University of Groningen

Time-varying output feedback stabilization of a class of nonholonomic Hamiltonian systems via canonical transformations

Fujimoto, Kenji; Scherpen, Jacquelien M.A.

Time-varying output feedback stabilization of a class of nonholonomic Hamiltonian systems via canonical transformations

Kenji Fujimoto[1]   and   Jacquelien M. A. Scherpen[2]

[1]Department of Systems Science
Graduate School of Informatics
Kyoto University
Uji, Kyoto 611-0011, Japan
fujimoto@i.kyoto-u.ac.jp

[2]Faculty Information Technology Systems
Department of Electrical Engineering
Delft University of Technology
2600 GA Delft, The Netherlands
J.M.A.Scherpen@its.tudelft.nl

Abstract

This paper is concerned with the output feedback stabilization of a class of nonholonomic systems in port-controlled Hamiltonian formulae via generalized canonical transformations. In order to obtain a dynamic feedback, an integrator is added to the system firstly. Then the generalized canonical transformation is utilized to let the integrator play the role of an estimator of the unmeasurable state based on passivity. This technique can derive a time-varying output feedback stabilizing controller under a certain assumption. Furthermore the effectiveness of the proposed technique is demonstrated via a well known knife edge example.

1 Introduction

Hamiltonian control systems [14, 8] are the systems described by Hamilton's canonical equations which represent general physical systems. Recently port-controlled Hamiltonian systems are introduced as the generalization of Hamiltonian systems [6]. They can represent Hamiltonian systems with a class of nonholonomic constraints [7, 5] as well as ordinary mechanical, electrical and electro-mechanical systems. The special structure of physical systems allows us to utilize the passivity which they innately possess and a lot of fruitful results were obtained so far. These methods are so called passivity based control [13, 9]. One of the advantages of passivity based control is output feedback control, see e.g. [9] for Euler-Lagrange systems and extended for Hamiltonian systems in [12], also a more general result can be found in [10]. It is usually difficult to stabilize a nonlinear system using output feedback because there is no efficient way of designing a state observer of nonlinear systems. Utilizing the intrinsic passive property of physical systems, however, it is easy to stabilize the system using only the information of output.

Generalized canonical transformations for port-controlled Hamiltonian systems were introduced as the generalization of conventional canonical transformations which are widely used for analysis in classical mechanics [1, 2]. This transformation preserves the structure of Hamiltonian systems and the passivity property of physical systems. Generalized canonical transformation gives us a general stabilization strategy for port-controlled Hamiltonian systems which is a natural generalization of passivity based control. As an application of this strategy, a time-varying stabilization method for nonholonomic port-controlled Hamiltonian systems was obtained which employs time-varying feedback and coordinate transformations [3, 4].

This paper is devoted to the output feedback stabilization of a class of nonholonomic Hamiltonian systems based on generalized canonical transformations. We utilize the dynamic extension as in [12, 10] to construct a dynamic compensator and canonical transformation to derive a time-varying output feedback controller for nonholonomic Hamiltonian systems. Firstly we refer to the generalized canonical transformation for port-controlled Hamiltonian systems [1], time-varying state feedback stabilization method [3, 4] for nonholonomic Hamiltonian systems. Secondly a framework of dynamic extension in order to obtain a dynamic compensator for nonholonomic Hamiltonian systems is derived. Finally the effectiveness of the proposed method is demonstrated via a numerical example.

2 Canonical transformations for Hamiltonian systems

2.1 Port-controlled Hamiltonian systems

A time-varying port-controlled Hamiltonian system with a Hamiltonian $H(x,t)$ is a system in the form of

$$\begin{cases} \dot{x} & = & J(x,t)\frac{\partial H(x,t)}{\partial x}^T + g(x,t)u \\ y & = & g(x,t)^T \frac{\partial H(x,t)}{\partial x}^T \end{cases} \quad (1)$$

with $u, y \in \mathbb{R}^m$, $x \in \mathbb{R}^n$ and a skew symmetric matrix J, i.e. $J = -J^T$ holds [1]. The following properties of

such systems are known which is a time-varying version of the result in [6].

Theorem 1 *[1] Consider the system (1). Suppose the Hamiltonian H satisfies $H(x,t) \geq H(0,t) = 0$ and $\partial H/\partial t \leq 0$. Then the input-output mapping $u \mapsto y$ of the system is passive with respect to the storage function H, and the feedback*

$$u = -C(x,t)\,y \tag{2}$$

with a matrix $C > 0$ renders $(u,y) \to 0$. Furthermore if H is a positive definite function and if the system is zero-state detectable, then the feedback (2) renders the system uniformly asymptotically stable.

The zero-state detectability and the positive definiteness of the Hamiltonian assumed in Theorem 1 do not always hold for general Hamiltonian systems. In such a case, the generalized canonical transformation is useful. It provides a generalization of the stabilization method of exploiting virtual potential energy [13].

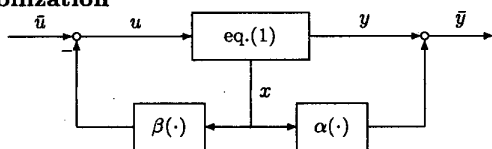## 2.2 Generalized canonical transformations and stabilization

Figure 1: Generalized canonical transformation

A generalized canonical transformation is a set of transformations

$$
\begin{aligned}
\bar{x} &= \Phi(x,t) \\
\bar{H} &= H(x,t) + U(x,t) \\
\bar{y} &= y + \alpha(x,t) \\
\bar{u} &= u + \beta(x,t)
\end{aligned}
\tag{3}
$$

which preserves the structure of port-controlled Hamiltonian systems given in (1). Here $\bar{x}$, $\bar{H}$, $\bar{y}$ and $\bar{u}$ denote the new state, the new Hamiltonian, the new output and the new input respectively. This transformation can be seen as in Figure 1 from the input-output point of view. The generalized canonical transformation is a natural generalization of a canonical transformation which is widely used for the analysis of conventional Hamiltonian systems in classical mechanics. The properties of such transformations are summarized as follows.

Theorem 2 *[1, 2] (i) Consider the system (1). For any functions $U(x,t) \in \mathbb{R}$ and $\beta(x,t) \in \mathbb{R}^m$, there exists a pair of functions $\Phi(x,t) \in \mathbb{R}^n$ and $\alpha(x,t) \in \mathbb{R}^m$ such*

that the set (3) yields a generalized canonical transformation. Φ yields a generalized canonical transformation if and only if

$$
\frac{\partial \Phi}{\partial (x,t)}
\left(
\begin{array}{c}
J\frac{\partial U}{\partial x}^T + K\frac{\partial H + U}{\partial x}^T + g\beta \\
-1
\end{array}
\right) = 0
\tag{4}
$$

holds with a skew-symmetric matrix $K(x,t)$. Further the change of output α and the matrices $\bar{J}$ and $\bar{g}$ are given by

$$
\begin{aligned}
\alpha &= g^T \frac{\partial U}{\partial x}^T & (5) \\
\bar{J} &= \frac{\partial \Phi}{\partial x}(J + K)\frac{\partial \Phi}{\partial x}^T & (6) \\
\bar{g} &= \frac{\partial \Phi}{\partial x}g. & (7)
\end{aligned}
$$

(ii) Transform the system (1) by the generalized canonical transformation with U and β such that $H + U \geq 0$. Then the new input-output mapping $\bar{u} \mapsto \bar{y}$ is passive with the storage function $\bar{H}$ if and only if

$$
\frac{\partial (H + U)}{\partial (x,t)}
\left(
\begin{array}{c}
J\frac{\partial U}{\partial x}^T + g\beta \\
-1
\end{array}
\right) \geq 0.
\tag{8}
$$

(iii) Suppose that (8) holds and that $H + U$ is positive definite. Then the feedback $\bar{u} = -C\,\bar{y}$ with $C > 0$ renders the system stable. Suppose moreover that the transformed system is zero-state detectable with respect to x. Then the feedback renders the system uniformly asymptotically stable.

Using the generalized canonical transformation, we can change the property of the system without changing the intrinsic passive property. The structure matrix $\bar{J}$ of the transformed system is given by (6). Therefore this transformation can also be used to change the structure matrix J.

3 State feedback stabilization

3.1 Hamiltonian systems with nonholonomic constraints

We consider a mechanical system with nonholonomic constraints which is described by a conventional Hamiltonian system with Lagrangian multipliers [7]

$$
\begin{cases}
\dot{q} &= \frac{\partial H(q,p_0)}{\partial p_0}^T \\
\dot{p}_0 &= -\frac{\partial H(q,p_0)}{\partial q}^T + A(q)\lambda + B(q)u \\
y &= B^T(q)\frac{\partial H(q,p_0)}{\partial p_0}^T \\
0 &= A^T(q)\frac{\partial H(q,p_0)}{\partial p_0}^T
\end{cases}
\tag{9}
$$

where $x = (q, p_0) \in \mathbb{R}^n \times \mathbb{R}^n$ is the state, the Hamiltonian $H(q, p_0) := (1/2)\, p_0^T M_0(q)^{-1} p_0$ describes the kinetic energy with a symmetric inertia matrix $M_0(q) >$

0, and $A(q)\lambda$ and $B(q)u$ denote the constraint force and the external force respectively with λ the Lagrangian multiplier.

It was shown in [7] that this Hamiltonian system with constraints can be described by a port-controlled Hamiltonian system

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & J_{12}(q) \\ -J_{12}^T(q) & J_{22}(q,p) \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q}^T \\ \frac{\partial H}{\partial p}^T \end{pmatrix} + \begin{pmatrix} 0 \\ G \end{pmatrix} u \\ y = G^T(q)\frac{\partial H}{\partial p}^T \end{cases} \quad (10)$$

with a Hamiltonian

$$H(q,p) = \frac{1}{2}p^T M(q)^{-1} p \quad (11)$$

$p \in \mathbb{R}^m$ and $M(q) \in \mathbb{R}^{m \times m}$. Here the skew-symmetric matrix J_{22} is very often replace by a negative semidefinite matrix in order to represent the friction of the original dynamics. In the sequel, we consider a stabilization of Hamiltonian systems in the form of (10).

3.2 Time-varying stabilization of nonholonomic Hamiltonian systems

This subsection refers to [3, 4] for the time-varying stabilization of a nonholonomic Hamiltonian system (10). The time-varying state feedback derived here is utilized for output feedback stabilization in the next section. We treat the stabilization of the systems in the form of (10), however we describe all results assuming $M = I$ and $G = I$ because of the simplicity. These assumptions can always be satisfied by transforming the system (10) into another Hamiltonian system employing a pair of coordinate and input transformations

$$\check{p} = M^{-\frac{1}{2}}(q)\, p \quad (12)$$
$$\check{u} = G(q)\, u \quad (13)$$

where $\check{p}$ and $\check{u}$ denote the new phase state and the new input respectively, and $M^{-\frac{1}{2}}$ denotes a matrix such that

$$M^{-\frac{T}{2}} M^{-\frac{1}{2}} = M^{-1}. \quad (14)$$

Theorem 3 *[3, 4] (i) Consider the system (10) with $M = I$ and $G = I$. Then the following generalized canonical transformation*

$$\begin{aligned} \bar{H} &= H(q,p) + \alpha^T p + \frac{1}{2}\alpha^T \alpha + U(\Psi(q,t)) \\ \bar{q} &= \Psi(q,t) \\ \bar{p} &= p + \alpha(q,t) \\ \bar{y} &= y + \alpha(q,t) \\ \bar{u} &= u + \left(J_{12}^T \frac{\partial U}{\partial q}^T + \frac{\partial \alpha}{\partial t} + \frac{\partial \alpha}{\partial q} J_{12} p - J_{22}\alpha \right) \end{aligned} \quad (15)$$

with a scalar function $U(\Psi(q,t)) \geq 0$ and a periodic odd function $\alpha(q,t) \in \mathbb{R}^m$ transforms the system into a pas-

sive time-varying port-controlled Hamiltonian system

$$\begin{cases} \begin{pmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \end{pmatrix} = \begin{pmatrix} 0 & \bar{J}_{12}(\bar{q},t) \\ -\bar{J}_{12}(\bar{q},t)^T & \bar{J}_{22}(\bar{q},\bar{p},t) \end{pmatrix} \begin{pmatrix} \frac{\partial \bar{H}}{\partial \bar{q}}^T \\ \frac{\partial \bar{H}}{\partial \bar{p}}^T \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} \bar{u} \\ \bar{y} = \frac{\partial \bar{H}}{\partial \bar{p}}^T = \bar{p} \end{cases} \quad (16)$$

with the Hamiltonian

$$\bar{H} = U(\bar{q}) + \frac{1}{2}\bar{p}^T \bar{p}. \quad (17)$$

Here the coordinate transformation $\bar{q} = \Psi(q,t)$ is a solution of a PDE

$$\frac{\partial \Psi}{\partial(q,t)} \begin{pmatrix} J_{12}\alpha \\ -1 \end{pmatrix} = 0 \quad (18)$$

and $\bar{J}$ is given by

$$\bar{J} = \left. \begin{pmatrix} 0 & \frac{\partial \Psi}{\partial q} J_{12} \\ -J_{12}^T \frac{\partial \Psi}{\partial q}^T & J_{22} \end{pmatrix} \right|_{x = \Phi^{-1}(\bar{x},t)}. \quad (19)$$

Furthermore if U is positive definite and the resulting system (16) is zero-state detectable with respect to the input-output mapping $\bar{u} \mapsto \bar{y}$, then the feedback

$$\bar{u} = -C(x,t)\, \bar{y} \quad (20)$$

with $C > 0$ renders the system uniformly asymptotically stable.

(ii) Suppose J_{12}^1 the first column of J_{12} satisfies

$$J_{12}^1 = (1, 0, \dots, 0). \quad (21)$$

Choose the function $\alpha_1(q,t)$ as

$$\alpha_1(q,t) = \frac{\partial h}{\partial t} \quad (22)$$

with a scalar periodic function $h(q_2, \dots, q_n, t)$ satisfying $h(0, \dots, 0, t) = 0$. Then the corresponding $\Psi(q,t)$ in (18) is given by

$$\Psi(q,t) = (q_1 + h(q_2, \dots, q_n, t), q_2, \dots, q_n). \quad (23)$$

Furthermore suppose $h(q_2, \dots, q_n, t)$ is chosen such that

$$\frac{\partial(\sum_{i=2}^n q_i^2)}{\partial q} J_{12}(q) = 0, \quad \frac{\partial^i h}{\partial t^i} = 0 \quad \Rightarrow \quad q = 0. \quad (24)$$

Then the feedback (20) with $U = \Psi^T \Psi$ renders the system uniformly asymptotically stable. A possible choice of $h(q_2, \dots, q_n, t)$ is

$$h(q_2, \dots, q_n, t) = \cos t \sum_{i=2}^n q_i^2. \quad (25)$$

The latter part of this theorem is based on [11] which is a time-varying stabilization method for driftless systems.

4 Output feedback stabilization

This section is devoted to the main result output feedback stabilization of nonholonomic Hamiltonian systems. Firstly how to derive a dynamic compensator using generalized canonical transformations is shown and then output feedback controller using a time-varying feedbacks is derived.

4.1 Dynamic extension

Consider the system (10) again and suppose $M = I$, $G = I$ and that we can only measure the state q. As in [12], add the integrator $r \in \mathbb{R}^m$ to the system:

$$
\begin{cases}
\begin{pmatrix} \dot{q} \\ \dot{p} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} 0 & J_{12} & 0 \\ -J_{12}^T & J_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q}^T \\ \frac{\partial H}{\partial p}^T \\ \frac{\partial H}{\partial r}^T \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} u \\ u_c \end{pmatrix} \\
\begin{pmatrix} y \\ y_c \end{pmatrix} = \begin{pmatrix} \frac{\partial H}{\partial p}^T \\ \frac{\partial H}{\partial r}^T \end{pmatrix}
\end{cases}
\tag{26}
$$

whose Hamiltonian is

$$
H = \frac{1}{2} p^T p. \tag{27}
$$

Here r is the state of the compensator. However it is not connected to the system (10) yet. The following lemma connects the dynamics of the original system and that of r-integrator via a generalized canonical transformation.

Lemma 1 *Consider the system (26) with the Hamiltonian (27) without loss of generality. Suppose $J_{22} = J_{22}(q)$. Then the generalized canonical transformation with input and output transformations*

$$
\begin{aligned}
\bar{H} &= H(q,p,r) + r^T r - r^T p \\
&= \tfrac{1}{2}(p-r)^T(p-r) + \tfrac{1}{2}r^T r
\end{aligned}
$$

$$
\begin{pmatrix} \bar{q} \\ \bar{p} \\ \bar{r} \end{pmatrix} = \begin{pmatrix} q \\ p - r \\ r \end{pmatrix}
$$

$$
\begin{pmatrix} \bar{y} \\ \bar{y}_c \end{pmatrix} = \begin{pmatrix} I & -I \\ 0 & I \end{pmatrix} \begin{pmatrix} y \\ y_c \end{pmatrix}
\tag{28}
$$

$$
\begin{pmatrix} \bar{u} \\ \bar{u}_c \end{pmatrix} = \begin{pmatrix} I & -I \\ 0 & I \end{pmatrix} \left\{ \begin{pmatrix} u \\ u_c \end{pmatrix} + \begin{pmatrix} J_{22}r \\ -J_{33}r \end{pmatrix} \right\}
$$

with a skew-symmetric (negative semidefinite) matrix $J_{33} \in \mathbb{R}^{m \times m}$ transforms the system into a port-controlled Hamiltonian system

$$
\begin{cases}
\begin{pmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{\bar{r}} \end{pmatrix} = \begin{pmatrix} 0 & J_{12} & J_{12} \\ -J_{12}^T & J_{22} & 0 \\ -J_{12}^T & 0 & J_{33} \end{pmatrix} \begin{pmatrix} \frac{\partial \bar{H}}{\partial \bar{q}}^T \\ \frac{\partial \bar{H}}{\partial \bar{p}}^T \\ \frac{\partial \bar{H}}{\partial \bar{r}}^T \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{u}_c \end{pmatrix} \\
\begin{pmatrix} \bar{y} \\ \bar{y}_c \end{pmatrix} = \begin{pmatrix} \frac{\partial \bar{H}}{\partial \bar{p}}^T \\ \frac{\partial \bar{H}}{\partial \bar{r}}^T \end{pmatrix} = \begin{pmatrix} \bar{p} \\ \bar{r} \end{pmatrix} = \begin{pmatrix} p - r \\ r \end{pmatrix}
\end{cases}
\tag{29}
$$

with a Hamiltonian

$$
\bar{H} = \frac{1}{2} \bar{p}^T \bar{p} + \frac{1}{2} \bar{r}^T \bar{r}. \tag{30}
$$

The proof is straightforwardly obtained from a direct calculation. Notice that if we choose $J_{33} = J_{22}$ in the dynamic extension (29), the state $\bar{r}$ can be regarded as an estimated value of $\bar{p}$ because both dynamics have the same form. In addition, both $\bar{p}$ and $\bar{r}$ affect the dynamics of $\bar{q} = q$. Hence $\bar{r}$ can be used for the stabilization of the whole system using a similar technique as in Theorem 3.

Remark 1 In Theorem 4 we utilize a combination of a generalized canonical transformation and an input transformation in the form of (28). This type of input transformations can be included in the definition of generalized canonical transformations. To this effect, the definition (3) should be replaced by

$$
\begin{aligned}
\bar{x} &= \Phi(x,t) \\
\bar{H} &= H(x,t) + U(x,t) \\
\bar{y} &= N(x,t)^T y + \alpha(x,t) \\
\bar{u} &= N(x,t)\, u + \beta(x,t)
\end{aligned}
\tag{31}
$$

with any nonsingular matrix $N(x,t)$.

Remark 2 J_{22} is a function of both q and p in general and the assumption $J_{22} = J_{22}(q)$ in Lemma 1 is restrictive. If $J_{22} = J_{22}(q,p)$, i.e., it is a function of p, then we can use the feedback in (28) with $J_{22} = J_{22}(q,r)$ instead of $J_{22}(q,p)$. This choice works well at least in the case J_{22} is negative definite. However the effectiveness of this strategy is not clear in general and it will be purchased in a future research.

4.2 Output feedback stabilization

The system (10) and the same problem setting as in the previous subsection is considered. This subsection derives an output feedback controller using time-varying feedback developed in Section 3.2.

Theorem 4 *(i) Consider the system (29) with the Hamiltonian (30). Suppose the assumptions in Lemma 1 hold. Then the generalized canonical transformation*

$$
\begin{aligned}
\hat{H} &= \bar{H}(\bar{q}, \bar{p}, \bar{r}) + r^T \alpha_c + \tfrac{1}{2}\alpha_c^T \alpha_c + U(\Psi(\bar{q},t)) \\
&= \tfrac{1}{2}\bar{p}^T \bar{p} + \tfrac{1}{2}(\bar{r} + \alpha_c)^T(\bar{r} + \alpha_c) + U(\Psi(\bar{q},t))
\end{aligned}
$$

$$
\begin{pmatrix} \hat{q} \\ \hat{p} \\ \hat{r} \end{pmatrix} = \begin{pmatrix} \Psi(\bar{q},t) \\ \bar{p} \\ \bar{r} + \alpha_c(\bar{q},t) \end{pmatrix}
$$

$$
\begin{pmatrix} \hat{y} \\ \hat{y}_c \end{pmatrix} = \begin{pmatrix} \bar{y} \\ \bar{y}_c \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha_c(\bar{q},t) \end{pmatrix}
$$

$$
\begin{pmatrix} \hat{u} \\ \hat{u}_c \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{u}_c \end{pmatrix} + \begin{pmatrix} J_{12}^T \frac{\partial U}{\partial \bar{q}}^T + J_{12}^T \frac{\partial \alpha_c}{\partial \bar{q}}^T(\bar{r} + \alpha_c) \\ J_{12}^T \frac{\partial U}{\partial \bar{q}}^T + \frac{\partial \alpha_c}{\partial t} + \frac{\partial \alpha_c}{\partial \bar{q}} J_{12}\bar{r} - J_{33}\alpha_c \end{pmatrix}
\tag{32}
$$

with a scalar function $U(\Psi(\bar{q}, t)) \geq 0$ *and a periodic odd function* $\alpha_c(\bar{q}, t) \in \mathbb{R}^m$ *transforms the system into a passive time-varying port-controlled Hamiltonian system*

$$
\begin{cases}
\begin{pmatrix} \dot{\hat{q}} \\ \dot{\hat{p}} \\ \dot{\hat{r}} \end{pmatrix} = \begin{pmatrix} 0 & \hat{J}_{12} & \hat{J}_{12} \\ -\hat{J}_{12}^T & \hat{J}_{22} & -\hat{J}_{32}^T \\ -\hat{J}_{12}^T & \hat{J}_{32} & \hat{J}_{33} \end{pmatrix} \begin{pmatrix} \frac{\partial \hat{H}}{\partial \hat{q}}^T \\ \frac{\partial \hat{H}}{\partial \hat{p}}^T \\ \frac{\partial \hat{H}}{\partial \hat{r}}^T \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{u}_c \end{pmatrix} \\
\begin{pmatrix} \hat{y} \\ \hat{y}_c \end{pmatrix} = \begin{pmatrix} \frac{\partial \hat{H}}{\partial \hat{p}}^T \\ \frac{\partial \hat{H}}{\partial \hat{r}}^T \end{pmatrix} = \begin{pmatrix} \hat{p} \\ \hat{r} \end{pmatrix} = \begin{pmatrix} p - r \\ r + \alpha_c(q, t) \end{pmatrix}
\end{cases}
$$

(33)

with a Hamiltonian

$$
\hat{H} = \bar{U}(\hat{q}) + \frac{1}{2}\hat{p}^T\hat{p} + \frac{1}{2}\hat{r}^T\hat{r}. \tag{34}
$$

The coordinate transformation $\hat{q} = \Psi(\bar{q}, t)$ *is a solution of a PDE (18) with* $\alpha(q, t) = \alpha_c(q, t)$, *and* $\hat{J}_{12}, \hat{J}_{22}, \hat{J}_{32}$ *and* $\hat{J}_{33}$ *are given by*

$$
\hat{J}_{12} := \frac{\partial \Psi}{\partial q} J_{12}, \quad \hat{J}_{22} := J_{22}, \quad \hat{J}_{32} := \frac{\partial \alpha_c}{\partial q} J_{12}, \quad \hat{J}_{33} := J_{33}.
$$

Furthermore if $\bar{U}$ *is positive definite and the resulting system (33) is zero-state detectable with respect to the input-output mapping* $(\bar{u}, \bar{u}_c) \mapsto \bar{y}_c$, *then the feedback*

$$
\begin{pmatrix} \bar{u} \\ \bar{u}_c \end{pmatrix} = \begin{pmatrix} 0 \\ -C(x, t)\,\bar{y}_c \end{pmatrix} \tag{35}
$$

with $C > 0$ *renders the system uniformly asymptotically stable.*

(ii) Suppose moreover that $J_{22} = 0$ *and that the* $J_{12}(q)$ *satisfies*

$$
J_{12}(q) = \begin{pmatrix} 1 & 0 \\ 0 & J_{122}(q) \end{pmatrix} \tag{36}
$$

with $J_{122}(q) \in \mathbb{R}^{(n-1) \times (m-1)}$. *Choose* $\alpha_c(q, t) = (\alpha_1(q_{2,n}, t), 0, \dots, 0)$ *such that the conditions (22) and (24) hold, and the positive definite function* $\bar{U}$ *such that*

$$
\bar{U}(\hat{q}) = \bar{U}_1(\hat{q}_1) + \bar{U}_2(\hat{q}_{2,n}) \tag{37}
$$

where $q_{2,n} := (q_2, q_3, \dots, q_n)$. *Then the system (33) is zero-state detectable with respect to the input-output mapping* $(\bar{u}, \bar{u}_c) \mapsto \bar{y}_c$, *i.e. the feedback (35) renders the system uniformly asymptotically stable.*

Proof. (i) Firstly apply Theorem 3 to the system (29). Then we obtain a generalized canonical transformation with

$$
\bar{\beta} = \begin{pmatrix} \frac{\partial \bar{U}}{\partial \bar{q}}^T \\ \frac{\partial \bar{U}}{\partial \bar{q}}^T + \frac{\partial \alpha_c}{\partial \bar{q}} J_{12}(\bar{p} + \bar{r}) - J_{33}\alpha_c \end{pmatrix}.
$$

However output feedback controller cannot use the information of $\bar{p}$. Hence we add another feedback with a

skew-symmetric matrix (which plays the role of K matrix in the whole generalized canonical transformation) in order preserve the Hamiltonian structure

$$
\begin{pmatrix} \hat{u} \\ \hat{u}_c \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{u}_c \end{pmatrix} + \bar{\beta} + \begin{pmatrix} 0 & J_{12}^T \frac{\partial \alpha_c}{\partial \bar{q}}^T \\ -\frac{\partial \alpha_c}{\partial \bar{q}} J_{12} & 0 \end{pmatrix} \begin{pmatrix} \bar{p} \\ \bar{r} + \alpha_c \end{pmatrix}
$$

$$
= \begin{pmatrix} \bar{u} \\ \bar{u}_c \end{pmatrix} + \begin{pmatrix} J_{12}^T \frac{\partial \bar{U}}{\partial \bar{q}}^T + J_{12}^T \frac{\partial \alpha_c}{\partial \bar{q}}^T (\bar{r} + \alpha_c) \\ J_{12}^T \frac{\partial \bar{U}}{\partial \bar{q}}^T + \frac{\partial \alpha_c}{\partial t} + \frac{\partial \alpha_c}{\partial \bar{q}} J_{12}\bar{r} - J_{33}\alpha_c \end{pmatrix}
$$

This feedback only contains the measurable signals $\bar{q}$ and $\bar{r}$, and is equivalent to the feedback in (32) so proves (i). (ii) Suppose $(\bar{u}, \bar{u}_c) \equiv 0$ and $\bar{y}_c \equiv 0$. Then (33) implies

$$
\begin{aligned}
0 \equiv \dot{\hat{r}} &= -J_{12}^T \frac{\partial \bar{U}(\hat{q})}{\partial \hat{q}}^T + \frac{\partial \alpha_c}{\partial \hat{q}} J_{12}\hat{p} + J_{33}\hat{r} \\
&= -\begin{pmatrix} \frac{\partial \bar{U}_1(\hat{q})}{\partial \hat{q}_1} \\ J_{122}^T \frac{\partial \bar{U}_2(\hat{q}_{2,n})}{\partial \hat{q}_{2,n}}^T \end{pmatrix} + \begin{pmatrix} \frac{\partial \alpha_1}{\partial \hat{q}} J_{12}\hat{p} \\ 0 \end{pmatrix}. (38)
\end{aligned}
$$

This reduces to

$$
J_{122}^T \frac{\partial \bar{U}_2(\hat{q}_{2,n})}{\partial \hat{q}_{2,n}}^T \equiv 0.
$$

This means

$$
\hat{p}_{2,m} \equiv 0 \tag{39}
$$

where $\hat{p}_{2,m} := (\hat{p}_2, \dots, \hat{p}_m)$ because of the block diagonal structure of J_{12} as in (36). It follows from (22) and (39) that

$$
\begin{aligned}
\hat{J}_{32}\,\hat{p} &= \begin{pmatrix} \frac{\partial \alpha_1}{\partial \hat{q}} J_{12}\hat{p} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \frac{\partial \alpha_1}{\partial \hat{q}_1}\hat{p}_1 + \frac{\partial \alpha_1}{\partial \hat{q}_{2,n}} J_{122}\hat{p}_{2,m} \\ 0 \end{pmatrix} \equiv 0.
\end{aligned}
$$

Further (38) implies

$$
\frac{\partial \bar{U}_1(\hat{q})}{\partial \hat{q}_1} \equiv 0.
$$

Therefore again from (33) we obtain

$$
\dot{\hat{p}} = \begin{pmatrix} \dot{\hat{p}}_1 \\ \dot{\hat{p}}_{2,m} \end{pmatrix} = \begin{pmatrix} \frac{\partial \bar{U}_1(\hat{q})}{\partial \hat{q}_1} \\ 0 \end{pmatrix} \equiv 0. \tag{40}
$$

The positive definiteness of the Hamiltonian implies

$$
\hat{p} = \hat{r} \equiv 0. \tag{41}
$$

It directly follows from the state-feedback case that

$$
\hat{q} \equiv 0. \tag{42}
$$

This means the zero-state detectability of the system (33) and completes the proof. $\qquad \square$
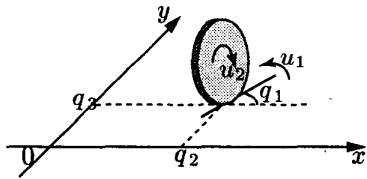
Figure 2: A rolling coin

4.3 Example

The well known "knife edge" example is considered here, see [15] for details. We consider a rolling coin on a horizontal plane depicted in Figure 2. Let x-y denote the Cartesian coordinate of the point of contact of the coin and the horizontal plane. Let q_1 denote the heading angle of the coin, and (q_2, q_3) the position of the coin in x-y plane. Furthermore let p_1 be the angular velocity with respect to the heading angle q_1, p_2 be the rolling angular velocity of the coin, u_1 and u_2 be the angular acceleration with respect to p_1 and p_2 respectively. Finally let all the parameters unity for simplicity, then the generalized Hamiltonian system (10) with $q = (q_1, q_2, q_3)$, $p = (p_1, p_2)$, $H = (1/2)\, p^T p$ and

$$J = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cos q_1 \\ 0 & 0 & 0 & 0 & \sin q_1 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -\cos q_1 & -\sin q_1 & 0 & 0 \end{pmatrix} \quad (43)$$

is obtained. We can easily check that the assumptions in Lemma 1 and Theorem 4 hold for this system.

For this system we choose the time-varying function

$$\alpha_1(q, t) = q_3 \sin t \quad (44)$$

with $U = (1/2)\, \bar{q}^T \bar{q}$ and $C = 5I$ and construct the feedback system according to Theorem 4. The response of q coordinate from the initial conditions $q(0) = (0, 0, 1)$, $p(0) = (0, 0)$ and $r(0) = (0, 0)$ of the resulting feedback system in simulation is shown in Figure 3. The solid line, dashed line and dashed and solid line denote q_1, q_2 and q_3 respectively. Although the convergence is slow and oscillatory as the usual time-varying state feedback, all states converges to the origin smoothly. This example exhibits the effectiveness of the proposed method.
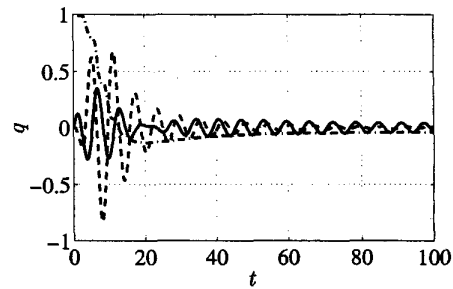
Acknowledgement

Figure 3: Response of q form the initial state $q(0) = (0, 0, 1)$.

References

[1] K. Fujimoto and T. Sugie. Canonical transformation and stabilization of generalized Hamiltonian systems. Submitted (1999); Preliminary version is in *Proc. 4th IFAC Symp. Nonlinear Control Systems*, pages 544–549, 1998.

[2] K. Fujimoto and T. Sugie. Stabilization of generalized Hamiltonian systems –approach by canonical transformations–. *Trans. Institute of Systems, Control and Information Engineers*, 11(11):623–629, 1998. (in Japanese).

[3] K. Fujimoto and T. Sugie. Time-varying stabilization of Hamiltonian systems via generalized canonical transformations. *Proc. IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control*, pages 68–73, 2000.

[4] K. Fujimoto and T. Sugie. Time-varying stabilization of nonholonomic Hamiltonian systems via canonical transformations. *Proc. American Control Conference*, pages 3269–3273, 2000.

[5] H. Khennouf, C. Canudas de Wit, A. J. van der Schaft, and J. Abraham. Preliminary results on asymptotic stabilization of Hamiltonian systems with nonholonomic constraints. *Proc. 34th IEEE Conf. on Decision and Control*, pages 4305–4310, 1995.

[6] B. M. J. Maschke and A. J. van der Schaft. Port-controlled Hamiltonian systems: modelling origins and system-theoretic properties. *IFAC Symp. Nonlinear Control Systems*, pages 282–288, 1992.

[7] B. M. J. Maschke and A. J. van der Schaft. A Hamiltonian approach to stabilization of nonholonomic mechanical systems. *Proc. 33rd IEEE Conf. on Decision and Control*, pages 2950–2954, 1994.

[8] H. Nijmeijer and A. J. van der Schaft. *Nonlinear Dynamical Control Systems*. Springer-Verlag, New York, 1990.

[9] R. Ortega, A. Loría, P. J. Nicklasson, and H. Sira-Rmírez. *Passivity-based Control of Euler-Lagrange Systems*. Springer-Verlag, London, 1998.

[10] R. Ortega, A. J. van der Schaft, B. M. J. Maschke, and G. Escobar. Stabilization of port-controlled Hamiltonian systems: passivity and energy-balancing. To appear in *Automatica* (1999); Also in *Proc. IEEE CDC*, 1999.

[11] J.-B. Pomet. Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift. *Systems & Control Letters*, 18:147–158, 1992.

[12] S. Stramigioli, B. M. J. Maschke, and A. J. van der Schaft. Passive output feedback and port interconnection. *Proc. 4th IFAC Symp. Nonlinear Control Systems*, pages 613–618, 1998.

[13] M. Takegaki and S. Arimoto. A new feedback method for dynamic control of manipulators. *Trans. ASME, J. Dyn. Syst., Meas., Control*, 103:119–125, 1981.

[14] A. J. van der Schaft. Stabilization of Hamiltonian systems. *Nonl. An. Th. Math. Appl.*, 10:1021–1035, 1986.

[15] A. J. van der Schaft. *L2-gain and Passivity Techniques in Nonlinear Control*, volume 218 of *Lecture Notes on Control and Information Science*. Springer-Verlag, Berlin, 1996.

2933