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Tire-Road Forces Estimation Using Extended Kalman Filter and Sideslip Angle Evaluation

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Abstract— Main task in driving safety is the understanding and prevention of risky situations. While looking closer at the accidents data analysis, it appears that vehicle loss of control represents a huge part of car accidents. Preventing such kind of accidents, using assistance systems needs several type of information about vehicle state and vehicle-road interaction phenomenon. Longitudinal velocity, acceleration and yaw rate are easily measured using low cost sensors that are actually mounted in standard on a large part of recent vehicles. However, other parameters, which have a major impact on vehicle dynamics, are more difficult to measure using vehicle industry technology sensors. These are for example the used friction coefficient and the sideslip angle. Using an appropriate vehicle model and available measurements, the vehicle state as well as the road/tire interaction forces are reconstructed by implementing an Extended Kalman Filter. Thereafter, we evaluate the used friction coefficient and the sideslip angle estimates. Simulation and estimation results are then compared to real measurements collected by an equipped test vehicle on Satory test track.

Keywords: Kalman Filtering, Vehicle Modeling, Tire/Road Forces Estimation, Friction Model, Sideslip Angle Estimation.

I. INTRODUCTION

Analysis of the number of people killed due to car accidents during these last decades, highlights a reduction in spite of the increase in the road traffic. This is on one side the result of successive transport policies, road infrastructure improvement and on the other side the result of safer vehicles and passive and active driving assistance systems. However, the number of accidents still remain high and certain types of accidents are more frequent and contribute to a large part to the number of death. According to France statistics, run-off-road accidents are generally more represented than other accidents. This fact is also true for all developed countries. Preventing this type of accidents requires several parameters which help to evaluate the dangerousness of the driving situation. A certain number of these parameters (lateral acceleration, yaw rate, ...) are relatively easy to measure with low cost sensors. They are actually implemented in most of recent vehicles and used in driving assistance system such as ESP (Electronic Stability Program). Howevers, others such as the sideslip angle and the road friction are still difficult to measure directly with cost effective sensors. The aim of this paper is to answer the question on how can the friction coefficient and the sideslip angle be deduced from the partly knowledge of the vehicle state.

Several works have already been conducted in order to estimate tire/road forces, sideslip angle and friction coefficient. In [1], Ray uses an Extended Kalman Filter (EKF) to estimate the dynamic state and tire/road forces for a nine degree of freedom (DOF) vehicle model. She identifies the friction coefficient in [2]. The friction coefficient is also estimated by LIU and PENG [3] using a Luenberger Observer for a 2 DOF vehicle model in order to evaluate the TTC coefficient (Time To Collision). In 2006, Gerard [4] estimates tire/road forces and friction coefficient using an EKF and an Unscented Kalman Filter (UKF). The estimation of the sideslip angle has been recently considered by Stephant using a Sliding Mode Observer for a nonlinear bicycle model [5].

In this paper, an estimation methodology of the vehicle state using an EKF is considered, The tire/road forces and the mobilized friction coefficient are evaluated, thus an estimation of the sideslip angle is proposed on the basis of the state estimation. The model used for the observer synthesis includes a nonlinear Dugoff tire forces model. The estimation results are validated by real measurements collected with an equipped vehicle prototype. In addition, the observer uses as input the signals and measures available in standard on vehicles equipped with driving assistance systems such as ABS and ESP.

This paper is organized as follows: First, a four wheel vehicle model is presented. This model is simple but is suitable for the problem under consideration. Section 3 is dedicated for the estimation method for the vehicle state and the tire/road forces. Section 4 proposes an estimation procedure for the friction coefficient while section 5 provides it for the sideslip angle. Along all the section, the estimation results are validated by comparison to real measurements collected using an equipped prototype vehicle running on the LIVIC test track in Satory in the city of Versailles, France.

II. FOUR WHEEL VEHICLE MODEL

A. Model Developpment

In this paper, we use a four wheel vehicle model [10] which is represented in the figure (1).

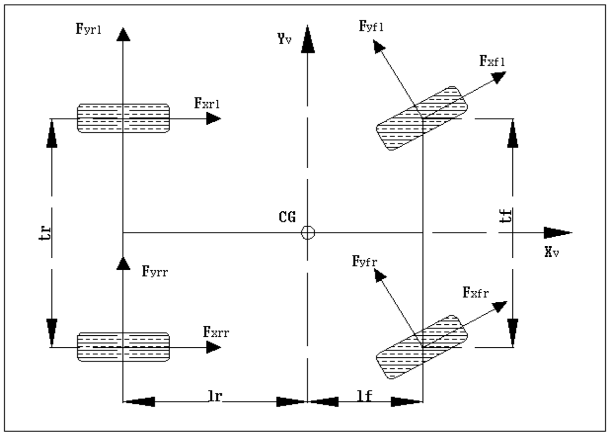

Fig. 1. Four wheels vehicle model

It is a simplified nonlinear vehicle model that considers the tire/road forces. The dynamic motion of the vehicle is modeled by three equations that represent respectively the longitudinal and lateral translational motion and the yaw rotational movement:

$$\left\{ \begin{array}{l} \dot{v}_x = \frac{1}{m}\sum F_{xi} + \dot{\psi} v_y \\ \dot{v}_y = \frac{1}{m}\sum F_{yi} - \dot{\psi} v_x \\ \ddot{\psi} = \frac{1}{I_z}\sum M_{zi} \end{array} \right. \tag{1}$$

where v_x is the longitudinal velocity, v_y the lateral one, $\dot{\psi}$ the yaw rate while I_z the moment of inertia. F_{xi}, F_{yi} and M_{zi} are respectively the longitudinal and lateral tire/road forces and the rotational moment around the vertical axis ($i = 1, 2, 3, 4$ represents the four wheel of the vehicle).

In the system (1), a tire/road force model is required in order. Several models exist in the litterature [6], [7], [8] and [9]. In [10] a comparative study between Dugoff model [6] and Pacejka [7] is presented. In [11] a comparaison is made among Kiencke model [8], Ben Amar [9] and Pacejka [7]. The Dugoff model is selected for this study for two main reasons: it needs a fewer number of parameters to evaluate the tire/road forces, and the formulation remains close to the linear formulation. The forces are given by:

$$\left\{ \begin{array}{l} F_{xi} = C_{xx}\frac{\lambda_i}{1-\lambda_i}k_i \\ F_{yi} = C_{yy}\frac{\tan\alpha_i}{1-\lambda_i}k_i \end{array} \right. \tag{2}$$

with:

$$k_i = \left\{ \begin{array}{lll} (2-\sigma_i)\sigma_i & \text{if} & \sigma_i < 1 \\ 1 & \text{if} & \sigma_i \geq 1 \end{array} \right. \tag{3}$$

$$\sigma_i = \frac{(1-\lambda_i)\,\mu_i F_{ni}}{2\sqrt{C_{xx}^2\lambda_i^2 + C_{yy}^2\tan^2\alpha_i}} \tag{4}$$

where λ_i and α_i represent respectively the longitudinal and lateral slip of each tire, C_{xx} and C_{yy} are respectively longitudinal and lateral stiffness, and F_{ni} is the normal force applied on each tire. λ_i, α_i and normal forces F_{ni} are given as follows:

$$\left\{ \begin{array}{l} \lambda_i = \frac{Rw_i - v_{pxi}}{\max(Rw_i, v_{pxi})} \\ \alpha_i = \delta_i - \arctan(\frac{v_{pyi}}{v_{pxi}}) \end{array} \right. \tag{5}$$

$$\left\{ \begin{array}{l} F_{nfl} = \frac{l_r mg}{2(l_f+l_r)} - \frac{hma_x}{2(l_f+l_r)} - \frac{l_r hma_y}{(l_f+l_r)S_b} \\ F_{nfr} = \frac{l_r mg}{2(l_f+l_r)} - \frac{hma_x}{2(l_f+l_r)} + \frac{l_r hma_y}{(l_f+l_r)S_b} \\ F_{nrl} = \frac{l_f mg}{2(l_f+l_r)} + \frac{hma_x}{2(l_f+l_r)} - \frac{l_f hma_y}{(l_f+l_r)S_b} \\ F_{nrr} = \frac{l_f mg}{2(l_f+l_r)} + \frac{hma_x}{2(l_f+l_r)} + \frac{l_f hma_y}{(l_f+l_r)S_b} \end{array} \right. \tag{6}$$

In equation (5) we use the longitudinal and lateral vehicle velocities at the tire/road contact point, v_{pxi} and v_{pyi} for each tire. These velocities are given using the following reference change.

$$v_{pi} = v_g + \Omega \wedge P_i \tag{7}$$

where v_g is the velocity vector at the center of gravity (CG) of the vehicle, Ω is the rotational velocity vector restricted in this case to the yaw motion along the z axis. P_i represents the vector position of each tire with reference to the center of gravity.

$$\Omega = \left[\begin{array}{ccc} 0 & 0 & \dot{\psi} \end{array} \right]^T, \; P_i = \left[\begin{array}{ccc} l_{xi} & l_{yi} & 0 \end{array} \right]^T \tag{8}$$

B. Model Simulation

Before use of the vehicle model developed above in order to estimate the tire/road forces and to evaluate the friction coefficient as well as the sideslip angle, a simulation is run in order to compare results obtained with model to real measurements collected with an equipped prototype vehicle running on the Satory test track of the Versailles city, France.

The following table summarizes the vehicle characteristics and parameter's numerical values.

m	vehicle mass	1550	kg
μ	friction coefficient	0.8	-
l_f	distance from CG to front axel	1.0065	m
l_r	distance from CG to rear axel	1.4625	m
t_f	front axel length	1.5	m
t_r	rear axel length	1.5	m
g	gravitational acceleration	9.81	m/s^2
I_z	moment of inertia	2200	kgm^2
R	tire radius	0.306	m
h	height of the CG	0.5	m
C_{xxi}	longitudinal stiffness of each tire	30000	N
C_{yyi}	lateral stiffness of each tire	57000	N

From the system (1), we consider the longitudinal and lateral velocity and the yaw rate as the components of the state vector:

$$x = \left[\begin{array}{ccc} v_x & v_y & r \end{array} \right]^T \text{ where } r = \dot{\psi} \tag{9}$$

The model can be functionally described by the block diagram given in figure (2), the input vector contains the steering angle δ and the four tires rotational velocity w_{ij} measured respectively by a steering angle sensor and the ABS sensors. Model output is the whole state vector and the longitudinal and lateral forces.

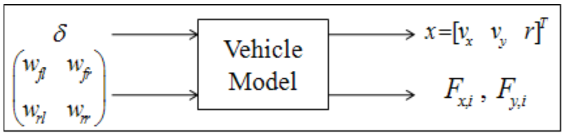

Fig. 2. Simulation bloc diagram

C. Results and Validation

Using the block diagram of figure (2), the model is simulated using the steering angle and the wheel speeds as input. The selected driving profile covers sufficient transient behavior and large lateral velocity values. The model state vector output is compared to real measurements collected with prototype vehicle. Figure (3) shows that the longitudinal and lateral vehicle velocity as well as the yaw rate are similar to the measurements, the simulation error is given in figure (4).

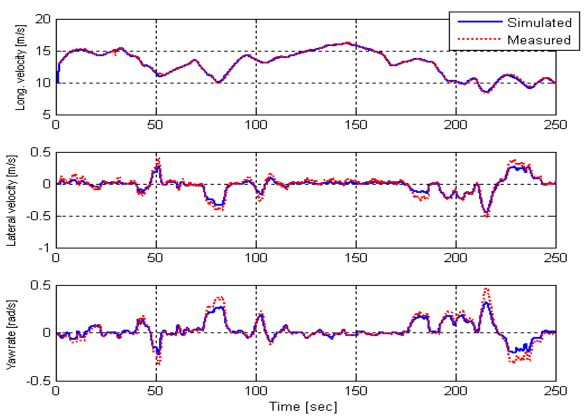

Fig. 3. Dynamic State Simulation

This simulation study proves that the mathematical model developed above gives a good description of the real dynamical evolution of the vehicle. Thus, this model can be used as a basis for the estimation of the dynamic state and the evaluation of the friction coefficient and sideslip angle.

III. VEHICLE DYNAMIC STATE AND TIRE/ROAD FORCES ESTIMATION

A. Extended Kalman Filter

The Extended Kalman Filter is dedicated to the estimation of the state vector of nonlinear systems [12], [13] and [14]. In order to develop directly a discrete-time EKF, the dynamic continous evolution of the vehicle (1) has to be discretize.

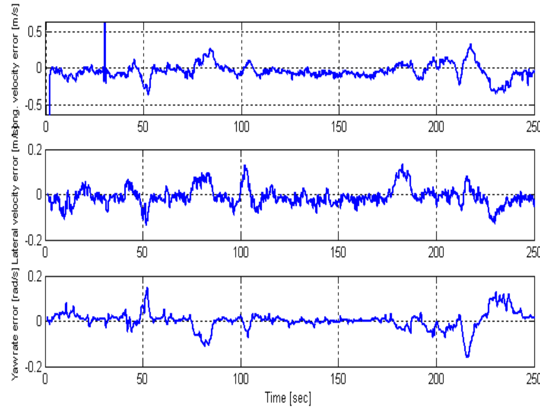

Fig. 4. Dynamic State Simulation Error

This discretization is performed by a forward Euler approximation. We get a nonlinear discrete-time system of the form:

$$\begin{cases} x_{k+1} = f(x_k) + g(x_k, u_k) + b_k \\ y_k = h(x_k) + w_k \end{cases} \quad (10)$$

where y_k is composed of the longitudinal velocity v_x and the yaw rate r, b_k is the dynamic noise vector and w_k is the measurement noise vector. Both are supposed to be non-intercorrelated, stationary, white and Gaussian with known covariances. The covariance of b_k (resp. w_k) is noted Q (resp R).

Under these hypothesis, an EKF can be applied to the estimation problem under consideration. It is worth mentioning that using this hypothesis, the state vector and the output vector are Gaussian even when they are conditioned on the measurements from time step 1 to time step k: $y_{1..}y_k$. We note $\hat{x}_{k|k} = E\left\{x_k \left| y_1 ... y_k \right.\right\}$ the mean of the state vector conditioned on the measurements from time step 1 to time step k and $P_{k|k} = E\left\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T \left| y_1 ... y_k \right.\right\}$ its covariance. The variables $\hat{x}_{k|k}$ and $P_{k|k}$ are also, respectively the estimate and estimation error covariance provided, at each time step, by the EKF.

The EKF algorithm is recursive and operates in two steps: a prediction step and an update step. The prediction step consists in the propagation of both the state estimate and the state estimation error covariance between two sampling instants. The update step occurs at each sampling time, and consists in correcting against the measurement, both the state forecast and the prediction error covariance.

- Prediction step:

$$\begin{array}{l} \hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}) + g(\hat{x}_{k-1|k-1}, u_{k-1}) \\ P_{k|k-1} = F_k P_{k-1|k-1} F^T + Q \end{array} \quad (11)$$

where $\hat{x}_{k|k-1}$ is the forecast, $P_{k|k-1}$ is the prediction error covariance and $F_k = \frac{\partial f(x)}{\partial (x)}\Big|_{x=\hat{x}_{k-1|k-1}}$ is the dynamic matrix resulting from the linearization of the state equation around the estimate $\hat{x}_{k-1|k-1}$.

- Update step:

$$
\begin{aligned}
K_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R)^{-1} \\
\hat{x}_{k|k} &= \hat{x}_{k-1|k-1} + K_k \left[y_k - h(\hat{x}_{k-1|k-1}) \right] \\
P_{k|k} &= P_{k|k-1} - K_k H_k P_{k|k-1}
\end{aligned}
\qquad (12)
$$

where $H_k = \frac{\partial f(x)}{\partial (x)}\big|_{x=\hat{x}_{k|k-1}}$ results from the linearization of the output equation around the forecast.

- Filter initialization:

Care must be given to the filtering initialization i.e. to the choice of $\hat{x}_{0|0}$ and $P_{0|0}$. Though in the case of linear Kalman filtering it may act on the convergence rate but not on the convergence property itself, an EKF may diverge depending on its initialization. It is thus recommended to use the available a priori knowledge as much as possible.

B. Estimation

The estimation scheme is given by the block diagram in figure (5). The following measurments are considered:

- Yaw rate, longitudinal and lateral accelerations measured by an inertial sensor.
- Rotational velocity for each tire given from the ABS.
- Steering angle measured by an optical sensor.

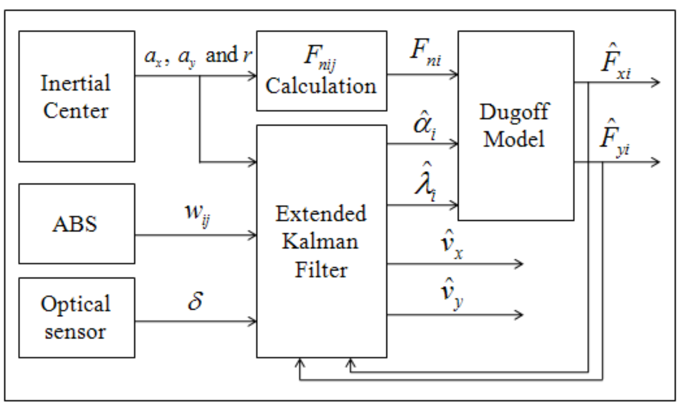

Fig. 5. Estimation bloc diagram

To ensure that parameters are observed using the two measurements set presented above, an observability study can reveal that our system is observable. This observability study is made by calculating the rank of the observability matrix which is given by Lie derivative for nonlinear system:

$$
L_f h(x) = \sum_{i=1}^{n} \frac{dh}{dx_i} f_i(x) = \frac{dh}{dx} f(x) \qquad (13)
$$

Where, iteratively:

$$
L_f^k h(x) = L_f(L_f^{k-1} h(x)) \text{ avec } L_f^0 h(x) = h(x) \qquad (14)
$$

The observability matrix for nonlinear system is then given by:

$$
Ob = \left[\begin{array}{cccc} dh & dL_f h & dL_f^2 h & \ldots \end{array} \right]^T \qquad (15)
$$

In figure (5) one can see the loop in which the estimation is spread out. The inertial sensor data are used to calculate the normal forces on each tire. Thereafter, all the measurements are used as input for the EKF to estimate the state, the tire/road forces and the sideslip angle at time sample k using the estimated tire/road forces at time sample $k-1$.

The dynamic state estimation results are given in figure (6) where the longitudinal velocity and the yaw rate are considered as input measurements while the lateral vehicle velocity measurement is only used for comparison. Thus, comparing with the test vehicle measurements, these results show that the estimation of the lateral velocity performs very well. The estimation error is given in figure (7).

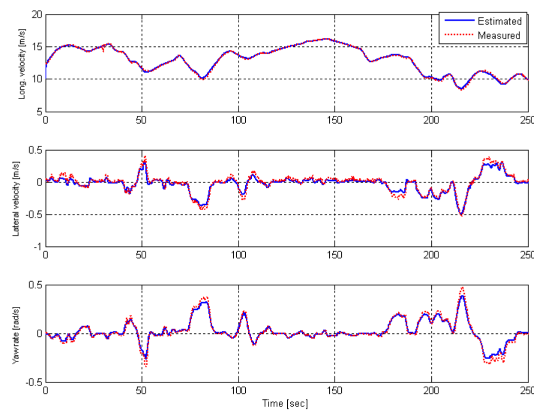

Fig. 6. Dynamic State Estimation

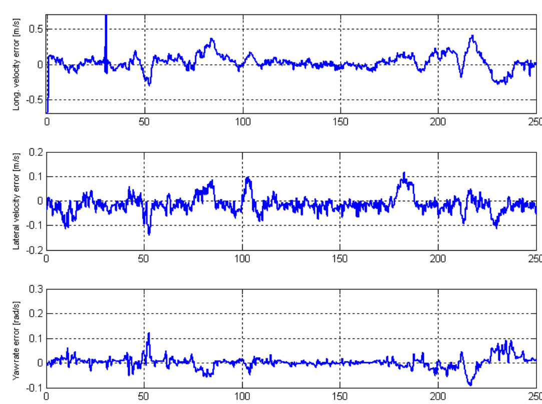

Fig. 7. Dynamic State Estimation Error

Now one can ensure a good estimation of the dynamic state and evaluate the longitudinal and lateral forces which are given in figure (8) and (9) respectively.

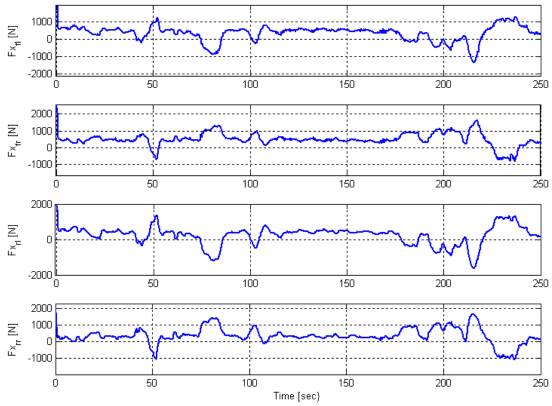

Fig. 8. Longitudinal Forces Estimation

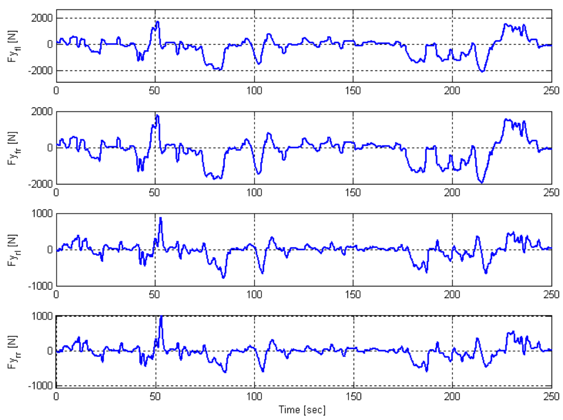

Fig. 9. Lateral Forces Estimation

IV. USED FRICTION COEFFICIENT ESTIMATION

Several approaches can be used to model friction. Number of them are based on detailed physical modeling while other are based on characteristic functions. A good summary of the main available models can be found in [15]. All the models define the friction coefficient μ, as the ratio between the friction forces and the vertical force. Thus one can have longitudinal and lateral friction coefficient referring to longitudinal and lateral forces.

$$\mu_x = \frac{F_x}{F_n} \;,\; \mu_y = \frac{F_y}{F_n} \tag{16}$$

Several works have already been conducted in order to estimate the used friction coefficient [2], [3] and [4]. Each time, one can consider different road conditions (wet, slippery, dry,...) to limit the friction coefficient (for exemple: 0.8 for dry road). Using the equation (16) the friction coefficient can be evaluated from the estimated longitudinal and lateral forces. Figure (10) and (11) represent, respectively, the estimation of the longitudinal and lateral friction coefficients. Analysis of these figures, one can find that μ is between 0 and 1 and it is less than 0.5, which represents that the estimated friction coefficient is the used friction that must be normally less than the available friction of the road (fixed at 0.8 for dry road). This estimation is important to evaluate the ratio of the used friction and then to know the remaining available one.

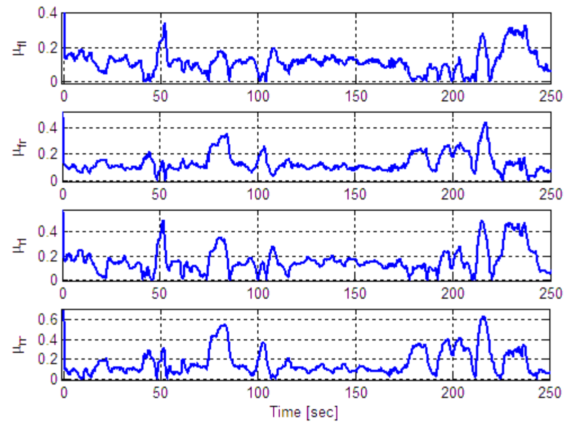

Fig. 10. Longitudinal Friction Coefficient

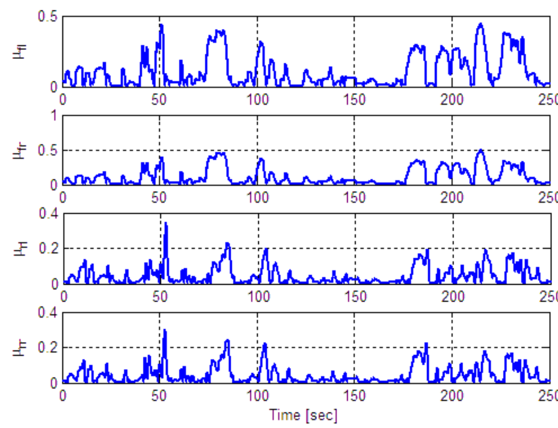

Fig. 11. Lateral Friction Coefficient

V. SIDESLIP ANGLE ESTIMATION

Braking and control systems must be able to stabilize the vehicle during cornering. When the vehicle is subjected to transversal forces, the tire torsional flexibility produces an aligning torque which modifies the original tire direction. The difference between a tire's longitudinal axis and tire speed is characterized by an angle known as "tire sideslip angle" β_i. The angle between the vehicle's longitudinal axis and the direction of vehicle speed is known as "sideslip angle" β. This is a significant signal in determining the stability of the vehicle [16], and it is at the origin of the main transversal force variable. Measuring sideslip angle, using the "correvit" sensor, would represent a disproportionate cost in the context on car industry and it must therefore be observed or estimated.

Given the estimated longitudinal and lateral vehicle velocity, v_x and v_y, at the CG, the sideslip angle is defined by:

$$\beta = \arctan\left(\frac{v_y}{v_x}\right) \quad (17)$$

Thus, figures (12) and (13) are the estimated sideslip angle and the estimation error respectively. As it is clear in figure (12) the estimated sideslip angle and the measured one are well merged.

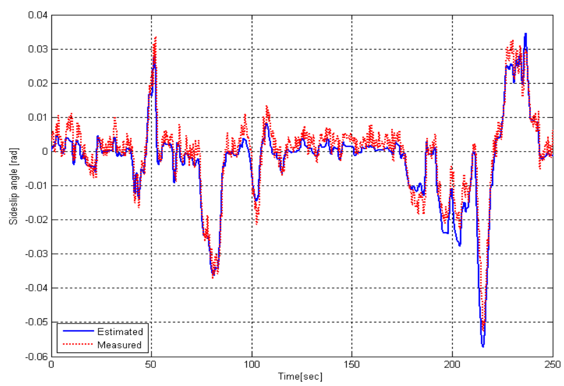

Fig. 12. Sideslip Angle Estimation

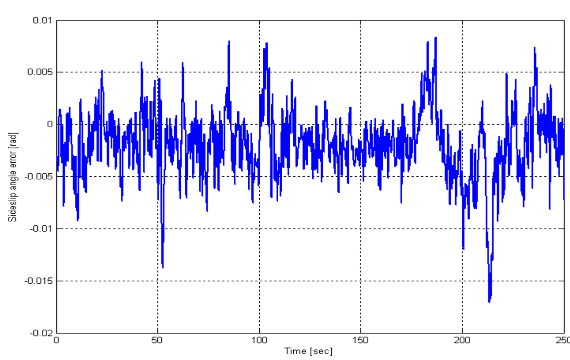

Fig. 13. Sideslip Angle Estimation Error

VI. CONCLUSION

This paper proposes a method to estimate the dynamic states and the tire/road forces in order to evaluate the sideslip angle and the mobilized friction coefficient that are among the most important parameters that influence run-off-road risk and vehicle stability. The setting of an estimator needs a vehicle model. A four wheel vehicle model is chosen because it is simple but sufficiently accurate for the considered application. After model validation on measurement set, the Extended Kalman Filter is used in order to estimate the vehicle dynamic state and the tire/road forces. Thereafter, we use the friction model to evaluate the friction coefficient according to the estimated longitudinal and lateral forces. The sideslip angle is also evaluated using the estimated longitudinal and lateral vehicle velocity.

Simulations showed that the estimation errors achieved by the Extended Kalman Filter are acceptable with a fast convergence by using only two measurements of the dynamic state vector: longitudinal velocity v_x and the yaw rate r. This estimator gives an idea on longitudinal and lateral tire/road forces and the friction coefficient. The sideslip angle is an important parameter to measure vehicle stability, this parameter is usually measured by a specific sensor that it is too expensive to be equipped on ordinary vehicle. Thus, estimating this variable is made in this paper using estimated dynamic state, and the estimation error is very small. So we can replace the expansive sensor by a virtual estimator that calculate the sideslip angle.

Future work will concern the estimation of other parameters known with a weak accuracy and are important for the knowledge of vehicle risks, like those of the vehicle, infrastructure and the behavior model of the driver. Several approach can be used, a potential one consists in regarding these parameters as additional states. However, it has to be checked that the newly obtained system remains observable.

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