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Time-Varying Multi-Objective Optimisation over Switching Graphs via Fixed-Time Consensus Algorithms

Zhongguo Li, Zhengtao Ding

***Abstract*—This paper considers distributed multi-objective optimisation problems with time-varying cost functions for network connected multi-agent systems over switching graphs. The scalarisation approach is used to convert the problem into a weighted-sum objective. Fixed-time consensus algorithms are developed for each agent to estimate the global variables, and drive all local copies of the decision vector to a consensus. The algorithm with fixed gains is first proposed, where some global information is required to choose the gains. Then, an adaptive algorithm is presented to eliminate the use of global information. The convergence of those algorithms to the Pareto solutions is established via Lyapunov theory for connected graphs. In case of disconnected graphs, the convergence to the subsets of the Pareto fronts is studied. Simulation results are provided to demonstrate the effectiveness of the proposed algorithms.**

***Index Terms*—Consensus, gradient descent, optimisation, distributed algorithm, multi-agent systems.**

I. Introduction

Recently, significant research efforts have been dedicated to network connected multi-agent systems to facilitate the development of large-scale and complex networks. The distributed agents are entitled to make local decisions using their private information and limited interactions with their neighbours. The communication topologies among the agents are modelled by graphs. Compared with the traditional centralised methods, distributed algorithms have a series of superior advantages, such as privacy protection, parallel computation, less communication and strong robustness (Cao et al., 2013).

Optimisation problems cover a large range of engineering and social applications, including optimal control of power systems (Yi et al., 2016; Yang et al., 2016), mobile robots (Wang and Xin, 2013) and game theory (Li and Ding, 2019). Those problems can be divided into two categories: single-objective optimisation problems (SOPs) and multi-objective optimisation problems (MOPs). Different from SOPs, there is no single optimum that can simultaneously minimise/maximise multiple conflicting objectives for MOPs. Generally speaking, most of the practical applications involve compromising among several objectives. For example, a resource allocation problem in a smart grid contains economic, environmental and technical objectives (Ren et al., 2010; Shi et al., 2018).

Many algorithms have been proposed to solve MOPs, including scalarisation approaches and population-based methods. Scalarisation approaches convert multiple objectives into one objective with some weighting parameters, including the weighted-sum approach (Johannes, 1984), and the weighted L_p preference-based method (Miettinen, 2012). Population-based methods have been extensively studied during the past two decades, including evolutionary algorithms and particle swarm algorithms, e.g., NSGA-II (Deb, 2005) and MOEA/D (Zhang and Li, 2007; Chen et al., 2017). Recently, particle swarm algorithms have demonstrated impressive success in some biomedical applications (Zeng et al., 2019; Zeng et al., 2016). One disadvantage of those algorithms is that establishing rigorous proof of convergence is usually difficult, and sometimes is given in a stochastic sense. Particularly, the performance of most population-based methods rely heavily on the selection of algorithms' parameters. Coding and tuning are usually difficult and time-consuming for many engineering applications. In addition, the aforementioned works are established using centralised approaches, which assume that the objective functions are all collected into one central node for computation. In multi-agent systems, where the cost functions are distributed across the agents, communication and computation requirements are highly demanding using centralised methods. Furthermore, privacy might be violated due to data transmission.

Distributed algorithms have been actively studied to solve SOPs, such as the subgradient method (Nedic and Ozdaglar, 2009), and the ADMM approach (Boyd et al., 2011) in discrete-time. More recently, a number of continuous-time algorithms have been proposed for distributed optimisation problems, e.g., Gharesifard and Corts (2014), Lin et al. (2017), Tran et al. (2017) and Garg et al. (2020). However, those works deal with SOPs, and only address time-invariant optimisation problems. Some recent studies that solve MOPs in distributed ways have been reported by Cao et al. (2017), Chen and Sayed (2013), Yang et al. (2018) and Li and Ding (2020). Chen and Sayed (2013) propose a gradient-based algorithm to search the Pareto solutions in discrete-time. A neurodynamic approach is introduced by Yang et al. (2018) based on the weighted-sum method, and the convergence is studied using consensus tools in continuous-time. Distributed multi-objective resource allocation problems are addressed by Li and Ding (2020) via an online L_p preference-based method, which does not

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need to specify the weighting parameters. In particular, fixed-time consensus based algorithms have been implemented for distributed optimisation (Ning et al., 2019; Ning et al., 2019).

Since none of the aforementioned literature can be implemented for MOPs with time-varying cost functions, and many engineering systems can be formulated as time-varying MOPs (Su, 2009; Fazlyab et al., 2018; Li et al., 2012), there is an urgent need to develop feasible algorithms for those problems. In this paper, we consider distributed MOPs with time-varying objectives using the scalarisation approach. A local copy of the decision vector is assigned to each agent such that the original problem can be reformulated as a group of local optimisation problems. When the local copies of the decision vector reach a consensus, the solution of the reformulated problem is also a Pareto optimum of the original problem. The weighted-sum approach is implemented to quantify the importance of each objectives. With some commonly-used assumptions, it is proved that all the Pareto solutions can be identified by varying the weighting parameters. A distributed scheme using fixed-time average consensus algorithms is presented with some *a prior* knowledge of the cost functions. Such consensus algorithms are implemented for the first/second order gradients, the time derivative of the first order gradient, and the decision variable as well. The time-varying consensus can be viewed as local estimates of those global variables. When all consensus are achieved within a fixed time, the distributed algorithm is then similar to those centralised algorithms except that local estimates are used in the distributed method without acquiring global information. The convergence of the proposed algorithm to the Pareto solution is established for any connected switching graphs via utilising fixed-time Lyapunov stability theory and convex analysis.

One limitation of our first algorithm is that the selection of the gains depends on some global information of the systems, which might be unavailable. To eliminate this requirement, we then propose a fully distributed and adaptive algorithm where the gains are learned during the optimisation process. Then, we further extend the results to disconnected graphs where Pareto fronts for different subgroups of the objectives can be identified with either the fixed-gain design or the adaptive-gain design. Therefore, the proposed algorithms are robust to the failures of communication links, even for disconnected graphs. In addition, disconnected switching graphs can be implemented to generate a variety of Pareto fronts for real application analysis. Detailed simulation results are provided to validate the effectiveness of those proposed algorithms.

The main contributions of our work can be summarised as follows. 1) Distributed optimisation for multi-agent systems with multiple time-varying objectives is studied using fixed-time consensus algorithms. 2) Algorithms with fixed gains and adaptive gains are proposed to solve distributed time-varying MOPs under switching graphs. 3) The deterministic convergence of those algorithms is established for connected graphs, and in case of disconnected graphs, subsets of the Pareto solutions can be obtained. Our work provides a distributed solution for a large range of engineering applications with time-varying objectives.

The remainder of this paper is organised as follows. The problem formulation and some general assumptions are presented in Section II. In Section III, distributed algorithms are proposed to solve the problem and convergence of those algorithms are established. Section IV shows the simulation results, and Section V draws the conclusion.

Notation: Let $\mathrm{diag}(a_1, \ldots, a_N)$ denote a diagonal matrix with $(a_1, \ldots, a_N)$ on the diagonal entries, and zero elsewhere. We denote $\mathrm{col}(a_1, \ldots, a_N)$ as a column vector consisting of $(a_1, \ldots, a_N)$ stacked on top of each other. For $x \in \mathbb{R}^n$, $\|x\|_2 \triangleq \sqrt{x^T x}$ (or simply $\|x\|$) denotes the Euclidean norm, and $\|x\|_1 \triangleq \sum_{i=1}^{n} |x_i|$ denotes 1-norm of x. The symbol $\otimes$ denotes Kronecker product. The component-wise signum function is represented by $\mathrm{sign}(\cdot)$. Let $\mathrm{sig}(x)^p = [\mathrm{sig}(x_1)^p, \ldots, \mathrm{sig}(x_n)^p]^T$, with $\mathrm{sig}(x_i)^p = \mathrm{sign}(x_i)|x_i|^p$, for $x \in \mathbb{R}^n$ and $p > 0$.

II. Problem Formulation

In this section, some basic definitions related to graphs are introduced. Then, we formulate the multi-objective optimisation problems of distributed multi-agent systems.

A. Graph Theory

For a piecewise constant switching signal, $\sigma(t) : [0, \infty) \rightarrow \mathcal{S} = \{1, \ldots, \chi\}$, and a set of χ graphs, $\mathcal{G}^s(\mathcal{V}, \mathcal{E}^s), s = 1, \ldots, \chi$, with $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}^s \in \mathcal{V} \times \mathcal{V}$, we define a switching graph sequence as $\mathcal{G}^\sigma(\mathcal{V}, \mathcal{E}^\sigma)$. The time index t has been omitted for notational conveniences. In a graph, $\mathcal{V}$ represents a set of N distinct vertices, denoting the agents in the network, and $\mathcal{E}^s = \{(i, j) : i, j \in \mathcal{V}\}$ denotes the set of edges, representing the communication channels among the agents. If an edge pair $(i, j) \in \mathcal{E}^s$, then agent i can communicate with agent j. A graph $\mathcal{G}^s(\mathcal{V}, \mathcal{E}^s)$ is said to be undirected if any edge pair $(i, j) \in \mathcal{E}^s$ implies $(j, i) \in \mathcal{E}^s$. The adjacent matrix of a graph is defined as $\mathcal{A}^s = [a_{ij}^s]_{N \times N}$, where $a_{ij}^s = 1$ if the edge pair $(j, i) \in \mathcal{E}^s$, and zero otherwise. The degree matrix is defined as $\mathcal{D}^s = \mathrm{diag}(d_1^s, \ldots, d_N^s)$, with $d_i^s = \sum_{j=1}^{N} a_{ij}^s$. Then, the Laplacian matrix is defined as $\mathcal{L}^s = \mathcal{D}^s - \mathcal{A}^s$. The neighbouring set of agent i is defined as $\mathcal{N}_i^s = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}^s\}$. In this paper, all the graphs are assumed to be undirected. For connected and undirected graphs, zero is a simple eigenvalue of the Laplacian matrix, and all other eigenvalues are positive real numbers (Ding, 2014). More definitions and properties related to graph theory can be found in Godsil and Royle (2001).

B. Problem Formulation

Consider a network of N agents, where each of them possesses a local objective, denoted as $f_i(z, t)$, with $z \in \mathbb{R}^n$ being the decision variable. Then, the network objective is to solve, in a distributed way,

$$\min_{z \in \mathbb{R}^n} F(z, t) = \{f_1(z, t), \ldots, f_N(z, t)\}. \tag{1}$$

For an MOP, it is assumed that the objectives are conflicting, that is, the objectives cannot be optimised simultaneously. The concept of Pareto optimality is used to describe the solutions, of which the formal definition is given as the following.

Definition 1 (Pareto optimality (Miettinen, 2012)): A decision vector $z^*(t) \in \mathbb{R}^n$ is the time-varying Pareto solution of problem (1), if for any time t there does not exist any $z(t) \in \mathbb{R}^n$ such that $f_i(z(t), t) \leq f_i(z^*(t), t)$ for all $i = 1, \ldots, N$, and $f_j(z(t), t) < f_j(z^*(t), t)$ for at least one index $j \in \mathcal{V}$.
Because the local objective $f_i(z, t)$ is privately known to the agent i only, distributed algorithms should be implemented to solve the problem. Introducing a local decision variable $x_i \in \mathbb{R}^n$ for agent i, problem (1) can be transferred to

$$\begin{aligned} \min_{x_1,\ldots,x_N \in \mathbb{R}^n} \quad & \{f_1(x_1, t), \ldots, f_N(x_N, t)\} \\ \text{subject to} \quad & \bar{\mathcal{L}}^\sigma \bar{x} = \mathbf{0} \end{aligned} \tag{2}$$

where $\bar{\mathcal{L}}^\sigma = \mathcal{L}^\sigma \otimes I_n$ and $\bar{x} = \operatorname{col}(x_1, \ldots, x_N)$. Note that $\bar{\mathcal{L}}^\sigma \bar{x} = \mathbf{0}$ indicates that all the local decision variable reach a consensus for connected graphs, and therefore, the reformulated problem (2) is equivalent to the original centralised problem (1). One of the commonly used approaches to search the Pareto solution is the scalarisation method, which assigns each cost function a positive weight ω_i. Then, the weighted problem is formulated as

$$\begin{aligned} \min_{x_1,\ldots,x_N \in \mathbb{R}^n} \quad & \sum_{i=1}^{N} \omega_i f_i(x_i, t) \\ \text{subject to} \quad & \bar{\mathcal{L}}^\sigma \bar{x} = \mathbf{0}. \end{aligned} \tag{3}$$

Now, we introduce some assumptions that are widely deployed in distributed optimisation, including the connectivity of the communication graph, and the convexity of the cost functions.

Assumption 1: The communication graphs $\mathcal{G}^s(\mathcal{V}, \mathcal{E}^s)$, $s = 1, \ldots, \chi$, are connected. The switching time between any two contiguous switching instances is greater than a positive threshold $\sigma_0 > 0$.

Assumption 2: The objective functions, $f_i(z, t), \forall i \in \mathcal{V}$, are strictly convex, and twice differentiable, with invertible Hessian matrix, defined as $[\nabla^2 f_i(z, t)]_{r,s} = \frac{\partial^2 f_i(z,t)}{\partial z_r \partial z_s}$, $\forall z, t$.

Assumption 3: The following items

$$\sup_{i,j \in \mathcal{V}} \left\| \frac{\partial}{\partial t} \nabla \omega_i f_i(x_i, t) - \frac{\partial}{\partial t} \nabla \omega_j f_j(x_j, t) \right\|$$

$$\sup_{i,j \in \mathcal{V}} \left\| \frac{\partial^2}{\partial t^2} \nabla \omega_i f_i(x_i, t) - \frac{\partial^2}{\partial t^2} \nabla \omega_j f_j(x_j, t) \right\|$$

$$\sup_{i,j \in \mathcal{V}} \left\| \frac{\partial}{\partial t} \nabla^2 \omega_i f_i(x_i, t) - \frac{\partial}{\partial t} \nabla^2 \omega_j f_j(x_j, t) \right\|$$

are all bounded.

Remark 1: Assumption 1 is to guarantee that information can be shared among the participants of the network at all time. Then, the consensus can be achieved such that the distributed problem (2) is equivalent to the original problem (1). To avoid Zeno behaviour, we assume the time length between two contiguous switching instances is lower bounded by σ_0. The connectivity of the graphs will be relaxed in Section III-C, where disconnected communication topologies are used to generate the Pareto fronts. The convexity in Assumptions 2 ensures that every Pareto solution can be obtained by the scalarisation method, as stated in the ensuing lemma. Assumption 3 implies that the changing rates of the gradients (first and second order), and the Hessian matrices with respect to t between any two cost functions are all bounded. This assumption is used to ensure that the consensus can be achieved within bounded time.

Lemma 1 (Miettinen (2012)): Let Assumptions 1 and 2 hold. For any non-negative weighting coefficients ω_i, and $\bar{\omega} = [\omega_i, \ldots, \omega_N]^T \neq \mathbf{0}$, the solution of the weighted problem (3) is a Pareto optimum of (1). Moreover, any Pareto solution of (1) can be found by the weighted problem in (3).

III. Algorithm Development and Convergence Analysis

In this section, we propose two distributed algorithms and then the convergence to the Pareto solution will be established using fixed-time Lyapunov theory for connected graphs. The results are further extended to disconnected graphs where subsets of the Pareto solutions can be derived.

A. Consensus-Based Pareto Solution Searching

The distributed algorithm for agent i is designed as

$$\begin{aligned} \dot{\rho}_i =& \alpha \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(\varrho_j - \varrho_i)^p + \beta \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(\varrho_j - \varrho_i)^q \\ &+ \sum_{j \in \mathcal{N}_i^\sigma} \gamma \operatorname{sign}(\varrho_j - \varrho_i) \\ \varrho_i =& \rho_i + \omega_i \nabla f_i(x_i, t) \end{aligned} \tag{4a}$$

$$\begin{aligned} \dot{\phi}_i =& \alpha \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(\varphi_j - \varphi_i)^p + \beta \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(\varphi_j - \varphi_i)^q \\ &+ \sum_{j \in \mathcal{N}_i^\sigma} \delta \operatorname{sign}(\varphi_j - \varphi_i) \\ \varphi_i =& \phi_i + \frac{\partial}{\partial t} \omega_i \nabla f_i(x_i, t) \end{aligned} \tag{4b}$$

$$\begin{aligned} \dot{\xi}_i =& \alpha \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(\varsigma_j - \varsigma_i)^p + \beta \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(\varsigma_j - \varsigma_i)^q \\ &+ \sum_{j \in \mathcal{N}_i^\sigma} \eta \operatorname{sign}(\varsigma_j - \varsigma_i) \\ \varsigma_i =& \xi_i + \nabla^2 \omega_i f_i(x_i, t) \end{aligned} \tag{4c}$$

$$\begin{aligned} \dot{x}_i =& \alpha \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(x_j - x_i)^p + \beta \sum_{j \in \mathcal{N}_i^\sigma} \operatorname{sig}(x_j - x_i)^q \\ &+ \sum_{j \in \mathcal{N}_i^\sigma} \kappa \operatorname{sign}(x_j - x_i) - \varsigma_i^{-1}(\tau \varrho_i + \varphi_i) \end{aligned} \tag{4d}$$

where ρ_i, ϕ_i, and ξ_i are intermediate states with initial values specified as $\rho_i(0) = \phi_i(0) = \xi_i(0) = 0$; ϱ_i, φ_i and ς_i are the consensus variables to estimate the global information; $\alpha, \beta, \gamma, \delta, \eta, \kappa$ are positive real constants to be designed later; and p, q are positive numbers satisfying $0 < p < 1$ and $q > 1$.

Remark 2: In (4a)-(4d), the local information required by agent i includes $\omega_i \nabla f_i(x_i, t), \frac{\partial}{\partial t} \omega_i \nabla f_i(x_i, t), \nabla^2 \omega_i f_i(x_i, t)$ and x_i, which are locally available to agent i. The shared information from its neighbours, $j \in \mathcal{N}_i^\sigma$, includes $\varrho_j, \varphi_j, \varsigma_j$ and x_j, which are exchanged through the communication graph $\mathcal{G}^\sigma$. Hence, the proposed algorithm is performed in a distributed manner. Some private information, for example, the local cost functions and their gradients, can be well-protected.

To show some insights into the proposed algorithm, let us examine the first part of the algorithm in (4a)-(4c), which is essentially formed by time-varying average consensus algorithms. It will be proved in Lemma 2, the consensus variables ϱ_i, φ_i and ς_i converge to the average of the time-varying reference signals in a fixed time T_1, that is,

$$\varrho_i(t) \to \frac{1}{N}\sum_{j=1}^{N} \nabla\omega_j f_j(x_j,t) \tag{5}$$

$$\varphi_i(t) \to \frac{1}{N}\sum_{j=1}^{N} \frac{\partial}{\partial t}\nabla\omega_j f_j(x_j,t) \tag{6}$$

$$\varsigma_i(t) \to \frac{1}{N}\sum_{j=1}^{N} \nabla^2\omega_j f_j(x_j,t). \tag{7}$$

With (5)-(7), the last term in (4d) can be rewritten as, for $t > T_1$,

$$\begin{aligned}-\varsigma_i^{-1}(\tau\varrho_i+\varphi_i) = &-\Big[\sum_{j=1}^{N}\nabla^2\omega_j f_j(x_j,t)\Big]^{-1}\\ &\times\Big[\tau\sum_{j=1}^{N}\nabla\omega_j f_j(x_j,t)+\sum_{j=1}^{N}\frac{\partial}{\partial t}\nabla\omega_j f_j(x_j,t)\Big].\end{aligned} \tag{8}$$

The first three terms in (4d) constitute another fixed-time consensus, with settling time being denoted as T_2, leading to $x_i = x_j$, for $t > T_1 + T_2$. Then, it follows that

$$\begin{aligned}\dot{x}_i(t) = &-\Big[\sum_{j=1}^{N}\nabla^2\omega_j f_j(x_j,t)\Big]^{-1}\\ &\times\Big[\tau\sum_{j=1}^{N}\nabla\omega_j f_j(x_j,t)+\sum_{j=1}^{N}\frac{\partial}{\partial t}\nabla\omega_j f_j(x_j,t)\Big].\end{aligned} \tag{9}$$

for $t > T_1 + T_2$. Finally, we obtain a time-varying convex optimisation algorithm (9) that shares a similar structure as the centralised methods (Su, 2009), of which the convergence will be established in Theorem 1.

Assumption 4: The coefficients in algorithm (4a)-(4d) satisfy

$$\frac{N-1}{2}\sup_{i,j\in\mathcal{V}}\Big\|\frac{\partial}{\partial t}\nabla\omega_i f_i(x_i,t)-\frac{\partial}{\partial t}\nabla\omega_j f_j(x_j,t)\Big\| < \gamma$$

$$\frac{N-1}{2}\sup_{i,j\in\mathcal{V}}\Big\|\frac{\partial^2}{\partial t^2}\nabla\omega_i f_i(x_i,t)-\frac{\partial^2}{\partial t^2}\nabla\omega_j f_j(x_j,t)\Big\| < \delta$$

$$\frac{N-1}{2}\sup_{i,j\in\mathcal{V}}\Big\|\frac{\partial}{\partial t}\nabla^2\omega_i f_i(x_i,t)-\frac{\partial}{\partial t}\nabla^2\omega_j f_j(x_j,t)\Big\| < \eta.$$

Note that Assumption 4 implies the bounds of the time derivatives are available to all agents. We will propose an adaptive algorithm to eliminate those global information. Now, we focus on the algorithm (4a)-(4d).

Lemma 2: Consider the following dynamics

$$\begin{aligned}\dot{\psi}_i =&\alpha\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sig}(z_j-z_i)^p+\beta\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sig}(z_j-z_i)^q\\ &+\sum_{j\in\mathcal{N}_i^\sigma}\gamma\,\mathrm{sign}(z_j-z_i)\\ z_i =&\psi_i+r_i\end{aligned} \tag{10}$$

where α,β and γ are positive constants; initial conditions are specified as $\psi_i(0)=0, \forall i\in\mathcal{V}$; and $0<p<1$ and $q>1$. If the communication graphs $\mathcal{G}^\sigma$ are connected, and $\gamma > \frac{(N-1)\bar{\mu}}{2}$, with $\bar{\mu} \geq \|\dot{r}_i(t)-\dot{r}_j(t)\|, \forall i,j\in\mathcal{V},\forall t$, then all the states z_i achieve a time-varying average consensus to $\frac{1}{N}\sum_{j=1}^{N} r_j(t)$ in a fixed time, bounded by

$$T_{\max} = \frac{1}{\alpha 2^{\frac{p-3}{2}}\lambda_2(\bar{\mathcal{L}}^\sigma)^{\frac{p+1}{2}}(1-p)}+\frac{N^{\frac{2q^2+q-3}{2q+2}}}{\beta 2^{\frac{q-3}{2}}\lambda_2(\bar{\mathcal{L}}^\sigma)^{\frac{q+1}{2}}(q-1)} \tag{11}$$

where $\lambda_2(\bar{\mathcal{L}}^\sigma)$ denotes the second smallest eigenvalue of the Laplacian matrix.

Proof: We denote the consensus error of agent i as $e_i = z_i - \frac{1}{N}\sum_{j=1}^{N} z_j$. Noticing that $\sum_{i=1}^{N}\psi_i(0)=\mathbf{0}$ and $\sum_{i=1}^{N}\dot{\psi}_i(t)=\mathbf{0}$, we have $\sum_{i=1}^{N}\psi_i(t)=\mathbf{0},\forall t$. From the second equation in (10), it follows $\sum_{i=1}^{N} z_i(t) = \sum_{i=1}^{N}\psi_i(t)+\sum_{i=1}^{N} r_i(t) = \sum_{i=1}^{N} r_i(t)$. If $e_i(t)\to\mathbf{0},\forall i\in\mathcal{V}$, it can be concluded that $z_i(t) \to \frac{1}{N}\sum_{j=1}^{N} z_j(t) = \frac{1}{N}\sum_{j=1}^{N} r_j(t)$. Now, consider a Lyapunov candidate as

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T e_i \tag{12}$$

of which the time derivative is given by

$$\begin{aligned}\dot{V}_1(t) =&\sum_{i=1}^{N} e_i^T\dot{e}_i\\ =&\sum_{i=1}^{N} e_i^T\dot{\psi}_i-\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T\dot{\psi}_j\\ &+\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(\dot{r}_i-\dot{r}_j)\end{aligned} \tag{13}$$

where the second equation follows from

$$\begin{aligned}\dot{e}_i =&\frac{1}{N}\sum_{j=1}^{N}(\dot{z}_i-\dot{z}_j)\\ =&\frac{1}{N}\sum_{j=1}^{N}(\dot{\psi}_i-\dot{\psi}_j)+\frac{1}{N}\sum_{j=1}^{N}(\dot{r}_i-\dot{r}_j).\end{aligned} \tag{14}$$

Since $\sum_{i=1}^{N} e_i = \sum_{i=1}^{N}(z_i - \frac{1}{N}\sum_{j=1}^{N} z_j(t)) = \sum_{i=1}^{N} z_i - \sum_{i=1}^{N} z_j = \mathbf{0}$, we have $-\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T\dot{\psi}_j = \mathbf{0}$. Now, substituting (10) into (13) yields

$$\begin{aligned}\dot{V}_1(t) =& \sum_{i=1}^{N} e_i^T \Big[\alpha \sum_{j\in\mathcal{N}_i^\sigma} \mathrm{sig}(z_j - z_i)^p \\ &+ \beta \sum_{j\in\mathcal{N}_i^\sigma} \mathrm{sig}(z_j - z_i)^q + \sum_{j\in\mathcal{N}_i^\sigma} \gamma\, \mathrm{sign}(z_j - z_i) \Big] \\ &+ \frac{1}{N} \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T (\dot{r}_i - \dot{r}_j) \\ =& \underbrace{\alpha \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} e_i^T\, \mathrm{sig}(z_j - z_i)^p}_{W_1} \\ &+ \underbrace{\beta \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} e_i^T\, \mathrm{sig}(z_j - z_i)^q}_{W_2} \\ &+ \underbrace{\gamma \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} e_i^T\, \mathrm{sign}(z_j - z_i)}_{W_3} \\ &+ \underbrace{\frac{1}{N} \sum_{i,j=1}^{N} e_i^T (\dot{r}_i - \dot{r}_j)}_{W_4}. \end{aligned} \tag{15}$$

For the first term W_1 in (15), it follows from the symmetrical property of undirected graphs that

$$\begin{aligned} W_1 =& \frac{\alpha}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma}^{N} e_i^T\, \mathrm{sig}(z_j - z_i)^p \\ &+ \frac{\alpha}{2} \sum_{j=1}^{N} \sum_{i\in\mathcal{N}_j^\sigma}^{N} e_j^T\, \mathrm{sig}(z_i - z_j)^p \\ =& -\frac{\alpha}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma}^{N} (e_i - e_j)^T\, \mathrm{sig}(e_i - e_j)^p \end{aligned} \tag{16}$$

where $e_j - e_i = z_j - z_i$ has been used to derive the last equation. Applying Lemma 3 in Zuo et al. (2018) yields

$$\begin{aligned} W_1 =& -\frac{\alpha}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|_{p+1}^{p+1} \\ \leq& -\frac{\alpha}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|^{p+1} \\ \leq& -\frac{\alpha}{2} \left(\sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|^2 \right)^{\frac{p+1}{2}}. \end{aligned} \tag{17}$$

By similar arguments, it can be obtained that

$$W_2 \leq -\frac{\beta}{2} N^{\frac{-2q^2-q+3}{2q+2}} \left(\sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|^2 \right)^{\frac{q+1}{2}}. \tag{18}$$

Due to the symmetricity of the graph, W_3 can be written as

$$\begin{aligned} W_3 =& \frac{\gamma}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} e_i^T\, \mathrm{sign}(z_j - z_i) \\ &+ \frac{\gamma}{2} \sum_{j=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} e_j^T\, \mathrm{sign}(z_i - z_j) \\ =& -\frac{\gamma}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} (e_i - e_j)^T\, \mathrm{sign}(e_i - e_j) \\ =& -\frac{\gamma}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|_1. \end{aligned} \tag{19}$$

For W_4, we have

$$W_4 = \frac{1}{2N} \sum_{i=1}^{N}\sum_{j=1}^{N} (e_i - e_j)^T (\dot{r}_i - \dot{r}_j). \tag{20}$$

Then, applying Cauchy-Schwarz inequality yields

$$\begin{aligned} W_4 \leq& \frac{1}{2N} \sum_{i=1}^{N}\sum_{j=1}^{N} \|e_i - e_j\| \|r_i - r_j\| \\ \leq& \frac{\bar{\mu}}{2N} \sum_{i=1}^{N}\sum_{j=1}^{N} \|e_i - e_j\|. \end{aligned} \tag{21}$$

Note that

$$\begin{aligned} \sum_{i=1}^{N}\sum_{j=1}^{N} \|e_i - e_j\| \leq& N \max_{i\in\mathcal{V}} \sum_{j=1, j\neq i}^{N} \|e_i - e_j\| \\ \leq& \frac{N(N-1)}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\| \\ \leq& \frac{N(N-1)}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|_1. \end{aligned} \tag{22}$$

Hence,

$$W_4 \leq \frac{(N-1)\bar{\mu}}{4} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i^\sigma} \|e_i - e_j\|_1. \tag{23}$$

Combing the results in (15)-(23) leads to

$$\begin{aligned}\dot{V}_1 \leq& -\frac{\alpha}{2}\bigg(\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i^\sigma}\|e_i-e_j\|^2\bigg)^{\frac{p+1}{2}}\\&-\frac{\beta}{2}N^{\frac{-2q^2-q+3}{2q+2}}\bigg(\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i^\sigma}\|e_i-e_j\|^2\bigg)^{\frac{q+1}{2}}\\&-\bigg(\frac{\gamma}{2}-\frac{(N-1)\bar{\mu}}{4}\bigg)\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i^\sigma}\|e_i-e_j\|_1\\\leq& -\frac{\alpha}{2}\bigg(2e^T\bar{\mathcal{L}}^\sigma e\bigg)^{\frac{p+1}{2}}-\frac{\beta}{2}N^{\frac{-2q^2-q+3}{2q+2}}\bigg(2e^T\bar{\mathcal{L}}^\sigma e\bigg)^{\frac{q+1}{2}}\\\leq& -\frac{\alpha}{2}\bigg(2\lambda_2(\bar{\mathcal{L}}^\sigma)V_1\bigg)^{\frac{p+1}{2}}\\&-\frac{\beta}{2}N^{\frac{-2q^2-q+3}{2q+2}}\bigg(2\lambda_2(\bar{\mathcal{L}}^\sigma)V_1\bigg)^{\frac{q+1}{2}}\\\leq& -\alpha 2^{\frac{p-1}{2}}\lambda_2(\bar{\mathcal{L}}^\sigma)^{\frac{p+1}{2}}V_1^{\frac{p+1}{2}}\\&-\beta N^{\frac{-2q^2-q+3}{2q+2}}2^{\frac{q-1}{2}}\lambda_2(\bar{\mathcal{L}}^\sigma)^{\frac{q+1}{2}}V_1^{\frac{q+1}{2}}.\end{aligned}\tag{24}$$

Invoking the fixed-time convergence theorem in Polyakov (2012), it can be concluded that $e_i, \forall i \in \mathcal{V}$ converge to zero in a fixed time, bounded by $T_{\max}$ in (11). This completes the proof. ■

Lemma 3: Under Assumptions 1-4, the average time-varying consensus of ϱ_i, φ_i and ς_i in (5)-(7) are obtained within a fixed time.

Proof: The results follow directly from Lemma 2. ■

Theorem 1: Let Assumptions 1-4 hold. The algorithm proposed in (4) solves the time-varying distributed problem (3). The solution trajectory $x_i(t)$ forms a Pareto optimum of the original time-varying multi-objective optimisation problem (1).

Proof: From Lemma 3, the algorithm in (4d) can be written as, for $t > T_1$,

$$\begin{aligned}\dot{x}_i =&\alpha\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sig}(x_j-x_i)^p+\beta\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sig}(x_j-x_i)^q\\&+\gamma_4\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sign}(x_j-x_i)-\bigg[\sum_{j=1}^{N}\nabla^2\omega_j f_j(x_j,t)\bigg]^{-1}\\&\times\bigg[\tau\sum_{j=1}^{N}\nabla\omega_j f_j(x_j,t)+\sum_{j=1}^{N}\frac{\partial}{\partial t}\nabla\omega_j f_j(x_j,t)\bigg].\end{aligned}\tag{25}$$

Notice that $\|\varsigma_i^{-1}(\tau\varrho_i+\varphi_i)-\varsigma_j^{-1}(\tau\varrho_j+\varphi_j)\|=0$ for $t > T_1$. It thus can be concluded a consensus of the state variable $x_i = x_j$ is obtained within a fixed time, T_2, by invoking Lemma 2. Note that, for $t \leq T_1 + T_2$, all the states remain bounded. Therefore, (25) reduces to

$$\begin{aligned}\dot{x}_i(t) =& -\bigg[\sum_{j=1}^{N}\nabla^2\omega_j f_j(x_j,t)\bigg]^{-1}\\&\times\bigg[\tau\sum_{j=1}^{N}\nabla\omega_j f_j(x_j,t)+\sum_{j=1}^{N}\frac{\partial}{\partial t}\nabla\omega_j f_j(x_j,t)\bigg]\end{aligned}\tag{26}$$

for $t > T_1 + T_2$. Now, consider a Lyapunov function

$$V_2(t)=\frac{1}{2}\bigg(\sum_{i=1}^{N}\nabla w_i f_i(x_i,t)\bigg)^T\bigg(\sum_{i=1}^{N}\nabla w_i f_i(x_i,t)\bigg).\tag{27}$$

The time derivative of $V_2(t)$ along with (26) is given by

$$\begin{aligned}\dot{V}_2(t) =&\bigg(\sum_{i=1}^{N}\nabla w_i f_i(x_i,t)\bigg)^T\bigg(\sum_{i=1}^{N}\nabla^2\omega_i f_i(x_i,t)\dot{x}_i\\&+\sum_{i=1}^{N}\frac{\partial}{\partial t}\nabla\omega_i f_i(x_i,t)\bigg)\\=&\bigg(\sum_{i=1}^{N}\nabla w_i f_i(x_i,t)\bigg)^T\bigg(-\tau\sum_{i=1}^{N}\nabla\omega_i f_i(x_i,t)\\&-\sum_{i=1}^{N}\frac{\partial}{\partial t}\nabla\omega_i f_i(x_i,t)+\sum_{i=1}^{N}\frac{\partial}{\partial t}\nabla\omega_i f_i(x_i,t)\bigg)\\=&-\tau\bigg(\sum_{i=1}^{N}\nabla w_i f_i(x_i,t)\bigg)^T\bigg(\sum_{i=1}^{N}\nabla\omega_i f_i(x_i,t)\bigg)\\=&-2\tau V_2(t).\end{aligned}\tag{28}$$

Hence, $\sum_{i=1}^{N}\nabla w_i f_i(x_i,t)$ converges to $\mathbf{0}$, which directly leads to the results. This completes the proof. ■

B. Adaptive Consensus-Based Pareto Solution Searching

In previous subsection, the parameters γ, δ and η are determined by the bounds of the time derivatives as shown in Assumption 4, which are global information. In order to release such restrictions, we propose an adaptive design in this subsection.

The adaptive parameters are designed as

$$\begin{cases}\dot{\gamma}_{ij}=\mathrm{sign}\left(\max_{s\in[t-\epsilon,t]}\|\varrho_j(s)-\varrho_i(s)\|\right)\\\dot{\delta}_{ij}=\mathrm{sign}\left(\max_{s\in[t-\epsilon,t]}\|\varphi_j(s)-\varphi_i(s)\|\right)\quad\forall(j,i)\in\mathcal{E}\\\dot{\eta}_{ij}=\mathrm{sign}\left(\max_{s\in[t-\epsilon,t]}\|\varsigma_j(s)-\varsigma_i(s)\|\right)\end{cases}\tag{29}$$

where $\epsilon > 0$ is an arbitrary small constant. The main structure of the adaptive algorithm is similar to previous fixed-gain design, and it is therefore omitted to avoid redundancy. Now, we show the convergence proof of the algorithm.

Lemma 4: Consider the following dynamics

$$\begin{aligned}\dot{\psi}_i =&\alpha\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sig}(z_j-z_i)^p+\beta\sum_{j\in\mathcal{N}_i^\sigma}\mathrm{sig}(z_j-z_i)^q\\&+\sum_{j\in\mathcal{N}_i^\sigma}\gamma_{ij}\,\mathrm{sign}(z_j-z_i)\\z_i =&\psi_i+r_i\\\dot{\gamma}_{ij} =&\mathrm{sign}\left(\max_{s\in[t-\epsilon,t]}\|z_j(s)-z_i(s)\|\right),\forall(j,i)\in\mathcal{E}\end{aligned}\tag{30}$$

where α, β are positive constants with initial condition $\psi_i(0) = \gamma_{ij}(0) = 0$. If the communication graph $\mathcal{G}^\sigma$ is connected, and $\|\dot{r}_i(t)-\dot{r}_j(t)\| < \bar{\mu}, \forall i,j \in \mathcal{V}, \forall t$, then all the states z_i achieve an average consensus to $\frac{1}{N}\sum_{k=1}^{N} r_k(t)$ in a fixed time.

Proof: The convergence can be divided into the following two cases. **Case 1:** The consensus of the states z_i is achieved within

$t \leq T_3 = \frac{(N-1)\bar{\mu}}{2}$, that is, $z_i(t) = z_j(t)$, for all $t > T_3$. Consequently, it follows that $z_i(t) = z_j(t) = \frac{1}{N}\sum_{k=1}^{N} r_k(t)$, since $\sum_{i=1}^{N} z_i(t) = \sum_{i=1}^{N} \psi_i(t) + \sum_{i=1}^{N} r_i(t)$ and $\sum_{i=1}^{N} \psi_i(t) = \mathbf{0}$.
Case 2: The consensus is not achieved within time $t \leq T_3 = \frac{(N-1)\bar{\mu}}{2}$. Then, we have $\max_{s\in[t-\epsilon,t]} \|z_j(s) - z_i(s)\| \neq \mathbf{0}$, and $\dot{\gamma}_{ij} = 1$ for $t < T_3$. Hence, it can be obtained that $\gamma_{ij}(t) > \frac{(N-1)\bar{\mu}}{2}$ for $t > T_3$. Following similar procedures in Lemma 2, it can be concluded that the consensus is achieved within $t < T_3 + T_1$. This completes the proof. ■

Theorem 2: Let Assumptions 1-3 hold. The algorithm in (4) with adaptive gains in (29) solves the time-varying distributed problem (3). The solution trajectory $x_i(t)$ constitutes a Pareto optimum of the original time-varying multi-objective optimisation problem (1).
Proof: The result directly follows from Lemma 4 and Theorem 1. ■

C. Pareto Solution Searching with Disconnected Graphs

In previous subsections, the communication graphs are assumed to be connected. In practice, the communication topology may be unstable and easily broken. Moreover, some subsystems might be intentionally disconnected from the network for maintenance. In this part, switching and disconnected graphs will be used to generate the Pareto fronts.

Apparently, if a graph is disconnected, it can also be considered as a set of d connected subgraphs. We denote the Laplacian matrices of the subgraphs as $\mathcal{L}_1^\sigma, \ldots, \mathcal{L}_d^\sigma$, where each of them consists of m_i agents with $\sum_{i=1}^{d} m_i = N$, and the objectives are reordered according to their communication subgroups. In addition, the weighting parameters ω_i for each subgroup are nonnegative with at least one element being positive.

Corollary 1: Under Assumptions 2-4, if the communication graph is disconnected, both the fixed-gain algorithm in (4) and the adaptive algorithm in (29) can solve the m_i-objective optimisation problem for each subgroup. Moreover, all the solutions of the subgroups are the Pareto solution of the original problem (1).
Proof: Note that each subgraph is connected, and therefore Theorems 1 and 2 remain valid. Now we consider the kth subgroup with m_k agents, denoted as $\mathcal{M}_k$, and each of them with objectives $f_{k,1}(x_{k,1}, t), \ldots, f_{k,m_k}(x_{k,m_k}, t)$. The subgroup's objective can be viewed as $\sum_{i=1}^{m_k} \omega_{k,i} f_{k,i}(x_{k,i}, t) + \sum_{j\in\mathcal{V}\setminus\mathcal{M}_k} 0 \times f_j(x_j, t)$, where $\omega_{k,i} \neq 0$ for at least one index i. According to Lemma 1, all the solutions of the disconnected subgroups are also Pareto solutions of the original network. This completes the proof. ■

Remark 3: With Corollary 1, we can generate the Pareto fronts for different sets of objective functions by switching the communication graphs. This is of great significance to analyse the properties of any combination of the cost functions, and to enrich the diversity of the solution set. Different from centralised optimisation where a global coordinator dominates the selection of the Pareto solution by enforcing its own preference on the networked agents, the distributed methods in this paper enable each agent to specify the local weights, by which the diversity of the Pareto solution is further improved.

Remark 4: The proposed algorithms for distributed network-connected systems can be implemented for a range of real-world applications, including power systems (Yang et al., 2016) and communication systems (Li et al., 2020). Since many practical applications have multiple time-varying objectives, the proposed method can quickly adjust the network strategy due to the utilisation of fixed-time consensus algorithms. In the simulation section, the algorithms will be implemented for an optimal charging problem of multiple electric vehicles.

Remark 5: Recent studies have been extensively focused on distributed single objective optimisation, see Ning et al. (2019) and Boyd et al. (2011). Another stream on MOPs mainly uses particle swarm optimization methods in centralised settings, as in Song et al. (2017) and Liu et al. (2019). The proposed method in this paper implements fixed-time consensus-based algorithms to solve distributed time-varying MOPs. The convergence of the proposed algorithms is established in a deterministic sense, which is different from the genetic and particle swarm algorithms. Distinct from the problem solved by Li and Ding (2020), time-varying objectives and switching graphs are considered in the work.

IV. Simulation

In this section, we will demonstrate the effectiveness of the proposed algorithms using two examples with detailed simulation results.

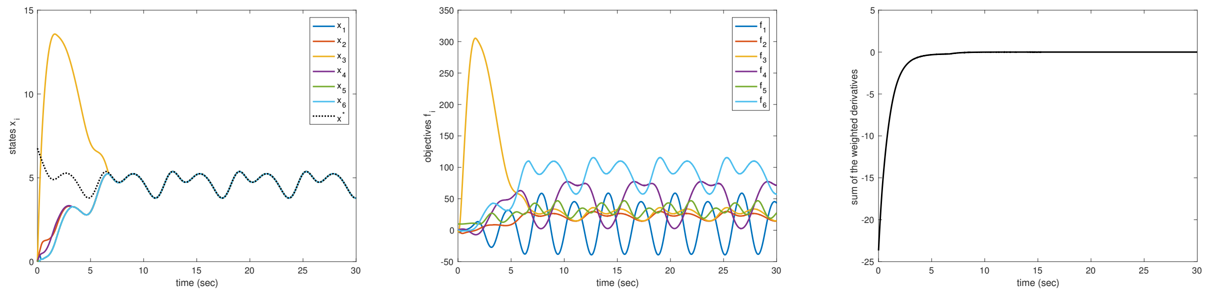
A. Numerical Example

Consider a network of 6 agents where each of them possesses a time-varying objective function, given by

$$\begin{aligned}
f_1(x_1, t) &= \frac{1}{2}x_1^2 - 10\cos(2t)x_1 + \sin(2t) \\
f_2(x_2, t) &= x_2^2 - 10e^{-t}x_2 + 5\cos(t) \\
f_3(x_3, t) &= 2x_3^2 - 100x_3 + 5\tanh(t) \\
f_4(x_4, t) &= 2x_4^2 - 10\sin(t)x_4 + 5\tanh(2t) \\
f_5(x_5, t) &= x_5^2 - 2\cos(3t)x_5 + 10 \\
f_6(x_6, t) &= 4x_6^2 - 20e^{-2t}x_6 + 20.
\end{aligned}$$

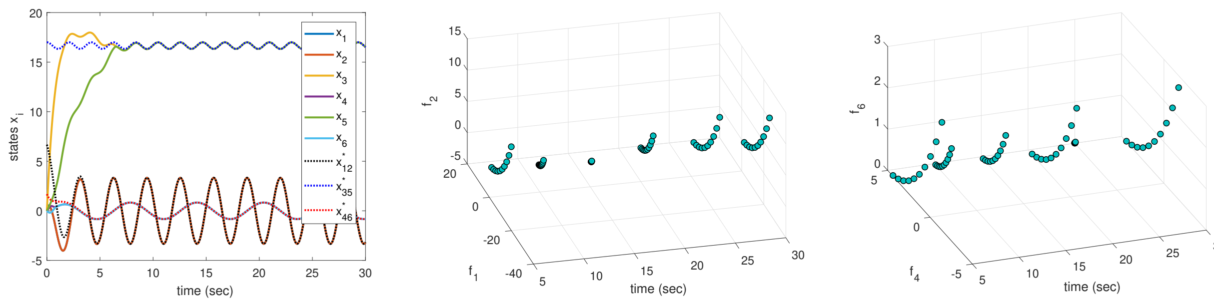
where $x_i \in \mathbb{R}, \forall, i = 1, \ldots, 6$. The communication graphs have been selected from a randomly generated set, satisfying Assumption 1. We implement the fixed-gain algorithm in (4). The parameters are set as follows: $\omega_i = \frac{1}{6}, i = 1, \ldots, 6$, $p = \frac{1}{3}$, $q = \frac{5}{3}$, $\alpha = \beta = 0.01, \gamma = 2$, and $\delta = \eta = \kappa = 1$.

The initial states of all agents are set to zero. The state trajectories are illustrated in Fig. 1a, where the time-varying Pareto solution $x^*(t)$ is shown in the dotted line. It can be observed that all the sates converge to the optimal solution after a few seconds. As the states change, the objective values can be calculated and plotted in Fig. 1b. Now, we demonstrate the obtained solution is indeed the Pareto optimum. To see this, the sum of the weighted derivatives $\sum_{i=1}^{N} \nabla\omega_i f_i(x_i, t)$ has been depicted in Fig. 1c. It is clear that the weighted derivatives converge to zero, which manifests that the time-varying trajectory is the Pareto solution. Though the objectives are time-varying, the distributed agents have been capable of



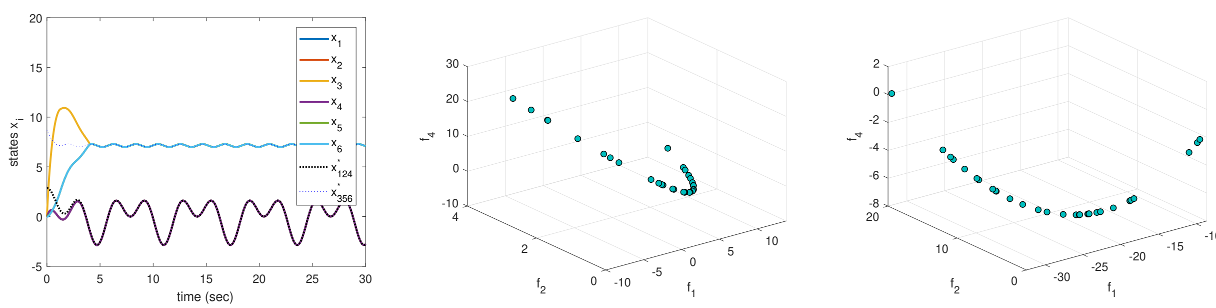
(a) (b) (c)

Fig. 1: The simulation results using distributed fixed-gain design: (a) state trajectories; (b) objective values; (c) convergence of the weighted derivatives.



(a) (b) (c)

Fig. 2: Simulation results under the first graph in Fig. 4: (a) Time-varying Pareto states with three separate equilibria; (b) Pareto fronts for agents 1 and 2; (c) Pareto fronts for agents 4 and 6.



(a) (b) (c)

Fig. 3: Simulation results under the first graph in Fig. 4: (a) Convergence of time-varying states with two separate equilibria; (b) Pareto fronts of agents 1, 2 and 4 at $t = 20$s; (c) Pareto fronts of agents 1, 2 and 4 at $t = 30$s.

tracking the optimal solution in a fast manner, due to the utilisation of fixed-time consensus algorithms. Similar results can be obtained using the adaptive design in (29), which have been omitted due to space limit. It has been shown in Lemma 1 that any Pareto solution of the original problem (1) can be obtained by varying the weighting parameters ω_i.

Now, we deploy the proposed algorithms for disconnected graphs to search the Pareto front. Fig. 2a displays the convergence of the state variables, where three time-varying Pareto solutions of the subgroups are obtained using the first communication topology. In Figs. 2b and 2c, the Pareto fronts are presented for the two subgroups in different time slots. For the second disconnected communication graph, the convergence of the decision variables is shown in Fig. 3a. The Pareto fronts can be obtained by changing the weighting parameters. Figs. 3b and 3c show, respectively, the Pareto fronts for the the subgroup with agents 1, 2 and 4 at time $t = 20$s and $t = 30$s.

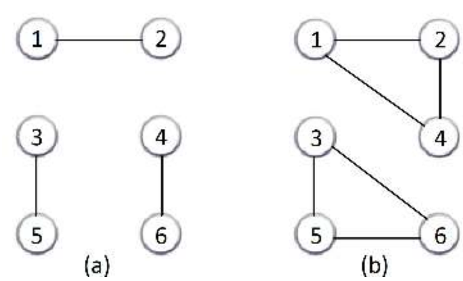


Fig. 4: The disconnected communication topology.

From above observations, the proposed algorithms can be implemented for disconnected graphs to identify the Pareto fronts, and separated subgroups are able to cooperatively

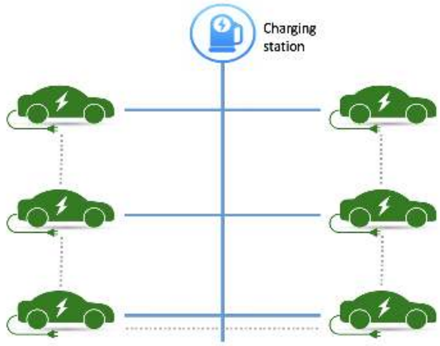


Fig. 5: Charging network of multiple electric vehicles, where solid line in blue denotes the power network and dotted line in grey denotes the communication channel.

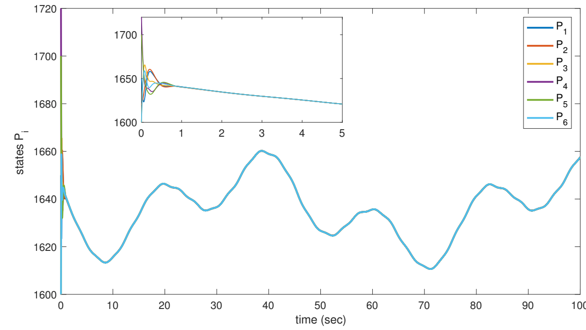


Fig. 6: Time-varying charging power of the electric vehicles.

optimise partial objectives of the network, which is important for applications with unstable communication channels. Moreover, by varying the communication topology, different Pareto optima can be derived, which is helpful for improving the diversity of the solutions.

B. Practical Application

In this subsection, we implement the proposed algorithms to an optimal charging problem of plug-in electric vehicles. Due to the increasing concerns over environmental degradation, electric vehicles have received significant research efforts, among which power charging management is one of the most important issues. As the number of electric vehicles increases, centralised methods are no longer feasible because of heavy communication and computation burden. Distributed algorithms developed in this paper can therefore serve as an alternative, which possesses a number of promising advantages, such as robustness and scalability.

We consider a group of electric vehicles at a charging station that aim to cooperatively optimise the charging power by using local communication with their neighbours. The objectives of the electric vehicles are formulated as

$$f_i(P_{EV}, t) = p_i(t)\left(\frac{P_{EV}}{R_i} - \frac{V_{o,i} + 2R_i I_i^{\text{ref}}}{2R_i^2}\sqrt{4R_i P_{EV} + V_{o,i}^2}\right)$$

where the variable P_{EV} is the parallel charging power to be optimised; $p_i(t)$ denotes the time-varying priority weight of the ith vehicle; R_i is the equivalent internal resistance; $V_{o,i}$ represents the open-circuit voltage; and I_i^{ref} is the desired charging current. Formulation of such objective functions has been studied in some recent works, for example, Xu (2014) and Zhao and Ding (2017). The simulation parameters in this paper are adopted from Xu (2014). The network structure and communication topology are shown in Fig. 5.

During the charging process, the priority of each vehicle may change as the charging status varies, and some coefficients can be time-varying, such as the desired charging current I_i^{ref}. In this case, the algorithms proposed in Gharesifard and Corts (2014) and Li and Ding (2020) dealing with time-invariant optimisation problems are infeasible. The time-varying optimisation developed in this paper can be utilised to obtain an optimal charging profile in an online manner. The simulation parameters are kept the same as the numerical case. Fig. 6 shows the charging power of the network, where consensus is achieved within 2 seconds. Due to the change of priority, the network can quickly adjust its power profile so as to optimise the time-varying objectives.

V. Conclusion

In this paper, time-varying multi-objective optimisation problems have been considered. A weighted sum approach is introduced to quantify different objectives. Distributed optimisation algorithms are proposed to solve those problems over switching graphs. With some knowledge of the system parameters, a fixed-gain algorithm is developed to achieve convergence to the Pareto solutions, where the average time-varying consensus plays a significant role in estimating the global variables. Then, an adaptive algorithm is designed, which overcomes the dependence on additional global information. When the graphs are disconnected, it is shown that the obtained solutions constitute the Pareto fronts for the original problem. The simulation results have been provided to validate the effectiveness of the algorithms.

Future work can be concentrated on dealing with different settings of communication topologies, e.g., directed graphs. Due to the existence of time delay and uncertainties in communication channels, future work may also consider robust and event-triggered optimisation strategies.

Notes on Contributors

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