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Tire/road friction coefficient estimation applied to road safety

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Abstract— **Recent statistics show that a large number of traffic accidents occur due to a loss of control on vehicle by the driver. This is mainly due to a loss of friction between tire and road. Many of these accidents could be avoided by introducing ADAS (Advanced Driver Assistance Systems) based on the detection of loss of tire/road friction. Friction (more specifically the maximum coefficient of friction) which is a parameter of tire/road interaction, mainly depends on the state of the road (dry, wet, snow, ice) and is closely related to the efforts at the tire level.**

In this paper, we propose, a new method for the estimation of the maximum tire/road friction coefficient, to automatically detect possible state of loss of friction which result in an abrupt change on the road state. This method is based on an iterative quadratic minimization of the error between the developed lateral force and the model of tire/road interaction. Results validate the application of the method.

I. INTRODUCTION

The various parameters of vehicle dynamics, such as tire/road interaction forces, side-slip angle, and friction, are essential for the development of driving support systems. The role of these systems is to warn the drivers of probable dangers due to critical driving situations. Some of these parameters can be measured and others, related to the tire/road interaction are not directly measurable. There exist on the market, sensors for measuring force/torque at tire (Kistler measuring wheels) or side slip angle, but the high cost of these sensors make their installation impossible on standard vehicles. Thus, it is necessary to reconstruct the unmeasurable variables for reasons of cost or physical possibility, developing statements of observers installed in the vehicles (also called virtual sensors). These observers use a model of knowledge associated with actual measurements. In this article, we are working on the robust observation of maximum lateral friction coefficient. This coefficient is a parameter of the vehicle dynamics closely related to the tire-road interaction, necessary for the calculation or observation of forces at the tires, and which characterizes the state of the road (dry, wet , snow or ice). The lateral friction created at the tire/road contact point, depends on several parameters, such as vertical and lateral forces at the tires, road conditions, and the cornering stiffness. An estimate of this parameter (adhesion or friction) in real time, while vehicle motion, and using easy access measurements (acceleration, speed, steering angle, etc.), can provide the driver (or the autopilot system) a warning of a possible loss of friction, that results in avoiding an imminent exit route. We could also estimate automatically during the motion of the vehicle, the type of the road on which it runs (dry, wet, snowy or icy), and optimize real-time observation of efforts at the tires in a closed loop.

The estimation of the lateral maximum friction coefficient is widely discussed in the literature using several tire models and several road states. Several publications estimate this coefficient but for a well-defined road states [2]. Others [1], [3], [4], estimate this coefficient for several road states. The estimation methods developed are mainly based on error optimization methods, as for the gradient descent method [1] and the least square recursive method [3]. In these works, tire/road forces are considered as known measured by an extremely costly sensors, or calculated by open loop methods. In this paper, we present a method to estimate the maximum lateral friction coefficient based on robust estimation of the tire/road forces, with non expensive measurements and using the iterative non-linear optimization method of Levemberg-Marquardt that, could work in real time once embedded in the vehicle. This method will be presented in details in the following sections, taking into account the error between the estimated lateral force (by observers during the vehicle motion) and that given by the tire/road-interaction theoretical-model of Dugoff.

As previously mentioned, the maximum friction is related to the forces developed at the tire/road. So it is an important parameter in the calculation of these forces, and the side-slip angle. In our previous work, we developed observers to estimate forces and side-slip angle [5], where the maximum friction was considered as a constant parameter and assumed to be known. The methodology for estimating the maximum friction online presented in this article is then related with our previous work, and it can improve these force observers coping with friction coefficient variations.

The paper is organized as follows. In section II, we define the maximum lateral coefficient of friction, its importance for the dynamics of vehicle and road safety and the possibility of its estimation from the estimated lateral forces at the contact point . In section III, we define the model of vehicle used and the Dugoff model for estimating lateral forces. In section IV, we present the iterative method for estimating the maximum lateral friction coefficient. Section V presents results that validate the performance of this method, followed by Section VI with the conclusion and the future works.

II. MAXIMUM LATERAL FRICTION COEFFICIENT

The lateral coefficient of friction is the ease with which a vehicle will skid on a road. It is the ratio between the lateral

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force and the normal force acting on the contact point [6].

$$\mu = \frac{F_{y_{ij}}}{F_{z_{ij}}} \tag{1}$$

With $F_{y_{ij}}$ the lateral force/tire and $F_{z_{ij}}$ the normal force/tire.

The maximum lateral friction coefficient is related to the maximum forces that the tires can provide. It corresponds to the value where the lateral force reaches saturation and there is a risk of an exit route. As shown in Figure 1, the

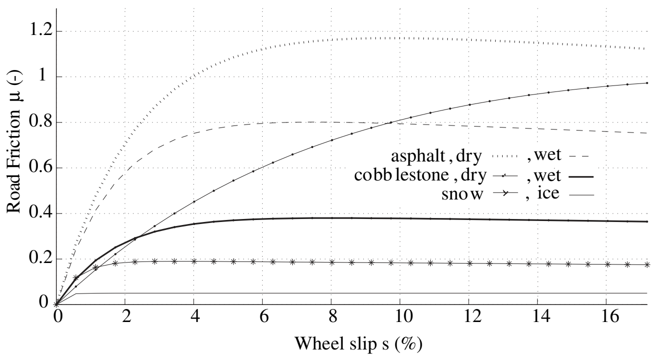

Fig. 1. Tire/Road interaction model. The maximum friction of coefficient is associated to the maximum point of each curve (at the point of the lateral force saturation) for different road states

maximum friction is largely related to road conditions and efforts between the tire and the road. It is an indication of road status and possible loss of adhesion. This parameter cannot be measured by conventional sensors, so it requires an estimation technique for its estimation based on available measurements and tire/road interaction models.

We present in Section IV a method for estimating this coefficient from the estimated forces at the contact point, and using a theoretical tire model (Dugoff model). In the next section, we present the modeling of the tire/road interaction and a reminder on the method of estimating efforts at the contact point.

III. Tire/Road Interaction and Forces Estimation

A. Dugoff Model

Modeling tire road forces is complex because of the interaction of many physical phenomena in a multitude of environmental and tires characteristics (applied load, tire pressure, road surface).

Several models have been used in the literature to model these forces. We distinguish two types, physical models that can characterize the tire/road contact surface [10] and empirical models that are derived from identification of parameters curves from experimental readings obtained on test [9].

The Dugoff model is suited for this study because it requires only a minor number of parameters to evaluate the lateral forces. It is a nonlinear model, whose simplified formula is:

$$F_{yij} = -C_{\alpha i} tan\alpha_{ij}.f(\lambda) \tag{2}$$

Where $C_{\alpha i}$ the cornering stiffness and $f(\lambda)$ given by the following equation:

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & \text{if } \lambda < 1 \\ 1, & \text{if } \lambda \geq 1 \end{cases} \tag{3}$$

$$\lambda = \frac{\mu_{max} F_{zij}}{2C_{\alpha_i} \left| tan\alpha_{ij} \right|} \tag{4}$$

With μ_{max} the maximum friction coefficient, α_{ij} the side slip angle and F_{zij} the normal force on the ij tire. This simplified model neglects the effect of longitudinal forces [8].

B. Tire/Road Forces Estimation

In our previous studies, we have developed observers that can estimate the tire-road forces in real time during the vehicle motion [7]. We will therefore use these results in estimating the maximum friction. Figure 2 presents the estimation process of lateral forces, vertical forces, and side-slip angle. Where a_x and a_y are respectively the longitudinal

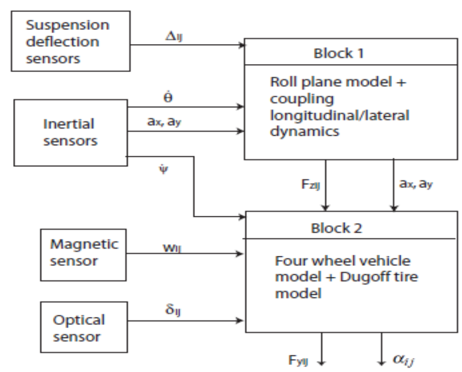

Fig. 2. Process estimation diagram

and lateral accelerations, $\dot{\psi}$ is the yaw rate, $\dot{\theta}$ is the roll rate, Δ_{ij} (i represents the front(1) or the rear(2) and j represents the left(1) or the right (2)) is the suspension deflection, w_{ij} is the wheel velocity, F_{zij} and F_{yij} are respectively the normal and lateral tire-road forces, α_{ij} is the side slip angle at the center of gravity (cog).

The estimation process consists of two blocks, and its role is to estimate side slip angle, normal and lateral forces at each tire/road contact point, which provides the input of the process of estimating the maximum friction described in Section IV. Modeling vehicle dynamics (roll model, 4-wheel dynamic model) is presented in detail in [7].

This estimation process requires the following measurements:

- yaw and roll rates measured by gyrometers,
- longitudinal and lateral accelerations measured by accelerometers,
- suspension deflections using suspension deflections sensors,
- steering angle measured by an optical sensor,
- rotational velocity for each wheel given by magnetic sensors.

The main objective of the first block is to provide the vehicle mass, the load transfer and vertical forces applied at the tire/road level, and the corrected lateral acceleration relative to vehicle roll, denoted a_y . The main objective of the second block is the estimation of the lateral forces and the side-slip angle. He uses the estimations of the first block. Two observation techniques were used in this process, the Extended Kalman Filter (EKF) and the UKF (Unscented Kalman Filter). More details can be found in our previous work [7].

IV. Estimation of the Maximum Lateral Friction Coefficient

A. Estimation Method

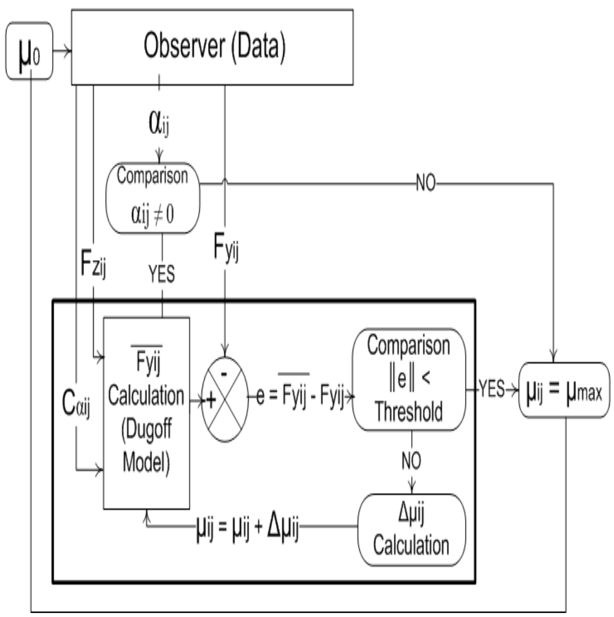

Fig. 3. Maximum lateral friction coefficient estimation method

The block diagram of Figure 3 shows the method used to estimate the maximum lateral friction coefficient for different road states. The first part (marked "Observer(Data)") estimate the lateral forces ($F_{y_{ij}}$), vertical forces ($F_{z_{ij}}$), side-slip angle (α_{ij}) and the cornering stiffness ($C_{\alpha_{ij}}$), and is presented in section III. A test is performed on the side-slip angle to see if it is different from zero (to avoid singularities). The second block at the bottom of the figure, shows the non-linear method of optimization of Levemberg-Marquardt. The objective of this method is to find the friction coefficient value which minimizes the error between the lateral force provided by the Dugoff model and lateral force estimated by our observers. The error between the value provided by the model and the value of the estimated force is evaluated and it is the stop criterion of the loop. If this error is below a certain threshold, then the friction coefficient used by the Dugoff model (eq. 2) is the one developed at the tire/road contact point.

The formalization of the optimization algorithm is based on three assumptions, which will be validated later in this section:

Hypothesis 1: The quasi-static Dugoff model given in the equation (2) can be used in the estimation of the lateral force at each tire.

Hypothesis 2: If Hypothesis 1 is verified, we assume that at every time k of the trajectory, it is possible to calculate an error $e_{ij} = \overline{F_{y_{ij}}} - F_{y_{ij}}$ where $\overline{F_{y_{ij}}}$ is the lateral force calculated with the Dugoff model and $F_{y_{ij}}$ the lateral force estimated by the observer.

Hypothesis 3: We assume that:

- The cornering stiffness is known;
- The lateral forces ($F_{y_{ij}}$) and vertical forces ($F_{z_{ij}}$) and the side-slip angle (α_{ij}) are estimated by the observers in Section III.

B. Hypothesis Validation

Before developing the algorithm for estimating the maximum friction coefficient, it seems essential to check the validity of assumptions made in the preceding paragraph. For this, we simulated a chicane trajectory to be traveled by a vehicle in the realistic simulator *CALLAS* (developed by SERA-CD). Figure 4, shows the evolution of the lateral force $F_{y_{11}}$ developed on the right front tire, for a lateral friction coefficient μ_{max} specified in the simulator (Speed = 60 km/h, $a_{y_{max}} = 4m/s^2$, $a_{x_{max}} = 0.02m/s^2$ ...). For each point of the trajectory, the lateral force calculated by Dugoff model, $F_{ydugoff_{11}}$, is also calculated with the same μ_{max}. The error $e_{11} = \overline{F_{ydugoff_{11}}} - F_{y_{11}}$, shown in figure 4 indicates that hypothesis 1 and 2 above are not rejected.

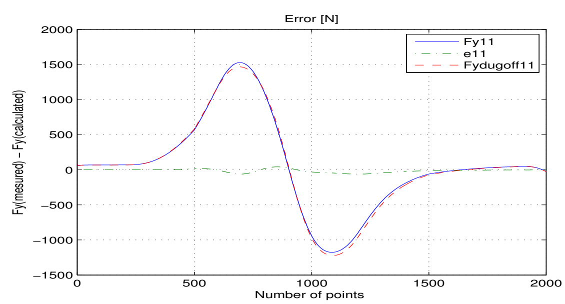

Fig. 4. Model Validation

C. Optimization Algorithm

The optimization method used is the method of Levemberg-Marquardt. The algorithm of Levenberg-Marquardt (LM) is an iterative technique to locate the minimum of a function with several variables, which is expressed as the sum of squares of real valued non-linear functions [8], [9]. It has become a standard technique for non-linear least-squares problems, widely adopted in a broad spectrum of disciplines [11].

The LM method can be thought of as a combination of steepest descent method and the method of Gauss-Newton [11], [12]. When the current solution is far from the correct one, the algorithm behaves like a steepest descent method: it is slow, but guaranteed to converge. When the current solution is close to the correct solution, it becomes a Gauss-Newton method.

The weighting function is minimized as follows:

$$\begin{aligned}L(\mu_{max}) &= \frac{1}{2}\sum_{k=1}^{n} e_k^2 = \frac{1}{2}\sum_{k=1}^{n}(\overline{F_{y_{ijk}}} - F_{y_{ijk}}) \\ &= \frac{1}{2}\sum_{k=1}^{n}(\frac{\mu_{max}^4 F_{z_{ijk}}^4}{(4C_{\alpha_{ijk}}\tan_{\alpha_{ijk}})^2} - 2\frac{\mu_{max}^3 F_{z_{ijk}}^3}{(4C_{\alpha_{ijk}}\tan_{\alpha_{ijk}})} \\ &+\mu_{max}^2 F_{z_{ijk}}^2) + (-2F_{y_{ijk}}(\frac{\mu_{max}^2 F_{z_{ijk}}^2}{(4C_{\alpha_{ijk}}\tan_{\alpha_{ijk}})} \\ &-\mu_{max}F_{z_{ijk}})) + F_{y_{ijk}}^2\end{aligned} \tag{5}$$

Where $\overline{F_{y_{ijk}}}$ the lateral force calculated with the Dugoff model and $F_{y_{ijk}}$ the lateral force estimated by the observers at section III. We consider its linearization with Taylor series development:

$$L(\hat{\mu}+\Delta\mu) = \left[L(\hat{\mu}) + L\mu * \Delta\mu + \frac{1}{2}L\mu\mu * \Delta\mu^2 + \theta^3\right] \tag{6}$$

Where $L(\hat{\mu}+\Delta\mu)$ the linear approximation of $L(\mu)$ around $\hat{\mu}$ and θ^3 the term of higher orders which is neglected by the approximation. In considering an infinitesimal perturbation of this function around $\Delta\mu$, we have:

$$\frac{\partial L(\hat{\mu}+\Delta\mu)}{\partial\Delta\mu} = L\mu + L\mu\mu * \Delta\mu \tag{7}$$

With $L_\mu = \frac{\partial L}{\partial\mu}$ et $L_{\mu\mu} = \frac{\partial^2 L}{\partial^2\mu}$. This gives,

$$L\mu\mu * \Delta\mu = -L\mu \tag{8}$$

$$L = \frac{1}{2}\varepsilon^T * \varepsilon \tag{9}$$

$$L\mu = \varepsilon_\mu^T * \varepsilon \tag{10}$$

$$L\mu\mu = \varepsilon_\mu^T * \varepsilon_\mu + \varepsilon_{\mu\mu}^T * \varepsilon \tag{11}$$

Where $\varepsilon = \left[e_{ij_1}, e_{ij_2}, \ldots, e_{ij_k}\right]$, the error vector and $\varepsilon_\mu = \left[\frac{\partial e_{ij_1}}{\partial\mu}, \frac{\partial e_{ij_2}}{\partial\mu}, \ldots, \frac{\partial e_{ij_k}}{\partial\mu}\right]$, the first derivative of the error vector. Assuming that ε_μ is a linear function, we can consider the following approximation:

$$L\mu\mu \approx \varepsilon_\mu^T * \varepsilon_\mu \tag{12}$$

and (8) becomes,

$$\varepsilon_\mu^T * \varepsilon_\mu * \Delta\mu = -\varepsilon_\mu^T * \varepsilon \tag{13}$$

Where

$$\Delta\mu = -(\varepsilon_\mu^T * \varepsilon_\mu)^{-1} * (\varepsilon_\mu^T * \varepsilon) \tag{14}$$

Equations (13) and (14) constitute the bloc "$\Delta\mu_{ij}$ calculation" in the loop of the optimization process of figure 3. Once $\Delta\mu$ is thus calculated, a new friction coefficient is finally determined by, $\mu_{t+1} = \mu_t + \Delta\mu$ where t indicates a step in the optimization loop shown in figure 3. From μ_{t+1}, the error vector ε is recalculated. The loop is stopped when the following condition: $\|\varepsilon\| < Threshold$ in verified.

V. RESULTS

The method represented in section IV, has been implemented and tested with the realistic simulator CALLAS on a chicane at a speed of 60 km/h, and with different road states (different μ_{max}).
The simulator CALLAS provide parameters of vehicle dynamics that constitute the inputs of the block " Observer (Data)" in Figure 3, which are essentially: suspension deflections, longitudinal and lateral accelerations , yaw rate, wheel speeds and steering angle (inputs for our forces observers in Figure 2). The true values of μ_{max}, used by the simulator in the generation of the dynamic parameters, are unknowns in the forces observers and optimization loop . The aim is thus to apply the method described in the previous section to estimate a value of μ_{max} which is closest to the true value.
We present here the application of the method of estimation of μ_{max} for a vehicle that rolls on four road states: dry, wet, snowy and icy. All results are related to the left front wheel of the vehicle. The initial value of μ_{max} in the optimization loop is initialized to 0.01. Subsequently, $F_{y_{11}}$ denotes the lateral force estimated by our forces observers with the parameters provided by CALLAS; $F_{ydugoff_{11}}$ denotes the lateral force estimated by the Dugoff model for a given μ_{max} , and e_{11} the error function.
Figure 5, 7,9 and 11 show the result of the evolution of the lateral force for respectively: dry, wet snowy and icy roads while vehicle motion. The left side shows the beginning of the optimization, the difference between the estimated force $F_{y_{11}}$ and the force estimated by the Dugoff model $F_{ydugoff_{11}}$ is due to the fact that original μ is far from the μ_{max} associated with this type of road. The right side shows the end of optimization. For a dry road, ($\mu_{max} = 0.9$ by CALLAS), results are shown in figures 5 and 6. We can see that

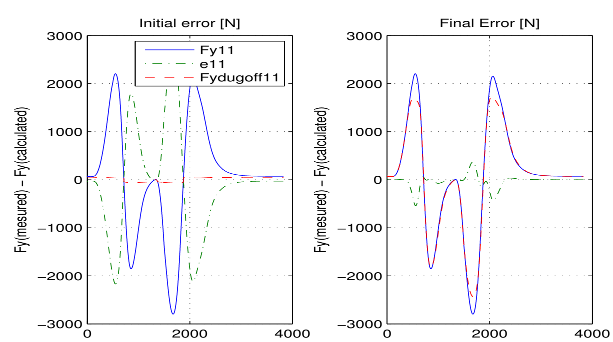

Fig. 5. Lateral force estimation on a dry road: (a) Beginning of the minimization , (b) End of the minimization

$F_{ydugoff_{11}}$ converges to $F_{y_{11}}$, and the error tends to zero with a mean value less than $200N$. The estimated friction coefficient μ converges to its true value. We also find a model rupture in the extreme parts of the curve, this is due to the strong solicitations at those corners. Figure 6 shows the evolution of the L (Eq. 5) (left) and μ (right).
For a wet road ($\mu = 0.6$), results are shown in figures 7 and 8. Figures 7 and 8 validate the estimation for a wet road.

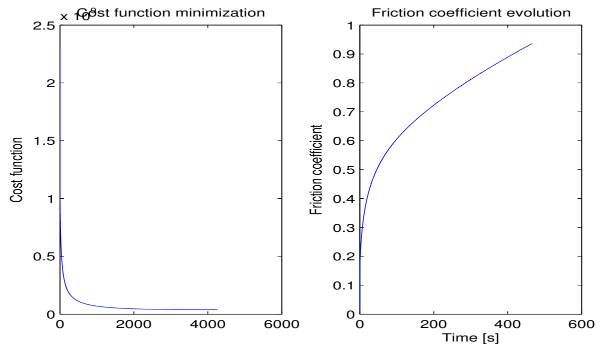

Fig. 6. (a)Weighting function evolution, (b) Friction coefficient evolution while optimization process on a dry road.

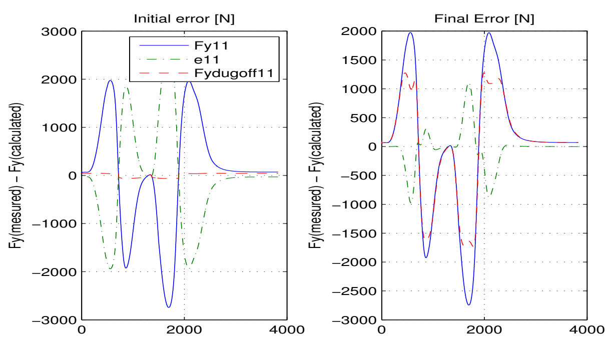

Fig. 7. Lateral force estimation on a wet road: (a) Beginning of the minimization , (b) End of the minimization

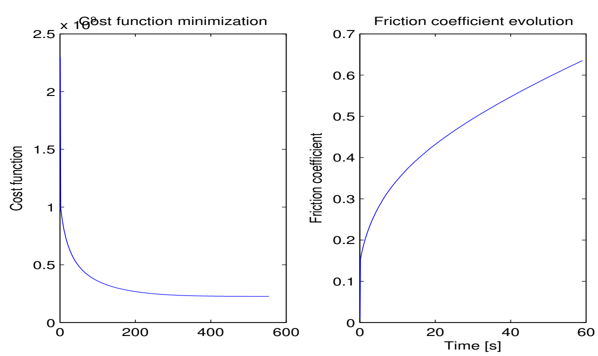

Fig. 8. (a)Weighting function evolution, (b) Lateral friction coefficient evolution while optimization process on a wet road.

Again, the average error after the minimization process is less than $200N$.

For a snowy road ($\mu = 0.2$), results are shown in figures 9 and 10. Figure 9 shows the effect of lack of friction and the dropout of the Dugoff model while cornering with strong solicitations. However, as shown in Figure 10, the estimated friction coefficient converges to its true value.

For an icy road ($\mu = 0.05$), results are shown in figures 11 and 12. Because of the extremely low friction, and the speed of $60km/h$, the dynamics of interaction exceeds the saturation limit for the lateral forces. We can deduce that the assumptions made in Section IV are within the limits of

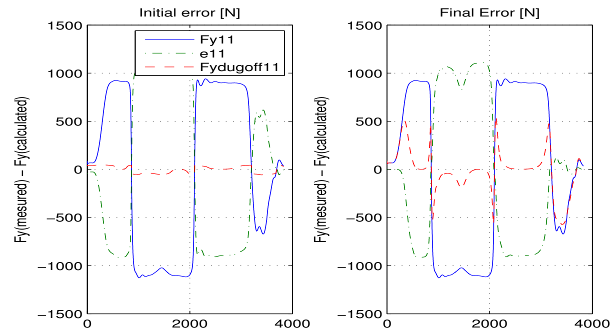

Fig. 9. Lateral force estimation on a snowy road: (a) Beginning of the minimization , (b) End of the minimization

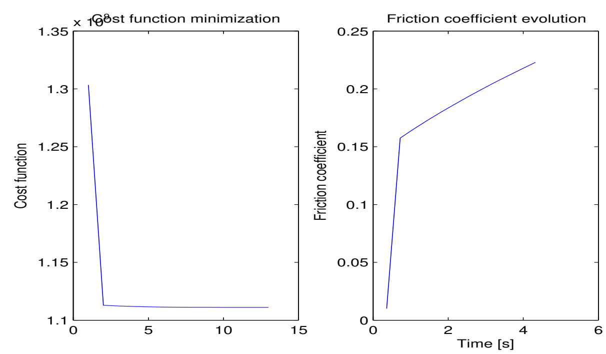

Fig. 10. (a)Weighting function evolution, (b) Lateral friction coefficient evolution while optimization process on a snowy road.

their validity. Nevertheless, the estimation system of μ_{max}

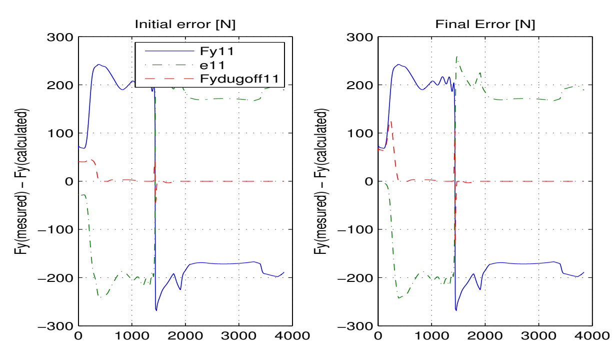

Fig. 11. Lateral force estimation on an icy road: (a) Beginning of the minimization , (b) End of the minimization

converges to its true value.

The results of estimating the maximum lateral friction coefficient and the average error are shown in table I, as follows: Associated data for the four types of roadways are thus concatenated as follows: Dry, Wet, Snowy and Icy.

A sliding window system is applied to the concatenated data. The step chosen is 2000 points, which corresponds to a distance of 78.26m over a total distance of 600m covered. The vehicle was moving at a speed of 60 km/h so the distance of each step will be crossed by car in 4.7 seconds. We applied our algorithm to the entire system, we had the following

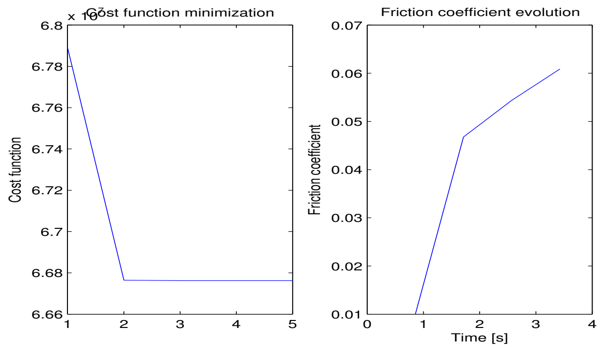

Fig. 12. (a)Weighting function evolution, (b) Lateral friction coefficient evolution while optimization process on an icy road.

TABLE I

ESTIMATION RESULTS AND AVERAGE ERROR

Road type	True μ	μ_{max} estimated	Average error(N)
Dry	0.9	0.9361	70.7425
Wet	0.6	0.6354	174.8348
Snow	0.2	0.2284	640.9552
Ice	0.05	0.0667	179.6127

results:

Figure 13 shows that the lateral friction coefficient converges

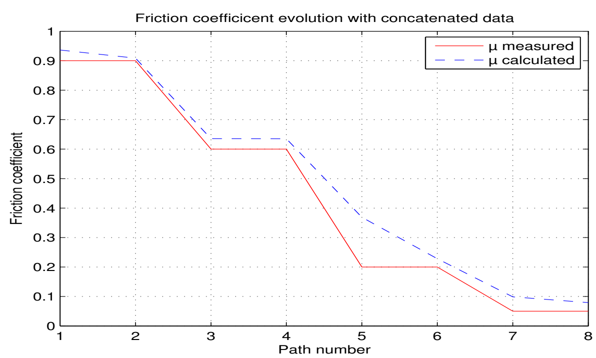

Fig. 13. Maximum lateral friction coefficient evolution over the different road status

to its true value.

The values of the lateral friction coefficient over the four road distances are illustrated in table II as follows:

Figures and table above shows that the maximum lateral

TABLE II

ESTIMATION RESULTS OVER THE FOUR ROAD STATES

True values	Estimated values
0.9000	0.9362
0.9000	0.9088
0.6000	0.6354
0.6000	0.6350
0.2000	0.3686
0.2000	0.2282
0.0500	0.0987
0.0500	0.0794

friction coefficient has converged to its true value for the four types of roadways.

VI. CONCLUSION AND FUTURE WORKS

In this article, we have presented a new method for estimating the maximum lateral friction coefficient. This method is based on a nonlinear optimization technique (Levemberg Marquardt) applied to an error function between the forces estimated by observers installed in the vehicle and those calculated by a theoretical tire/road interaction model (Dugoff model). The method was tested using data from a realistic simulator of vehicle dynamics (CALLAS) for four different road states (dry, wet, snowy and icy). For these four types, the true friction coefficient was considered satisfactory. However, the method reached the limit of its validity for the snowy and icy roads.

Having this coefficient estimated, we can implement a system for automatically detecting the road state to allow the calculation of risk indicators of exit route (accident), and thus be able to warn the driver.

Following this work, we plan to validate the method embedded on a real vehicle, and to integrate the multi-model approach to estimate the maximum lateral friction coefficient.

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