r}\right] p_D. \tag{8}$$

Thus, the equity value is given by

$$E(X) = X - \frac{(1 - \eta)c}{r} + \left[(1 - \eta)\frac{c}{r} - X_D\right] p_D. \tag{9}$$

Applying Itô's lemma, we obtain the stochastic process for the equity value as follows:

$$\frac{dE_t}{E_t} = \mu_{E,t} + \frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\sqrt{X_t}dW_{1t} + \frac{1}{E_t}\frac{\partial E_t}{V_t}\sigma\sqrt{V_t}dW_{2t}, \tag{10}$$

where $\mu_{E,t}$ is the instantaneous equity return. Given the specification in equation (**??**), we obtain the model-implied equity volatility as

$$\sigma_{E,t} = \sqrt{\left[\left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}\right] V_t}. \tag{11}$$

As is clear from equation (**??**), we can solve for the firm's equity value once p_D is found. Under the risk-neutral measure Q,

$$p_D = E^Q[e^{-r\tau}], \tag{12}$$

with $\tau = \inf\{s \geqslant 0, X_s \leqslant X_D\}$. To solve for p_D, we need to compute the probability density function of the stopping time τ under measure Q. We solve for the default probability by applying Fortet's lemma.

Recently, Longstaff and Schwartz (1995) introduce this approach into the finance literature to solve for the default probability in a stochastic interest rate setting. Collin-Dufresne and Goldstein (2001) extend Fortet's equation to the case where the state variables (leverage ratio and interest rate) follow a general two-dimensional Gaussian Markov process. In both the Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001), the state variables are assumed not only to be Markov, but also Gaussian. However this is not the case given our volatility dynamics. In order to apply Fortet's equation to our framework, we first have to solve for the joint probability density of the asset value and asset variance. Next we briefly outline the steps involved.

First, define $z_t = \ln(X_t/X_D)$, a distance to default and $p(z_t, V_t, t|z_0, V_0, 0)$ the transition density function conditional on log asset value being z_0 and asset variance being V_0 at the outset. Further, denote $H(z_\tau, V_\tau, \tau|z_0, V_0, 0)$ the probability density that the first passage time of the log asset value to z_D is τ and the asset variance takes value V_τ at τ. Since z_t and V_t follow a two-dimensional Markov process in the stochastic volatility model, applying the Fortet's lemma, we obtain for $z_0 > z_D > z_t$,[9]

$$p(z_t, V_t, t|z_0, V_0, 0) = \int_0^t d\tau \int_0^\infty dV_\tau H(z_\tau = z_D, V_\tau, \tau|z_0, V_0, 0) p(z_t, V_t, t|z_\tau, V_\tau, \tau). \tag{13}$$

The probability density $H(z_\tau, V_\tau, \tau|z_0, V_0, 0)$ is implicit in equation (**??**), which we first discretize and then use a recursive algorithm to solve for numerically. We discretize time T into n_T equal subperiods and define $t_j = j\frac{T}{n_T} = j\triangle t$ with $j\epsilon\{1, 2, \cdots, n_T\}$. Let the maximum and minimum for the asset variance be $\bar{V}$ and $\underline{V}$, respectively. We discretize the variance V into n_V equal increments and denote $V_i = \underline{V} + i\triangle V$ with $i\epsilon\{1, 2, \cdots, n_V\}$ and $\triangle V = \frac{\bar{V} - \underline{V}}{n_V}$. Furthermore, we define $q(V_i, t_j) = \triangle t \cdot \triangle V \cdot H(z_{t_j = z_D}, V_{t_j} = V_i, t_j|z_0, V_0, 0)$. Note that $H(z_{t_j = z_D}, V_{t_j} = V_i, t_j|z_0, V_0, 0)$ is the probability density that the default time is t_j and asset variance is V_i at default. Then, the discretized version of equation (**??**) is

$$p(z_{t_j}, V_i, t_j|z_0, V_0, 0) = \sum_{m=1}^{j} \sum_{u=1}^{n_V} q(V_u, t_m) p(z_{t_j}, V_i, t_j|V_u, t_m), \forall i\epsilon\{1, 2, \cdots, n_V\}. \tag{14}$$

Given the joint transition density of z_t and V_t, we obtain $q(V_i, t_j)$ recursively as follows:

$$q(V_i, t_1) = \triangle V p(z_{t_1}, V_i, t_1|z_0, V_0, 0),$$

$$q(V_i, t_j) = \triangle V \left[p(z_{t_j}, V_i, t_j|z_0, V_0, 0) - \sum_{m=1}^{j-1} \sum_{u=1}^{n_V} q(V_u, t_m) p(z_{t_j}, V_i, t_j|V_u, t_m) \right], \forall j\epsilon\{2, 3, \cdots, n_T\}.$$

The probability that the default (first passage) time is less than T is given by

$$Q(z_0, V_0, T) = \sum_{j=1}^{n_T} \sum_{i=1}^{n_V} q(V_i, t_j). \tag{15}$$

Therefore, given the joint transition density function of z_t and V_t, we can apply Fortet's lemma to solve for the default probability. In the next subsection, we detail the procedure to solve for the

[9] One main intuition behind the Fortets lemma is that given a continuous process, if it starts at z_0 which is higher than a fixed boundary (z_D), it has to cross the boundary to reach a point below the boundary (zt).

joint transition density of z_t and V_t: first, by solving for the joint characteristic function and then using inverse Fourier to back out the transition density.

Define $\Psi(t)$ as the joint characteristic function of (z_T, V_T) conditional on (z_t, V_t) at $t < T$, i.e., $\Psi(t) \equiv E_t^Q \left[e^{i(\varphi_1 z_T + \varphi_2 V_T)} | z_t, V_t\right] = \Psi(\varphi_1, \varphi_2; z_t, V_t, h)$, where $h = T - t$. Zhylyevskyy (2010) shows that $\forall \sigma > 0$, the solution for $\Psi(t)$ is given by

$$\Psi(\varphi_1, \varphi_2; z_t, V_t, h) = e^{f_1(h;\varphi_1,\varphi_2) + f_2(h;\varphi_1,\varphi_2)V_t + i\varphi_1 z_t}, \tag{16}$$

where

$$f_1(h; \varphi_1, \varphi_2) = h\left(r - \delta - \frac{\kappa^*\theta^*\rho}{\sigma}\right) i\varphi_1 + \frac{\kappa^*\theta^*}{\sigma^2}\left[h\kappa^* + h\sqrt{G} + 2ln\frac{H+1}{He^{h\sqrt{G}}+1}\right], \tag{17a}$$

$$f_2(h; \varphi_1, \varphi_2) = \frac{1}{\sigma^2}\left[\kappa^* - i\rho\sigma\varphi_1 - \sqrt{G}\frac{He^{h\sqrt{G}}-1}{He^{h\sqrt{G}}+1}\right], \tag{17b}$$

with

$$G(\varphi_1) = \sigma^2(1-\rho^2)\varphi_1^2 + (\sigma^2 - 2\rho\sigma\kappa^*)i\varphi_1 + {\kappa^*}^2,$$

$$H(\varphi_1, \varphi_2) = -\frac{i\rho\sigma\varphi_1 - \kappa^* - \sqrt{G} + i\sigma^2\varphi_2}{i\rho\sigma\varphi_1 - \kappa^* + \sqrt{G} + i\sigma^2\varphi_2}.$$

Let a_1, a_2, b_1 and b_2 be "large" in absolute value. Further, define $\triangle_1 = \frac{b_1 - a_1}{N_1}$, $\triangle_2 = \frac{b_2 - a_2}{N_2}$, $\varphi_{j_1} = a_1 + j_1\triangle_1$ and $\varphi_{j_2} = a_2 + j_2\triangle_2$, where $j_1 = 0, 1, \cdots, N_1$ and $j_2 = 0, 1, \cdots, N_2$. Zhylyevskyy (2010) applies a kernel-smoothed bivariate fast Fourier transformation and obtains the conditional joint density of (z_T, V_T) as follows:

$$f(z_T, V_T; z_t, V_t, h) \cong \frac{1}{4\pi^2}\triangle_1\triangle_2 W(\triangle_1 z_T, \triangle_2 V_T)\sum_{j_2=0}^{N_2}\sum_{j_1=0}^{N_1} e^{-i(z_T\varphi_{j_1} + V_T\varphi_{j_2})}\Psi(\varphi_{j_1}, \varphi_{j_2}), \tag{19}$$

where

$$W(\triangle_1 z_T, \triangle_2 V_T) = \int_{a_2}^{b_2}\int_{a_1}^{b_1} e^{-i(\triangle_1 z_T x + \triangle_2 V_T y)} K(x, y)dxdy,$$

and

$$K(x, y) = \begin{cases} \frac{(1-|x|)^2(1-|y|)^2}{x^2y^2 + x^2(1-|y|)^2 + (1-|x|)^2y^2 + (1-|x|)^2(1-|y|)^2}, & \text{if } |x| \le 1 \text{ and } |y| \le 1, \\ 0 & \text{elsewhere.} \end{cases}$$

3 Comparative Statics

Introducing stochastic volatility into a credit risk model adds three new potential channels for asset risk to influence credit spreads. First, the very fact that volatility is random may impact spreads directly. As we shall see, this is particularly true for short-term credit spreads. Second, volatility may be correlated with shocks to asset value. For example, there may be a "leverage effect" or asymmetry at the unlevered volatility level - that is, the asset risk may increase as the value decreases. This would work over and above the traditional financial leverage effect that is already present in Merton (1974) and subsequent models. Third, volatility risk may be systematic and carry a risk premium that is eventually reflected in credit spreads. In what follows, we address each of these channels. First, we study them in a comparative statics setting. We acknowledge up front the limitations of such an exercise, which does not require the model to match empirical moments. However, it does help crystallize the economic intuition for the different effects. Later, we reconsider the impact of stochastic volatility in a calibration experiment akin to that designed by Huang and Huang (2003).

3.1 Stochastic volatility and the term structure of credit spreads

The most obvious potential channel through which our model can influence credit spreads relative to existing models is the randomness in the volatility itself. Uncertainty about the volatility level generates fatter tails in the asset value distribution, which, all else equal, increases the likelihood of distressed scenarios and thus spreads. Figure 1 demonstrates this effect by retracing the yield spread curves for different levels of the volatility of the asset variance, nesting the constant volatility case, corresponding to the original Black and Cox (1976) model. Panel A plots the yield spread curves in basis points, whereas Panel B plots the ratios of the spread curves relative to the constant volatility case. Note that the effect can be quite significant but is more so for maturities less than 10 years. This is even more noticeable in the lower panel of the figure that reproduces the same data in terms of ratios of spreads to the Black and Cox (1976) case. For maturities less than 7 years, it is quite straightforward for the model to more than double spreads. The relative effect dissipates further out on the term structure and seems to reach stable levels after the 10-year tenor - a spread increases in the range of 10% to 20% of the constant volatility spread.

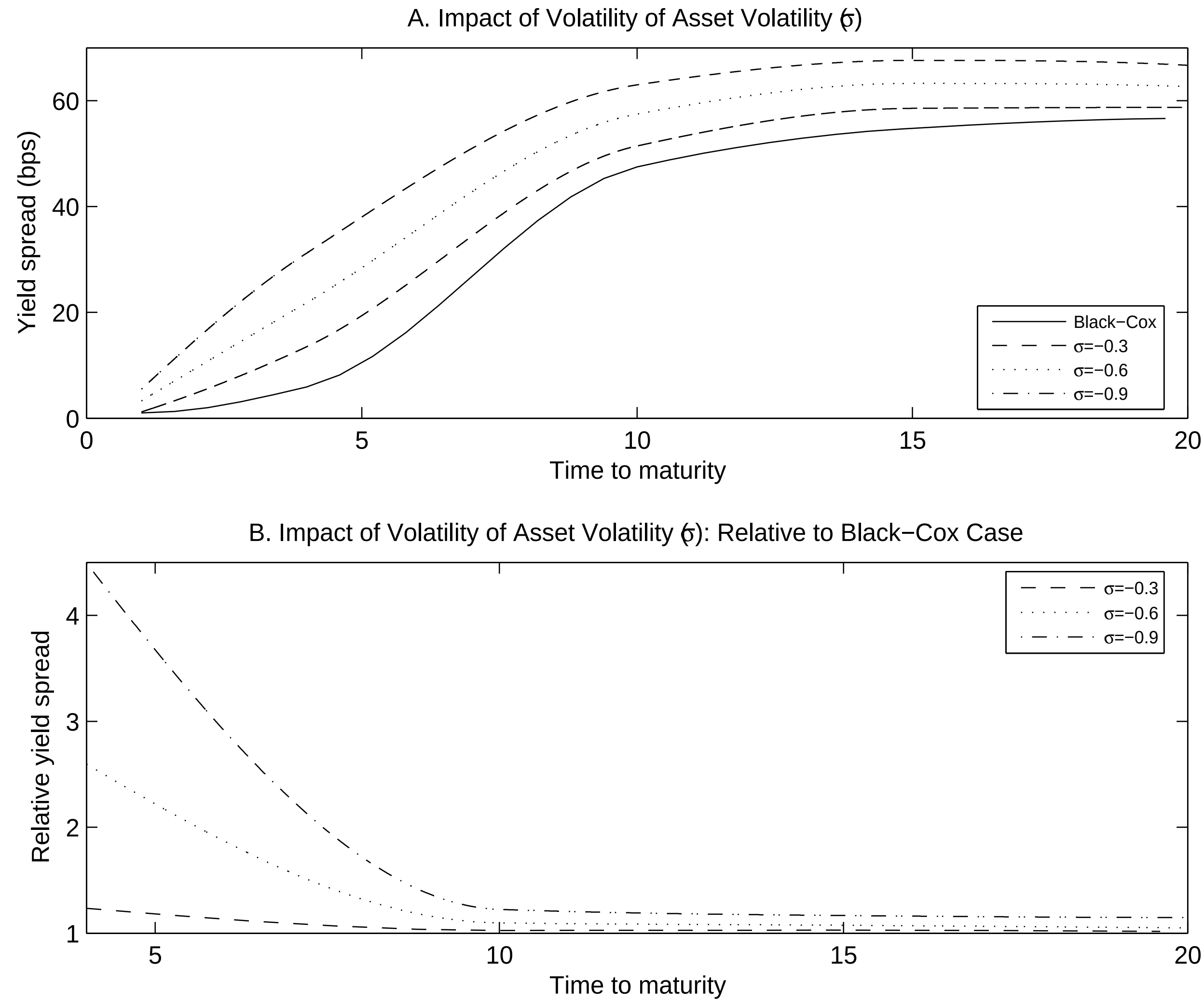

Figure 1. The impact of the volatility of volatility (σ) on the yield spread. This figure shows the impact of the volatility of volatility on the yield spread when the market price of volatility risk is zero. In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\rho = -0.1$, $\theta = 0.21^2$.

3.2 Asymmetric asset volatility and credit spreads

Since Black (1976) and Christie (1982), the question remains whether the observed negative correlation between equity prices and equity volatilities is a purely financial effect. More recently, Choi and Richardsson (2009) study firm-level returns and document a degree of negative correlation between asset values and asset volatilities. We now consider the comparative statics of the parameter that governs this asymmetry in our setting, ρ the instantaneous correlation between shocks to asset value and volatility.

Figure 2 visualizes the relationship between asset volatility asymmetry and spreads. The second panel reports ratios of spreads for varying levels of ρ to the spread in the constant volatility level. For this experiment, we make conservative assumptions regarding the volatility of asset risk, setting $\sigma = 0.3$ and $\kappa = 4$. The case where $\rho = 0$ corresponds to a Black & Cox model extended to allow for asset-risk dynamics independent of asset value dynamics. The resulting spreads are barely higher than in the constant volatility case.

Setting the asymmetry parameter to $\rho = -0.3$, which implies positive shocks to asset risk on average when asset values suffer negative shocks, has a limited effect on spreads, in the range of 2-10 basis points. For short term spreads (less than five years), this amounts to a non-trivial relative increase - spreads are approximately doubled. The absolute size of the increase is relatively stable so that for longer tenors such as 15-20 years, the percentage stabilizes around 5%. Increasing the correlation between asset volatility and value shocks to $\rho = -0.6$ provides a more significant boost in spreads. With this level of correlation, 5-year spreads essentially triple as compared to the zero correlation case. At 15 years, spreads increase by about a fifth of the no-asymmetry spreads.

The pattern for the relative spread increases is quite similar to the one reported for the volatility of volatility parameter σ. It seems that the impact of asymmetry might be quantitatively slightly more important than volatility risk itself, although not dramatically. However, given that in a situation where a model is implemented empirically it is faced with matching several moments of the data, this result is limited to a ceteris paribus setting. We will return below to whether this holds in a calibration setting below. In addition, we will estimate the amount of asymmetry that best describes firm specific time series of equity volatilities and default swap spreads.

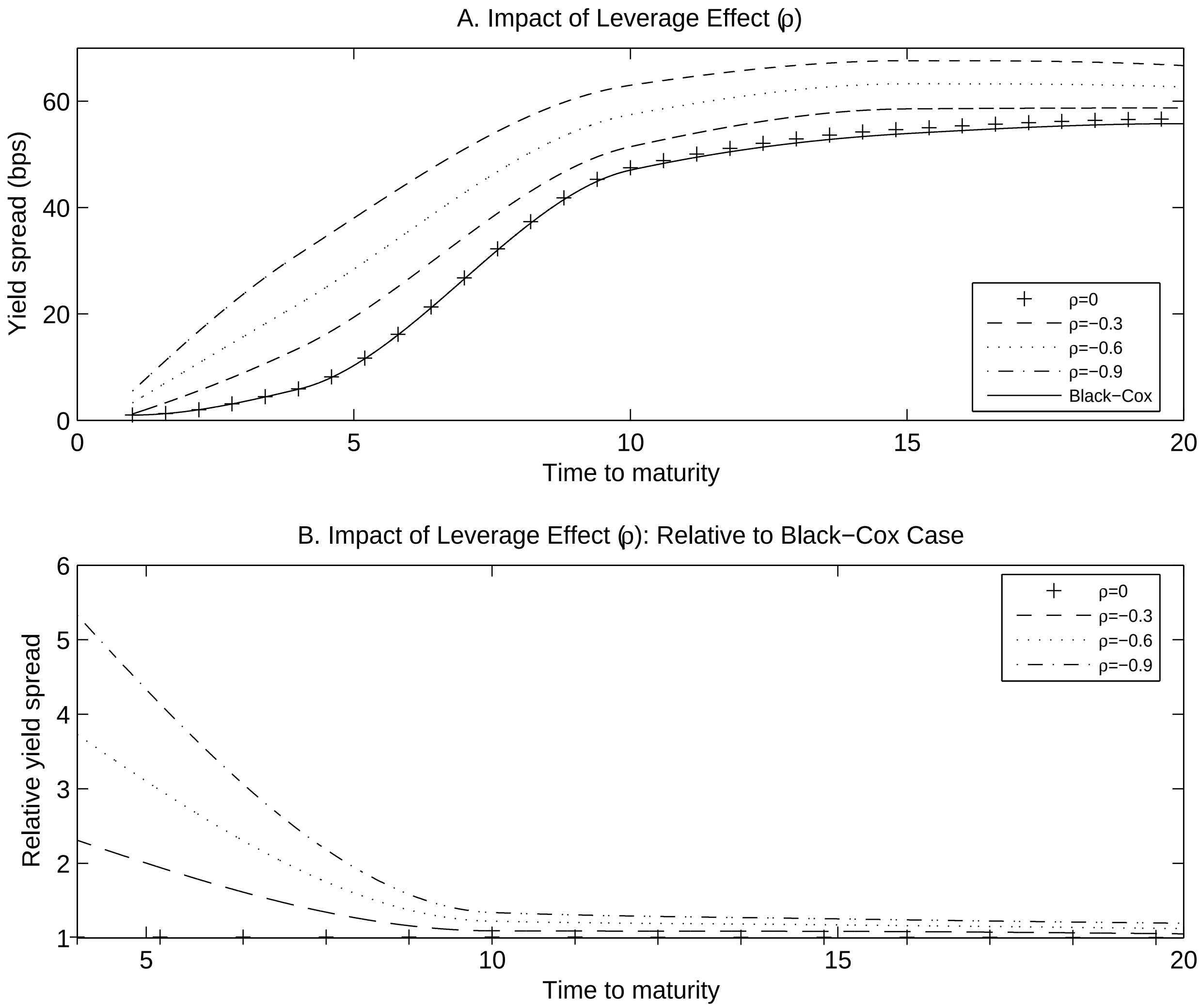

Figure 2. The impact of the leverage effect (ρ) on the yield spread. This figure shows the impact of the leverage effect on the yield spread when the market price of volatility risk is zero. In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\sigma = 0.3$, $\theta = 0.21^2$.

3.3 Asset risk premia

Finally, Figure 3 illustrates the impact of volatility risk premia on credit spreads without any asymmetry effect ($\rho = 0$). As can be seen, the effect of the risk premium parameter k is first-order. Spreads can be be increased dramatically by allowing for systematic asset volatility risk. For example, when $k = 3$, spreads more than triple for some maturity segments.

In a relative sense, it is clear from the lower panel of Figure 3 that, like in the case of volatility risk (σ) and asymmetry (ρ), the effect dominates the shorter part of the term structure, up to about 10 years. However, the sheer magnitude of the impact makes the effect significant for all maturities. While short-term spreads can be inflated tenfold, long-term spreads can easily double, if not triple

Obviously, the difficulty at this stage will be to determine reasonable values for the unlevered volatility risk premium. We address this below, and we will see that this third effect of stochastic volatility on credit spreads is in fact the dominant one, and will survive the calibrations to empirical moments.

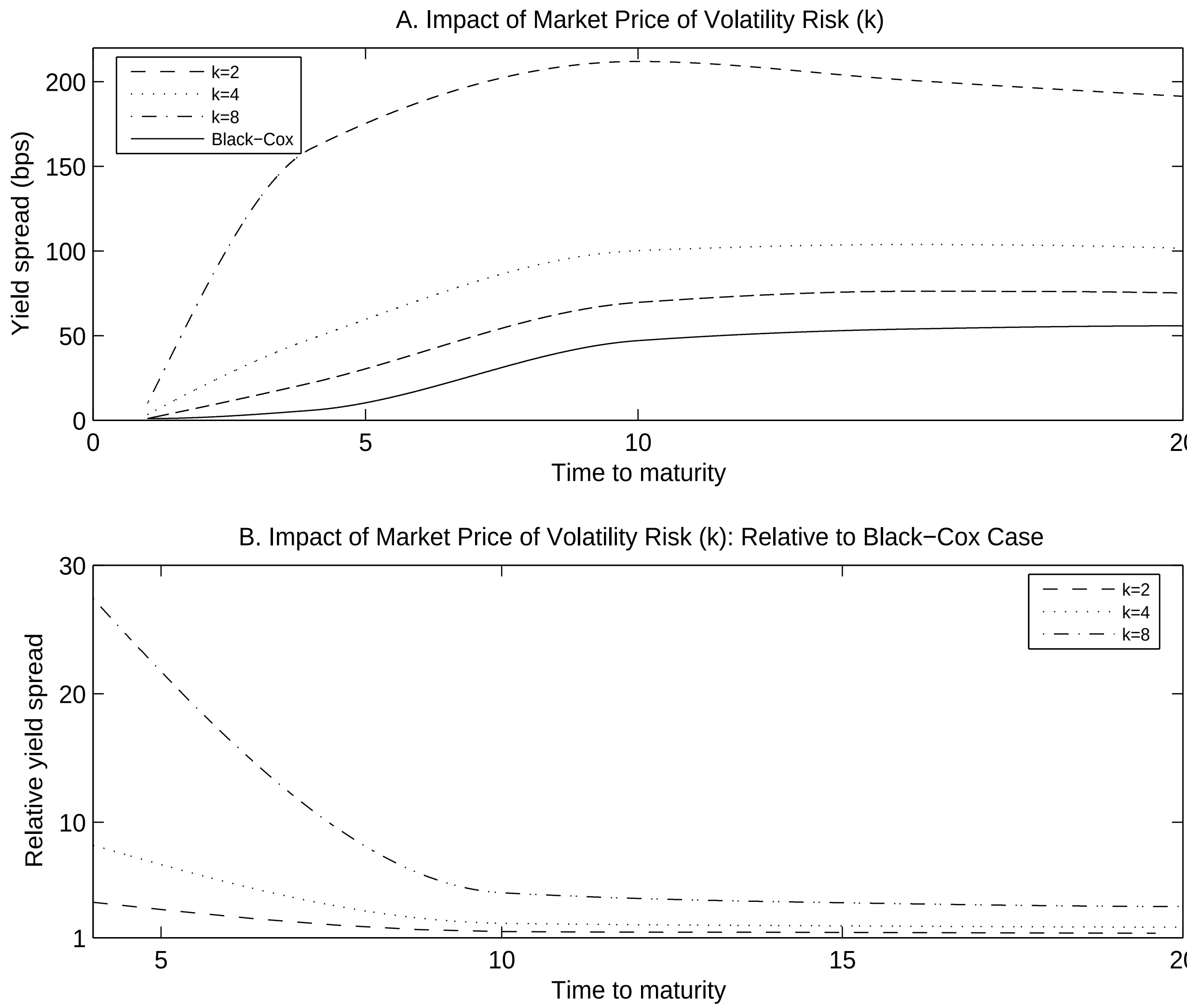

Figure 3. The impact of the market price of volatility risk (k) on the yield spread. This figure shows the impact of the market price of volatility risk on the yield spread in the absence of leverage effect ($\rho = 0$). In Panel A, the Y-axis illustrates the absolute value of the yield spread, which is calculated as the difference between the bond yield and risk-free rate. The solid curve corresponds to the Black-Cox (1976) setting, where the asset volatility is a constant. In Panel B, the values in the Y-axis are normalized by (or relative to) the corresponding values from the Black-Cox case. The initial asset value $X_0 = 100$, the default boundary $X_B = 35$, the initial asset volatility is 21%, the yearly interest rate is 8% and the asset payout ratio is 6%. The other parameter values for the stochastic volatility model are: $\kappa = 4$, $\sigma = 0.3$, $\theta = 0.21^2$.

4 Stochastic Asset Volatility and the credit spread puzzle

We have now documented that in a comparative static setting, three channels exist that may have an important effect on credit spreads: the volatility of asset risk itself, a "leverage" effect at the unlevered firm level and risk premia associated with shocks to asset risk. All three effects have the potential to help structural models achieve the levels of credit spreads necessary to address what has recently become known as the credit spreads puzzle. Huang and Huang (2003) calibrate a selection of different structural models to historical default rates, recovery rates, equity risk premia and leverage ratios. They find that all models are consistently incapable of simultaneously matching credit spreads while calibrated to these four moments. Their results are striking since they compare models with quite different features: stochastic interest rates, time-varying leverage, jump risk, counter-cyclical risk premia, endogenous default and strategic debt service. None of these extensions of the basic Merton (1974) framework is able to more than marginally bring market and model spreads closer to each other. This finding forms the basis for the credit spread puzzle.

We now ask whether the three channels through which stochastic asset volatility may influence spreads in our model, can help reconcile model with market spreads on average. In order to do so, we perform a calibration experiment closely following the methodology used in Huang and Huang (2003). In other words, we require our model to match, for different rating categories, the following observables

1. The historical default probability
2. The equity risk premium
3. The leverage ratio
4. The recovery rate

Table 1 reports on this exercise. We assume values for the additional parameters to be $\rho = -0.1, \sigma = 0.3, \kappa = 4$, and $k = 7$. The asymmetry is chosen to be modest as reported by Choi and Richardssson (2009). By means of comparison, in equity markets, Heston (1993) uses $\rho = -0.5$, while Broadie Chernov and Johannes (2009) use $\rho = -0.52$ and Eraker Johannes and Polson (2003) find values for ρ between -0.4 and -0.5. Pan (2002) uses a value for $\lambda_V = k\sigma$ equal to 7.6, while Bates (2006) uses a λ_V equal to 4.7. Our choice of k implies a volatility risk premium $\lambda_V = 2.1$. Bates (2006) documents estimates of κ in the range of 2.8 and 5.9 for a selection of models, whereas Pan (2002) estimates values ranging between 5.3 and 7.

For the Aaa category we are able to explain about 96% of average historical spread levels. For Aa to Baa we explain between 73% and 82% while for the two lowest we actually overestimate spreads by 13% and 24% respectively. This compares to 16% for Aaa, 29% for Baa and a maximum of 83% for B rated firms in Huang and Huang (2003). It is clear from this table that for this set of parameters, we can address the spread underestimation for high and low rating categories and reduce it significantly for the intermediate ones.[10] The exact numbers are sensitive to whether we consider 4 or 10 year spreads and we see that the hardest spreads to fit are high grade and short term. Nonetheless, the overall impression remains. Our framework, for reasonable and conservative inputs, does not suffer from the same systematic underestimation problem that all the models studied in Huang and Huang (2003) are subject to.

Unfortunately, we do not have rating-specific estimates of the new parameters related to our stochastic volatility model. We cannot claim that our model is able to match historical spreads given historical moment restrictions, only that it has little difficulty in reaching the required spread levels. To highlight the marginal importance of stochastic volatility in our model, the top panel in Table 2 repeats the calibration exercise in Table 1 with all parameters related to time-varying volatility set to zero ($\sigma = \rho = \kappa = k = 0$). The model thus recovered, corresponding to Black and Cox (1976), behaves very similarly to those studied in Huang and Huang (2003). For high grade bonds, the model cannot explain any significant part of the spread and reaches a maximum of 68% for the lowest grade bonds. This clearly highlights that various aspects of time-varying volatility are at the heart of the improved performance of our model in relation to historical spreads. This stands in sharp contrast to the previous literature.

However, the lack of granularity of our volatility parameter estimates remains. To address this, we carry out a "comparative statics" analysis of the calibration in Table 1. The intent is to understand the relative contribution of each of the three parameters to the improved spread fitting ability of the model. As noted above, panel A in Table 2 defines the benchmark, by shutting down stochastic asset volatility altogether. This benchmark corresponds to the credit spread puzzle as presented in Huang and Huang (2003). Panel B opens up for stochastic volatility by setting $\sigma = 0.3$ but without any asymmetry (or "leverage" effect at the asset level) or risk premium for asset volatility risk. In this case, the predicted spreads remain very similar to the constant volatility case and the credit spread puzzle remains.

Thus uncorrelated and fully idiosyncratic asset risk is not the channel that allows our model to generate sufficient yield spreads. This may seem counterintuitive given that the comparative statics

[10] Note that there is always some value for the variance risk premium that will fit the spread exactly. We return to that exercise below.

discussed above and depicted in Figure 2 appear to permit stochastic asset risk significant leeway in influencing credit spreads. The reason for this perhaps surprising result can be traced back to the design of the calibration experiment: the four moment conditions (default losses and rates, leverage and equity risk premia) work to cancel out the effect of more pronounced tails in the asset value distribution. For a given level of volatility risk, the increased spread that would result in a ceteris paribus exercise is mitigated by the requirement to fit the moments in the calibration. In particular, the model will tend to produce lower asset volatility levels for the high grade scenarios where the underestimation of spreads is the most severe.

Panel C of Figure 2, adds a modest amount of asymmetry to the scenario summarized in panel B. Again, the effect, which was significant in the comparative statics above (see Figure 1), is cancelled out by the requirements to fit the moments in the calibration. Thus a leverage effect at the asset value level with fully idiosyncratic asset risk does not help explain the puzzle documented in Huang and Huang (2003). Finally, Panel D of Figure 2 adds a risk premium to asset volatility. This effect is not constrained by the four moments used in the calibration. The spread explanation percentages increase significantly to between 75% and 128%, which, while not fitting spreads within each rating category accurately, does remove any systematic underestimation of spreads. In summary, it appears that the market price of asset volatility risk is one channel through which a structural credit risk model's ability to explain market spreads can be significantly improved.

So far, we have based our analysis very closely on the HH calibration as it forms the basis for the credit spread puzzle. However, although we have established that our framework is not subject to the limitation of generating insufficient spreads, we still face the problem of risk premium estimation. Although, at this stage, a full-fledged firm level estimation of risk premia is beyond our paper's scope, we will attempt to better understand the required volatility risk premia. In a first step, we simply ask what levels of asset volatility risk premia would be necessary to explain market spreads in the HH calibration.

Table 3 reports our results for an exercise where we augment the moment conditions used in HH by a requirement to also fit historical spreads (in addition to default losses, probabilities, equity risk premia and leverage ratios).[11] Rather than report the parameter k directly, which has no obvious intuitive empirical counterpart, we report λ_V as well as the square root of the ratio of the three month expected risk-adjusted and historical volatilities respectively. This ratio, $\frac{E^Q(V_t)}{E(V_t)}$, is intended

[11]It is important to note that the two types of risk premia present in this calibration are not distinct. The level of the asset volatility premium impacts the asset return risk premium which is matched to historical equity risk premia. Thus, we are not merely adding a free parameter which trivially fits the new moment condition. All five moment conditions are satisfied simultaneously.

to provide a quantity similar in spirit to observable ratios of option-implied and historical equity volatilities.

We find that 3 month risk adjusted volatilities need to be need to be between 22% and 53% higher that their historical counterparts, in order to fit historical credit spread data for ten year bonds across the rating categories. This wedge for 4 years bonds lies between 7% and 54%. The risk premium parameter λ_V ranges from -1.23 to -2.55 for the ten year spreads and between 0.45 and -2.55 for 4 year bonds. We are not aware of any empirical estimates of this quantity for individual firms. As mentionned above, for equity indices, λ_V has been found to be greater in absolute terms (-7.6 in Pan (2002) and -4.7 in Bates (2006)). Given that our estimates are for an unlevered volatility, risk premia should be lower in absolute terms. In addition, it has been shown that measures of implied volatilities tend to be lower relative to historical volatility for individual stocks (see e.g. Carr (2008)).

Another interesting finding that emerges from Table 3 is the pattern of the risk premia across ratings. The required risk premium is higher for the higher grade firms than for speculative grade firms. The ratio of risk-adjusted to historical volatilities is in fact monotonically increasing in credit quality. This implies that higher grade firms are relatively more sensitive to systematic shocks to volatility. A similar point has been made recently for the structured credit markets. Coval Jurek and Stafford (2009) show that prices in long dated index options markets imply proportionally much higher risk premium components in senior than junior CDO tranches. For single-name securities grouped into credit rating categories, Huang and Huang (2003) document that it is harder to explain higher grade spreads with constant volatility structural models. Berndt et al (2008) and Elkamhi and Ericsson (2008) show that ratios of risk-adjusted to historical default probabilities are indeed increasing in credit quality. In sum, these are consistent with the average economic state in which a highly rated fixed income instrument defaults being worse than the average economic state in which a lower rated security defaults. Along the same line of reasoning, the higher the systematic risk of a firm, the greater the ratio of its risk-adjusted volatility over its historical volatility.

We note that the model fits spreads with limited impact on the most significant free parameter in the calibration we (like Huang and Huang (2003)) use - the asset volatility level. In panel A of Table 2, which corresponds to our version of the HH calibration with constant asset risk, implied asset volatilities range between 25% and 35%. This is comparable to the numbers reported in the base case by HH which range between 25% and 40%. It should be noted that these estimates are in fact quite high. Indeed, they are comparable to the estimates of *equity* volatilities across rating categories reported by Schaefer and Strebulaev (2008) which range from 25% for AAA firms to 42% (61%) for BB (B) firms respectively. In contrast, Schaefer and Strebulaev (2008) report asset volatilites averaging 22% for most rating categories and increasing to 28% for B firms. In other words, when confronted with the four moment conditions used so far, the model remains unable to produce reasonable asset and equity volatilities.

To address this problem, we modify the calibration by requiring that our model also fit historical equity volatilities, leaving the model with a total of six moments to match. Table 4 provides evidence from this modified calibration exercise. Our stochastic volatility model now simultaneously fits

1. default probabilities,
2. recovery rates,
3. leverage ratios,
4. equity risk premia,
5. equity volatilities,
6. and credit spreads.

The model is able to fit the four HH moments in addition to the historical equity volatilities and spreads, while retaining the values previously assumed for ρ and σ, while letting k and κ float. Again, instead of reporting the implied k values, we present the ratio of expected three month volatilities under the risk-adjusted and historical probability measures respectively.

The calibration produces quite reasonable implied asset volatility levels ranging from 21% for rating categories Aaa to Baa and 25% for Ba and 37% for B. These figures are much closer to those reported by Schaefer and Strebulaev although a little higher for B firms. Thus it appears that our model does not need to systematically suggest unrealistically high levels of asset volatility to fit the required moments, in contrast to the constant volatility models studied in HH. Furthermore, the link between ratings and risk-adjustment noted above survives. Ratios of expected volatilities and the risk premium parameter λ_V decrease as credit quality deteriorates. The levels remain similar.

We have thus shown that a stochastic volatility model with priced volatility risk is able to match all the moments in Huang and Huang (2003) in addition to credit spreads and historical equity volatilities for some level of $\frac{\theta^*}{\theta}$, the ratio of long term volatilities under the risk-adjusted and historical probability measures. However, we have so far placed the full burden of explaining spreads beyond a simple structural model on the presence of a variance risk premium. But corporate bond spreads may contain compensation for other risks not captured by our model. A strong candidate missing factor is the illiquidity of corporate bond markets. If illiquidity is an important determinant of bond spreads then our estimates of required volatility risk adjustments may be excessive. To understand how important such an effect might be, we follow Almeida and Phillippon (2007) in their liquidity correction of the Huang and Huang (2003) rating based scenario. We assume that the AAA rated one year yield spread contains negligible compensation for default risk and can be thought of mainly as compensating for illiquidity relative to a one-year government bond. We then substract this yield spread from our previous spreads to obtain rough estimates of default risk only spreads. Clearly this approach is simplistic as the liquidity spread may well depend on the credit rating (see. e.g. Ericsson and Renault (2006) and Xiong and He (2010)). However, the objective here is merely to gauge the quantitative impact on required variance risk premia of a reasonable level of liquidity adjustment, not a precise estimation of risk premia per rating category.

Table 5 reports our findings. There is a negligible impact on the implied asset volatilities. The significant, and expected, effect is to reduce the required variance risk premia. For 10 year bonds, the required ratio of expected variances now ranges from 1.22 to 1.39 (compared to the previous range of 1.23 to 1.56), while the range of lambda is -1.21 to -1.76 (compared to -1.26 to -2.34). The effect is strongest for higher rated bonds, which is intuitive since it is those spreads that are adjusted the most in a relative sense.

Table 6 summarizes the variance risk premium adjustments across ratings, calibration scenarios and risk adjustment metrics. The table reports on the ratio of long term means ($\frac{\theta^*}{\theta}$), ratios of risk-adjusted to historical expected variances with 1 and 3 month horizons, as well as the risk premium parameter λ_V directly. One clear pattern is that the risk-adjustment (as a ratio) depends critically on the horizon of the metric. The ratio $\frac{\theta^*}{\theta}$ has a perpetual horizon as it measures the wedge between the long run mean levels for the variance dynamics under the risk-adjusted and historical probability measures respectively. It tends to be higher than the ratio of expected variances at 3 months, which in turn is higher than that for a one month horizon.

Han and Zhou (2010) find ratios of risk-adjusted variances (measured as one month model-free option implied variances)in a large panel of individual firms to be on average 38% higher than their physical counterparts. This is higher than our typical risk adjustment which at the one month horizon lies between 10% and 20% for 10 year bonds (6% to 15% for 4 year bonds with the liquidity correction). On the other hand estimates of variance risk premia in Carr and Wu (2008), which are estimated using variance swap returns, come in lower at about 4% on average. Although the variance risk premia our model requires to explain the cross-section of historical credit spreads do not appear unreasonable, we leave further empirical work in this direction to future work.

All ratios decrease as credit quality deteriorates, as does λ_V, reflecting, as discussed above, that risk premia are proportionally more important to explain credit spreads for higher grade firms. Requiring the model to fit equity volatility has a positive effect on the required ratio $\frac{\theta^*}{\theta}$ for higher rated firms, while it has little effect on the other metrics.

5 Specification tests - constant vs. Stochastic asset volatility

5.1 Data

The CDS spread is the premium paid to insure the loss of value on the underlying realized at pre-defined credit events. This contrasts with the yield spread of a corporate bond, which reflects not only default risk but also the risk-free benchmark yield, the differential tax treatment and liquidity of corporate bonds vs. Treasury bonds. Further, while bonds age over time, CDS spreads are quoted daily for a fixed maturity. In addition, CDS contracts trade on standardized terms and while CDS and bond spreads are quite in line with each other in the long run, in the short run CDS spreads tend to respond more quickly to changes in credit conditions. For all these reasons, it is plausible that the CDS spread is a cleaner and more timely measure of the default risk of a firm than bond spreads. As a result, they may be better suited for specification tests of a structural credit risk model.

We collect single-name CDS spreads from a comprehensive database compiled by Markit. Daily CDS spreads reflect the average quotes contributed by major market participants. This database has already been cleaned to remove outliers and stale quotes. We require that two or more banks should have contributed spread quotes in order to include an observation (Cao, Yu, and Zhong, 2010). The data sample that is available to us include only the firms that constituted the CDX index from January 2002 to March 2008.

Our sample includes US dollar-denominated five-year CDS contracts written on senior unsecured debt of US firms. While CDS contracts range between six months and thirty years to maturity, we use the 1, 3, 5,7 and 10 years only because that are relatively more liquid than other maturities (6 months, 2 years, 20 years)

The range of restructurings that qualify as credit events vary across CDS contracts from no restructuring (XR) to unrestricted restructuring (CR). Modified restructuring (MR) contracts that limit the range of maturities of deliverable instruments in the case of a credit event are the most popular contracts in the United States. We therefore include only US dollar-denominated contracts on senior unsecured obligations with modified restructuring (MR), which also happen to be the most liquid CDS contracts in the US market (Duarte, Young, and Yu, 2007).

Together with the pricing information, the dataset also reports average recovery rates used by data contributors in pricing each CDS contract. In addition, an average rating of Moody's and S&P ratings as well as recovery rates are also included. Following Huang and Zhou (2008) we perform our test on monthly data. Their sample is restricted to 36 monthly intervals because their sample ends in 2004. Instead, we require that the CDS time series has at least 62 consecutive monthly observations to be included in the final sample. Another filter is that CDS data have to match equity price (CRSP), equity volatility computed from (TAQ) and accounting variables (COMPUSTAT). We also exclude financial and utility sectors, following previous empirical studies on structural models. After applying these filters, we are left with 49 entities in our study.

In testing structural models, the asset return volatility is unobserved and is usually backed out from the observed equity return volatility. Traditionally, researchers use a rolling window of daily returns volatility to proxy for equity volatility. In order to benchmark our results to the specification tests of alternative models covered in Huang and Zhou (2008) we use a more accurate measure of equity volatility from high-frequency data. Following Huang and Zhou(2008) we use bi-power variation to compute volatility. As shown by Barndorff-Nielsen and Shephard (2003), such an estimator of realized equity volatility is robust to the presence of rare and large jumps. The data on high frequency prices are provided by the NYSE TAQ (Trade and Quote) data base, which includes intra-day (tick-by-tick) transaction data for all securities listed on NYSE, AMEX, and NASDAQ. The monthly realized variance is the sum of daily realized variances, constructed from the squares of log intra-day 5-minute returns. Then, monthly realized volatility is the square-root of the annualized monthly realized variance.

5.2 GMM Estimation of the Model

Let $cds(t, t+T)$ and $cds^{obs}(t, t+T)$ denote the model-implied and empirically observed CDS spreads of a CDS contract at time t for which the maturity date is $t+T$ respectively. Let $\sigma_{E,t}$ and $\sigma_{E,t}^{obs}$ denote the model-implied and empirically observed equity volatilities at time t respectively. Following the literature, the solution for the model-implied CDS spread is given by

$$cds(t, t+T) = \frac{(1-R)\sum_{i=1}^{4T} B(t, t+T_i)\left[Q(t, t+T_i) - Q(t, t+T_{i-1})\right]}{\sum_{i=1}^{4T} B(t, t+T_i)\left[1 - Q(t, t+T_i)\right]/4}, \tag{20}$$

where R is the recovery, $B(t, t+T_i)$ is the default-free discount function and $Q(t, t+T_i)$ is the risk-neutral default probability. As shown in the model section, the model-implied equity volatility is given by

$$\sigma_{E,t} = \sqrt{\left[\left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}\right]V_t}. \tag{21}$$

We define $f_t(\Theta)$ as the overidentifying restrictions, which is given by

$$f_t(\Theta) = \begin{bmatrix} cds(t, t+T_1) - cds^{obs}(t, t+T_1) \\ \ldots\ldots \\ cds(t, t+T_j) - cds^{obs}(t, t+T_j) \\ \sigma_{E,t} - \sigma_{E,t}^{obs} \end{bmatrix}, \tag{22}$$

where $\Theta = (\rho, X_D, \kappa, \theta, \sigma)$ is the parameter vector to be estimated. [12] The term structure of CDS spread includes five maturities:1, 2, 3, 5, 7 and 10 years. Thus we apply the seven moment conditions (from six CDS spreads and equity volatility) to estimate five parameter values in the GMM test. Given that the model is correctly specified, we obtain that $E[f_t(\Theta)] = 0$. We define the sample mean of the moment conditions as $g_{\bar{T}}(\Theta) = 1/\bar{T}\sum_{t=1}^{\bar{T}} f_t(\Theta)$, where $\bar{T}$ is the number of time series observations. Following Hansen (1982), the GMM estimator is given by

$$\hat{\Theta} = \arg\min g_{\bar{T}}(\Theta)' W(\bar{T}) g_{\bar{T}}(\Theta), \tag{23}$$

where $W(\bar{T})$ is the asymptotic covariance matrix of $g_{\bar{T}}(\Theta)$.[13] With some regularity conditions, the GMM estimator $\hat{\Theta}$ is $\sqrt{T}$ consistent and asymptotically normally distributed given that the model is correctly specified (null hypothesis). The J-statistics is given by

$$J = \bar{T} g_{\bar{T}}(\hat{\Theta})' W(\bar{T}) g_{\bar{T}}(\hat{\Theta}). \tag{24}$$

The J-statistics is asymptotically distributed as a Chi-square with the degree of freedom being equal to the difference between the number of moment conditions and the length of Θ, which is equal to one in our setup.

We estimate our model in two steps. First, we set the initial asset volatility ($\sqrt{V_t}$) at each observation date t to be $(1 - leverage_t)\sigma_{E,t}^{obs}$ and then obtain one GMM estimator ($\widehat{\Theta}$) for Θ. In the second step, we obtain the updated asset volatility ($\sqrt{V_t^{update}}$) at each observation date t such that the model implied equity volatility to be equal to the observed one, i.e., $\sigma_E^{model}(V_t^{update}, \hat{\Theta}) = \sigma_{E,t}^{obs}$. Then use V_t^{update} to obtain the updated GMM estimator ($\hat{\Theta}^{update}$) for Θ.

[12] There are two latent variables in the estimation: asset value and asset variance at each observation date. Following Huang and Zhou (2008), we back out the asset value from the observed leverage ratio, which is defined as the ratio of face value over the asset value. We estimate the initial asset variance in the two steps to be discussed later in this section.

[13] Following Newey and West (1987), we use a heteroskedasticity robust estimator for $W(\bar{T})$.

5.3 Results

Table 8 reports summary statistics for the 49 firms (4116 default swap quotes in total) in our sample which spans the period 2002-2008 and contains firms that are also part of the sample in the Huang and Zhou (2008) study we rely on as a benchmark. This choice of data is intentional to permit a better comparison - so that any differences in our results are more likely to indicate differences across models rather than data sample.

Rating-based averages for equity volatilities range from 22% to 41% and leverage ratios from 26% to 77%. Asset payout rates are also quite similar to those in Huang and Zhou (2008) typically just above 2%. CDS spreads are similar as well. Panel C in Table 8 reports on the standard deviations of CDS spreads.

Table 9 contains the parameter estimates resulting from the GMM implementation. We note first that the degree of volatility asymmetry - that is, the correlation between shocks to asset values and asset variances (ρ) is similar across firms and ratings and averages -0.58. This is similar to values reported in the literature on equity volatilities (see Eraker Johannes and Polson (2003) and the discussion in section 4 above). This is higher than the value we assumed in the comparative statics above (-0.1) and provides evidence of a "leverage" effect at the asset value level. In other words, the asymmetry observed in equity markets stems both from mechanical changes in financial leverage as stock prices fluctuate and the negative correlation between the levels of asset values and volatility. Table 12 converts the asset value asymmetry into an equity leverage effect (Appendix C provides the necessary derivations). We find that for most rating categories, the instantaneous correliton between asset value and variance shocks is lower than for equity and equity variance shocks. However, the magnitude of this difference is small. This suggests that financial leverage only plays a minor role in the asymmetry observed at the equity return level.

The estimated speed of mean reversion under the risk-adjusted probability measure (κ^*) ranges from 0.5 and 1.2 averaged within rating categories, lower than the levels we used in the comparative statics. Given that higher mean reversion speeds will tend to reduce variance volatility, our assumptions in the comparative statics, like those make for the correlation parameter, also appear conservative. The asset variance volatility parameter (σ), is estimated to values in the range of 24% to 45%, averaging 37%, somewhat higher than our choice of parameter value in the comparative statics (30%).

We find that the default boundary is estimated to between 62% and 75% of the book value of debt. This entails that a BBB firm, whose default boundary is 67% of debt, would default at an asset value level of about 32% of its current non-distressed value. This is broadly consistent with estimates in Davydenko (2007) and Warner (1977). Firms often operate at significantly negative net worth levels before defaulting, reflecting the valuable optionality of equity when faced with financial distress. Note that the ratio of the default point to liabilities is smaller (greater) for the lower (higher) grade firms - a B (AA) firm defaults at 75% (62%) of book debt where leverage is around 77% (26%). Thus they would default at an asset value level 42% (84%) lower than current value.

Long run risk-adjusted variance levels are estimated to lie between 3% and 9%, corresponding to volatility levels of 17% and 30% respectively.

The reported mean J-stats in Table 9 are well below the critical values at conventional significance levels. This stands in stark contrast with the findings in Huang and Zhou (2008) who find that almost all the models they study: the Merton (1974), Black and Cox (1976), and Longstaff and Schwartz (1995) are consistently rejected whereas the Collin-Dufresne and Goldstein (2001) model is rejected in half of the cases. The last two columns of Table 9 report on the number of firms for which our model can be rejected. At the 1% (5%) level only 1 (3) firm out of 49 leads to a rejection of the model.

To provide a more specific benchmark by which to judge these results, Table 10 reports on the same exercise with the stochastic asset volatility channel turned off. These results are similar to the findings of Huang and Zhou (2008) for the same model. Here, the Black and Cox (1976) model can be rejected for 45 out of the 49 firms (compared to at best 87 out of 93, in HZ). Clearly, the addition of stochastic volatility renders the rejection of the model significantly harder. Note that the number of free parameters is greater when we introduce stochastic asset risk, and that this will make a rejection harder. A fairer comparison in this regard is the Collin-Dufresne Goldstein (2001) model (CDG) evaluated in Huang and Zhou (2008), which has the same number of additional parameters with respect to the Black and Cox (1976) model. In that case between and 67% and 75% of the firms lead to non rejections, compared to between 94% and 98% in Table 9. The CDG model yields somewhat lower pricing errors on the defaults swaps but, not surprisingly, faces more resistance in fitting the time series of equity volatilities.

The pricing errors are reported in Panels B and C of Table 9. The first finding is that spreads are underestimated by between 3 (A rated firms) and 61 (B rated firms) basis points with an average of 18. The direction of the bias is reminiscent of the findings across 29 of 35 rating model combinations in Huang and Zhou (2009). However, the level is significantly lower than for the Merton, Black and Cox, Longstaff and Schwartz models as reported by HZ. For the Collin-Dufresne Goldstein model, we have already noted a slightly better performance. For the BB (B) rating categories we underestimate spreads by 43 (171) basis points while Huang and Zhou find -143 (-518) basis points respectively. Moreover the dispersion of the errors is smaller. Huang and Zhou report absolute pricing errors in the range 13 to 1381 basis points across ratings (averaging 101 basis points) for the Black and Cox model. In contrast, we find a range between 7 and 96 basis points with average of 26 basis points.

A better understanding of the findings can be had by comparing to the Black & Cox (1976) model estimated on our sample (by shutting down stochastic asset risk). Our overall average underestimation with stochastic volatility is by 18 basis points (or 10 percent of the average total spread) as compared to 48 basis points (or 65% of the average total spread) with constant volatility. The average absolute pricing error is 26 basis points with stochastic volatility and 50 basis points without. Not surprisingly, the model with stochastic asset volatility does a much better job at fitting the time series of equity volatilities, generating average pricing errors (absolute pricing errors) of 15 (35) basis points as compared to -396 (1085) basis points.

Figure 4 summarizes average model implied and market spreads for the sample by rating groupings. In comparison to Huang and Zhou (2009), these figures provide a much more encouraging summary of the model's performance. Note also that one of the conclusions in Huang and Zhou is that their model finds it hard to fit both CDS *and* equity volatility time series. Our model, of course, matches equity volatilities by construction, but it appears that in doing so, it is also better able to fit the price of default insurance.

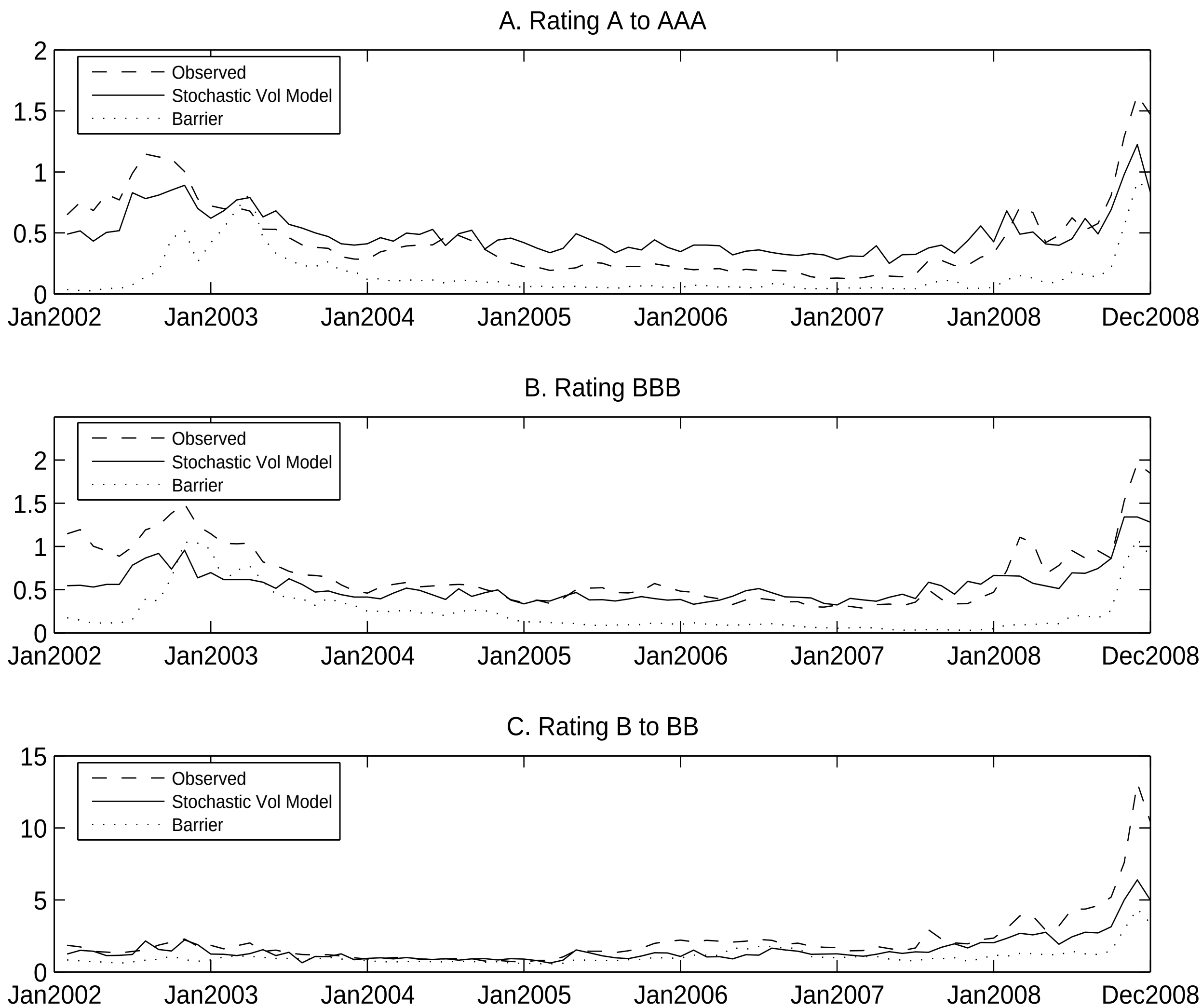

Figure 4. Observed and model-implied 5-year CDS spreads. This figure shows the time series of observed 5-year CDS spreads and those estimated from the stochastic volatility model and the Black-Cox (1976) model. One unit in the Y axis corresponds to 100 basis points.

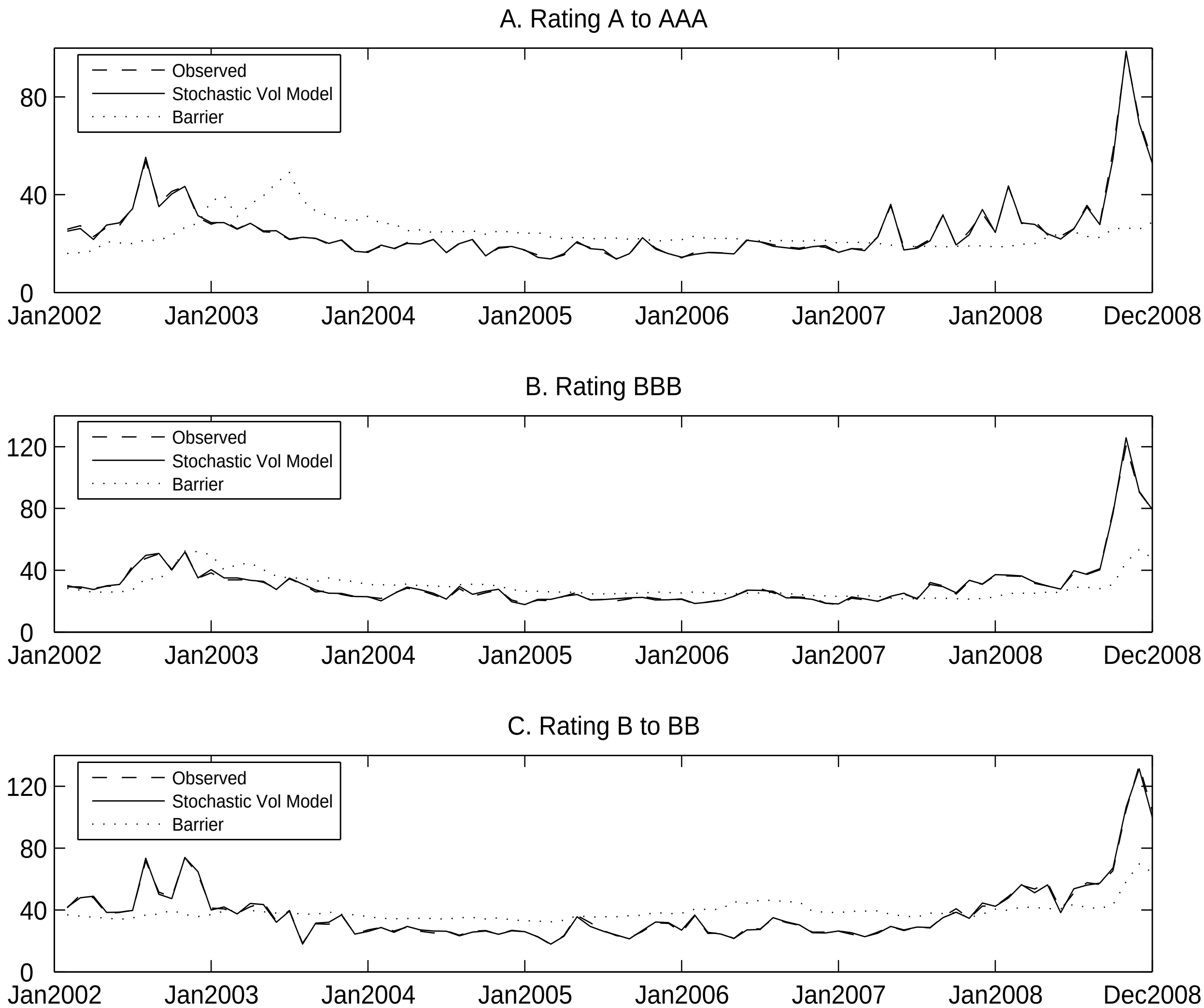

Figure 5. Observed and model-implied equity volatility. This figure shows the time series of the realized volatility, which is estimated from 5-minute intraday stock returns, and the model-implied equity volatility from the stochastic volatility model and the Black-Cox (1976) model.

6 Concluding remarks

We have developed and studied a first-passage time structural credit risk model with stochastic volatility as a means of addressing the credit spread puzzle documented in Huang and Huang (2003) and further studied in Collin Dufresne Goldstein (2009). We find that, in a comparative static setting, such a model has various ways of generating higher credit spreads than constant volatility models, but in a calibration setting, the key driver of spreads ends up being the volatility risk premium.

Having found that our model is able to generate sufficiently high credit spreads to not be subject to the credit spread puzzle, we consider the levels and patterns of volatility risk premia that are necessary to resolve the puzzle. The levels are quite plausible and the pattern is interesting. For high grade firms, the risk-adjustment needs to be proportionally higher than for lower grade firms. An Aaa firm will likely encounter financial difficulties only subsequent to a massive systematic shock to volatility echoing the findings of Coval Jurek and Stafford (2008). In the context of default swaps and corporate bonds respectively, Berndt et al (2008) and Elkamhi and Ericsson (2008) show that the risk adjustment ratios are indeed increasing in credit quality. Their results indicate that the average economic state in which a highly rated bond defaults is worse than the average economic state in which a lower rated security is likely to default. Similarly, the higher the systematic risk a firm has, the greater the ratio of its risk-adjusted volatility over its objective volatility. This translates to a downward sloping curve which links the risk adjustment ratio to the credit quality.

We extend the calibration method of Huang and Huang (2003) and find that our model, is able to fit their four moments as well as both spread levels *and* historical equity volatility levels quite easily, something earlier models have been incapable of.

Having thus evaluated the cross sectional properties of spreads implied by our model, we proceed to also study the ability of our model to explain jointly dynamics of credit spreads and equity volatilities, a task which has been shown to be out of the reach of constant volatility structural credit risk models. By construction, our model fits equity volatilities well while the fit for CDS prices is much improved relative to the findings for constant volatility models studied in Huang and Zhou (2009). In addition, this exercise provides interesting empirical evidence on the dynamics of firms' unlevered assets. We find evidence of a significant non-financial leverage effect - asset value and variances shocks are significantly negatively correlated.

The technical contribution of our paper, closed form analytics for a first passage time stochastic volatility model has many obvious applications in the credit risk literature. More generally, we believe there are numerous applications in the real options literature, where investment and volatily are closely related.

Appendix A: details on the calibration procedure

We calibrate the model in three different ways. In the first one, we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability. In the second one, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk and asset risk premium to match the target leverage ratio, equity premium, cumulative default probability and historical average yield spread. In the third one, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average yield spread and equity volatility. For all the three calibration, we assume that the firm recovers 51.31% of the face value given default.

To calibrate our model, we need to specify the asset premium and the leverage ratio. The asset premium is given by

$$\pi_X = (1 - L)\pi_E + L\pi_D, \tag{25}$$

where π_E is the equity premium, π_D is the bond risk premium and L is the firm's leverage ratio. We use the yield spread of the corporate bond over a comparable default-free bond as a proxy for the bond risk premium. The leverage ratio is given by

$$L = P/X, \tag{26}$$

where P is the face value and X is the unlevered asset value.

Also to calibrate the model to the term structure of yield spreads, we need to price a corporate bond with finite maturity. For a corporate bond with maturity T and semi-annual coupon payment, the bond price is given by

$$D_{0,T} = \frac{c}{2}\sum_{i=1}^{2T-1}\left[1 - \omega Q(0,T_i)\right]/(1+r)^{T_i} + \left(P + \frac{c}{2}\right)\left[1 - \omega Q(0,T)\right]/(1+r)^{T}, \tag{27}$$

where P is the face value of the bond, c is the annual coupon payment, T_i is the ith coupon date, ω is loss rate given default, and $Q(0,T_i)$ is the risk-neutral default probability before time T_i. We assume that the corporate bond is priced at par. Thus we can back out the annual coupon payment from equation (**??**). Also the bond yield is the same as the coupon rate.

Finally, when we perform the second and third calibrations as mentioned earlier, we use equation (**??**) to calibrate the model to the historical average equity volatility for different credit ratings.

Appendix B: a Solution for the Default Probability: One Special Case

We denote $S(z,V,h)$ as the risk-neutral probability that the log asset value z has never crossed the default boundary $z_D = 0$ before $T = t + h$, given that $z_t = z$ and $V_t = V$. Obviously, the default probability $Q(z,V,h) = 1 - S(z,V,h)$. Below we show the procedure to obtain the closed-form solution for S when S satisfies the smooth pasting condition at $z = 0$.

Define the default time as $\tau_t = inf\{s \geq t, z_s \leq z_D\}$. Then $P(\tau_t < T | z_t = z, V_t = V) = 1 - S(z,V,h)$. Since we can rewrite $S(z,V,h)$ as a conditional expectation, it follows a martingale and satisfies the following backward Kolmogorov equation.

$$S_h = \frac{1}{2}VS_{zz} + \rho\sigma VS_{zV} + \frac{1}{2}\sigma^2 VS_{VV} + (r - \frac{1}{2}V)S_z + \kappa^*(\theta^* - V)S_V, \tag{28}$$

with the initial condition $S(z,V,0) = 1$ and the boundary condition $S(0,V,h) = 0$. Define $\tilde{S}(\omega,V,h) = \int_0^\infty e^{-\omega z}S(z,V,h)dz$ with the real part of ω being positive. Given the solution for $\tilde{S}$, we obtain that $S(z,V,h) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} e^{z\omega}\tilde{S}(\omega,V,h)d\omega$ with C being a positive constant. To solve for $\tilde{S}(\omega,V,h)$, we use the following equations:

$$\tilde{S}(\omega,V,0) = \int_0^\infty e^{-\omega z}dz = \frac{1}{\omega}, \tag{29a}$$

$$\int_0^\infty e^{-\omega z}\frac{\partial S}{\partial z}dz = \omega\tilde{S}, \tag{29b}$$

$$\int_0^\infty e^{-\omega z}\frac{\partial^2 S}{\partial z \partial V}dz = \omega\frac{\partial \tilde{S}}{\partial V}, \tag{29c}$$

$$\int_0^\infty e^{-\omega z}\frac{\partial^2 S}{\partial z^2}dz = \omega^2\tilde{S}. \tag{29d}$$

When getting equation (**??**), we assume that the "smooth pasting" (*we need to find the correct wording or the economic intuition here since it is the same as in Leland (1994)*) condition is satisfied by S at $z = 0$, i.e., $\frac{\partial S}{\partial z}|_{z=0} = 0$. Applying the transform $\int_0^\infty e^{-\omega z} \cdot dz$ on both sides of equation (**??**) and plugging equations (**??**)-(**??**), we obtain that

$$\frac{\partial \tilde{S}}{\partial h} = \frac{1}{2}\sigma^2 V\frac{\partial^2 \tilde{S}}{\partial V^2} + [\kappa^*\theta^* + (\rho\sigma\omega - \theta^*)V]\frac{\partial \tilde{S}}{\partial V} + \left[\omega r + \left(\frac{1}{2}\omega^2 - \frac{1}{2}\omega\right)V\right]\tilde{S}. \tag{30}$$

Guessing that the solution for $\tilde{S}$ is $\tilde{S}(\omega,V,h) = \frac{1}{\omega}e^{-A(\omega,h)-B(\omega,h)V}$, we obtain

$$-A' - B'V = \frac{1}{2}\sigma^2 VB^2 - [\kappa^*\theta^* + (\rho\sigma\omega - \kappa^*)V]B + \omega r + \left(\frac{1}{2}\omega^2 - \frac{1}{2}\omega\right)V. \tag{31}$$

Thus A and B satisfy

$$-A' = -\kappa^*\theta^* B + \omega r, \tag{32a}$$

$$-B' = \frac{1}{2}\sigma^2 B^2 - (\rho\sigma\omega - \kappa^*)B + \frac{1}{2}\omega^2 - \frac{1}{2}\omega, \tag{32b}$$

with $A(\omega,0) = 0$ and $B(\omega,0) = 0$. Note that we use $'$ to denote the first-order derivative. For example, $A' = \frac{dA}{dh}$ and $B' = \frac{dB}{dh}$. We first solve for B from equation (**??**) and then solve for A from equation (**??**). Essentially, equation (**??**) is a Riccati equation. Let $B(h) = \frac{q'(h)}{q(h)} \cdot \frac{2}{\sigma^2}$ and plug it into equation (**??**), we obtain

$$q'' - (\rho\sigma\omega - \kappa^*)q' - \frac{1}{2}\sigma^2\left(-\frac{1}{2}\omega^2 + \frac{1}{2}\omega\right)q = 0.$$

Thus the general solution for q is $q(h) = C_1 e^{\lambda_1 h} + C_2 e^{\lambda_2 h}$, where C_1 and C_2 are constants, and λ_1 and λ_2 solve

$$\lambda^2 - (\rho\sigma\omega - \kappa^*)\lambda - \frac{1}{4}\sigma^2(-\omega^2 + \omega) = 0.$$

Thus $\lambda_{1,2} = \frac{\rho\sigma\omega - \kappa^* \pm d}{2}$ with $d = \sqrt{(\rho\sigma\omega - \kappa^*)^2 + \sigma^2(-\omega^2 + \omega)}$. The solution for $B(\omega,h)$ is

$$B(\omega,h) = \frac{2}{\sigma^2} \cdot \frac{C_1\lambda_1 e^{\lambda_1 h} + C_2\lambda_2 e^{\lambda_2 h}}{C_1 e^{\lambda_1 h} + C_2 e^{\lambda_2 h}}. \tag{33}$$

Since $B(\omega,0) = 0$, we obtain that $C_2 = -C_1\lambda_1/\lambda_2$. Plugging equation the expression for C_2 into equation (**??**), we obtain

$$B(\omega,h) = \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot \frac{1 - e^{-dh}}{1 - ge^{-dh}}, \tag{34}$$

with $d = \sqrt{(\rho\sigma\omega - \kappa^*)^2 + \sigma^2(-\omega^2 + \omega)}$ and $g = \frac{\rho\sigma\omega - \kappa^* + d}{\rho\sigma\omega - \kappa^* - d}$. Now we solve for $A(\omega,h)$. First, plugging equation (**??**) into equation (**??**) yields

$$A' = \kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot \frac{1 - e^{-dt}}{1 - ge^{-dt}} - \omega r. \tag{35}$$

Let $u(h) = e^{-dh}$, then $u' = -du$. Plugging $A' = A'_u \cdot u'$ into equation (**??**) yields

$$A'_u \cdot u' = -A'_u du = \kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot \frac{1 - e^{-dt}}{1 - ge^{-dt}} - \omega r.$$

Thus

$$A'_u = \kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2 d} \cdot \left(-\frac{1}{u} + \frac{1 - g}{1 - gu}\right) + \frac{\omega r}{du}. \tag{36}$$

The solution for equation (**??**) is

$$\begin{aligned} A &= \kappa^*\theta^* \cdot \tfrac{\rho\sigma\omega - \kappa^* + d}{\sigma^2 d} \cdot \left[-ln(u) + \tfrac{(1-g)ln(1-gu)}{-g}\right] + \tfrac{\omega r}{d} ln(u) + C_3 \\ &= \kappa^*\theta^* \cdot \tfrac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot h + \kappa^*\theta^* \cdot \tfrac{\rho\sigma\omega - \kappa^* + d}{\sigma^2 d} \cdot \tfrac{g-1}{g} ln\left(1 - ge^{-dh}\right) - \omega r h + C_3, \end{aligned} \tag{37}$$

where C_3 is a constant. Since $A(0) = 0$, we obtain $C_3 = -\kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2 d} \cdot \frac{g-1}{g} ln(1 - g)$. Thus

$$A(\omega,h) = -\omega r h + \kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot h + \kappa^*\theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2 d} \cdot \frac{g - 1}{g} ln\left[\frac{1 - ge^{-dh}}{1 - g}\right]. \tag{38}$$

Since $(\rho\sigma\omega - \kappa^* + d)\frac{g-1}{g} = (\rho\sigma\omega - \kappa^* + d)\frac{2d}{\rho\sigma\omega - \kappa^* - d} \cdot \frac{\rho\sigma\omega - \kappa^* - d}{\rho\sigma\omega - \kappa^* + d} = 2d$, we rewrite the solution for A as

$$A(\omega, h) = -\omega r h + \kappa^* \theta^* \cdot \frac{\rho\sigma\omega - \kappa^* + d}{\sigma^2} \cdot h + \frac{2\kappa^*\theta^*}{\sigma^2} ln \left[\frac{1 - ge^{-dh}}{1-g} \right]. \tag{39}$$

Thus the solution for $S(z, V, h)$ is given by

$$S(z, V, h) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} e^{z\omega} \tilde{S}(\omega, V, h) d\omega, \tag{40}$$

with $\tilde{S}(\omega, V, h) = \frac{1}{\omega} e^{-A(\omega,h) - B(\omega,h)V}$, and the solutions for $A(\omega, h)$ and $B(\omega, h)$ are given by equations (**??**) and (**??**).

Appendix C: Volatility asymmetry at equity and asset value levels

In this appendix, we translate the correlation between asset value and variance shocks to a correlation between equity and equity variance shocks. The stochastic process that the unlevered asset value follows is given by

$$\frac{dX_t}{X_t} = (\mu - \delta)dt + \sqrt{V_t} dW_1, \tag{41}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_2, \tag{42}$$

where δ is the firm's payout ratio and $E(dW_1 dW_2) = \rho dt$. Applying Itô's lemma, we obtain that

$$\frac{dE_t}{E_t} = \mu_{E,t} + \frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\sqrt{X_t} dW_{1t} + \frac{1}{E_t}\frac{\partial E_t}{V_t}\sigma\sqrt{V_t} dW_{2t}, \tag{43}$$

where $\mu_{E,t}$ is the instantaneous equity return. The equity variance is given by

$$V_{E,t} = C \cdot V_t, \tag{44}$$

where $C = \left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}$. Applying Itô's lemma to $V_{E,t}$ given by equation (**??**), we obtain that

$$dV_{E,t} = \mu_{V_E,t} dt + \sigma V_{E,t} dW_{2t}. \tag{45}$$

Given the specification of the processes for equity and equity variance as in equations (**??**) and (**??**), we obtain that the correlation between equity and equity variance is

$$\rho_{E,t} = \left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\rho + \frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right)/C, \tag{46}$$

where $C = \left(\frac{X_t}{E_t}\frac{\partial E_t}{\partial X_t}\right)^2 + \left(\frac{\sigma}{E_t}\frac{\partial E_t}{\partial V_t}\right) + \rho\sigma\frac{X_t}{E_t^2}\frac{\partial E_t}{X_t}\frac{\partial E_t}{\partial V_t}$.

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Table 1: **Calibrated Yield Spreads for Different Credit Ratings and Maturities.** This table reports the calibration results for the model in which the firm value follows the stochastic volatility process. The yield spread is calculated as the difference between the bond yield and risk-free rate. Following Huang and Huang (2003), we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability (columns 2-4). Given default, the firm recovers 51.31% of the face value. The average yield spreads in the last column are the historically observed yield spreads for the bonds as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$, $\kappa = 4$ and $k = 7$.

Panel A: Maturity = 10 years

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.3	60.3	63	95.7
Aa	21.2	5.60	0.99	27.4	67.5	91	74.2
A	32.0	5.99	1.55	24.8	89.4	123	72.7
Baa	43.3	6.55	4.39	25.3	159.8	194	82.4
Ba	53.5	7.30	20.63	32.3	361.2	320	112.8
B	65.7	8.76	43.91	39.8	584.7	470	124.4

Panel B: Maturity = 4 years

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.04	35.9	20.9	55	38.0
Aa	21.2	5.60	0.23	33.4	53.5	65	82.3
A	32.0	5.99	0.35	28.9	70.3	96	73.2
Baa	43.3	6.55	1.24	28.5	150.1	158	95.0
Ba	53.5	7.30	8.51	34.7	434.3	320	135.7
B	65.7	8.76	23.32	40.5	796.3	470	169.4

Panel C: Maturity = 1 year

Credit Rating	Target Leverage Ratio (%)	Target Equity Premium (%)	Target Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aa	21.2	5.60	0.03	53.5	12.4		
A	32.0	5.99	0.01	41.4	3.5		
Baa	43.3	6.55	0.12	40.7	57.8		
Ba	53.5	7.30	1.29	45.1	303.1		
B	65.7	8.76	6.47	50.1	944.8		

Table 2: **Calibrated 10-year Yield Spreads for Different Credit Ratings and Parameter Values.** This table reports the calibration results for different parameter values of the stochastic volatility model. The yield spread is calculated as the difference between the bond yield and risk-free rate. Following Huang and Huang (2003), we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability (columns 2-4). Given default, the firm recovers 51.31% of the face value. The average yield spreads in the last column are the historically observed yield spreads for the bonds as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean.

Panel A: $\rho = 0$, $\sigma = 0$, $\kappa = 0$ and $k = 0$

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.9	0	63	0
Aa	21.2	5.60	0.99	28.1	0	91	0
A	32.0	5.99	1.55	25.0	5.9	123	4.8
Baa	43.3	6.55	4.39	24.7	32.9	194	16.9
Ba	53.5	7.30	20.63	30.1	148.3	320	46.3
B	65.7	8.76	43.91	35.2	320.8	470	68.3

Panel B: $\rho = 0$, $\sigma = 0.3$, $\kappa = 4$ and $k = 0$

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.5	0	63	0
Aa	21.2	5.60	0.99	27.6	0	91	0
A	32.0	5.99	1.55	24.9	3.7	123	3.0
Baa	43.3	6.55	4.39	25.0	31.8	194	16.4
Ba	53.5	7.30	20.63	31.4	151.1	320	47.2
B	65.7	8.76	43.91	38.6	341.1	470	72.6

Panel C: $\rho = -0.1$, $\sigma = 0.3$, $\kappa = 4$ and $k = 0$

Credit Rating	Target: Leverage Ratio (%)	Target: Equity Premium (%)	Target: Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.2	0	63	0
Aa	21.2	5.60	0.99	27.4	0	91	0
A	32.0	5.99	1.55	24.6	3.2	123	2.7
Baa	43.3	6.55	4.39	24.8	31.0	194	16.0
Ba	53.5	7.30	20.63	31.2	149.8	320	46.8
B	65.7	8.76	43.91	38.5	340.3	470	72.4

Panel D: $\rho = 0$, $\sigma = 0.3$, $\kappa = 4$ and $k = 7$

Credit Rating	Target: Leverage Ratio (%)	Target: Equity Premium (%)	Target: Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	31.6	61.7	63	97.9
Aa	21.2	5.60	0.99	27.7	69.1	91	75.9
A	32.0	5.99	1.55	25.1	91.8	123	74.6
Baa	43.3	6.55	4.39	25.3	157.1	194	81.0
Ba	53.5	7.30	20.63	32.8	371.3	320	116.0
B	65.7	8.76	43.91	40.6	599.3	470	127.5

Table 3: **Calibrated Market Price of Volatility Risk for Different Credit Ratings and Maturities.** This table reports the calibrated asset volatility and ratio of the risk-neutral over the objective expected asset variance of 3-month maturity (denoted Q/P) for the stochastic volatility model. We calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk and asset risk premium to match the target leverage ratio, equity premium, cumulative default probability and historical average yield spread(columns 2-4). Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$, and $\kappa = 4$.

Panel A: Maturity = 10 years

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Average Yld. Spread (bps)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.77	63	31.6	1.534	-2.55
Aa	21.2	5.60	0.99	91	27.4	1.503	-2.43
A	32.0	5.99	1.55	123	24.1	1.494	-2.40
Baa	43.3	6.55	4.39	194	25.3	1.486	-2.37
Ba	53.5	7.30	20.63	320	32.3	1.353	-1.83
B	65.7	8.76	43.91	470	39.8	1.222	-1.23

Panel B: Maturity = 4 years

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Average Yld. Spread (bps)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.04	55	35.6	1.535	-2.55
Aa	21.2	5.60	0.23	65	33.7	1.447	-2.22
A	32.0	5.99	0.35	96	28.6	1.436	-2.17
Baa	43.3	6.55	1.24	158	28.5	1.425	-2.12
Ba	53.5	7.30	8.51	320	34.6	1.285	-1.53
B	65.7	8.76	23.32	470	40.5	1.074	-0.45

Table 4: **New Calibration Results for Different Credit Ratings and Maturities.** This table reports the calibrated asset volatility and ratio of the risk-neutral over the objective expected asset variance of 3-month maturity (denoted Q/P) for the stochastic volatility model. We calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average yield spread and equity volatility (columns 2-5). Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$ and $\sigma = 0.3$.

Panel A: Maturity = 10 years

	Target							
Credit Rating	Lev. Ratio (%)	Equity Prem. (%)	Cumulative Def. Prob. (%)	Average Yld. Sprd (bps)	Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.77	63	24.5	21.6	1.560	-2.34
Aa	21.2	5.60	0.99	91	26.5	21.3	1.511	-2.14
A	32.0	5.99	1.55	123	28.5	21.3	1.508	-2.16
Baa	43.3	6.55	4.39	194	31.8	21.5	1.452	-1.92
Ba	53.5	7.30	20.63	320	42.7	25.8	1.387	-1.68
B	65.7	8.76	43.91	470	53.8	37.6	1.228	-1.26

Panel B: Maturity = 4 years

	Target							
Credit Rating	Lev. Ratio (%)	Equity Prem. (%)	Cumulative Def. Prob. (%)	Average Yld. Sprd (bps)	Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.04	55	24.5	21.8	1.481	-1.98
Aa	21.2	5.60	0.23	65	26.5	22.4	1.441	-1.88
A	32.0	5.99	0.35	96	28.5	22.1	1.424	-1.86
Baa	43.3	6.55	1.24	158	31.8	22.4	1.411	-1.80
Ba	53.5	7.30	8.51	320	42.7	25.4	1.234	-1.02
B	65.7	8.76	23.32	470	53.8	37.3	1.168	-0.84

Table 5: **New Calibration Results for Different Credit Ratings and Maturities with Liquidity Correction.** This table reports the calibrated asset volatility and ratio of the risk-neutral over the objective expected asset variance of 3-month maturity (denoted Q/P) for the stochastic volatility model. We calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average credit spread and equity volatility (columns 2-5). The credit spread for each credit rating is calculated as the yield spread minus the 1-year AAA yield spread (51 bps) as in Almeida and Philippon (2007). Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$ and $\sigma = 0.3$.

Panel A: Maturity = 10 years

	Target							
Credit Rating	Lev. Ratio (%)	Equity Prem. (%)	Cumulative Def. Prob. (%)	Average Credit Sprd (bps)	Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.77	12	24.5	21.6	1.391	-1.76
Aa	21.2	5.60	0.99	40	26.5	21.3	1.365	-1.64
A	32.0	5.99	1.55	72	28.5	21.3	1.358	-1.63
Baa	43.3	6.55	4.39	143	31.8	21.5	1.326	-1.47
Ba	53.5	7.30	20.63	269	42.7	25.8	1.309	-1.40
B	65.7	8.76	43.91	419	53.8	37.6	1.222	-1.21

Panel B: Maturity = 4 years

	Target							
Credit Rating	Lev. Ratio (%)	Equity Prem. (%)	Cumulative Def. Prob. (%)	Average Credit Sprd (bps)	Equity Vol. (%)	Implied Asset Vol. (%)	Ratio of Exp. Asset Var. (Q/P)	λ_V
Aaa	13.1	5.38	0.04	4	24.5	21.8	1.447	-1.87
Aa	21.2	5.60	0.23	14	26.5	22.4	1.433	-1.86
A	32.0	5.99	0.35	45	28.5	22.1	1.420	-1.85
Baa	43.3	6.55	1.24	107	31.8	22.4	1.406	-1.78
Ba	53.5	7.30	8.51	269	42.7	25.4	1.278	-1.19
B	65.7	8.76	23.32	419	53.8	37.3	1.152	-0.77

Table 6: **Calibration Results for the Market Price of Volatility Risk for Different Credit Ratings and Maturities.** This table reports the calibrated results for the market price of volatility risk for the stochastic volatility model. In columns 2 and 6, we report the ratio of the risk-neutral long-run mean over the objective long-run mean for asset variance. In columns 3, 4, 7 and 8, we report ratios of the risk-neutral over the objective expected asset variance (of 3-month maturity in columns 3 and 7 and of 1-month maturity in columns 4 and 8). In the 5-target calibration, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk and asset risk premium to match the target leverage ratio, equity premium, cumulative default probability and historical average yield spread. In the 6-target calibration, we calibrate the initial asset value, long-run mean of asset volatility, market price of volatility risk, asset risk premium and mean-reversion parameter to match the target leverage ratio, equity premium, cumulative default probability, historical average yield spread and equity volatility. Given default, the firm recovers 51.31% of the face value. The yield spread is calculated as the difference between the bond yield and risk-free rate. The average yield spreads are the same as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$ & $\kappa = 4$ for 5-target calibration, and $\rho = -0.1$ & $\sigma = 0.3$ for 6-target calibration.

Panel A: Maturity = 10 years

	5-target Case				6-target Case			
Credit	Ratio of	Ratio of Exp. Asset Var.		λ_V	Ratio of	Ratio of Exp. Asset Var.		λ_V
Rating	LR Mean	3 Months	1 Month		LR Mean	3 Months	1 Month	
Aaa	2.756	1.534	1.200	-2.55	7.693	1.560	1.192	-2.34
Aa	2.548	1.503	1.190	-2.43	6.668	1.511	1.175	-2.14
A	2.496	1.494	1.187	-2.40	5.382	1.508	1.175	-2.16
Baa	2.454	1.486	1.184	-2.37	5.001	1.452	1.157	-1.92
Ba	1.843	1.353	1.139	-1.83	3.572	1.387	1.136	-1.68
B	1.444	1.222	1.092	-1.23	1.460	1.228	1.094	-1.26

Panel B: Maturity = 4 years

	5-target Case				6-target Case			
Credit	Ratio of	Ratio of Exp. Asset Var.		λ_V	Ratio of	Ratio of Exp. Asset Var.		λ_V
Rating	LR Mean	3 Months	1 Month		LR Mean	3 Months	1 Month	
Aaa	2.759	1.535	1.200	-2.55	9.819	1.481	1.163	-1.98
Aa	2.241	1.447	1.172	-2.21	4.665	1.441	1.153	-1.88
A	2.190	1.436	1.168	-2.40	3.475	1.424	1.150	-1.86
Baa	2.139	1.425	1.164	-2.13	3.472	1.411	1.145	-1.80
Ba	1.619	1.285	1.115	-1.53	2.480	1.234	1.083	-1.02
B	1.127	1.074	1.032	-0.45	1.455	1.168	1.065	-0.84

Panel C: Calibration results with liquidity correction

	Maturity = 10 years				Maturity = 4 years			
Credit	Ratio of	Ratio of Exp. Asset Var.		λ_V	Ratio of	Ratio of Exp. Asset Var.		λ_V
Rating	LR Mean	3 Months	1 Month		LR Mean	3 Months	1 Month	
Aaa	2.833	1.391	1.141	-1.76	6.194	1.447	1.154	-1.87
Aa	2.745	1.365	1.132	-1.64	4.207	1.433	1.151	-1.86
A	2.538	1.358	1.130	-1.63	3.372	1.420	1.149	-1.85
Baa	2.485	1.326	1.118	-1.47	3.372	1.406	1.144	-1.78
Ba	2.386	1.309	1.112	-1.40	3.204	1.278	1.097	-1.19
B	1.460	1.222	1.091	-1.21	1.393	1.152	1.059	-0.77

Table 7: **Calibrated Yield Spreads for Different Credit Ratings and Maturities for the Stochastic Model with Merton Default.** This table reports the calibration results for the model in which the firm value follows the stochastic volatility process and the default only occurs at maturity. The yield spread is calculated as the difference between the bond yield and risk-free rate. Following Huang and Huang (2003), we calibrate the initial asset value, long-run mean of asset volatility and asset risk premium to match the target leverage ratio, equity premium and cumulative default probability (columns 2-4). Given default, the firm recovers 51.31% of the face value. The average yield spreads in the last column are the historically observed yield spreads for the bonds as reported in Huang and Huang (2003). We choose the initial asset volatility to the same as the long-run mean. The other parameter values are: $\rho = -0.1$, $\sigma = 0.3$, $\kappa = 4$ and $k = 7$.

Panel A: Maturity = 10 years

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.77	33.6	54.9	63	87.1
Aa	21.2	5.60	0.99	30.1	63.3	91	69.6
A	32.0	5.99	1.55	27.1	77.1	123	62.7
Baa	43.3	6.55	4.39	28.9	144.6	194	74.5
Ba	53.5	7.30	20.63	39.8	355.1	320	111.0
B	65.7	8.76	43.91	57.6	665.2	470	141.5

Panel B: Maturity = 4 years

	Target						
Credit Rating	Leverage Ratio (%)	Equity Premium (%)	Cumulative Default Prob. (%)	Implied Asset Vol. (%)	Calculated Yld. Spread (bps)	Average Yld. Spread (bps)	% of Spread due to Default
Aaa	13.1	5.38	0.04	36.1	9.4	55	17.1
Aa	21.2	5.60	0.23	34.8	43.3	65	66.6
A	32.0	5.99	0.35	30.2	54.2	96	56.5
Baa	43.3	6.55	1.24	31.2	137.1	158	86.8
Ba	53.5	7.30	8.51	39.5	392.8	320	122.8
B	65.7	8.76	23.32	53.2	829.7	470	176.5

Table 8: **Summary Statistics by Ratings.** This table reports the summary statistics on the CDS spreads and the underlying firms from January 2002 to December 2008. Equity volatility is estimated using 5-minute intraday returns. Leverage ratio is calculated as the ratio of the total liabilities over the total asset, which is the sum of the total liability and equity market value. Asset payout ratio is the weighted average of dividend payout and interest expense over the total asset.

Panel A: Firm Characteristics

Credit Rating	Firms	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)
AAA [a]	1	24.29	63.67	2.83
AA	1	21.99	25.81	1.36
A	16	27.19	37.95	2.33
BBB	22	27.72	48.51	2.12
BB	7	35.18	51.01	2.50
B	2	41.24	77.10	2.66

Panel B: CDS Spreads (%)

Credit Rating	1-year	2-year	3-year	5-year	7-year	10-year
AAA	0.45	0.49	0.52	0.58	0.60	0.63
AA	0.12	0.14	0.17	0.22	0.25	0.30
A	0.28	0.33	0.37	0.45	0.51	0.57
BBB	0.40	0.47	0.54	0.67	0.75	0.85
BB	0.75	0.86	1.00	1.35	1.44	1.57
B	3.73	4.22	4.49	4.80	4.78	4.77

Panel C: CDS Spreads Std. Dev. (%)

Credit Rating	1-year	2-year	3-year	5-year	7-year	10-year
AAA	1.16	1.12	1.08	0.98	0.90	0.85
AA	0.16	0.16	0.17	0.18	0.17	0.17
A	0.39	0.40	0.40	0.39	0.37	0.36
BBB	0.49	0.49	0.50	0.49	0.47	0.45
BB	0.86	0.89	0.88	0.88	0.83	0.85
B	7.78	7.02	6.40	5.70	5.17	4.70

[a] Note that the only AAA-rated company in our sample is GE.

Table 9: **Specification Test of the Stochastic Volatility Model.** This table reports the GMM test results of the stochastic volatility model. Panel A reports the parameter estimates, p-values (in parentheses) and J-statistic. Panel B reports the average and absolute pricing errors of CDS spreads and equity volatility. Panel C reports the average and absolute percentage pricing errors of CDS spreads and equity volatility. The pricing errors are calculated as the time-series average, absolute, average percentage and absolute percentage differences between the model-implied and observed values.

Panel A: Parameter Estimates

Credit Rating	ρ	κ^*	σ	$X_B/100$	$\theta^*(\%)$	J-stat	Proportion of Not Rejected	
							Sig. level=0.01	0.05
Overall	-0.58	0.98	0.37	0.67	5.34	1.637	48/49	46/49
	(0.007)	(0.008)	(0.007)	(0.013)	(0.011)			
AAA	-0.47	1.2	0.24	0.70	3.06	0.924	1/1	1/1
	(0.006)	(0.008)	(0.005)	(0.014)	(0.008)			
AA	-0.54	0.8	0.36	0.62	6.40	0.672	1/1	1/1
	(0.008)	(0.012)	(0.005)	(0.018)	(0.011)			
A	-0.56	1.2	0.42	0.68	4.58	1.008	17/17	16/17
	(0.006)	(0.009)	(0.007)	(0.014)	(0.007)			
BBB	-0.61	0.9	0.31	0.67	5.20	2.352	21/22	20/22
	(0.008)	(0.007)	(0.008)	(0.010)	(0.011)			
BB	-0.59	0.8	0.45	0.70	6.92	1.260	6/6	6/6
	(0.007)	(0.009)	(0.004)	(0.015)	(0.012)			
B	-0.65	0.5	0.32	0.75	9.24	1.092	2/2	2/2
	(0.012)	(0.009)	(0.007)	(0.018)	(0.017)			

Panel B: Average and Absolute Pricing Errors

Credit	Average Pricing Error (%)		Absolute Pricing Error (%)	
Rating	CDS Spreads	Equity Volatility	CDS Spreads	Equity Volatility
Overall	-0.18	0.15	0.26	0.35
AAA	-0.15	0.03	0.18	0.14
AA	-0.04	0.08	0.07	0.12
A	-0.03	0.12	0.17	0.06
BBB	-0.13	0.10	0.19	0.55
BB	-0.59	0.36	0.55	0.45
B	-0.61	0.23	0.96	0.34

Panel C: Average and Absolute Percentage Pricing Errors

Credit	Average Percentage Pricing Error (%)		Absolute Percentage Pricing Error (%)	
Rating	CDS spreads	Equity Volatility	CDS spreads	Equity Volatility
Overall	-10.16	0.40	29.61	0.96
AAA	-27.42	0.12	32.90	0.58
AA	-20.02	0.36	35.04	0.55
A	-7.16	0.41	36.65	0.20
BBB	-11.16	0.28	25.25	1.55
BB	-19.28	0.78	24.43	0.97
B	22.34	0.56	35.06	0.82

Table 10: **Specification Test of the Black and Cox (1976) Model.** This table reports the GMM test results of the Black and Cox (1976) model. Panel A reports the parameter estimates, p-values (in parentheses) and J-statistic. Panel B reports the average and absolute pricing errors of CDS spreads and equity volatility. Panel C reports the average and absolute percentage pricing errors of CDS spreads and equity volatility. The pricing errors are calculated as the time-series average, absolute, average percentage and absolute percentage differences between the model-implied and observed values.

Panel A: Parameter Estimates

Credit Rating	Asset Vol(%)	$X_B/100$	J-stat	Proportion of Not Rejected	
				Sig. level=0.01	0.05
Overall	12.41	1.31	15.112	4/49	1/49
	(0.008)	(0.036)			
AAA	10.01	1.067	11.413	0/1	0/1
	(0.004)	(0.016)			
AA	11.75	1.795	8.487	1/1	0/1
	(0.010)	(0.051)			
A	10.15	1.586	16.396	0/16	0/16
	(0.008)	(0.042)			
BBB	12.45	1.201	15.263	2/22	0/22
	(0.008)	(0.034)			
BB	14.35	1.125	11.545	1/7	1/7
	(0.007)	(0.031)			
B	20.17	0.804	20.822	0/2	0/2
	(0.006)	(0.028)			

Panel B: Average and Absolute Pricing Errors

Credit Rating	Average Pricing Error (%)		Absolute Pricing Error (%)	
	CDS Spreads	Equity Volatility	CDS Spreads	Equity Volatility
Overall	-0.48	-3.96	0.50	10.85
AAA	-0.27	3.04	0.29	10.35
AA	0.08	-1.56	0.27	8.41
A	-0.31	-5.01	0.31	11.22
BBB	-0.43	-1.81	0.43	8.14
BB	-0.74	-8.09	0.76	12.99
B	-1.77	-9.46	2.20	31.94

Panel C: Average and Absolute Percentage Pricing Errors

Credit Rating	Average Percentage Pricing Error (%)		Absolute Percentage Pricing Error (%)	
	CDS spreads	Equity Volatility	CDS spreads	Equity Volatility
Overall	-65.44	-6.56	67.31	29.51
AAA	-25.11	17.63	40.69	51.64
AA	-61.11	6.55	65.67	35.91
A	-73.02	-4.64	73.73	32.32
BBB	-68.09	-9.53	68.08	24.15
BB	-62.17	-3.74	64.16	32.61
B	-9.36	-17.69	32.64	41.08

Table 11: **Summary Statistics of Individual Names.** This table reports the ratings, 5-year CDS spread, equity volatility, leverage ratio and asset payout ratio for each of the 49 firms.

Company Name	Last Rating	5-year CDS(%)	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout(%)
Amgn Inc	A	0.392	26.94	15.99	0.42
Arrow Electronics Inc	BBB	1.357	35.48	55.89	1.76
Boeing Co	A	0.441	29.89	50.51	1.58
Baxter Intl Inc	A	0.343	26.66	26.42	1.59
Bristol Myers Squibb Co	A	0.297	27.28	25.09	3.95
Boston Scientific Corp	BB	0.816	35.41	24.53	0.75
Conagra Inc	BBB	0.440	21.10	41.21	3.56
Caterpillar Inc	A	0.359	26.43	54.47	2.45
Cigna Corp	BBB	0.680	30.36	77.62	0.27
Comcast Corp New	BBB	1.138	31.93	61.15	1.91
Campbell Soup Co	A	0.277	21.98	32.10	2.49
Computer Sciences Corp	BBB	0.613	28.78	46.44	1.17
Du Pont Co	A	0.275	24.43	36.84	2.81
Deere & Co	A	0.394	29.40	58.44	2.61
Disney Walt Co	A	0.492	27.27	34.68	1.49
Dow Chemical Co	A	0.658	27.37	45.31	3.21
Darden Restaurants Inc	BBB	0.689	32.41	31.42	1.69
Devon Energy Corp	BBB	0.553	33.31	44.82	1.87
Eastman Kodak Co	B	2.136	32.64	60.35	2.12
Eastman Chemical Co	BBB	0.658	29.04	53.05	3.13
Ford Motor Co	B	7.471	41.29	93.85	3.21
Fortune Brands Inc	BBB	0.570	21.48	36.96	2.31
General Electric Co	AAA	0.584	24.29	63.68	2.83
General Mills Inc	BBB	0.432	17.76	40.72	3.03
Goodrish Corp	BBB	0.780	28.43	52.39	2.55
Honeywell Intl Inc	A	0.359	28.39	40.17	2.10
Intl Paper Co	BBB	0.932	28.74	56.38	3.08
Ingersoll Rand Co	BBB	0.414	28.12	39.39	1.91
J C Penny Co Inc	BB	2.09	39.93	50.98	1.94
Kraft Foods Inc	BBB	0.464	21.12	52.20	2.76
Kroger Company	BBB	0.641	29.25	52.34	1.93

Company Name	Last Rating	5-year CDS(%)	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout(%)
Marriott Intl Inc New	BBB	0.894	27.06	32.10	1.29
Mcdonalds Corp	A	0.285	25.45	25.70	2.31
Mckesson Inc	BBB	0.564	27.67	53.14	0.76
Northrop Grumman Corp	BBB	0.459	20.99	46.26	2.08
Newell Rubbermaid Inc	BBB	0.572	26.50	41.99	3.03
Olin Corp	BB	1.247	37.83	48.51	3.11
Radioshack Corp	BB	1.136	37.90	29.09	1.64
Raytheon Co	BBB	0.599	28.09	43.06	2.33
Sherwin Williams Co	A	0.486	29.66	31.15	1.99
Supervalu Inc	BB	1.526	27.78	60.52	3.21
Safeway Inc	BBB	0.646	30.68	48.76	2.01
A T&T Corp	A	1.249	28.29	43.81	3.50
Target Corp	A	0.392	30.47	35.16	1.48
Temple Inland Inc	BB	1.411	31.01	81.40	3.75
Tyson Foods Inc	BB	1.254	36.37	62.01	3.11
Verizon Communications Inc	A	0.621	25.19	51.37	3.40
Whirlpool Corp	BBB	0.683	31.38	59.95	2.16
Wal Mart Stores Inc	AA	0.215	21.97	25.81	1.36

Table 12: **Equity Leverage Effect (ρ_E).** This table reports the calculated leverage effect of equity (column 10) for each credit rating. We report the parameter values used to calculate the equity leverage effect in column 2-9.

Credit Rating	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)	ρ	κ^*	σ	$X_B/100$	θ^* (%)	ρ_E
AAA	24.29	63.67	2.83	-0.47	1.2	0.24	0.70	3.06	-0.57
AA	21.99	25.81	1.36	-0.54	0.8	0.36	0.62	6.40	-0.53
A	27.19	37.95	2.33	-0.56	1.2	0.42	0.68	4.58	-0.69
BBB	27.72	48.51	2.12	-0.61	0.9	0.31	0.67	5.20	-0.68
BB	35.18	51.01	2.50	-0.59	0.8	0.45	0.70	6.92	-0.67
B	41.24	77.10	2.66	-0.65	0.5	0.32	0.75	9.24	-0.74