

TMD measurements at CLAS

Harut Avakian (JLab)

3rd Workshop on the QCD Structure of the Nucleon (QCD-N'12) October 22-26, 2012
Bilbao, Spain



Outline

Transverse structure of the nucleon and partonic correlations

- Introduction
- Hard scattering processes and correlations between spin and transverse degrees of freedom
- k_T -effects with polarized SIDIS
- Higher twist effects in SIDIS
- Summary

Structure of the nucleon

PDFs, $q(x)$: Probabilities to find a quark with a fraction x of proton momentum P

$$\begin{aligned}
 f_1(x) &= \text{circle with dot} = \text{circle with R} + \text{circle with L} \\
 &= \text{circle with dot and up arrow} + \text{circle with dot and down arrow} \\
 S_L g_1(x) &= \text{circle with R and right arrow} - \text{circle with L and right arrow} \\
 S_T^\alpha h_1(x) &= \text{circle with dot and up arrow} - \text{circle with dot and down arrow}
 \end{aligned}$$

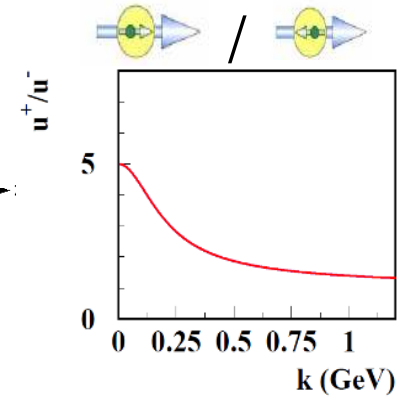
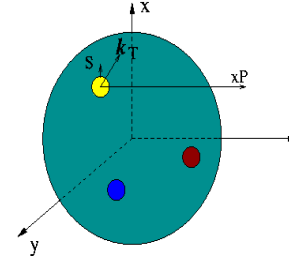
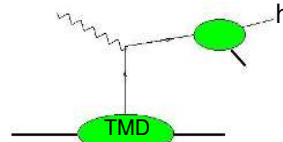
$$h_1(x, \vec{k}_T)$$

$$H_T(x, \vec{r}_T)$$

Detection of final state particles in semi-inclusive and hard exclusive processes allows access also to transverse distribution of quarks.

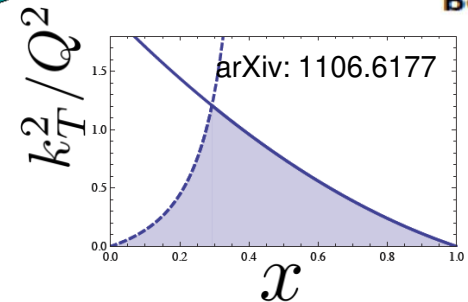
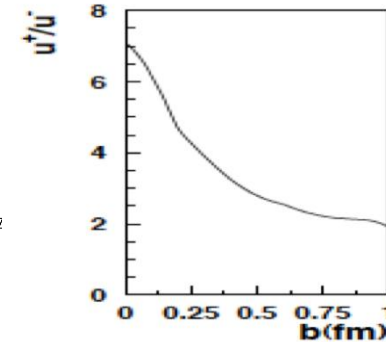
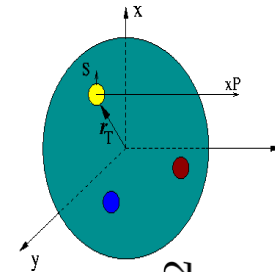
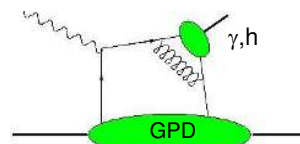
Collinear analysis of observables in semi-inclusive and hard exclusive processes will be sensitive to integration region over transverse degrees of freedom due to correlations of x , spin and transverse degrees of freedom.

Semi-Inclusive processes and transverse momentum distributions



B.Pasquini et al

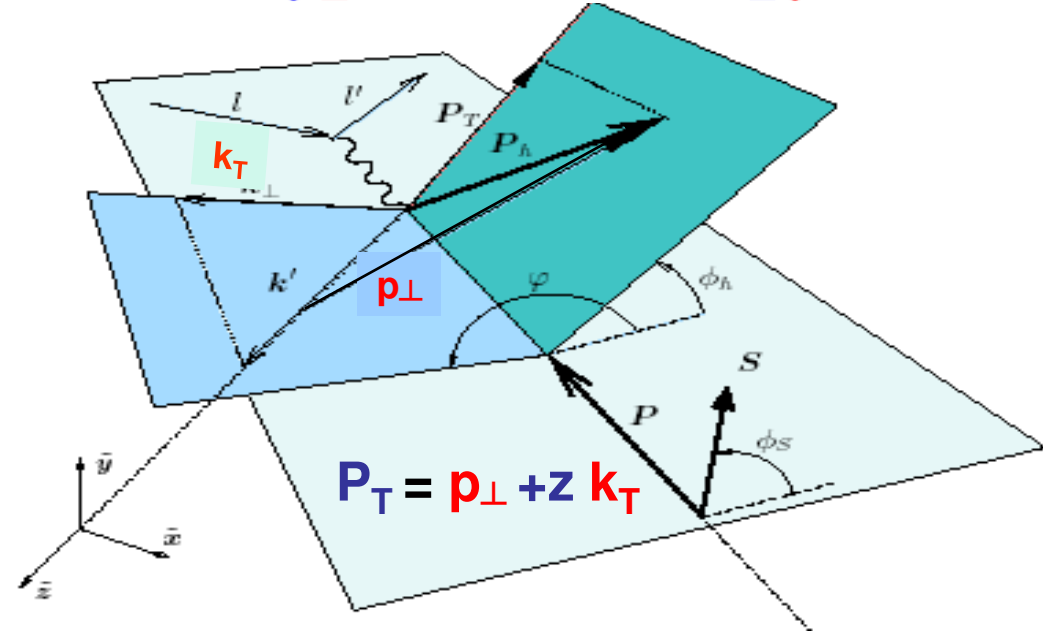
Hard exclusive processes and spatial distributions of partons



SIDIS: partonic cross sections

$$\begin{aligned} \nu &= (qP)/M \\ Q^2 &= (k - k')^2 \\ y &= (qP)/(kP) \\ x &= Q^2/2(qP) \\ z &= (qP_h)/(qP) \end{aligned}$$

$$\sigma = F_{UU} + P_t F_{UL}^{\sin \phi} \sin 2\phi + P_b F_{LU}^{\sin \phi} \sin \phi \dots$$



Transverse momentum of hadrons in SIDIS provides access to orbital motion of quarks

$$d\sigma^{\gamma^* H \rightarrow h X} \propto \sum e_q^2 \int d^2 \vec{k}_T d^2 \vec{p}_\perp f^{H \rightarrow q}(x, \vec{k}_T) D^{q \rightarrow h}(z, \vec{p}_\perp) \delta^{(2)}(z \vec{k}_T + \vec{p}_\perp - \vec{P}_T)$$



$$d\sigma^h \propto \sum f^{H \rightarrow q}(x) d\sigma_q(y) D^{q \rightarrow h}(z)$$

Azimuthal moments in SIDIS

quark polarization

$$\frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} =$$

$$\left[\frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \right.$$

$$\left. + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \right.$$

$$\left. + S_{\parallel} \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \right.$$

$$\left. + S_{\parallel} \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \right.$$

$$\left. + |S_{\perp}| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \right.$$

$$\left. + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \right.$$

$$\left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right]$$

$$\left. + |S_{\perp}| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \right.$$

$$\left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right\},$$

N/q	U	L	T
U	f_1		h_1^{\perp}
L		g_1	h_{1L}^{\perp}
T	f_{1T}^{\perp}	g_{1T}	h_1, h_{1T}^{\perp}

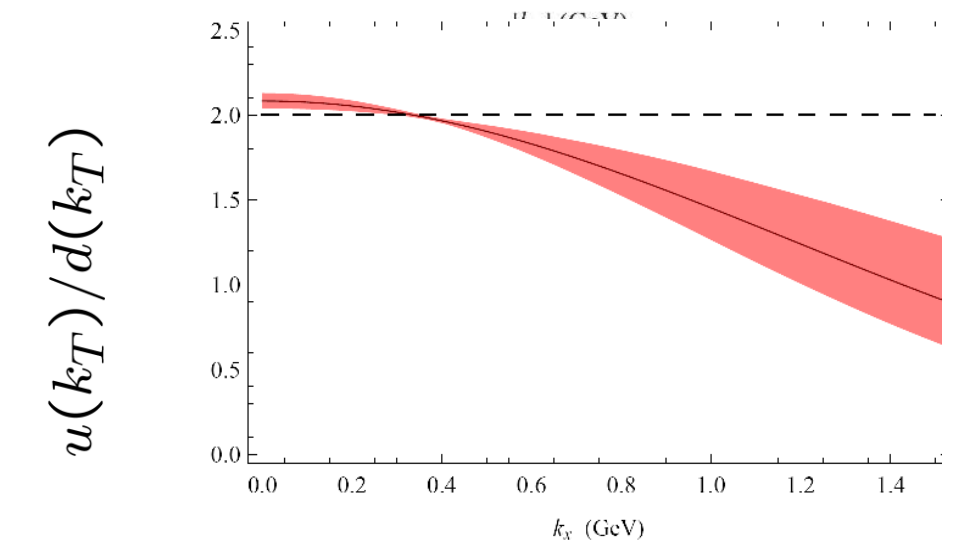
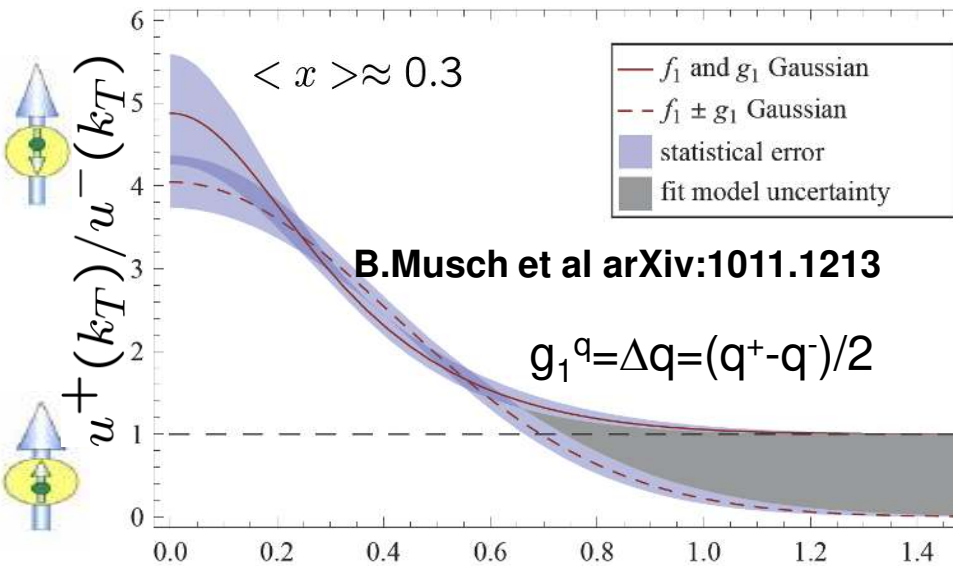
Higher Twist PDFs

N/q	U	L	T
U	f^{\perp}	g^{\perp}	h, e
L	f_L^{\perp}	g_L^{\perp}	h_L, e_L
T	f_T, f_T^{\perp}	g_T, g_T^{\perp}	$h_T, e_T, h_T^{\perp}, e_T^{\perp}$

$D_1^{q \rightarrow h}$ | $H_1^{\perp q \rightarrow h}$

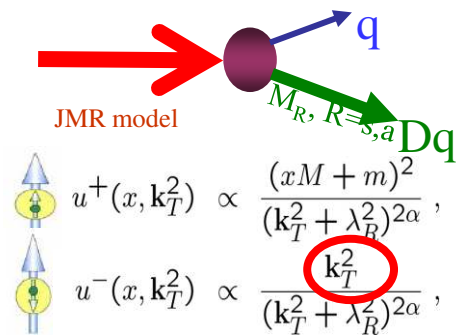
Experiment for a given target polarization measures all moments simultaneously

Quark distributions at large k_T : lattice



$$f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$
~~$$g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$~~

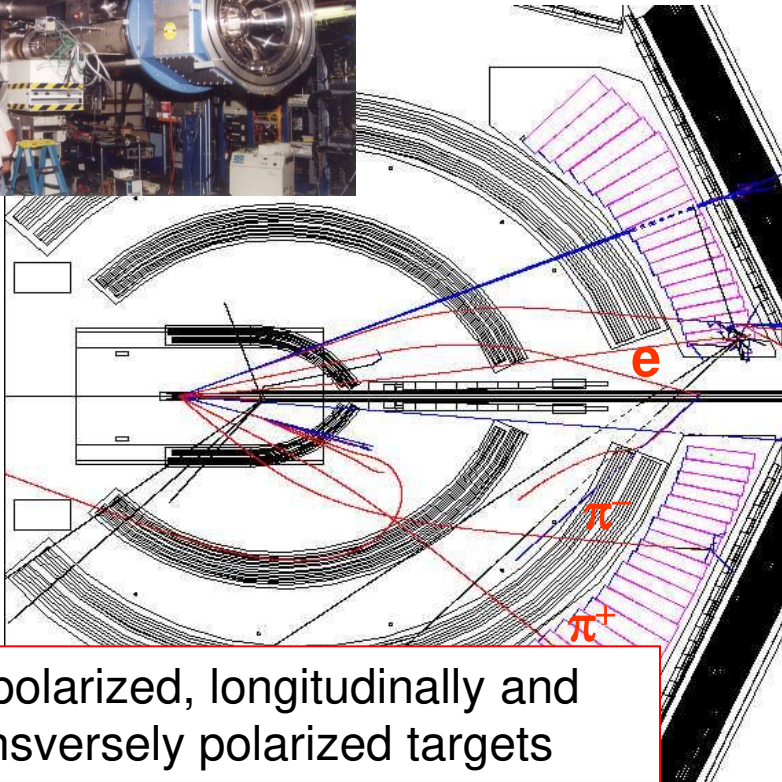
Higher probability to find a quark anti-aligned with proton spin at large k_T



Higher probability to find a d-quark with at large k_T

k_T -distributions of TMDs may depend on flavor and spin

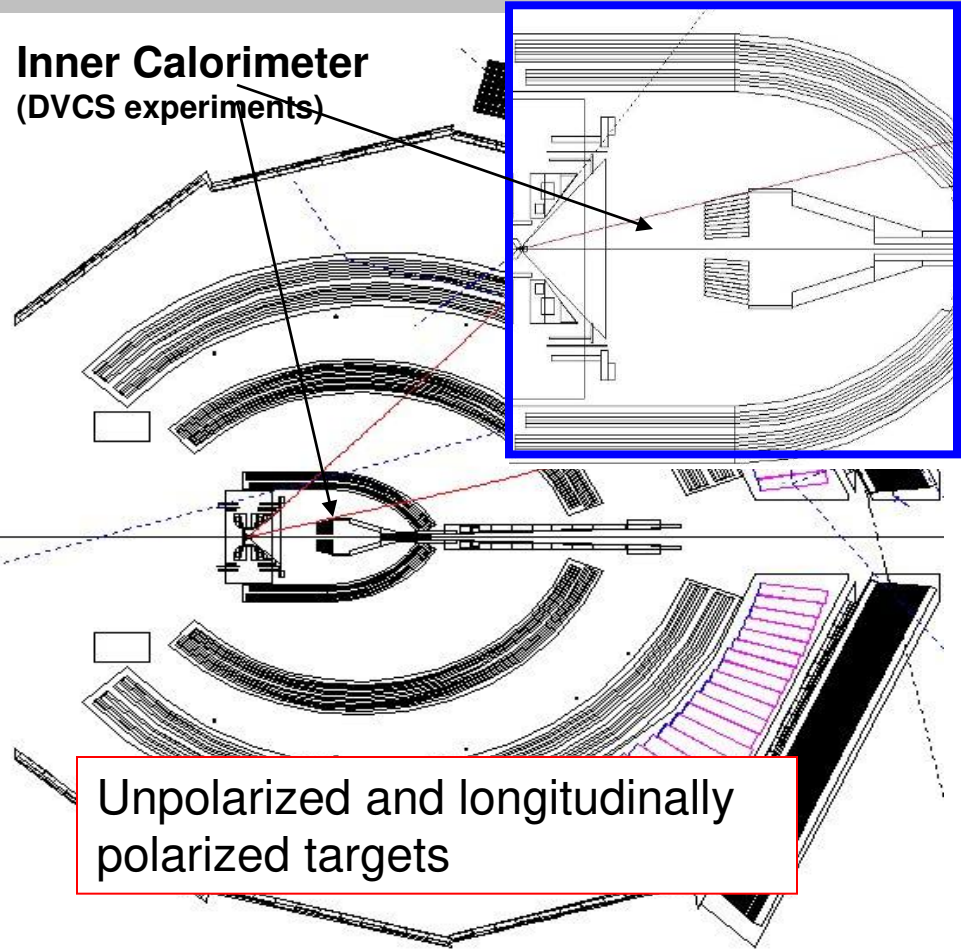
$ep \rightarrow e' \pi X$ CLAS configurations



Unpolarized, longitudinally and transversely polarized targets

- Polarized NH₃/ND₃ (no IC, ~5 days)
- Unpolarized H (with IC ~ 60 days)
- Polarized NH₃/ND₃ with IC 60 days
10% of data on carbon
- Polarized HD-Ice (no IC, 25 days)

Inner Calorimeter (DVCS experiments)



Unpolarized and longitudinally polarized targets

- **Polarizations:**
- **Beam: ~80%**
- **NH₃ proton 80%, ND₃ ~30%**
- **HD (H-75%, D-25%)**

A₁ – P_T dependence

Z/g	U	L	T
U	f ₁	g₁	h ₁ [⊥]
L			h _{1L} [⊥]
T	f _{1T} [⊥]	g _{1T}	h ₁ h _{1T} [⊥]

$$A_1(x, z, P_T) = A_1(x, z) \frac{\langle P_T^{2,unp} \rangle}{\langle P_T^{2,pol} \rangle} \exp(-P_T^2 / \langle P_T^{2,pol} \rangle - P_T^2 / \langle P_T^{2,unp} \rangle)$$

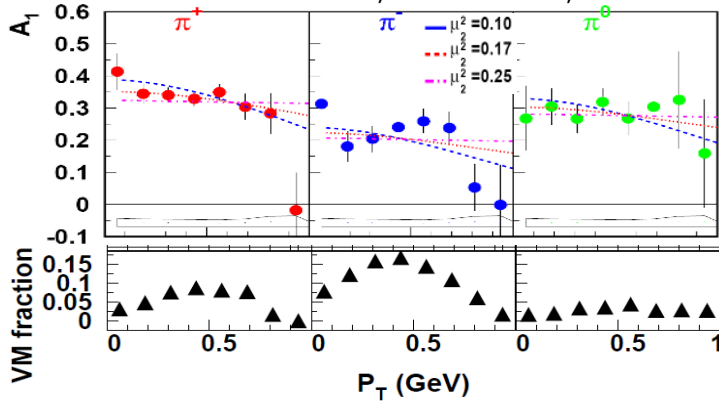
$$\mu_0^2 = 0.25 \text{ GeV}^2$$

$$\mu_D^2 = 0.2 \text{ GeV}^2$$

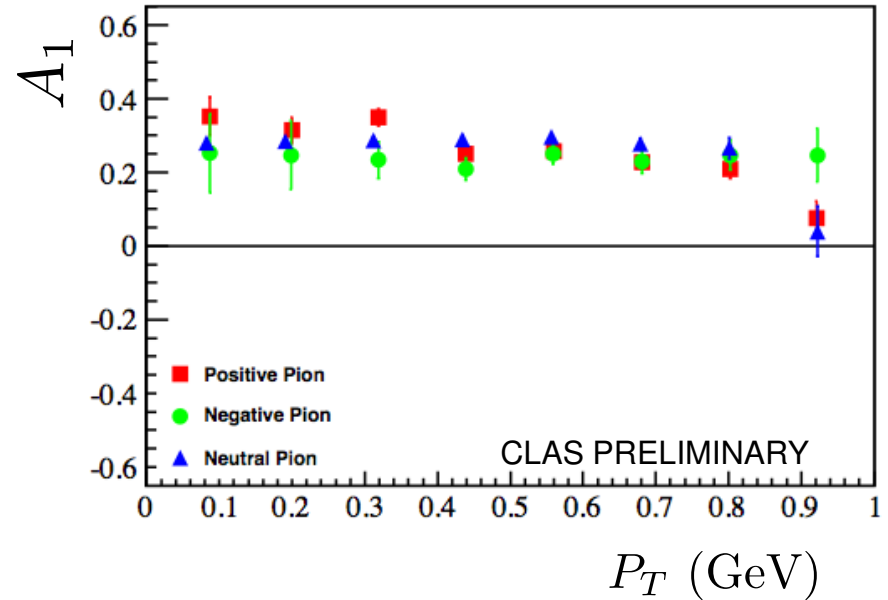
M. Anselmino et al PRD74:074015, 2006

0.4 < z < 0.7

H. A. & CLAS Coll., PRL.105:262002, 2010



S. Skoirala

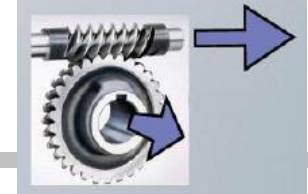


B. Musch et al arXiv:1011.1213

$$\mu_2^2 / \mu_0^2 = 0.692 \pm 0.039 \pm 0.045$$

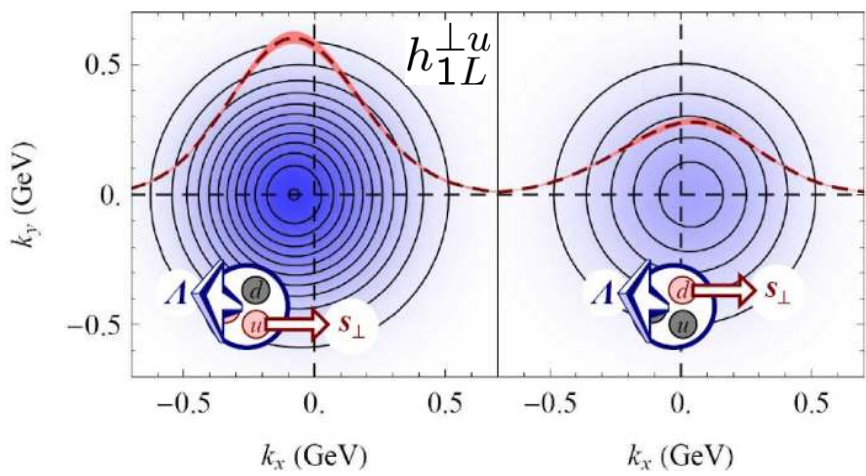
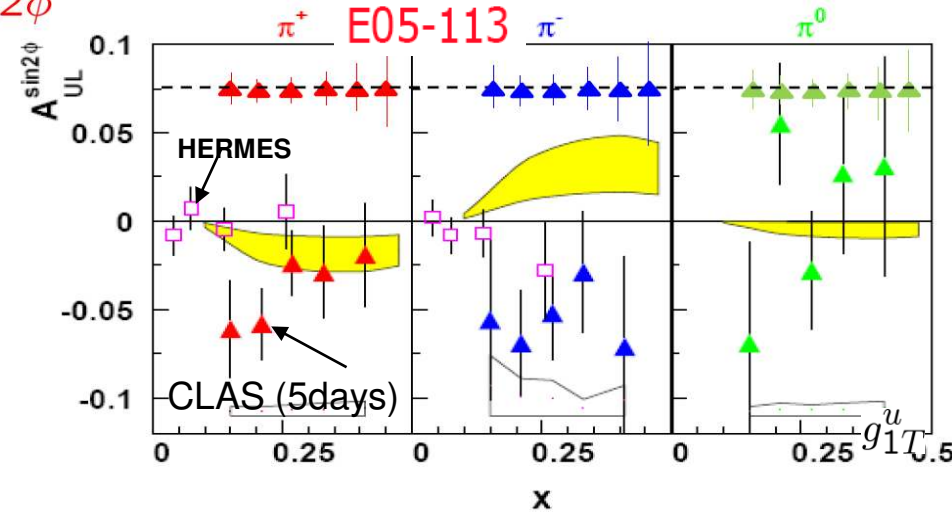
π^+ A₁ suggests broader k_T distributions for f₁ than for g₁
 The new data is consistent with old measurements, now available in several bins in x

Kotzinian-Mulders Asymmetries

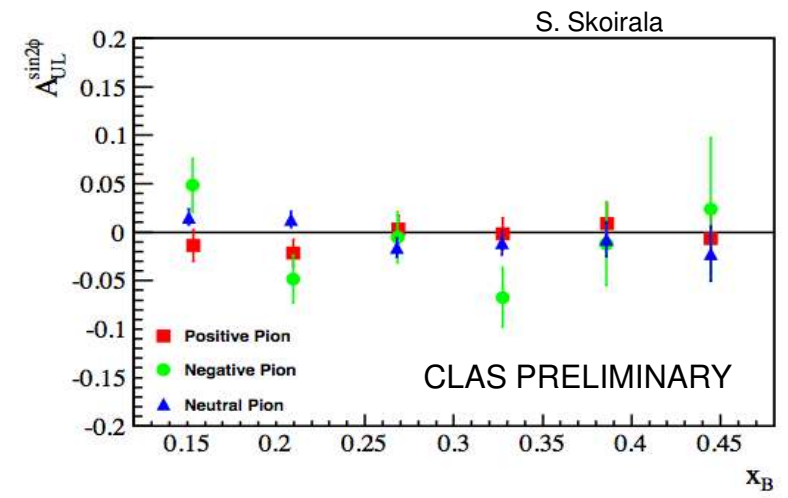


Z \ g	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

$$A_{UL}^{\sin 2\phi} \sim h_{1L}^\perp H_1^\perp \sin 2\phi$$



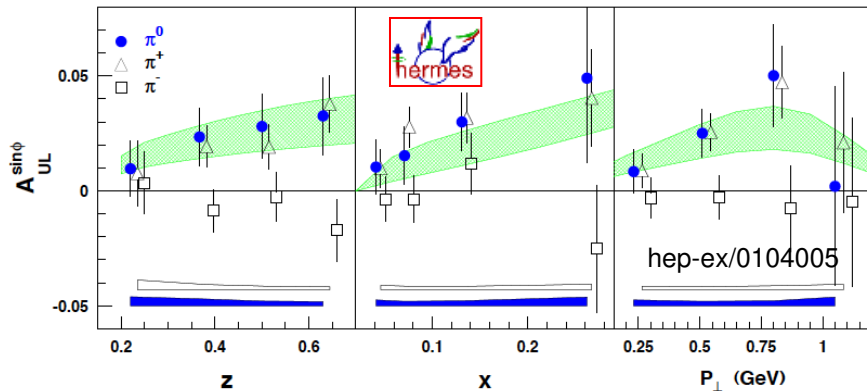
B.Musch arXiv:0907.2381



Worm gear TMDs are unique (no analog in GPDs)

Logitudinally polarized Target SSA

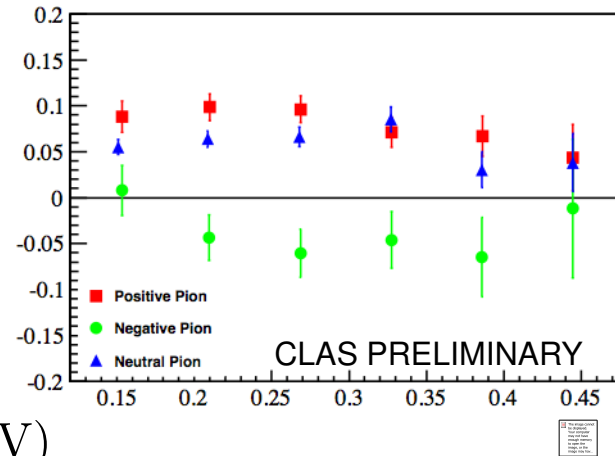
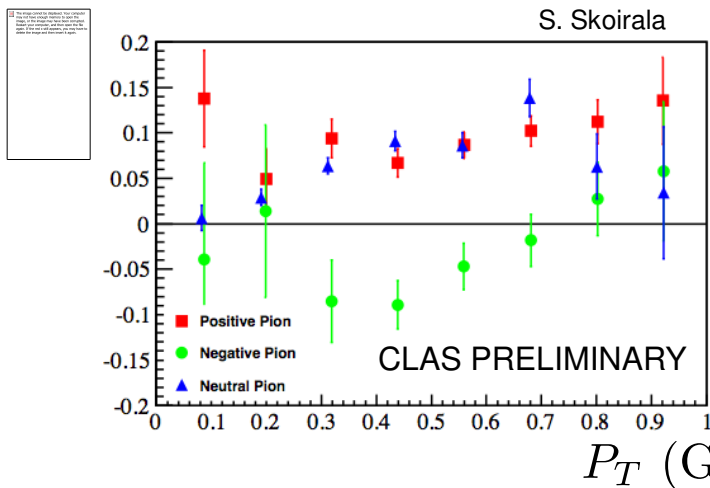
$$A_{UL}(\phi) = \frac{1}{P_t} \frac{N^+ - N^-}{N^+ + N^-}$$



N/q	U	L	T
L			h_L

$$A_{UL}^{Collins} \sim h_L H_1^\perp$$

Kotzinian et al (1999)

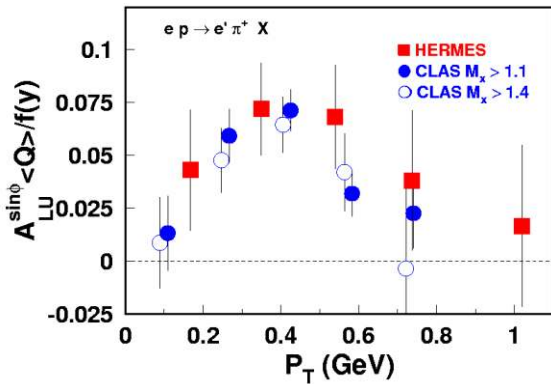


$$A_{UL}^{\sin\phi} \propto f_L^\perp D_1$$

The $\sin\phi$ moments of π^0 and π^+ of target SSA comparable, while π^- SSA seem to have an opposite to $\pi^{0/+}$ sign at CLAS (also HERMES)

Longitudinally Polarized Beam SSA

$A_{LU}^{\sin\phi}$ CLAS @4.3 GeV (2003)



$$A_{LU}(\phi) = \frac{1}{P_b} \frac{N^+ - N^-}{N^+ + N^-}$$

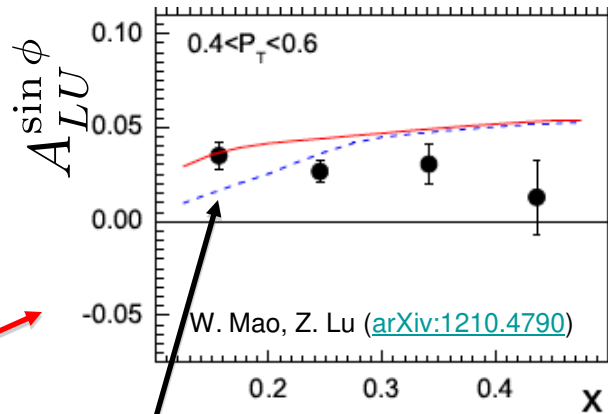
HT function related to force on the quark. Burkardt (2008), Qiu(2011)

N/q	U	L	T
U			e

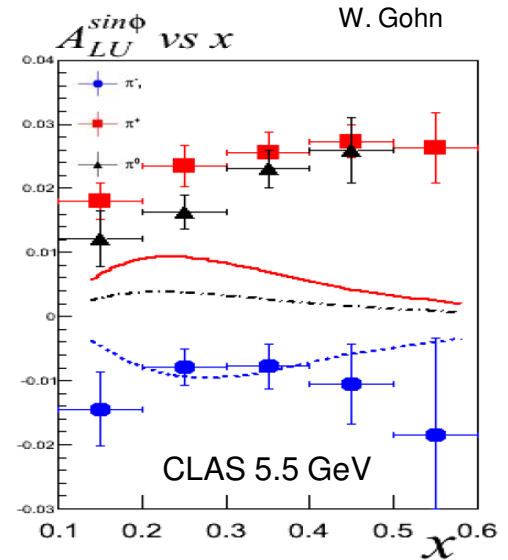
$$A_{LU}^{Collins} \sim eH_1^\perp$$

Efremov et al (2003)

$g^\perp D_1$

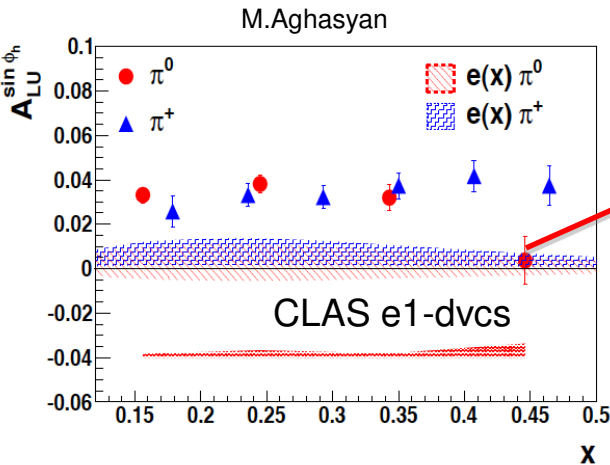


x vs k_T correlations matter



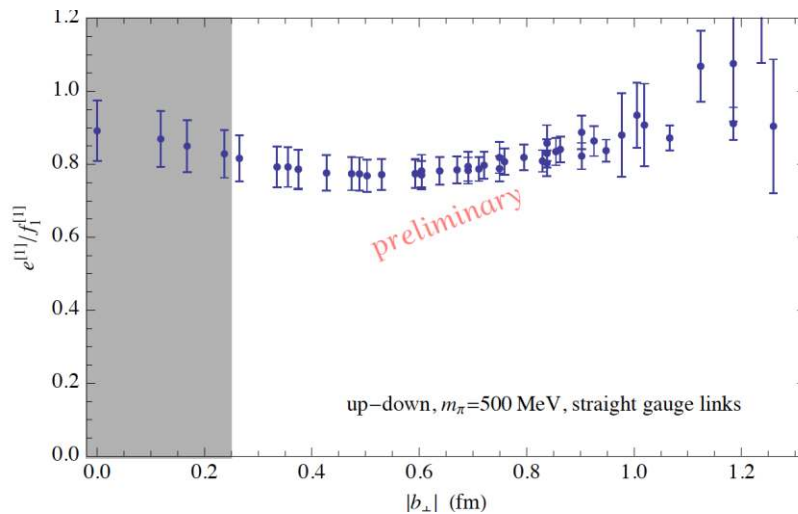
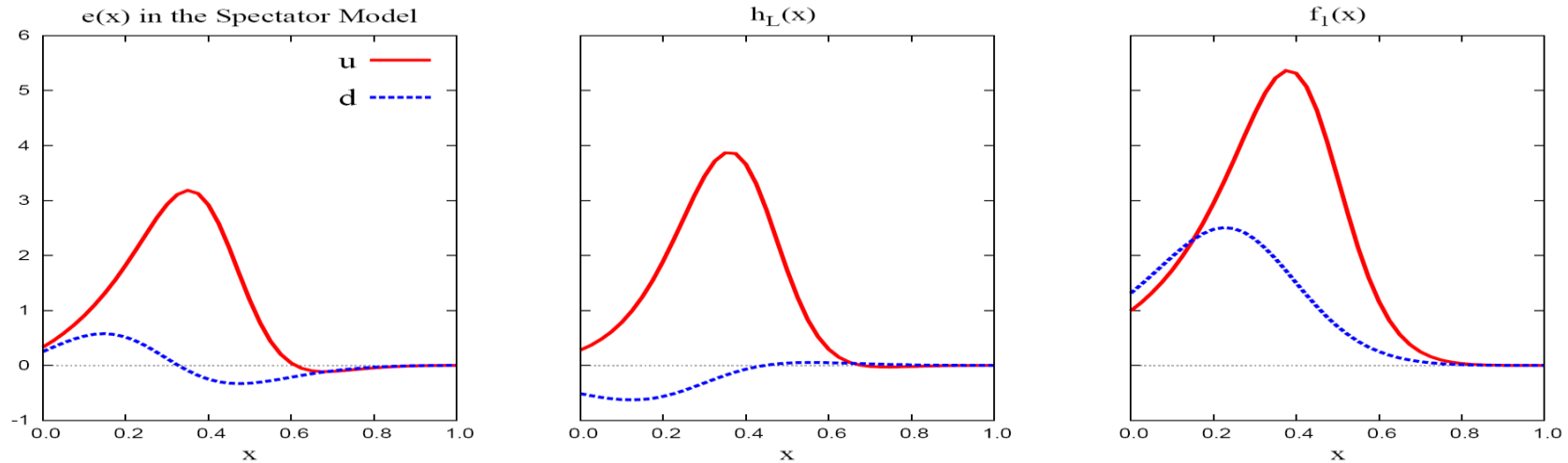
W. Gohn

Collins type contribution may be dominant for π^-



Sivers type contribution may be dominant for π^0

Model predictions: unpolarized target



Lattice provides important cross check with data and models for all HT TMDs (Musch et al, arXiv:1011.1213)

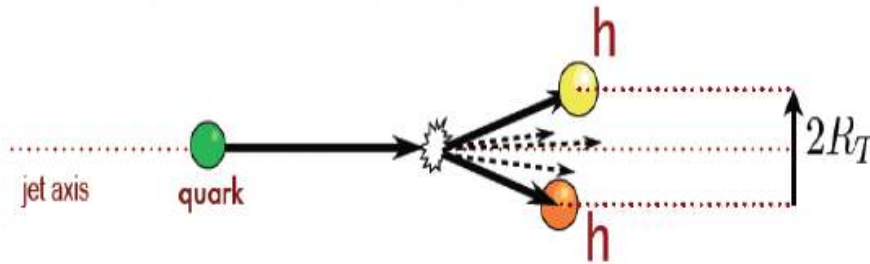
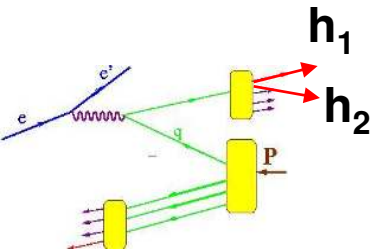
$$f_1^{[1]}(k_{\perp}^2) \equiv \int_{-1}^1 dx f_1(x, k_{\perp}^2)$$

$$\frac{e^{[1]}}{f_1^{[1]}} = \frac{\tilde{A}_1}{\tilde{A}_2}$$

- Models agree on a large beam SSA for $\pi\pi$ pair production
- Lattice results for u-d can be directly compared to models and data.

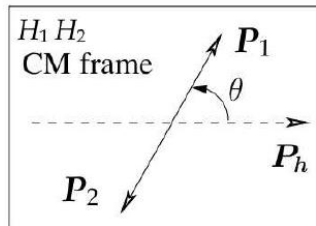
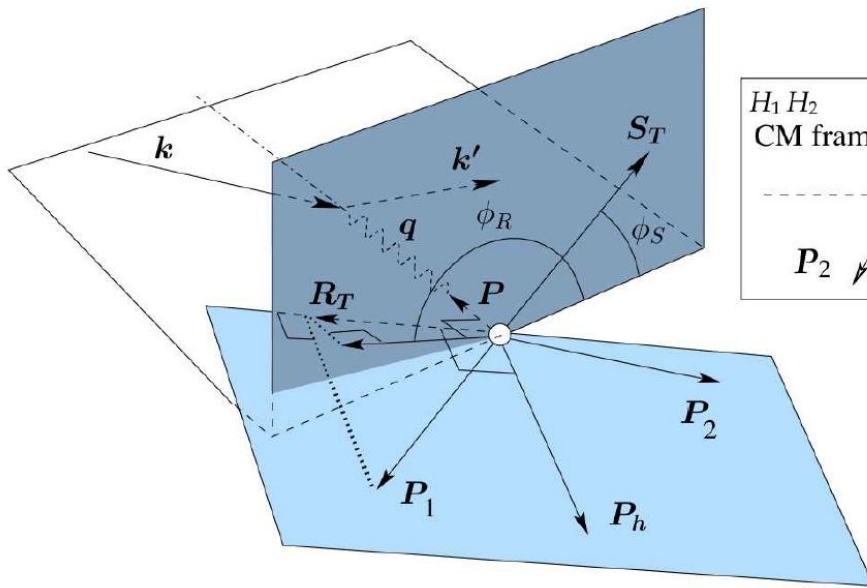
Dihadron production kinematics

z_1, z_2 - fractions of energy carried by a hadrons



◆ DIFF

$$D_1^{q \rightarrow h_1, h_2}(z_1, z_2, R_T^2)$$



$$\phi_R \equiv \frac{(q \times k) \cdot R_T}{|(q \times k) \cdot R_T|}$$

$$R_T = R - (R \cdot \hat{P}_h) \hat{P}_h$$

$$R \equiv (P_1 - P_2)/2$$

- Factorization proven
- Evolution known
- Extracted at BELLE for $\pi\pi$ pairs, planned for πK pairs

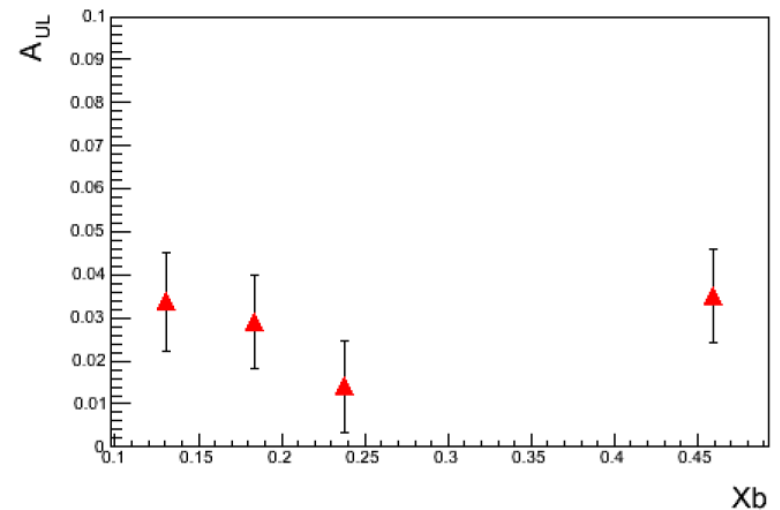
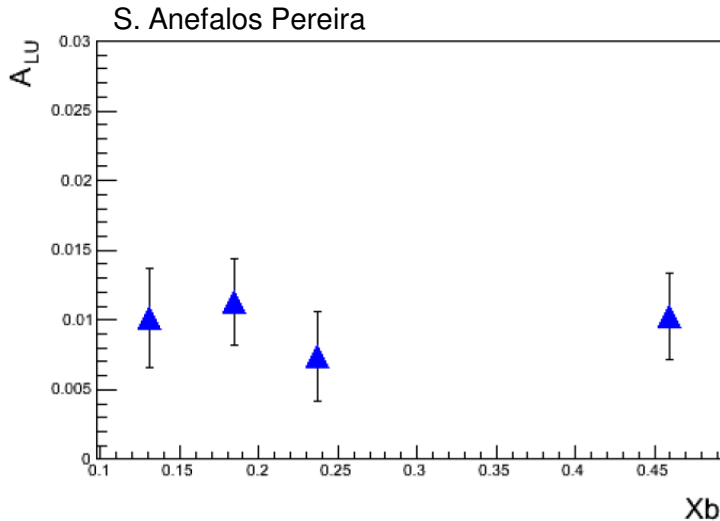
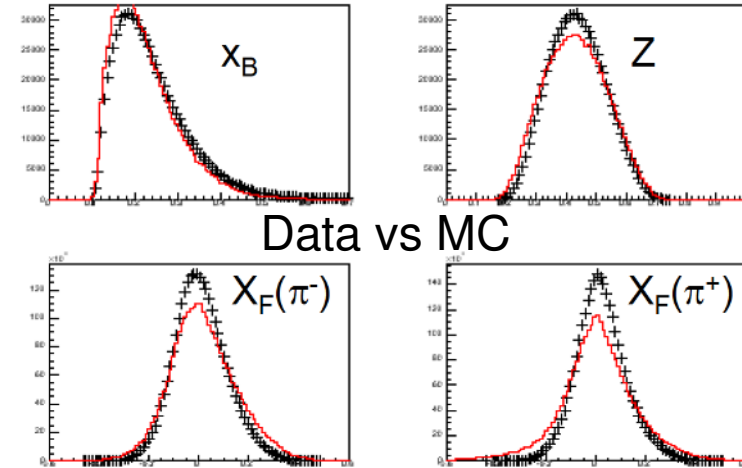
Dihadron productions offers exciting possibility to access HT pdfs as we deal with the product of functions instead of convolution

Dihadron production kinematics

$$F_{LU}^{\sin \phi_R} = -x \frac{|R| \sin \theta}{Q} \left[\frac{M}{M_h} x e^q(x) H_1^{\langle q \rangle}(z, \cos \theta, M_h) + \frac{1}{z} f_1^q(x) \tilde{G}^{\langle q \rangle}(z, \cos \theta, M_h) \right],$$

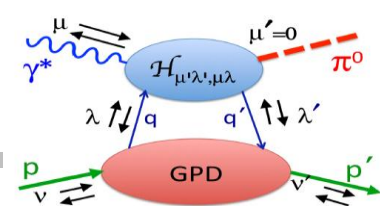
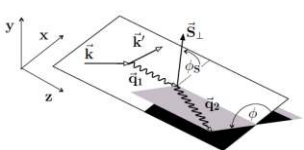
$$F_{UL}^{\sin \phi_R} = -x \frac{|R| \sin \theta}{Q} \left[\frac{M}{M_h} x h_L^q(x) H_1^{\langle q \rangle}(z, \cos \theta, M_h) + \frac{1}{z} g_1^q(x) \tilde{G}^{\langle q \rangle}(z, \cos \theta, M_h) \right]$$

$$F_{UU,T} = x f_1^q(x) D_1^q(z, \cos \theta, M_h)$$



Dihadron productions offers exciting possibility to access HT parton distribution surviving k_T -integration

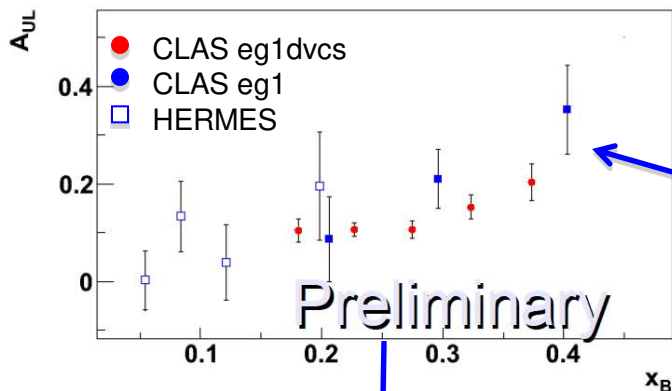
3D structure: GPDs



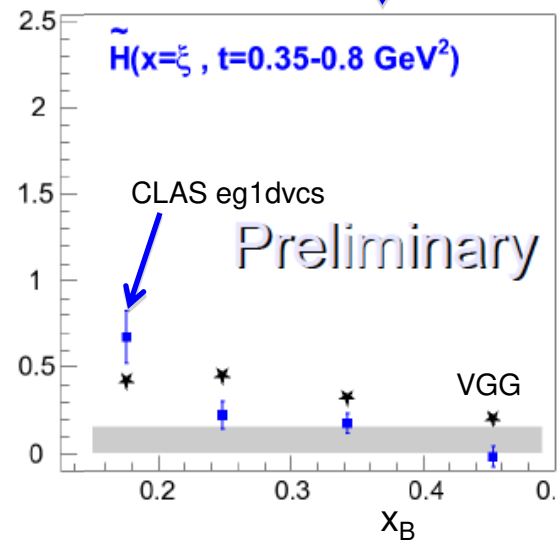
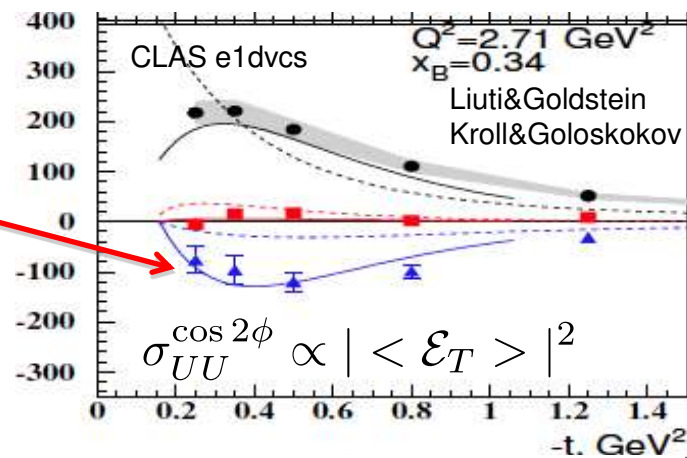
$$\sigma_{UL}^{\sin\phi} \sim F_1 \tilde{H} + \xi(F_1 + F_2) \mathcal{H} \quad ep \rightarrow e' p \gamma$$

$ep \rightarrow e' p \gamma$

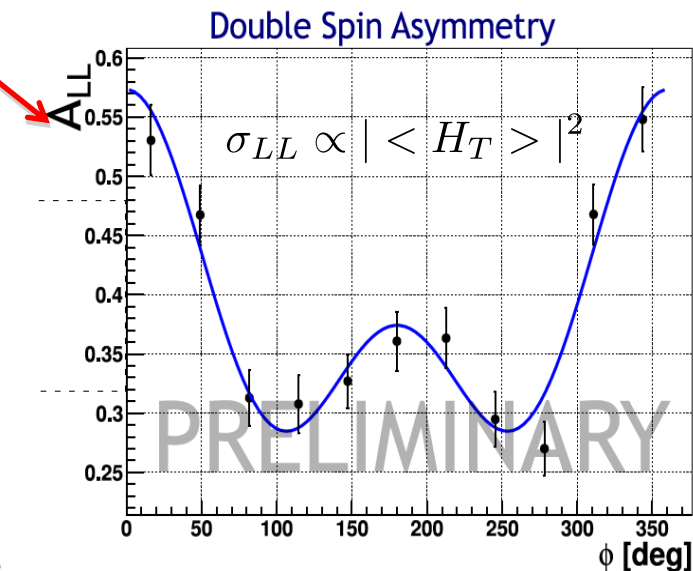
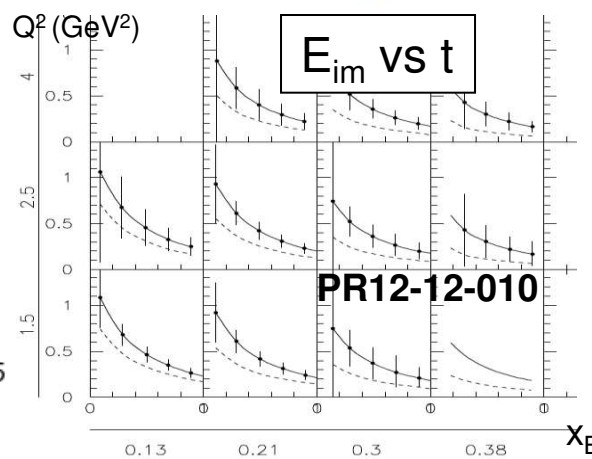
$ep \rightarrow e' p \pi^0$



	U	L	T
U	H		\mathcal{E}_T
L		\tilde{H}	
T	E		H_T, \tilde{H}_T



$$\sigma_{UT}^{\cos\phi} \sim \frac{1-x}{2-x} \frac{t}{M^2} F_2 \mathcal{H} + \frac{t}{4M^2} (2-x) F_1 \mathcal{E}$$



Spin-azimuthal asymmetries in hard exclusive photon (DVCS) and hadron (DVMP) production give access to underlying GPDs and PDFs

Summary

- Measurements of azimuthal dependences of double and single spin asymmetries in hard scattering (SIDIS, DVMP) indicate that there are significant correlations between spin and transverse distribution of quarks
- Current JLab data are consistent with a partonic picture and measurements performed at higher energies
- Sizable higher twist asymmetries measured both in SIDIS and exclusive production indicate the quark-gluon correlations may be significant at moderate Q^2

*Jlab measurements at 6 GeV provide important input for model independent flavor decomposition of TMDs and GPDs
tools are required to extract the 3D PDFs in multidimensional space*

Support slides....

The Multi-Hall SIDIS Program at 12 GeV

M. Aghasyan, K. Allada, H. Avakian, F. Benmokhtar, E. Cisbani, J-P. Chen, M. Contalbrigo, D. Dutta, R. Ent, D. Gaskell, H. Gao, K. Griffioen, K. Hafidi, J. Huang, X. Jiang, K. Joo, N. Kalantarians, Z-E. Meziani, M. Mirazita, H. Mkrtchyan, L.L. Pappalardo, A. Prokudin, A. Puckett, P. Rossi, X. Qian, Y. Qiang, B. Wojtsekhowski
for the Jlab SIDIS working group

The complete mapping of the multi-dimensional SIDIS phase space will allow a comprehensive study of the TMDs and the transition to the perturbative regime.

Flavor separation will be possible by the use of different target nucleons and the detection of final state hadrons.

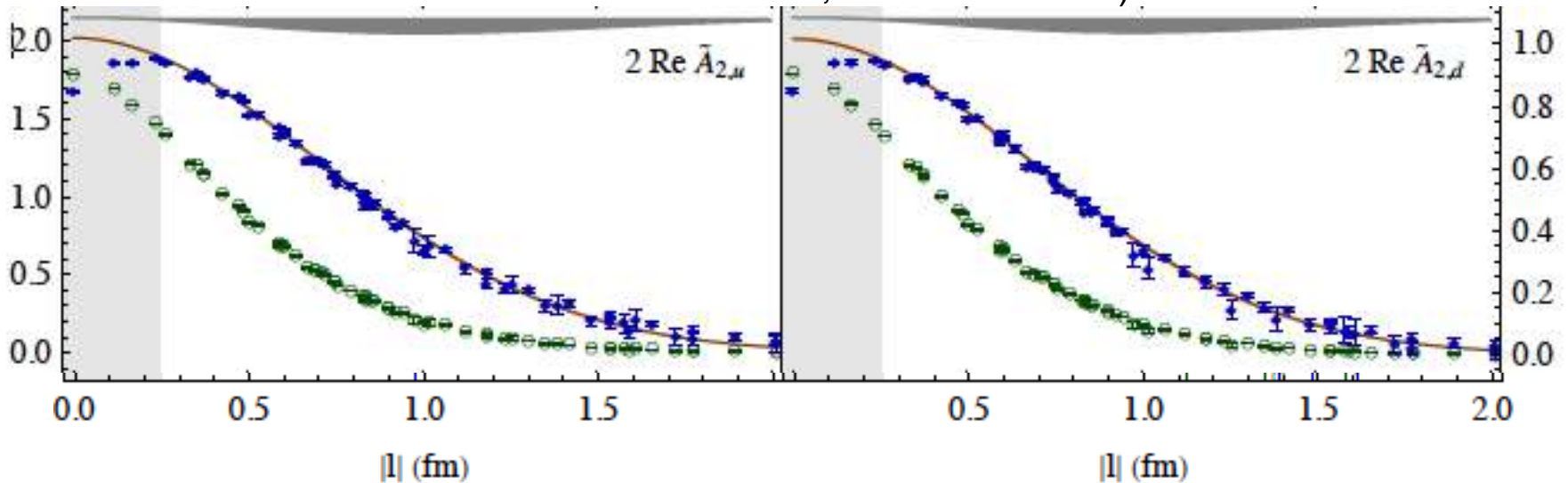
Measurements with pions and kaons in the final state will also provide important information on the hadronization mechanism in general and on the role of spin-orbit correlations in the fragmentation in particular.

Higher-twist effects will be present in both TMDs and fragmentation processes due to the still relatively low Q^2 range accessible at JLab, and can apart from contributing to leading-twist observables also lead to observable asymmetries vanishing at leading twist. These are worth studying in themselves and provide important information on quark-gluon correlations.

Lattice calculations and b_T -space

(PDFs in terms of Lorenz invariant amplitudes
Musch et al, arXiv:1011.1213)

\tilde{A}_i

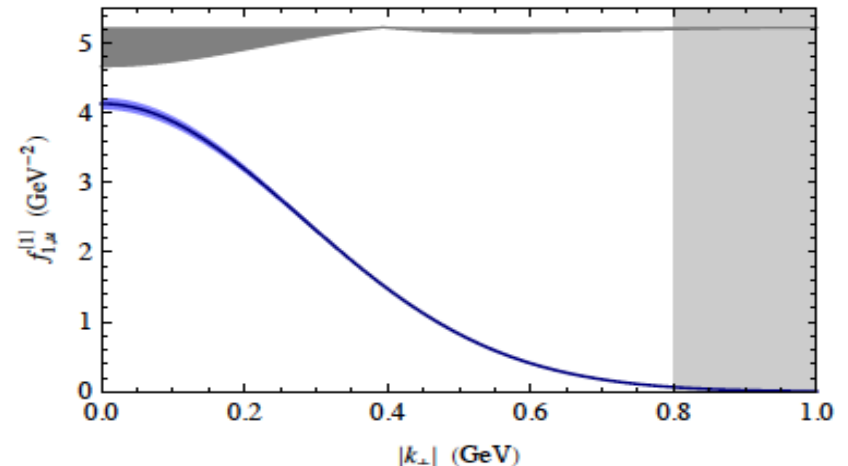


C_2

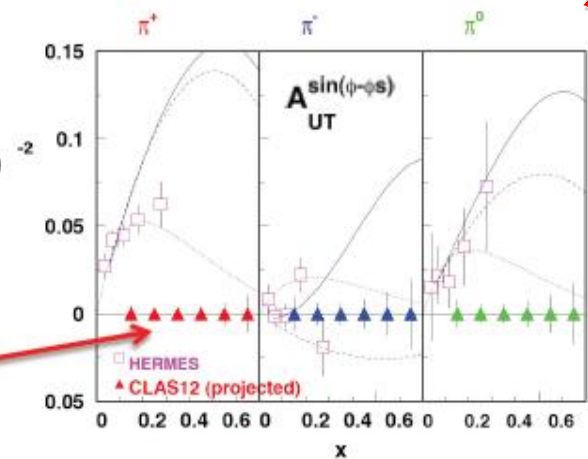
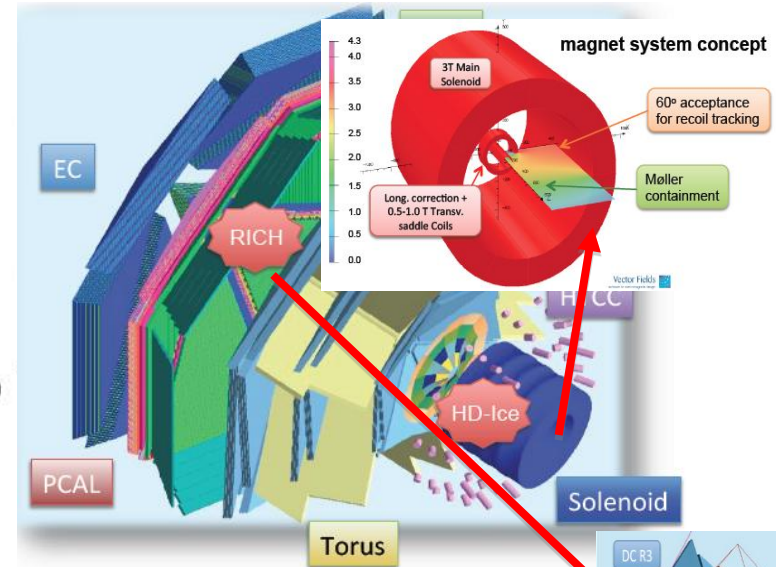
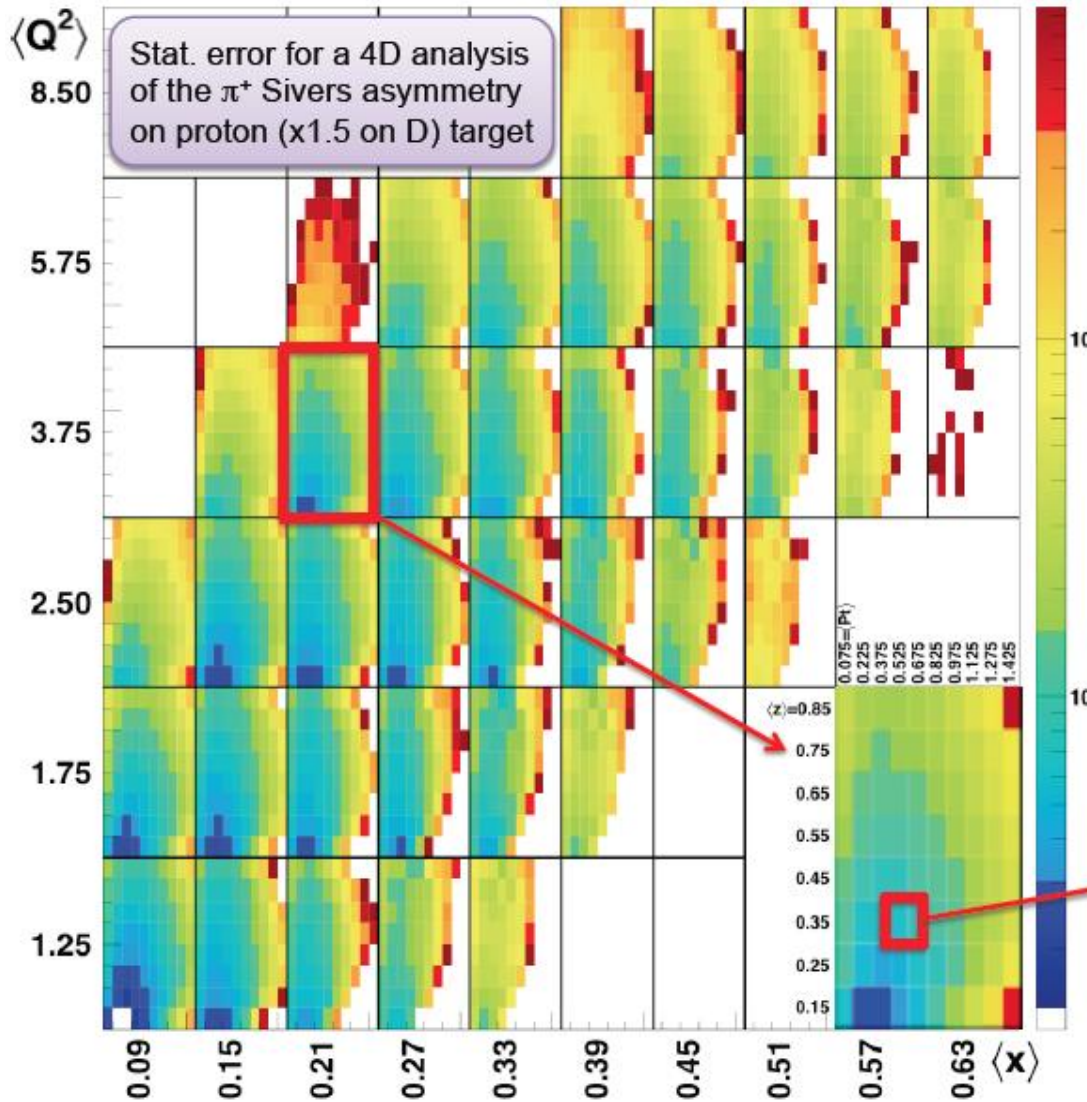
σ_2

$\tilde{A}_{2,u}$	$2.0186 \pm 0.0063 \pm 0.0008$	$1.001 \pm 0.010 \pm 0.068$
$\tilde{A}_{2,d}$	$1.0171 \pm 0.0064 \pm 0.0005$	$0.975 \pm 0.012 \pm 0.063$

$$f_1^{[1]}(k_\perp^2) = \frac{C_2 \sigma_2^2}{4\pi} e^{-\frac{k_\perp^2}{(2/\sigma_2)^2}}$$

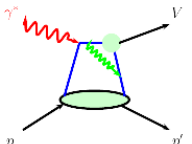


CLAS12 A_{UT} with transverse proton target



Curves from hep-ph/0507266 and hep-ph/0507181

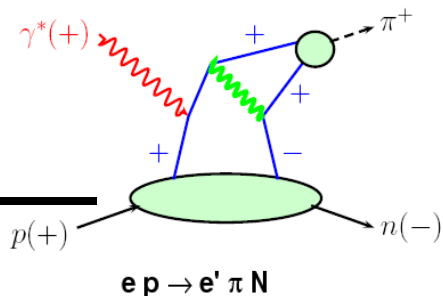
SSAs in exclusive pion production



Transverse photon matters

Ahmad, Liuti & Goldstein: arXiv:0805.3568
 Gloskokov & Kroll : arXiv:0906.0460

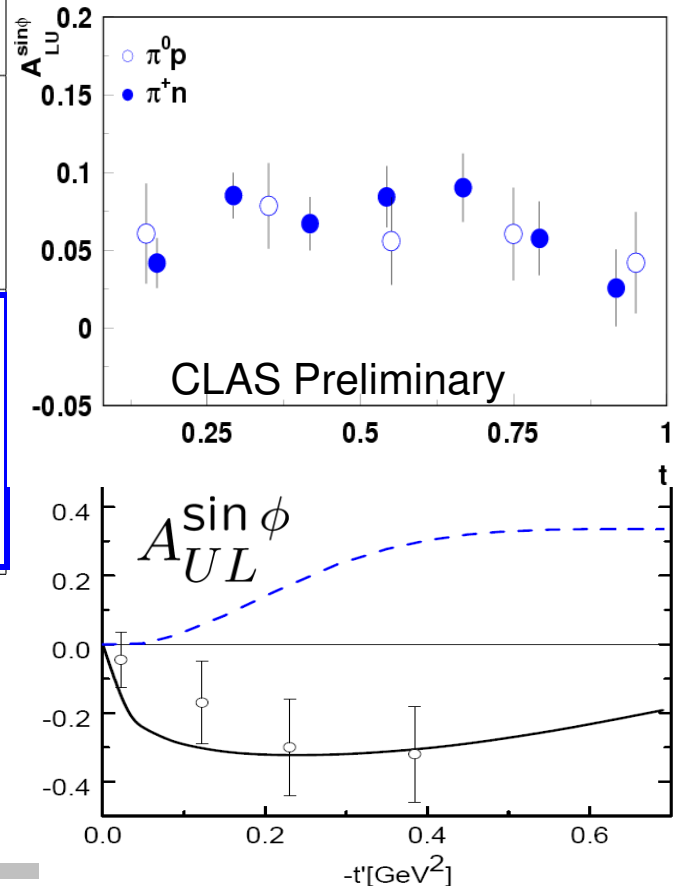
$$\mathcal{M}_{0-,++}^{twist-3} \approx e_0 \sqrt{1 - \xi^2} \int_{-1}^{+1} d\bar{x} \mathcal{H}_{0-,++} [H_T^{(3)} \leftarrow \dots]$$

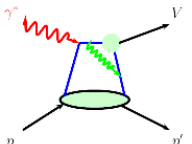


observable	dominant interf. term	amplitudes	low t' behavior
$A_{UT}^{\sin(\phi - \phi_s)}$	LL	$\text{Im}[\mathcal{M}_{0-,0+}^* \mathcal{M}_{0+,0+}]$	$\propto \sqrt{-t'}$
$A_{UT}^{\sin(\phi + \phi_s)}$	TT	$\text{Im}[\mathcal{M}_{0-,++}^* \mathcal{M}_{0+,++}]$	$\propto \sqrt{-t'}$
$A_{UT}^{\sin(3\phi - \phi_s)}$	TT	$\text{Im}[\mathcal{M}_{0-, -+}^* \mathcal{M}_{0+, -+}]$	$\propto (-t')^{(3/2)}$
$A_{UT}^{\sin \phi_s}$	LT	$\text{Im}[\mathcal{M}_{0-,++}^* \mathcal{M}_{0+,0+}]$	const.
$A_{UL}^{\sin \phi}$	LT	$\text{Im}[\mathcal{M}_{0-,++}^* \mathcal{M}_{0-,0+}]$	$\propto \sqrt{-t'}$
$A_{LU}^{\sin \phi}$	LT	$\text{Im}[\mathcal{M}_{0-,++}^* \mathcal{M}_{0-,0+}]$	$\propto \sqrt{-t'}$
$A_{LL}^{\cos \phi}$	LT	$\text{Re}[\mathcal{M}_{0-,++}^* \mathcal{M}_{0-,0+}]$	$\propto \sqrt{-t'}$

$$A_{LU}^{\sin \phi} / A_{UL}^{\sin \phi} \approx \sqrt{(1 - \epsilon) / (1 + \epsilon)}$$

- HT SSAs are expected to be very significant
- Wider coverage (CLAS12, EIC) would allow measurements of Q^2 dependence of HT SSAs



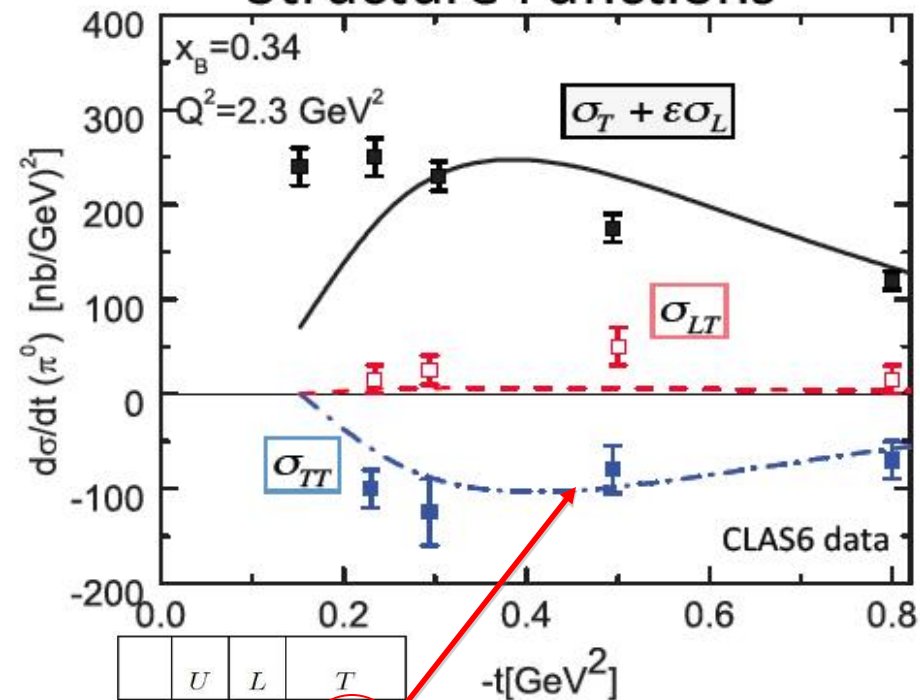


Exclusive π^0 production

Recent progress with GPD-based description

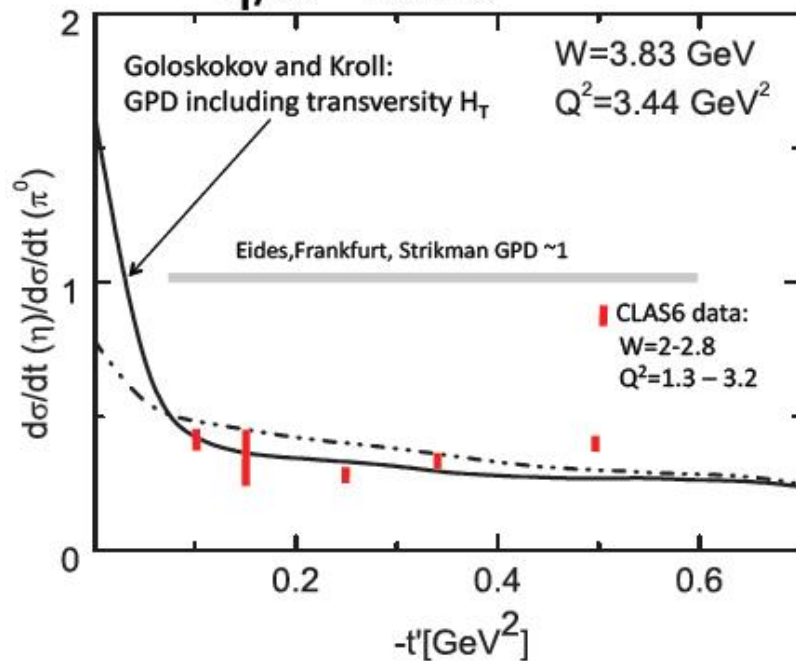
- Goloskokov&Kroll, Goldstein&Liuti. Include **transversity GPDs** H_T and $\varepsilon_T = 2\tilde{H}_T + E_T$. Dominate in CLAS kinematics. Successfully described data.

Structure Functions



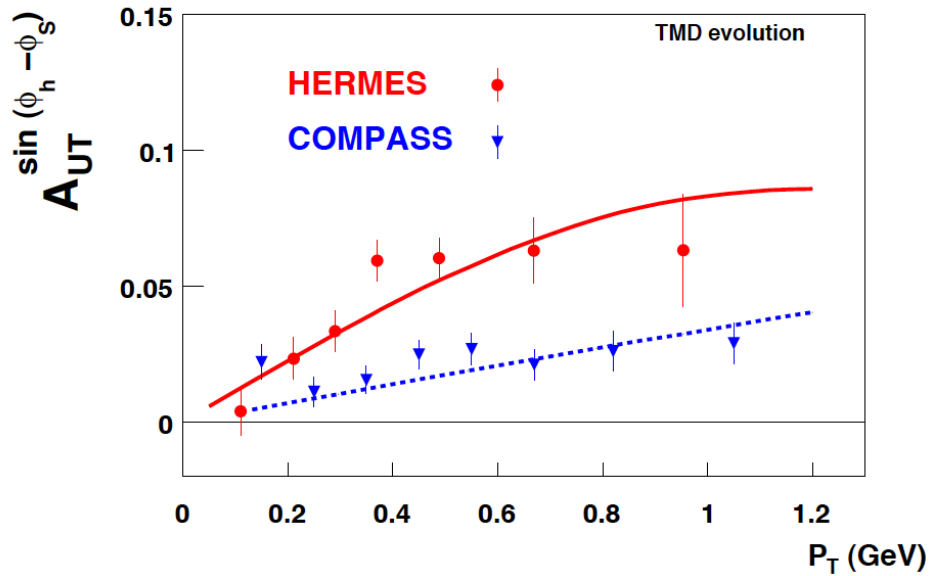
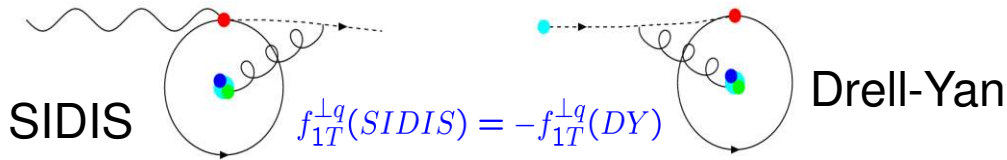
	U	L	T
U	H		ε_T
L		\tilde{H}	
T	E		H_T, \tilde{H}_T

η/π^0 Ratio



Goloskokov&Kroll
 H_T and \bar{E}_T dominate

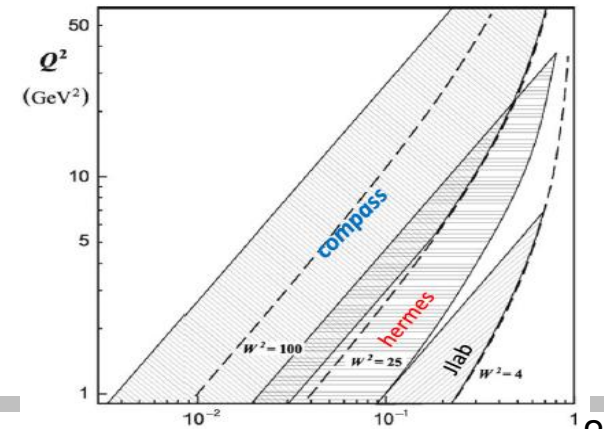
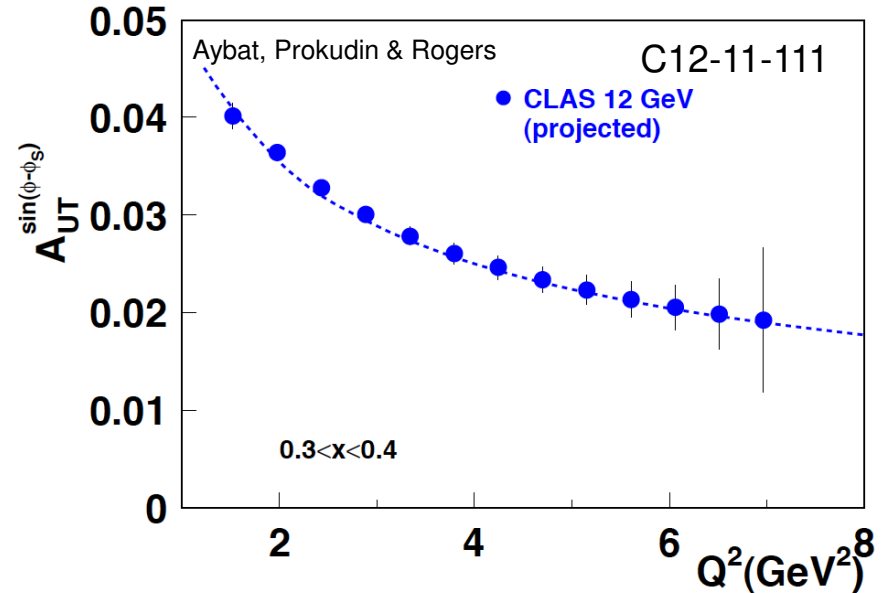
Sivers TMD evolution



TMD Evolution may explain existing differences between HERMES and COMPASS .

Aybat, Prokudin & Rogers : arXiv:1112.4423

Comparison of JLab12 data with HERMES and COMPASS will pin down the Q^2 evolution of Sivers asymmetry.



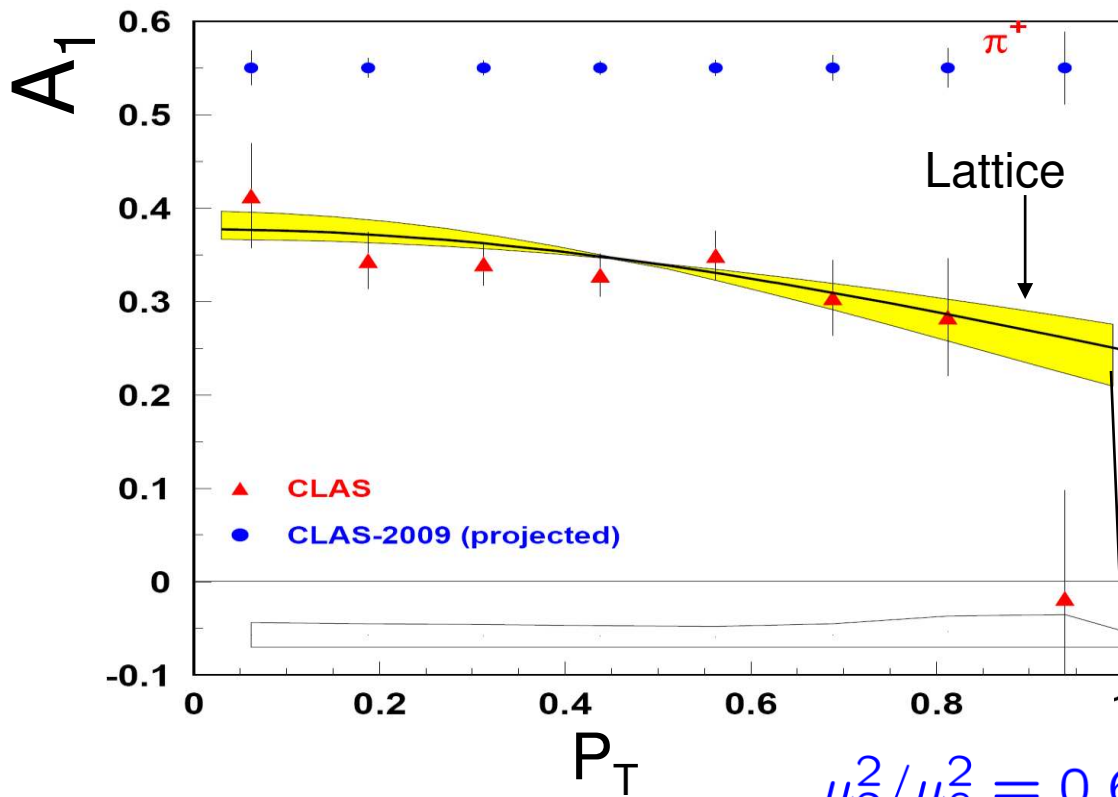
	U	L	T
U	f_1	g_1	h_1^\perp
L		g_1^\perp	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}^\perp	$h_1 h_{1T}^\perp$

A_1 P_T -dependence

arXiv:1003.4549

$$A_1(\pi) \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^{q \rightarrow \pi}(z)}{\sum_q e_q^2 f_1^q(x) D_1^{q \rightarrow \pi}(z)}$$

$$A_1(x, z, P_T) = A_1(x, z) \frac{\langle P_T^{2,unp} \rangle}{\langle P_T^{2,pol} \rangle} \exp(-P_T^2 / \langle P_T^{2,pol} \rangle - P_T^2 / \langle P_T^{2,unp} \rangle)$$

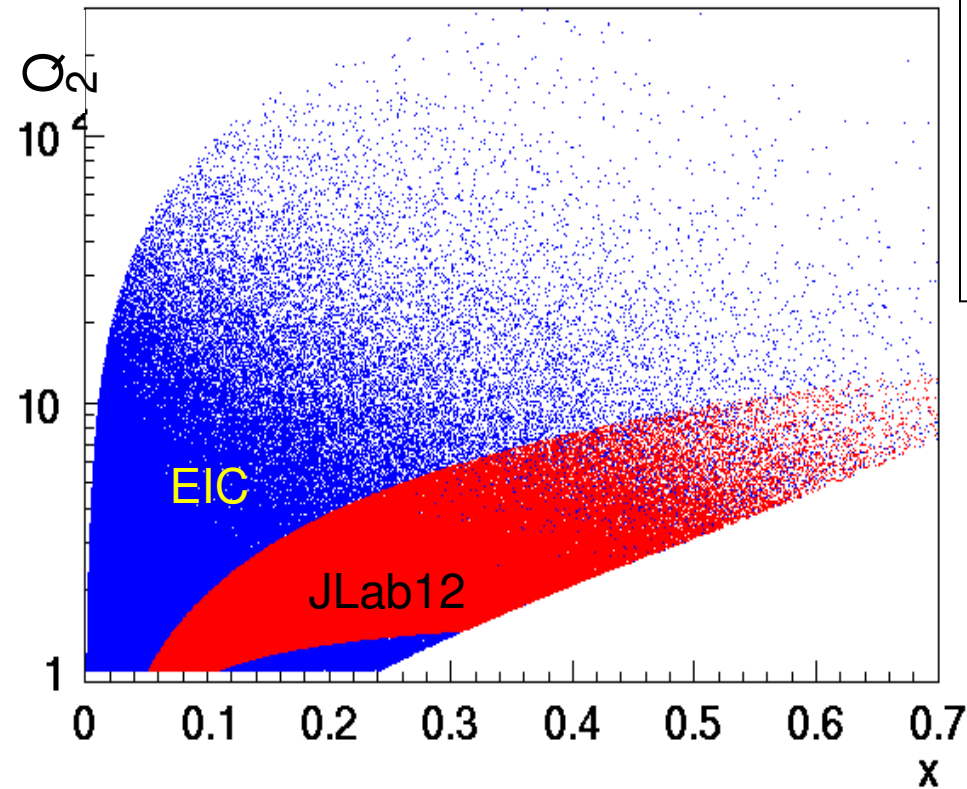


B.Musch et al arXiv:1011.1213

$$\mu_2^2 / \mu_0^2 = 0.692 \pm 0.039 \pm 0.045$$

CLAS data consistent with model predictions and lattice predicting that width of g_1 is less than the width of f_1

From JLab12 to EIC



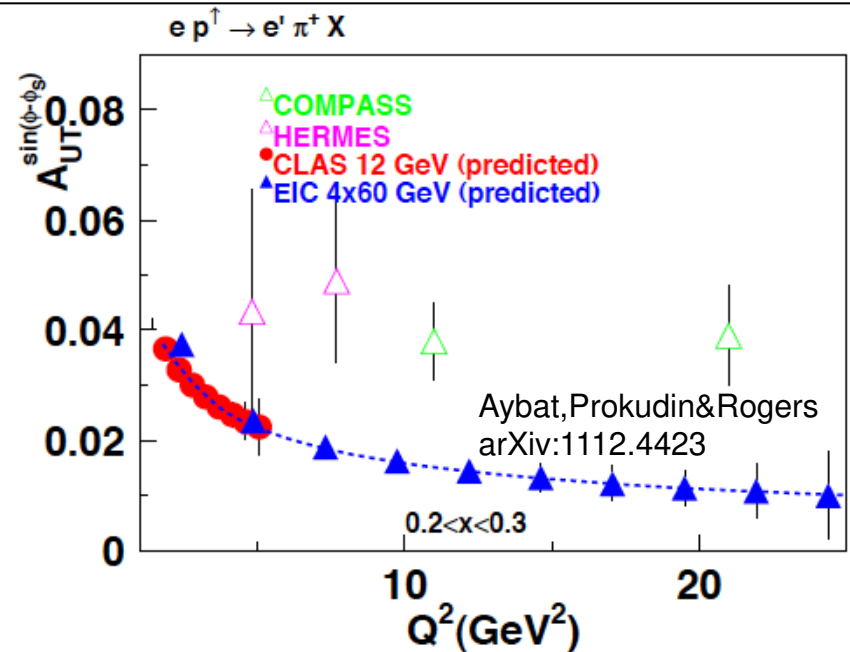
$$A_{UT}^{\sin(\phi-\phi_S)} = \frac{\sum_q e_q^2 f_{1T}^{\perp q} D_1^q}{\sum_q e_q^2 f_1^q D_1^q}$$

JLab@12GeV (25/50/75)

→ $0.1 < x_B < 0.7$: valence quarks

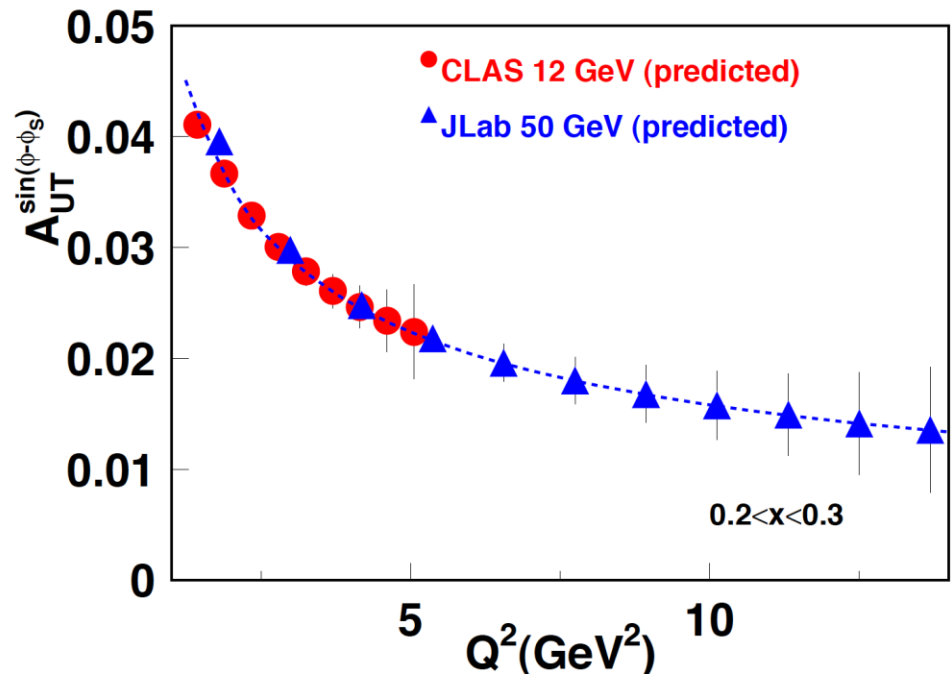
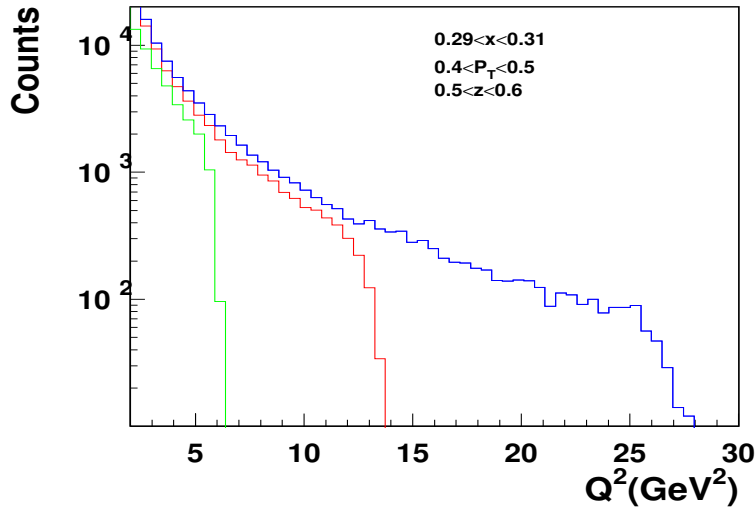
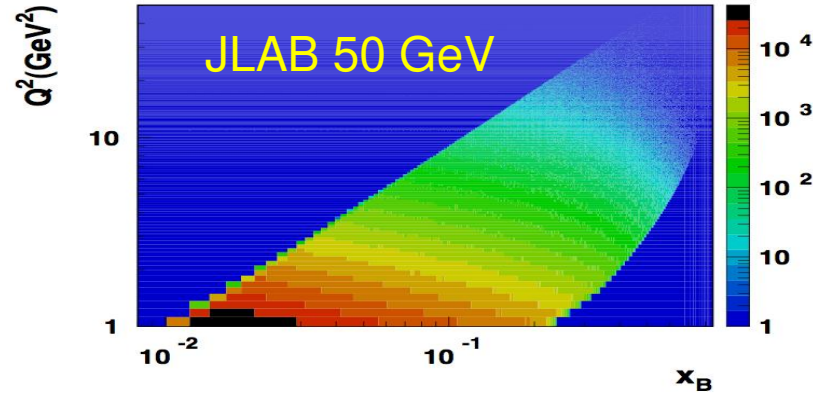
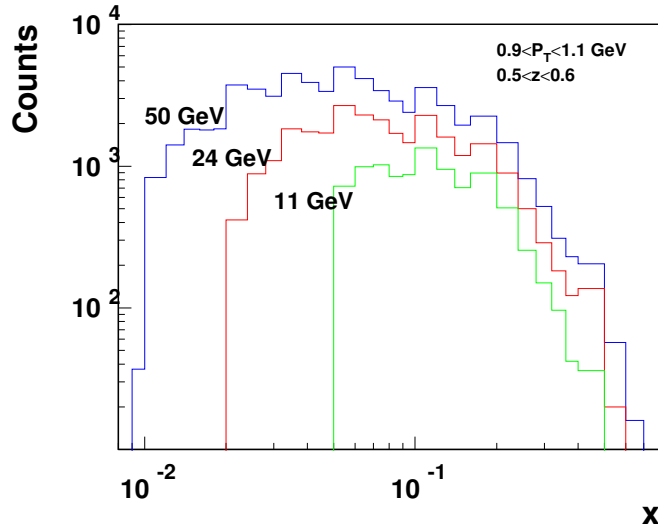
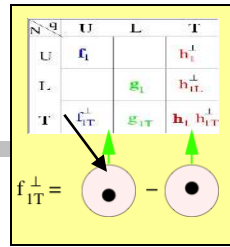
EIC $\sqrt{s} = 140, 50, 15$ GeV

→ $10^{-4} < x_B < 0.3$: gluons and quarks, higher P_T and Q^2 .



- Study of high x domain requires high luminosity, low x higher energies
- Wide range in Q^2 is crucial to study the evolution
- Overlap of EIC and JLab12 in the valence region will be crucial for the TMD program

$ep \rightarrow e' \pi^+ X$ From JLab12 to JLab50

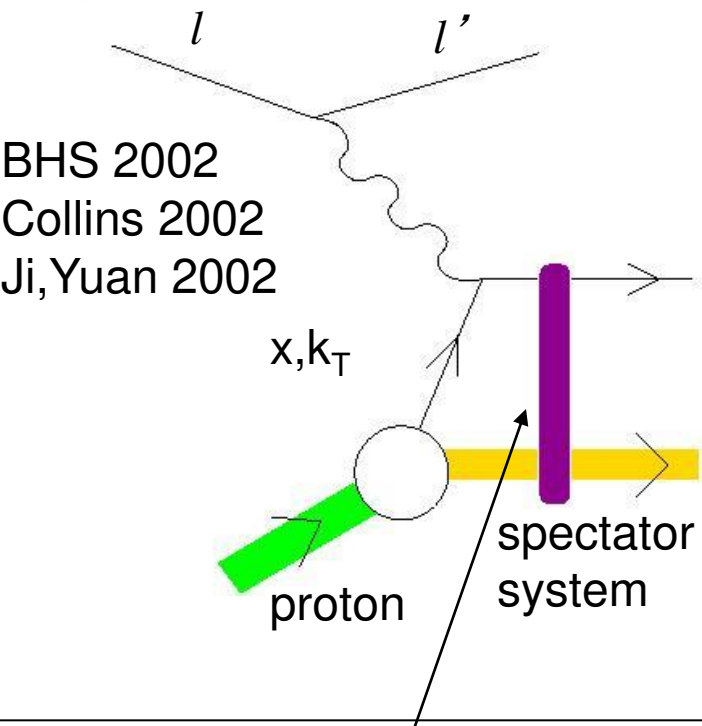


For a given lumi (30min of runtime) and given bin in hadron z and P_T , higher energy provides higher counts and wider coverage in Q^2 , allowing studies of Q^2 evolution of 3D partonic distributions in a wide Q^2 range.

k_T and FSI

- Factorization proven for small k_T (Ji, Ma, Yuan 2005)
- Medium modifications of k_T PDFs (Tang, Wang, Zhou 2008)
 - Complete definition of TMDs (Collins 2011 “Foundation of Perturbative QCD”)
 - Evolution of TMDs, (Collins, Aybat, Rogers 2011)
 - TMDs on Lattice, (Musch, Haegler et al. 2011)
- Color Lorentz Force acting on ejected quark, torque along trajectory (Burkardt 2008, 2012)
- k_T -dependent flavor decomposition (BGMP procedure, 2011)

$$f_q^N(x, \vec{k}_T)$$



BHS 2002
Collins 2002
Ji, Yuan 2002

soft gluon exchanges included in the distribution function (gauge link)

- Experiments consistent with evolution on $\langle k_T^2 \rangle$ increasing with Q^2 .
- What is the source of the k_T (dynamical vs static)?
- What is the role of FSI and how they modify in medium

Forces and binding effects in the partonic medium

$$xe = x\tilde{e} + \frac{m}{M} f_1$$

Interaction dependent parts

$$xh_L = x\tilde{h}_L + \frac{p_T^2}{M^2} h_{1L}^\perp + \frac{m}{M} g_{1L}$$

“Wandzura-Wilczek approximation” is equivalent to setting functions with a tilde to zero.

N/q	U	L	T
U			e
L			h_L
T		g_T	

$$e_2 \equiv \int_0^1 dx x^2 \tilde{e}(x)$$

Quark polarized in the x-direction with k_T in the y-direction

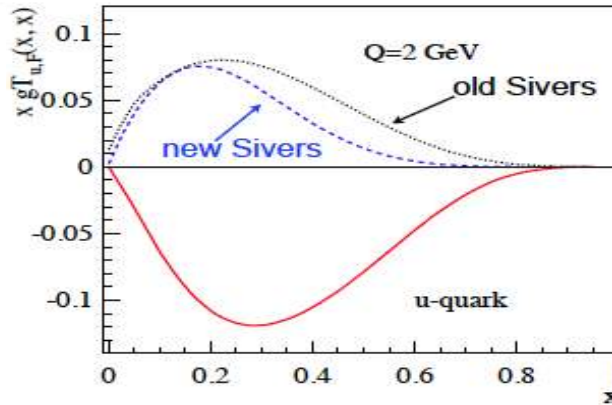
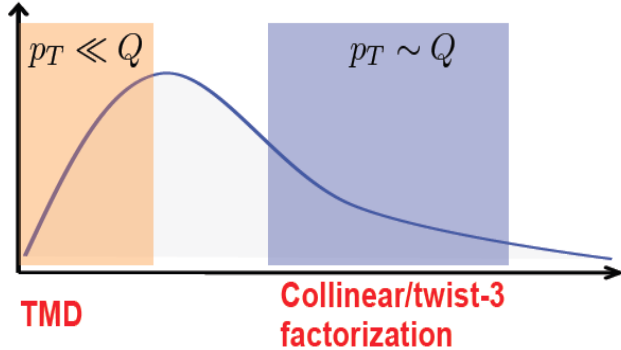
$\frac{M^2}{2} e_2$ **Boer-Mulders Force** on the active quark right after scattering (t=0)

current quark masses

Interpreting HT (quark-gluon-quark correlations) as force on the quarks (Burkardt hep-ph:0810.3589)

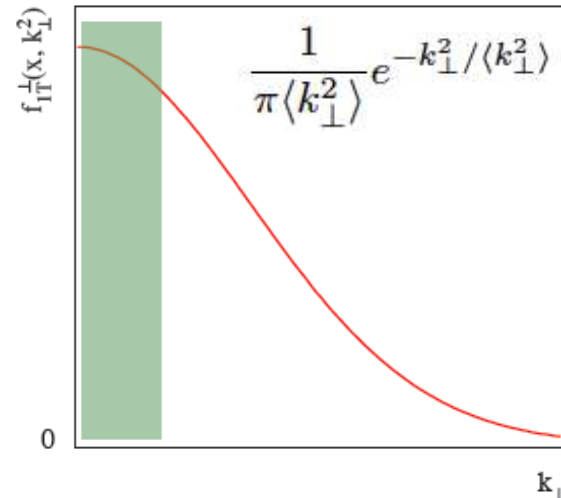
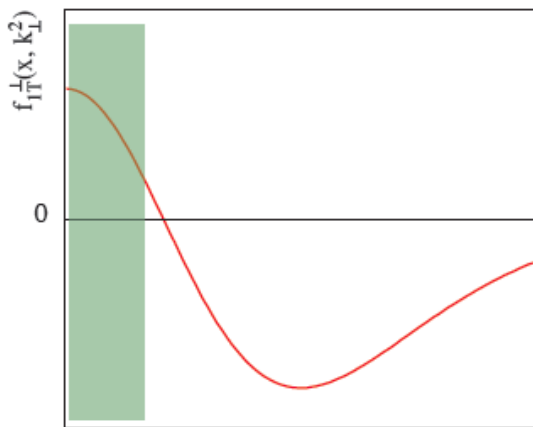
k_T -dependence of TMDs

- Transition from low p_T to high p_T



Directly obtained ETQS functions are opposite in sign to those from k_T moments “**sign mismatch**”

Sivers function extracted assuming k_T distribution is gaussian



- With orbital angular momentum TMD can't be gaussian
- How to measure k_T -dependences of TMDs

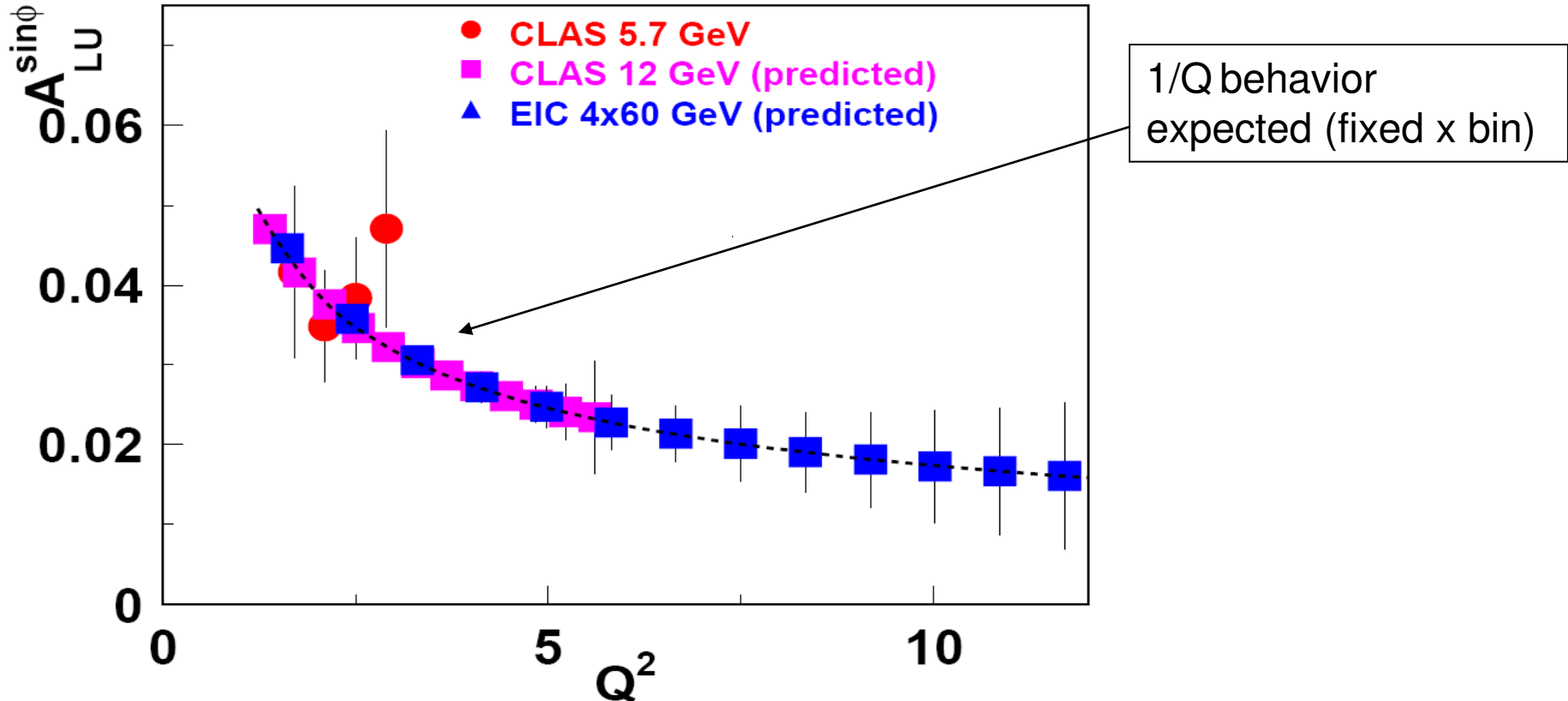
(Z. Kang et al, 2011)

Q²-dependence of beam SSA

$$\sigma_{LU(UL)}^{\sin\phi} \sim F_{LU(UL)} \sim 1/Q \text{ (Twist-3)}$$

$$A_{LU} \propto g^\perp(x) D_1(z)$$

$$\vec{e} p \rightarrow e' \pi^+ X$$



Study for Q² dependence of beam SSA allows to check the higher twist nature and access quark-gluon correlations.

P_T -dependence studies at Hall-C

H. Mkrtchyan(DIS2011)

Experiment E00-108

Beam energy 5.5 GeV

4 cm LH2 and LD2 targets

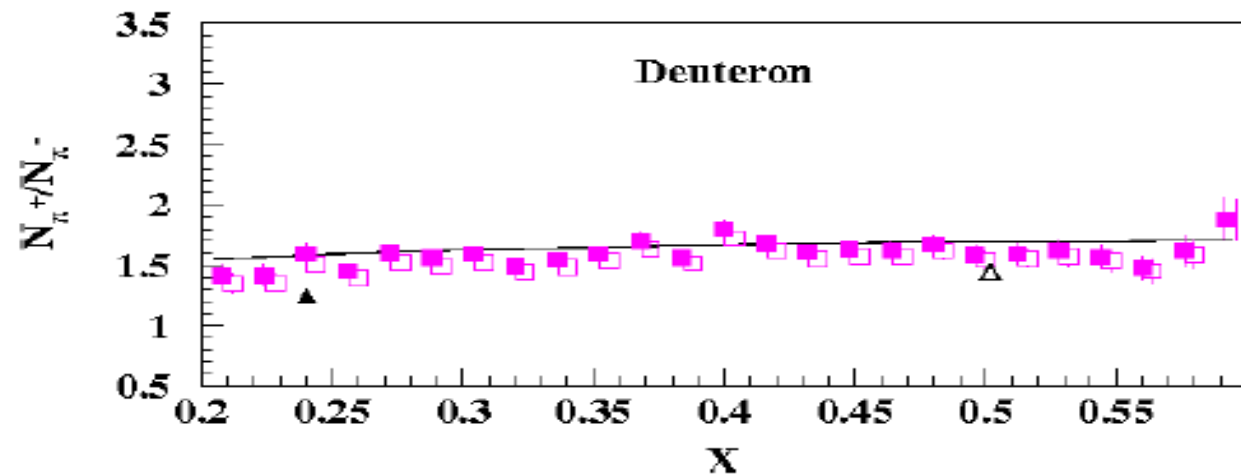
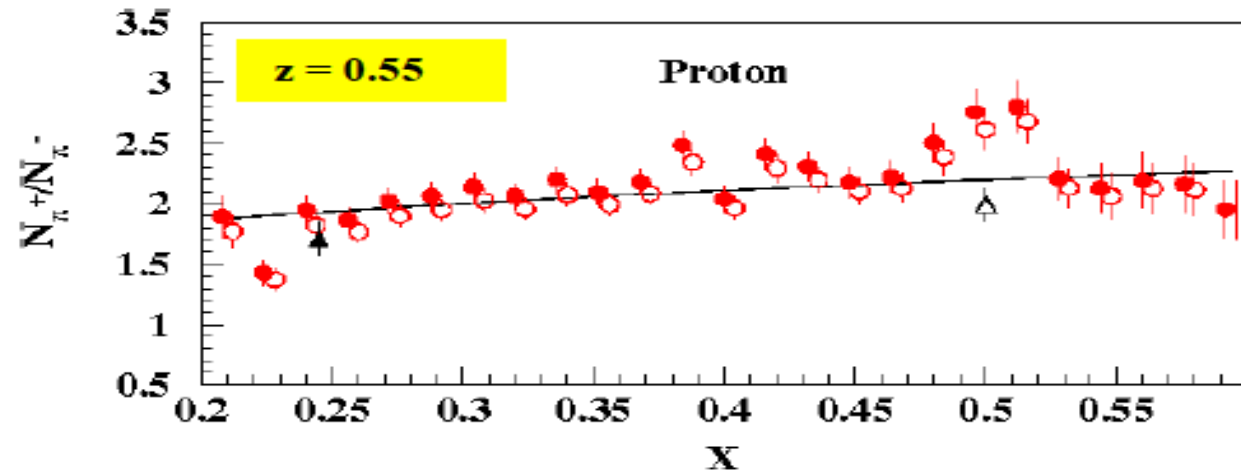
$$\sigma_d^{\pi^+} \propto (4D^+ + D^-)(u + d)$$

$$\sigma_d^{\pi^-} \propto (4D^- + D^+)(u + d)$$

$$\frac{\sigma_d^{\pi^+}}{\sigma_d^{\pi^-}} = \frac{4D^+ + D^-}{4D^- + D^+}$$

$$D^-/D^+ = (4 - r) / (4r - 1)$$

$$r = \sigma_d(\pi^+) / \sigma_d(\pi^-)$$

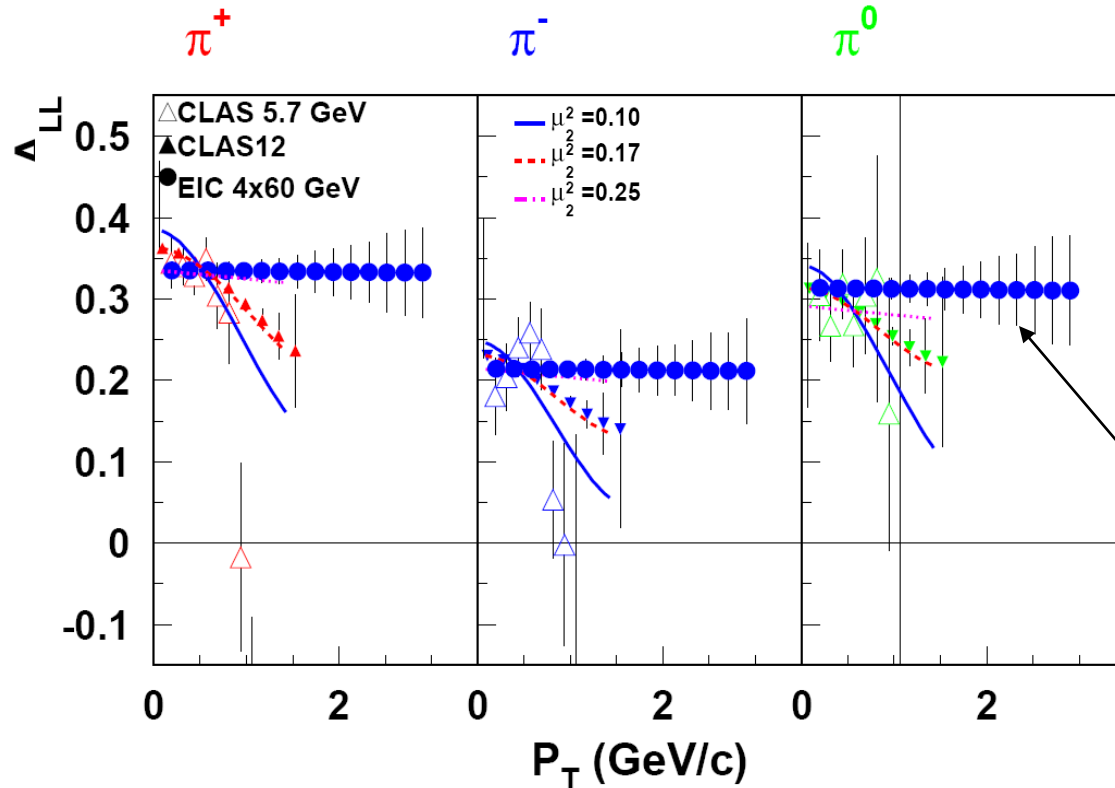


x-dependence of π^+/π^- ratio is in good agreement with the quark parton model predictions (lines CTEQ5M+BKK).

A₁ P_T-dependence in SIDIS

$$A_1(\pi) \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^{q \rightarrow \pi}(z)}{\sum_q e_q^2 f_1^q(x) D_1^{q \rightarrow \pi}(z)} e^{-z^2 P_T^2 \frac{(\mu_0^2 - \mu_2^2)}{(\mu_D^2 + z^2 \mu_0^2)(\mu_D^2 + z^2 \mu_2^2)}}$$

M. Anselmino et al
hep-ph/0608048



$$f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

$$D_1^q(z, p_T) = D_1(z) \frac{1}{\pi \mu_D^2} \exp\left(-\frac{p_T^2}{\mu_D^2}\right)$$

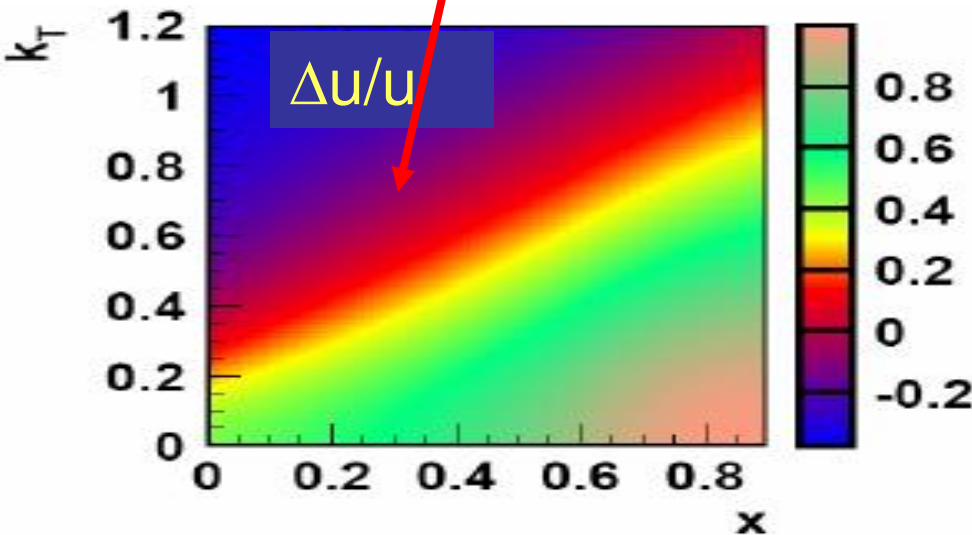
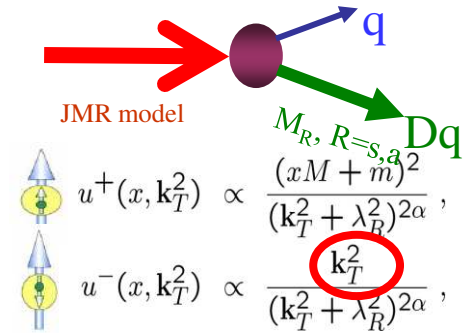
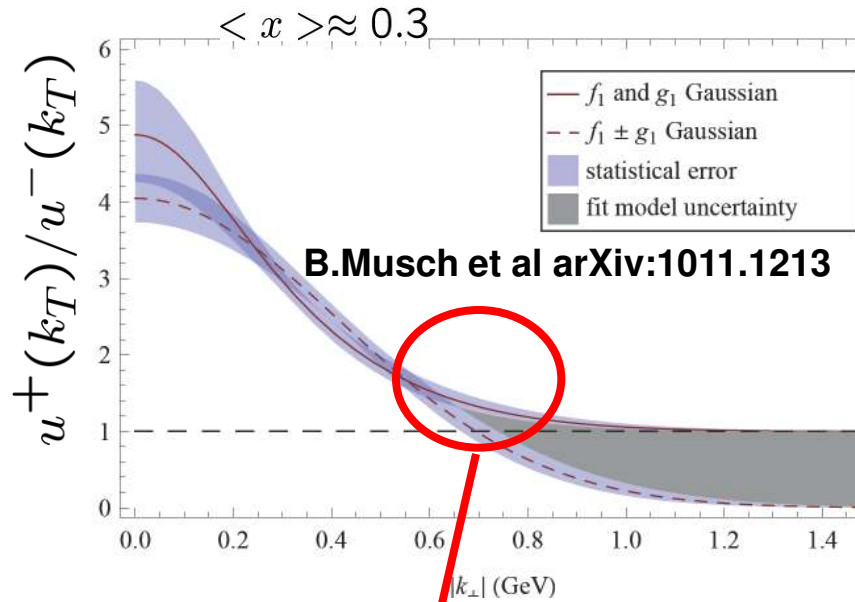
$\mu_0^2 = 0.25 \text{ GeV}^2$
 $\mu_D^2 = 0.2 \text{ GeV}^2$

Perturbative limit calculations available for $g_1^q(x, k_T), f_1(x, k_T)$:

J. Zhou, F. Yuan, Z. Liang: arXiv:0909.2238

- $A_{LL}(\pi)$ sensitive to difference in k_T distributions for f_1 and g_1
- Wide range in P_T allows studies of transition from TMD to perturbative approach

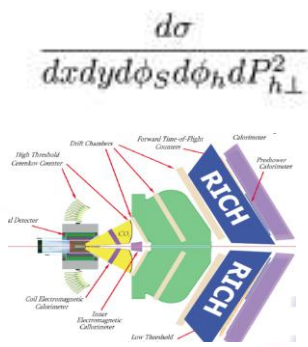
Quark distributions at large k_T : models



Sign change of $\Delta u/u$ consistent between lattice and diquark model

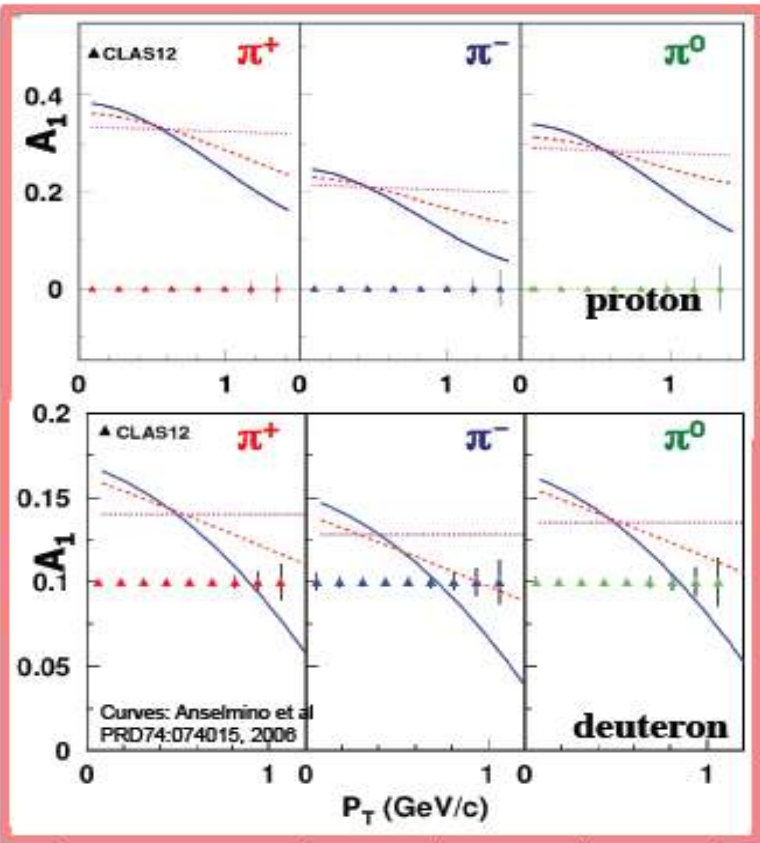
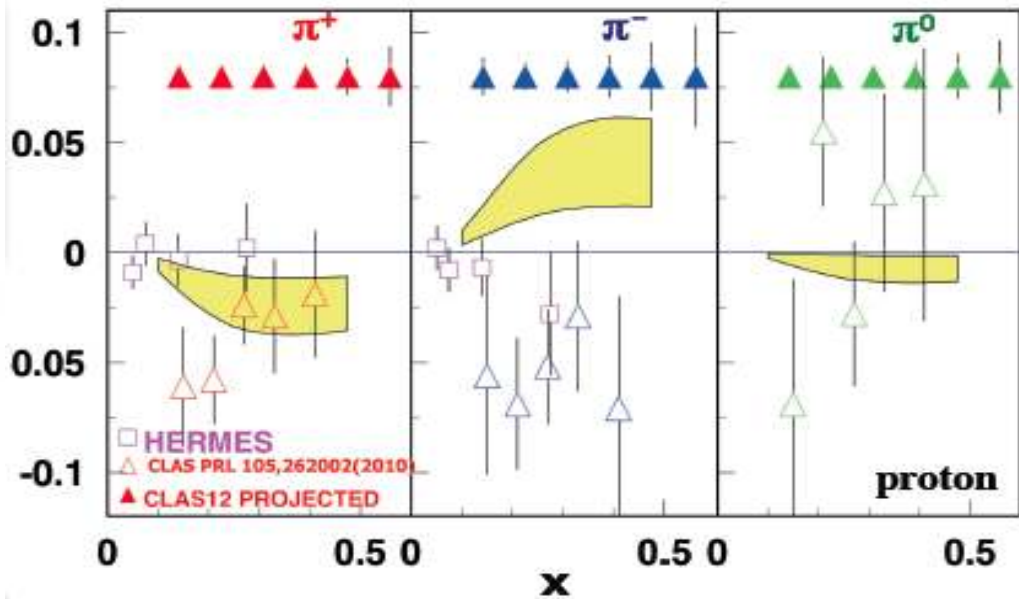
E12-07-107: Studies of Spin-Orbit Correlations with Longitudinally Polarized Target

$N \backslash q$	U	L	T
L		g_{1L}	h_{1L}^\perp



$$\frac{d\sigma}{dx dy d\phi_S d\phi_h dP_{h,\perp}^2} \propto S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \epsilon \sin(2\phi_h) F_{UL}^{\sin(2\phi_h)} \right] + S_L \lambda_\epsilon \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_h) F_{LL}^{\cos(\phi_h)} \right]$$

Annotations: $h.t.$ (highlighted in red circles) is placed above the first term and below the second term. $h_{1L}^\perp \otimes H_1^\perp$ is circled in red above the first term. $g_{1L} \otimes D_1$ is circled in red below the second term.

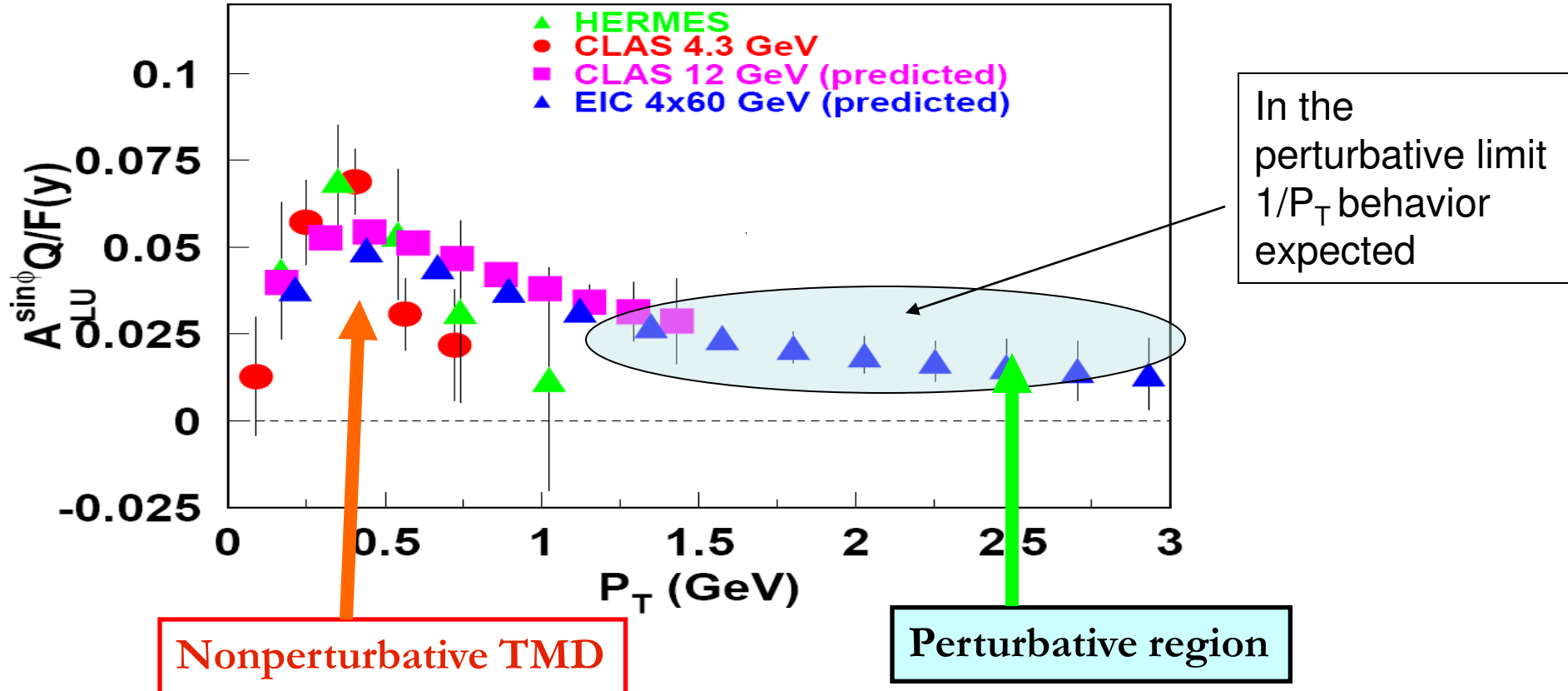


- A_1 P_T -dependence provides access to helicity dependence of k_T -distributions of quarks
- p & d data required for P_T -dependence flavor decomposition

P_T -dependence of beam SSA

$$\sigma^{\sin\phi}_{LU(UL)} \sim F_{LU(UL)} \sim 1/Q \text{ (Twist-3)}$$

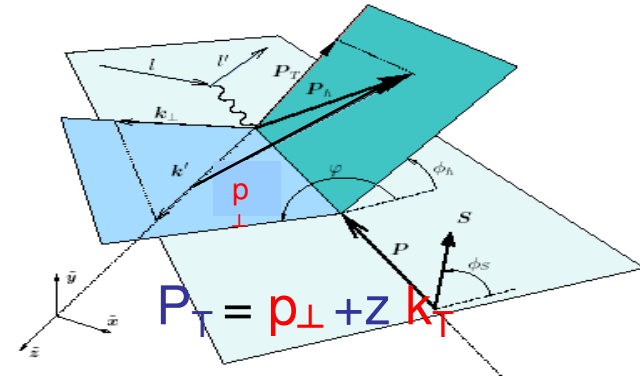
$$A_{LU} \propto g^\perp(x) D_1(z)$$



Study for SSA transition from non-perturbative to perturbative regime.
EIC will significantly increase the P_T range.

FAST-MC for CLAS12

SIDIS MC in 8D $(x, y, z, \phi, \phi_S, p_T, \lambda, \pi)$

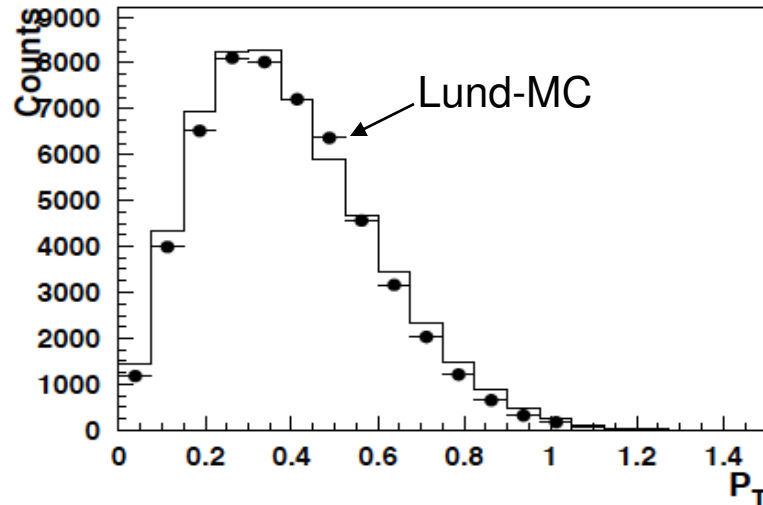
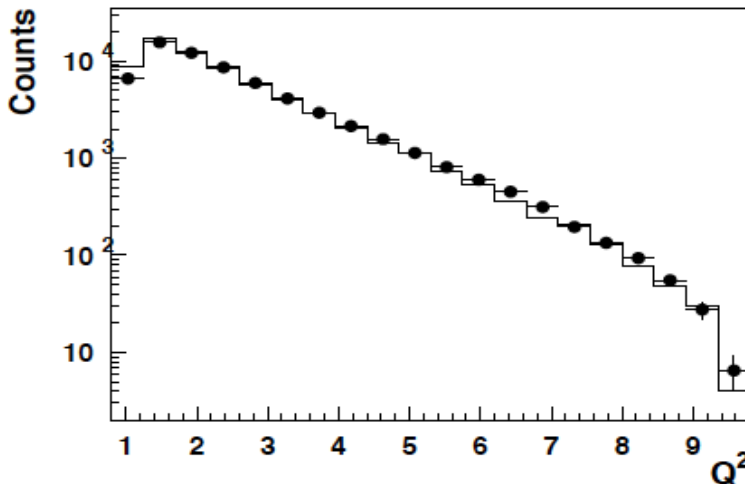


Simple model with 10% difference between f_1 (0.2GeV^2) and g_1 widths with a fixed width for D_1 (0.14GeV^2)

CLAS12 acceptance & resolutions

$$f_q(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

Events in CLAS12

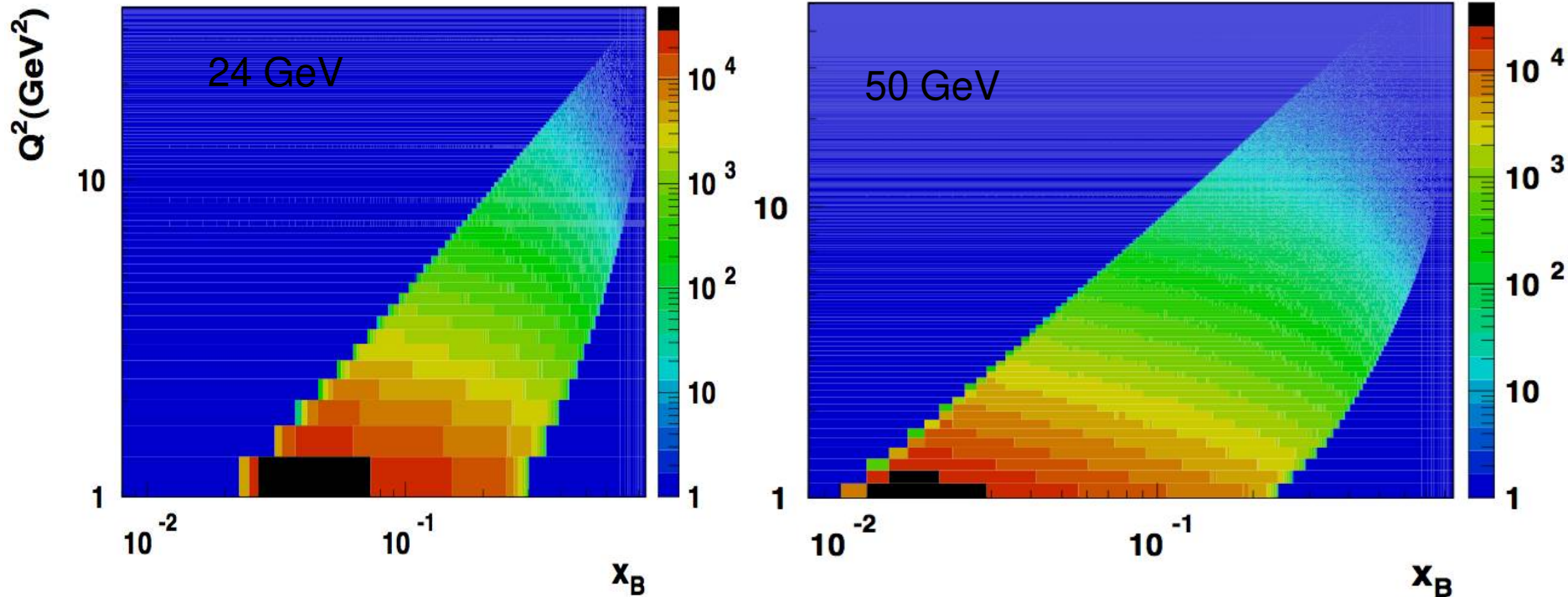


Reasonable agreement of kinematic distributions with realistic LUND

simulation

H. Avakian, QCD-N'12, Oct 23

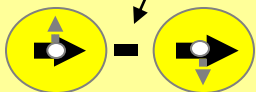
$ep \rightarrow e' \pi^+ X$ Kinematic coverage



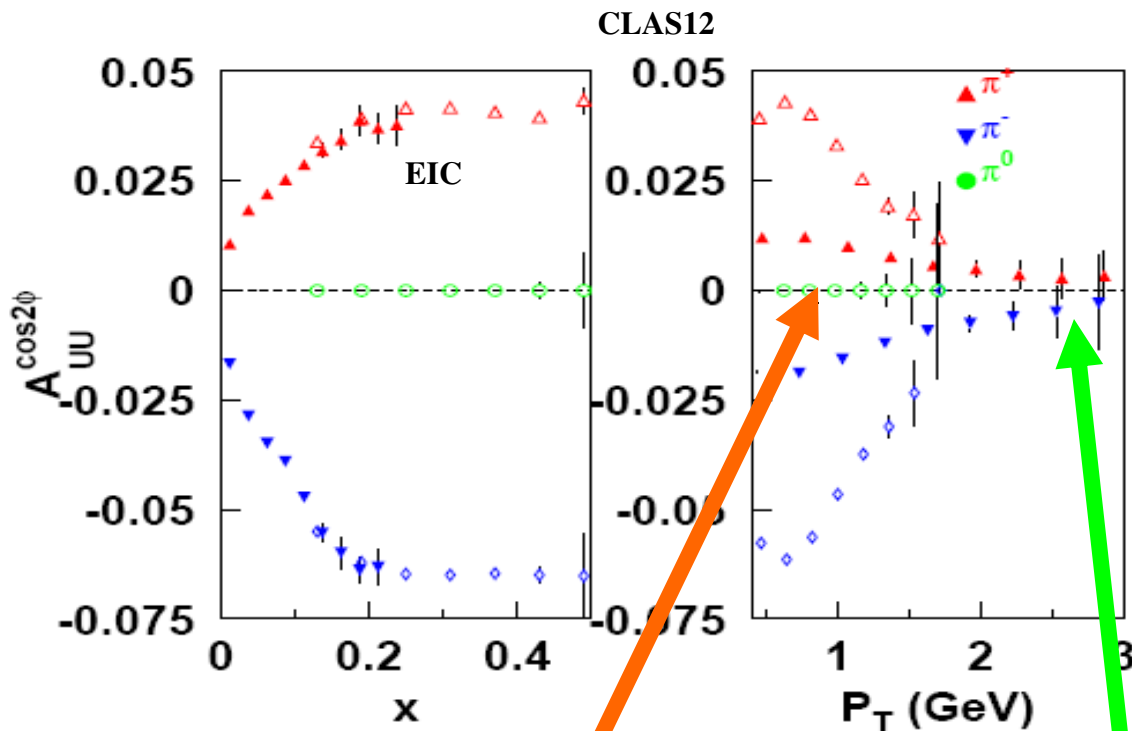
For a given lumi (30min of runtime with $L=10^{35} \text{cm}^{-2} \text{s}^{-1}$) and given bin in hadron z and P_T , higher energy provides higher counts and wider coverage in x and Q^2

Boer-Mulders Asymmetry with CLAS12 & EIC

Z/A	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}^\perp	$h_{1T}^\perp, h_{1TT}^\perp$



Transversely polarized quarks in the unpolarized nucleon



$$\sin(\phi_C) = \cos(2\phi_h)$$

$$A_{UU}^{\cos 2\phi} \propto h_1^\perp(x, k_T) H_1^\perp(x, k_T)$$

$$\langle \cos 2\phi \rangle |_{P_{h\perp} \gg \Lambda_{\text{QCD}}} \propto \frac{1}{P_{h\perp}^2}$$

Nonperturbative TMD

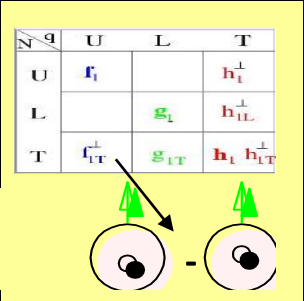
Perturbative region

Perturbative limit calculations available for $f_1(x, k_T), h_1^\perp(x, k_T)$

J.Zhou, F.Yuan, Z Liang: arXiv:0909.2238

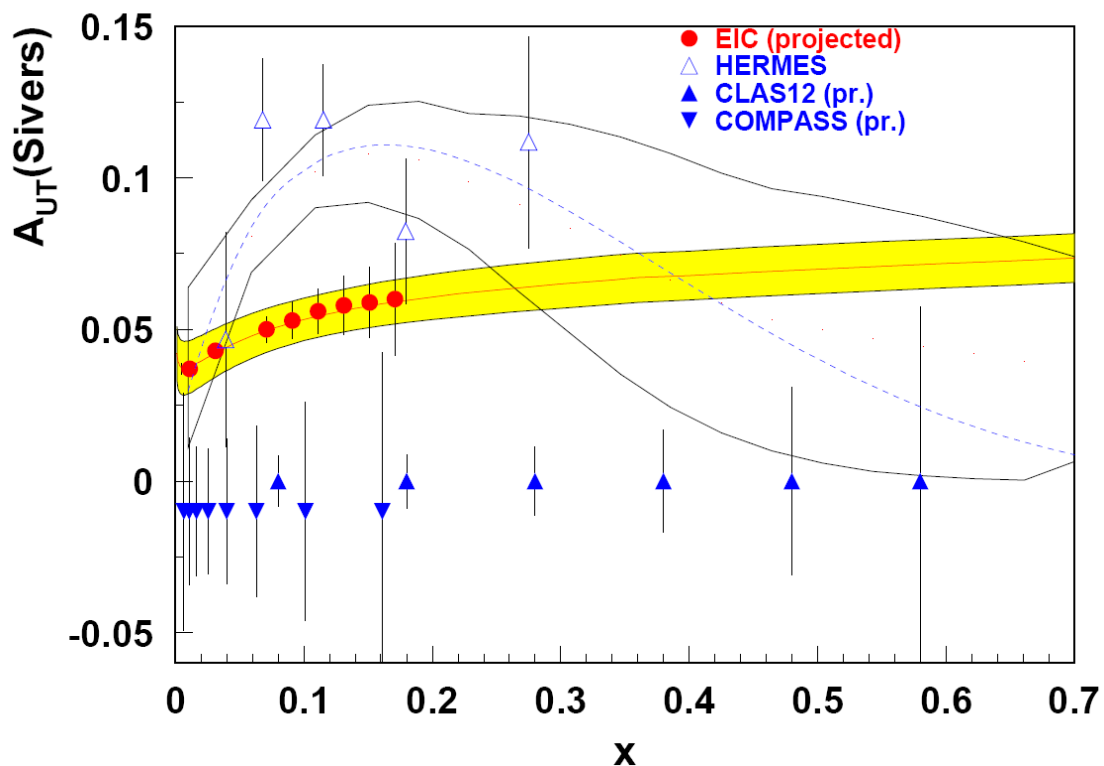
CLAS12 and EIC studies of transition from non-perturbative to perturbative regime will provide complementary info on spin-orbit correlations and test unified theory (Ji et al)

Sivers effect: Kaon electroproduction

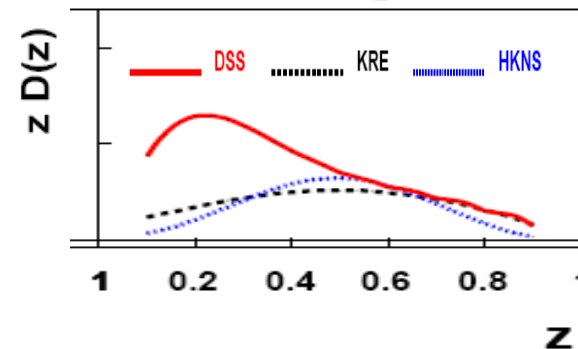
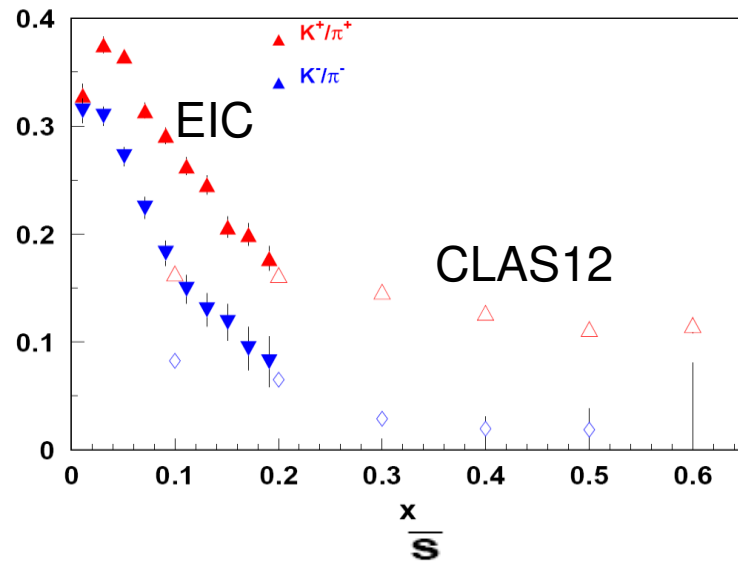


$$A_{UT}^{\sin(\phi-\phi_S)} = \frac{\sum_q e_q^2 f_{1T}^{\perp q} D_1^q}{\sum_q e_q^2 f_1^q D_1^q}$$

$ep \rightarrow e' K^+ + X$



$K^+ \sim u\bar{s}$ $K^- \sim s\bar{u}$



- At small x of EIC Kaon relative rates higher, making it ideal place to study the Sivers asymmetry in Kaon production (in particular K^-).
- Combination with CLAS12 data will provide almost complete x -range.

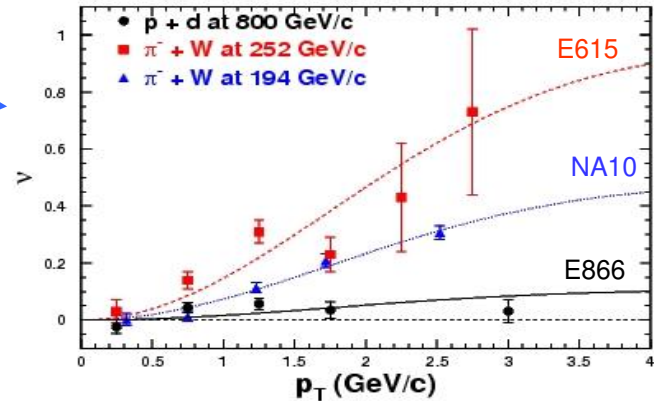
TMD Correlation Functions in other experiments

Z \ q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1^\perp h_{1T}^\perp

BOER-MULDERS
Spin Orbit effect

$$v \approx h_{1q}^\perp \times h_{1\bar{q}}^\perp$$

hp \rightarrow $\mu\mu X$



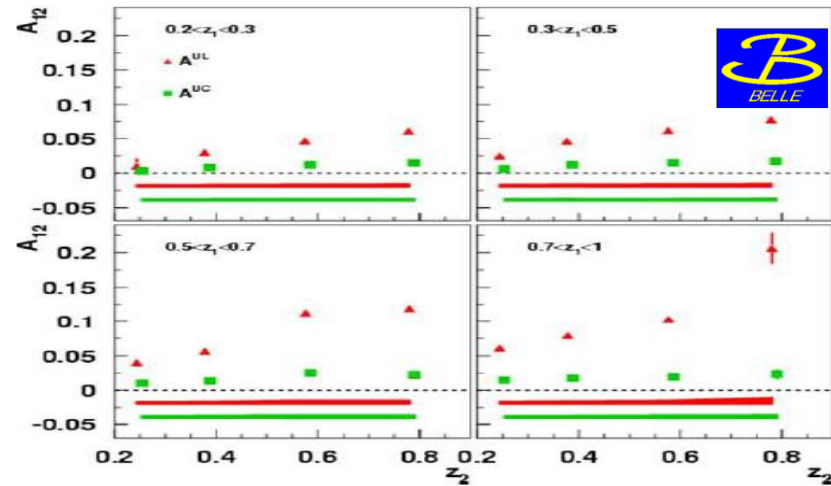
Fragmentation
Functions (FF)

q/h	U
U	D_1
T	H_1^\perp

COLLINS
Quark spin probe

$$A_{12} \approx H_{1q}^\perp \times H_{1\bar{q}}^\perp$$

ee \rightarrow $\pi\pi X$

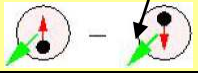


In di-hadron case H_1^\perp

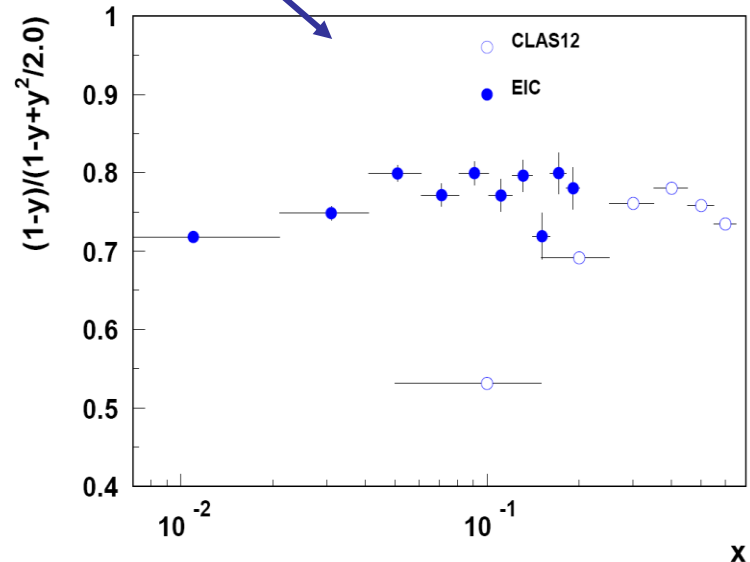
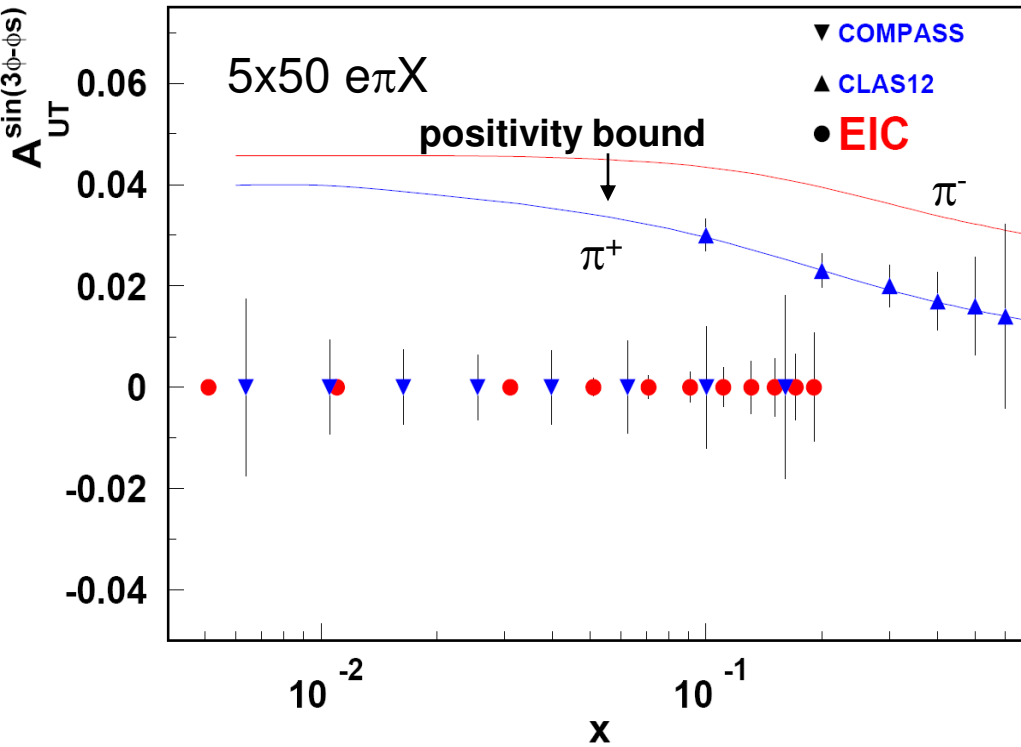
Interference Fragmentation Function (IFF)

Pretzelosity @ EIC

Z^q	U	L	T
U	f_1		h_{1T}^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_{1T} h_{1L}^\perp$

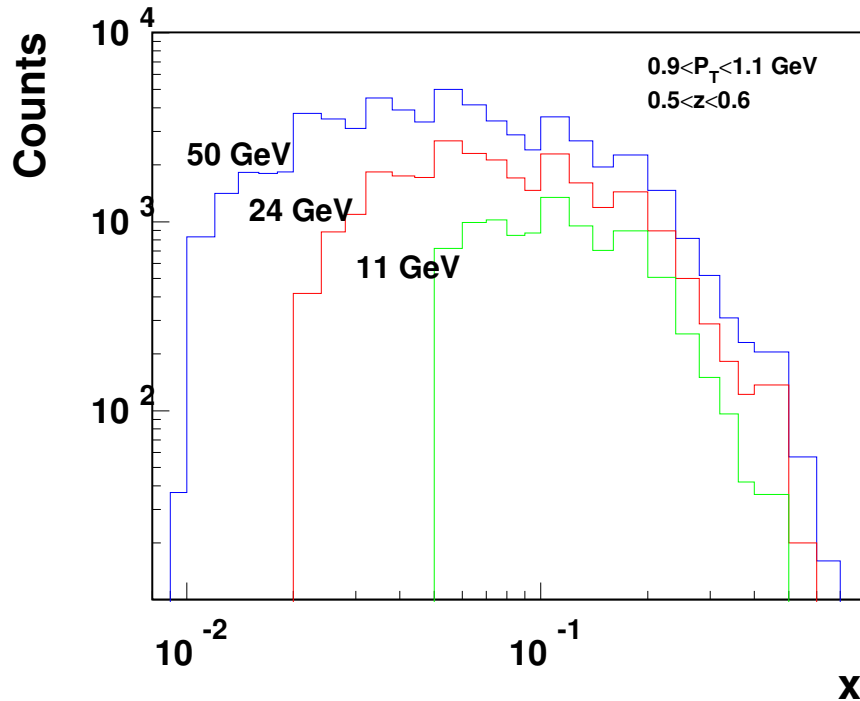


$$A_{UT}^{\sin(3\phi - \phi_S)} \propto \frac{1-y}{1-y+y^2/2} \frac{\sum_q e_q^2 h_{1T}^{\perp(1)q} H_1^{\perp q}}{\sum_q e_q^2 f_1^q D_1^q}$$

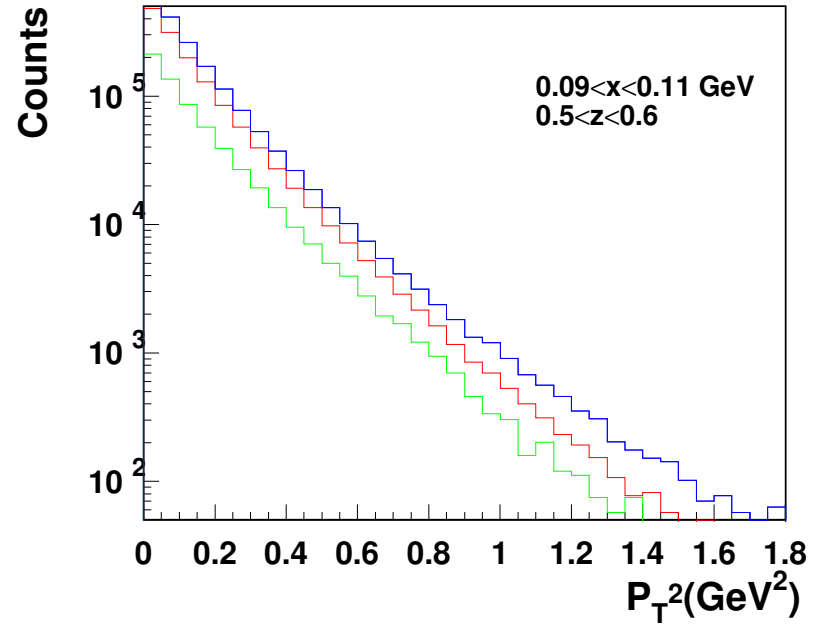


• EIC measurement combined with CLAS12 will provide a complete kinematic range for pretzelosity measurements

$ep \rightarrow e' \pi^+ X$ Kinematic coverage



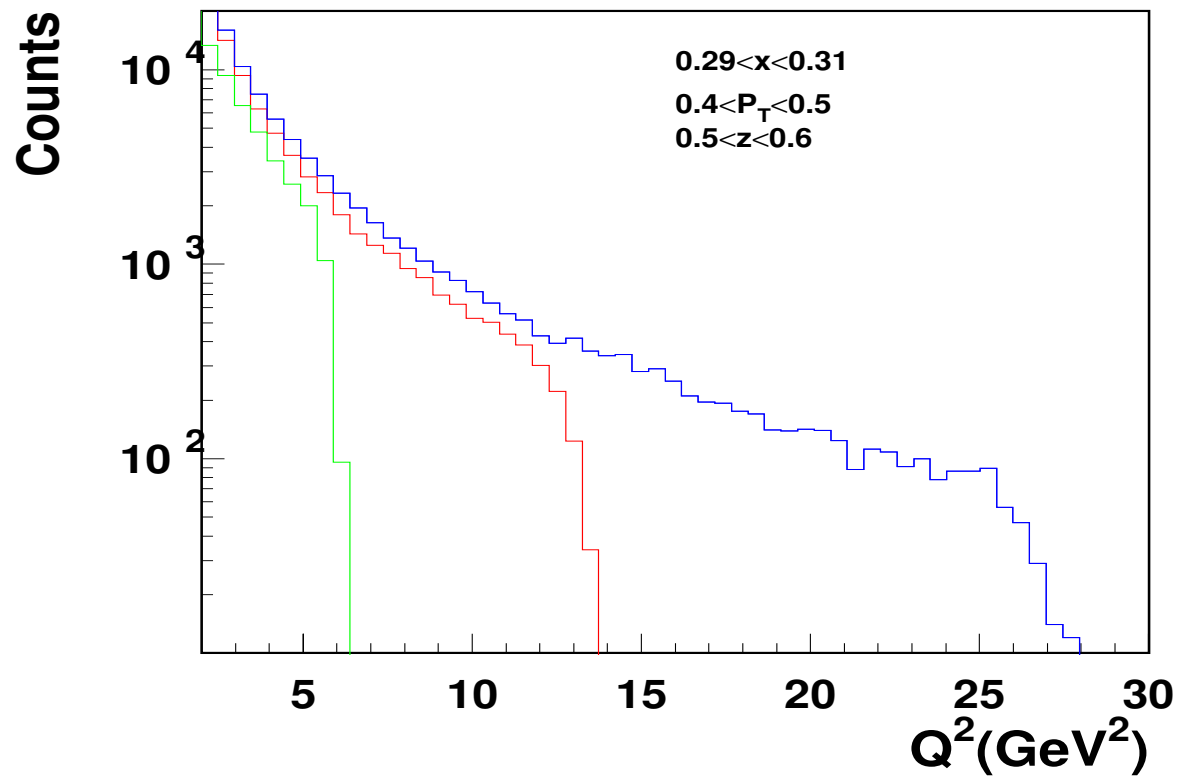
Wider x range allow studies of transverse distributions of sea quarks and gluons



Wider P_T range will be important in extraction of k_T -dependences of PDFs

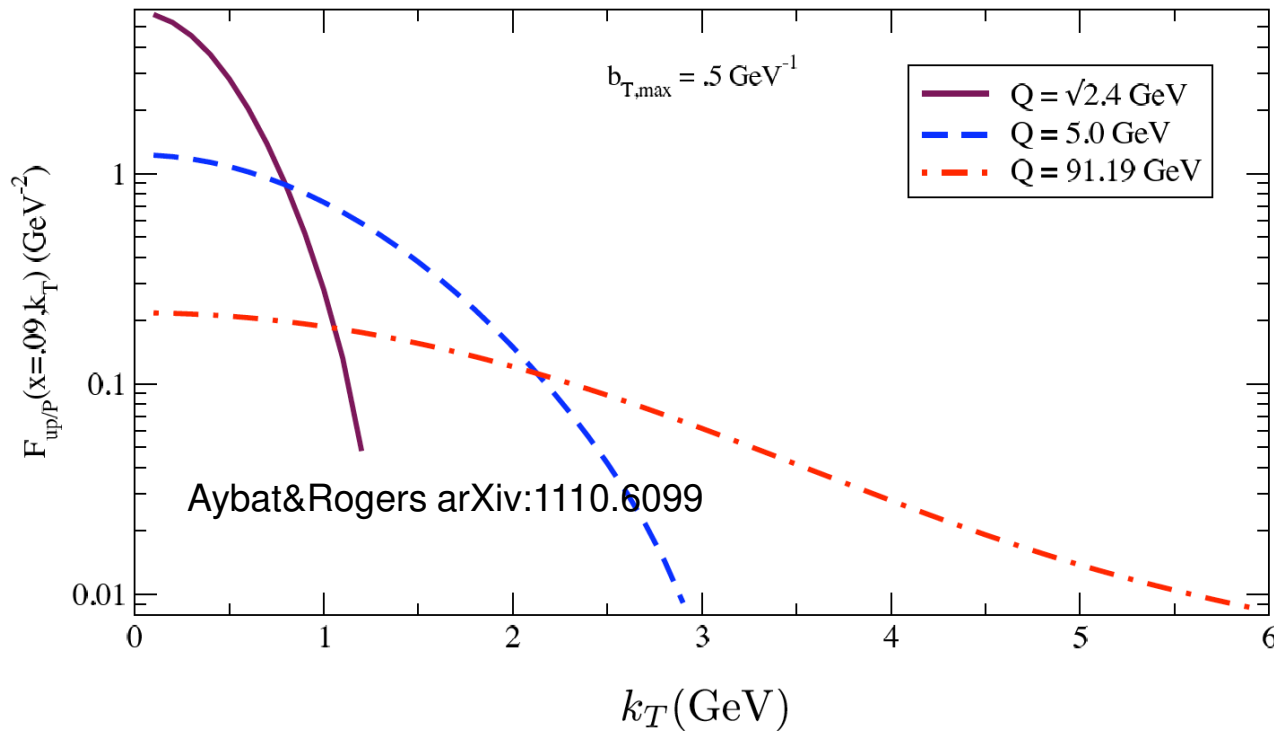
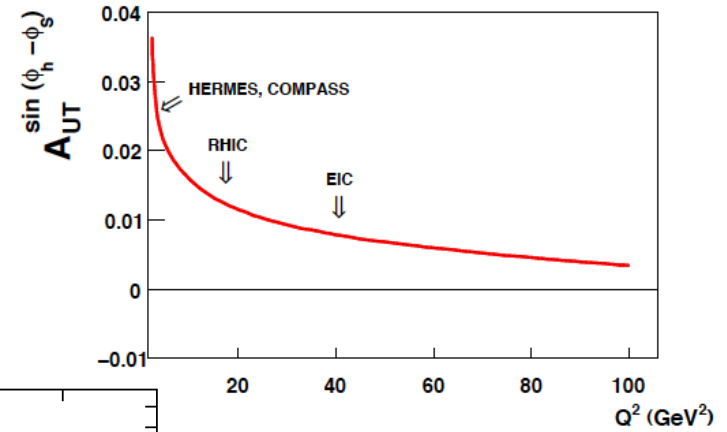
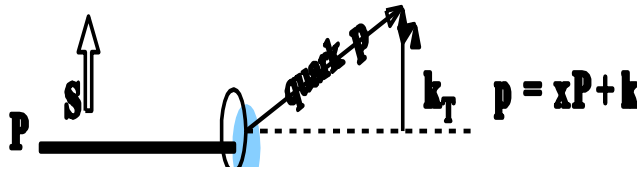
For a given lumi (30min of runtime with 10^{35}) and given bin in hadron z and P_T , higher energy provides higher counts and wider coverage in x and P_T to allow studies of correlations between longitudinal and transverse degrees of freedom

$ep \rightarrow e' \pi^+ X$ Kinematic coverage



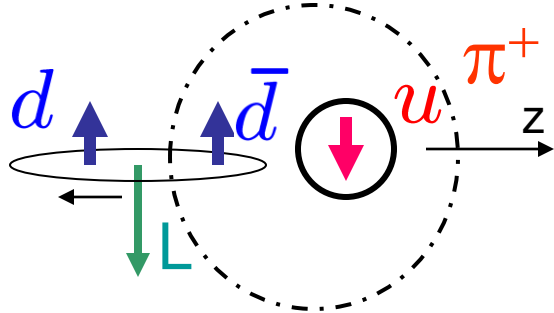
For a given lumi (30min of runtime) and given bin in hadron z and P_T , higher energy provides higher counts and wider coverage in Q^2 , allowing studies of Q^2 evolution of 3D partonic distributions in a wide Q^2 range.

Evolving TMD PDFs



Collins effect

Simple string fragmentation (Artru model)

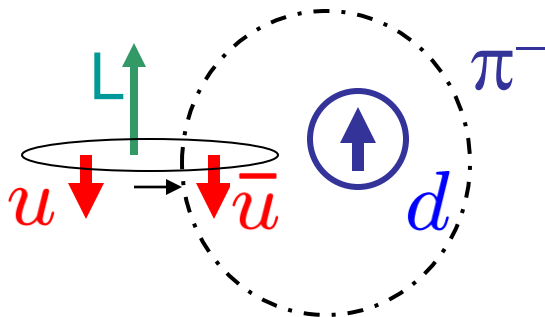


Leading pion out of page (\bar{d} - direction)

$$h_1 H_1^\perp u \rightarrow \pi^+$$



d kicked in the opposite to the leading pion (into the page)



Sub-leading pion opposite to leading (double kick into the page)

$$H_1^\perp u \rightarrow \pi^- > H_1^\perp u \rightarrow \pi^+$$

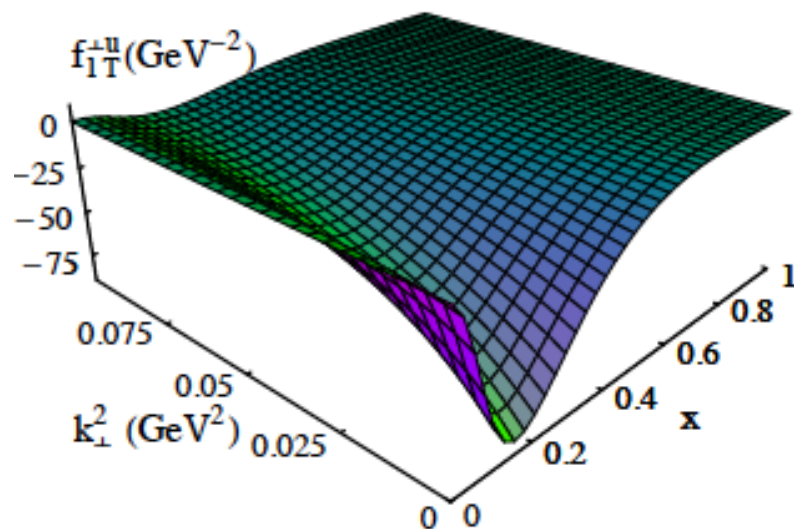
If unfavored Collins fragmentation dominates measured π^- vs π^+ , why K^- vs K^+ is different?

SIDIS ($\gamma^* p \rightarrow \pi X$) : Transversely polarized target

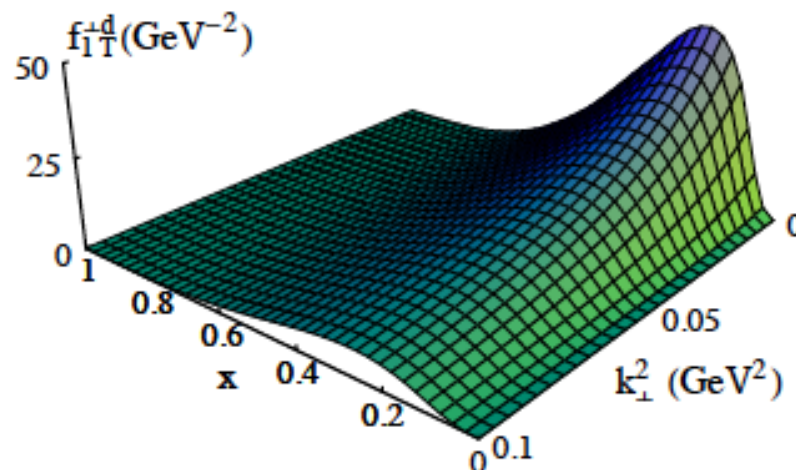
Azimuthal moments in pion production in SIDIS

- $\sin(\phi - \phi_S)$ (Sivers function f_{1T}^\perp) and relation with GPDs
- $\sin(\phi + \phi_S)$ (Collins function H_1^\perp and transversity h_1)
- $\sin(3\phi - \phi_S)$ (Collins function H_1^\perp and pretzelosity h_{1T}^\perp)

$N \backslash q$	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



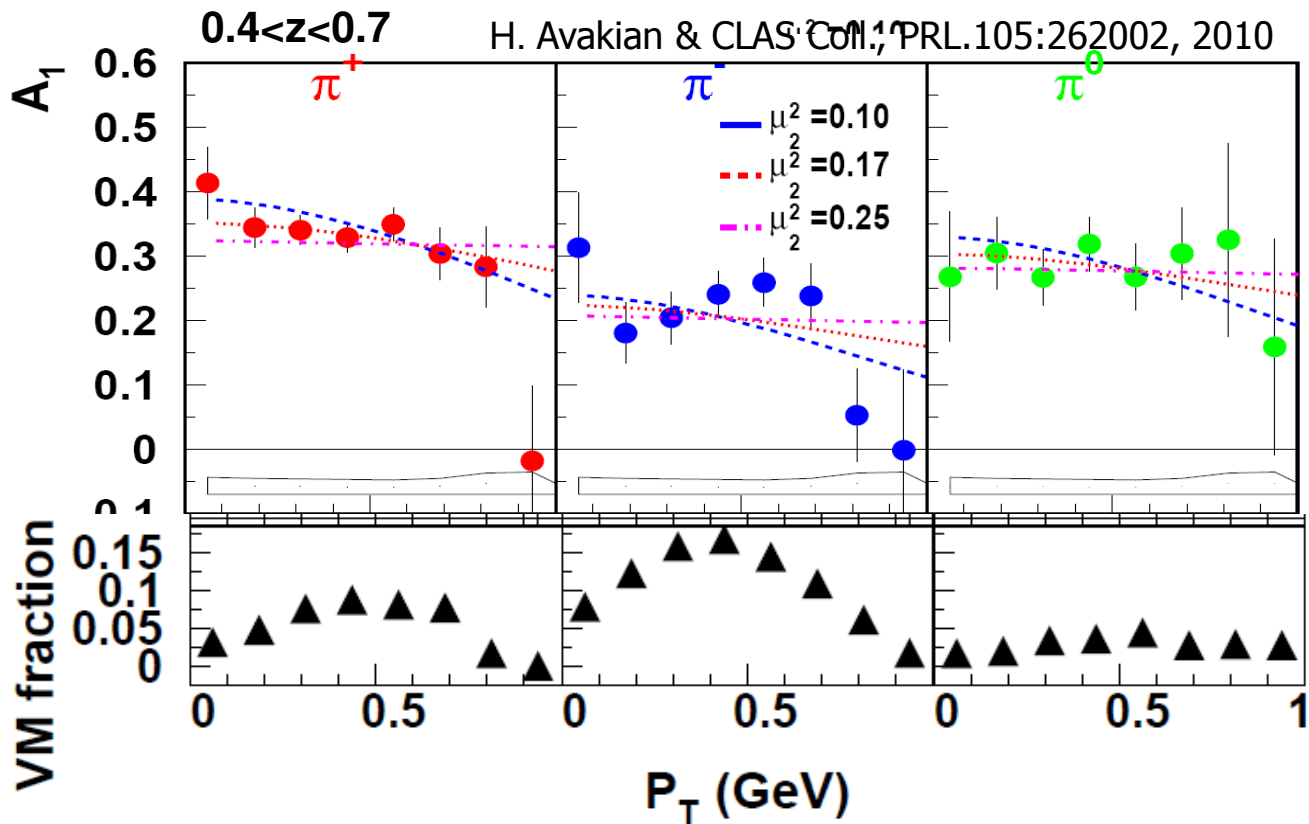
Pasquini and Yuan, Phys.Rev.D81:114013,2010



$A_1 - P_T$ dependence

Z/q	U	L	T
U	f_1	g_1	h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

$$A_1(x, z, P_T) = A_1(x, z) \frac{\langle P_T^{2,unp} \rangle}{\langle P_T^{2,pol} \rangle} \exp(-P_T^2 / \langle P_T^{2,pol} \rangle - P_T^2 / \langle P_T^{2,unp} \rangle)$$



M. Anselmino et al PRD74:074015, 2006

$$f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

$$\langle P_T^2(z) \rangle = z^2 \mu_{0/2}^2 + \mu_D^2$$

π^+ A_1 suggests broader k_T distributions for f_1 than for g_1

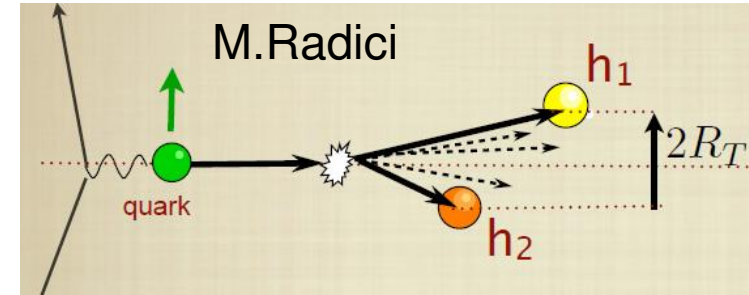
π^- A_1 may require non-Gaussian k_T -dependence for different helicities and/or flavors

HT-distributions and dihadron SIDIS

Compare single hadron and dihadron SSAs

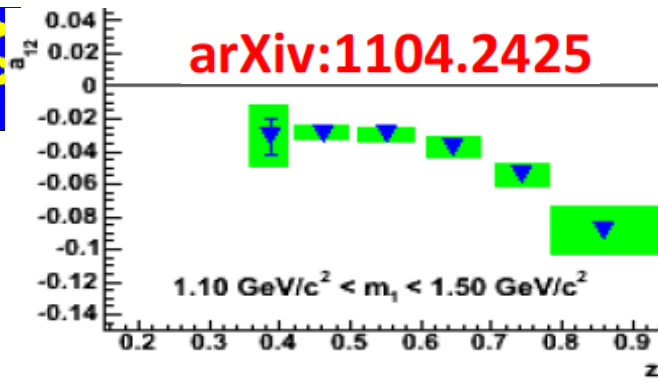
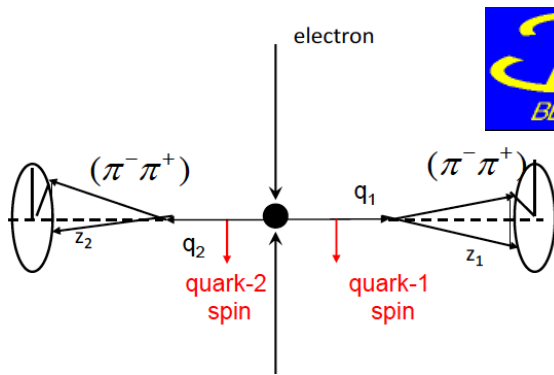
$$\frac{M}{M_h} x e(x) H_1^{\triangleleft}(z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^{\triangleleft}(z, \zeta, M_h^2)$$

$$\frac{M}{M_h} x h_L(x) H_1^{\triangleleft}(z, \zeta, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}^{\triangleleft}(z, \zeta, M_h^2)$$

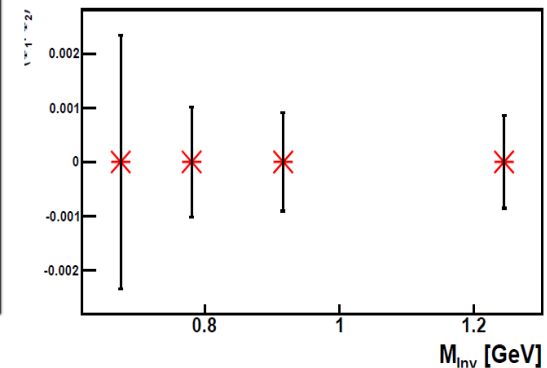


Only 2 terms with common unknown HT $\tilde{G} \sim$ term!

Aurore Courtoy/Anselm Voosen - Spin session



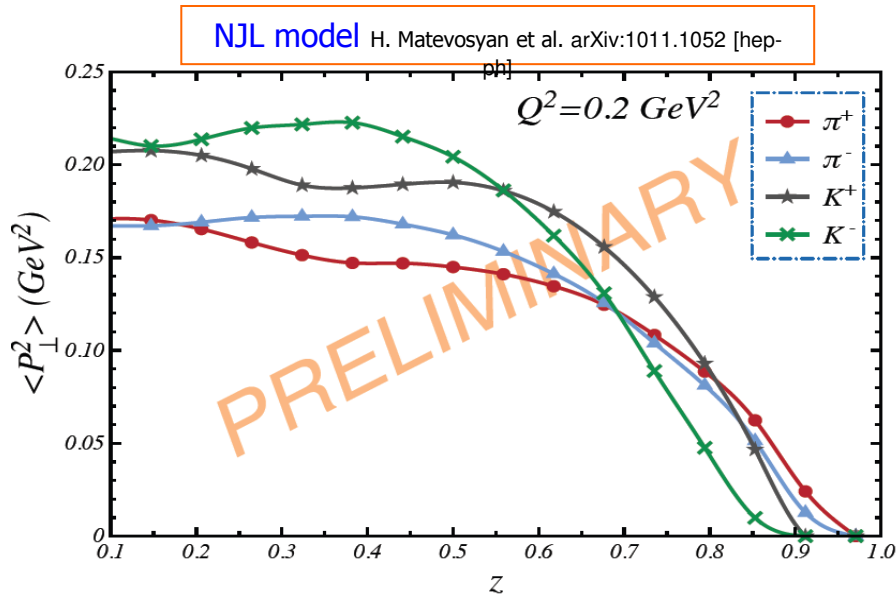
Projections for (π^+K^-) ($K^+\pi^-$) for 580 fb^{-1}



• Higher twists in dihadron SIDIS collinear (no problem with factorization)

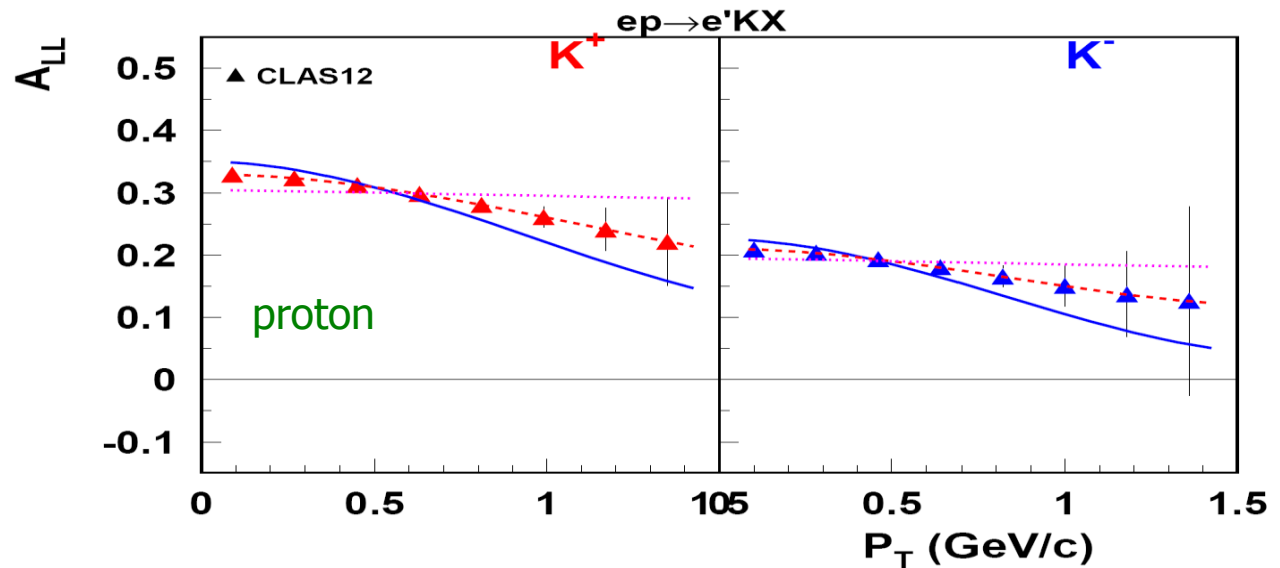
• Bell can measure $K^+\pi^-$ dihadron fragmentation functions

Transverse momentum distributions of partons



$$\langle P_T^2 \rangle \approx z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle$$

Transverse momentum distributions in hadronization may be flavor dependent
 \Rightarrow measurements of different final state hadrons required



Collins effect: from asymmetries to distributions

$Z \backslash q$	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1^\perp, h_{1T}^\perp

need H_1^\perp

$$F \equiv \sigma_{UL}^{\sin 2\phi}, \sigma_{UU}^{\cos 2\phi}, \dots$$

$$\frac{H_1^{u/K+} - H_1^{u/K-}}{H_1^{u/\pi+} - H_1^{u/\pi-}} = \frac{15}{4} \frac{F_p^{K+} - F_p^{K-}}{3(F_p^{\pi+} - F_p^{\pi-}) + (F_d^{\pi+} - F_d^{\pi-})}$$

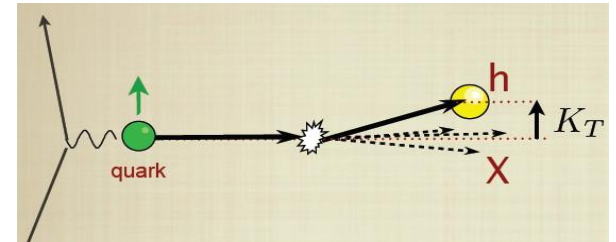
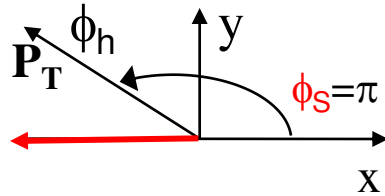
Combined analysis of Collins fragmentation asymmetries from proton and deuteron and for π and K may provide independent to e^+e^- (BELLE/BABAR) information on the underlying Collins function.

Chiral odd HT-distribution

How can we separate the HT contributions?

$$F_{LU}^{\sin \phi}$$

$$F_{UL}^{\sin \phi}$$

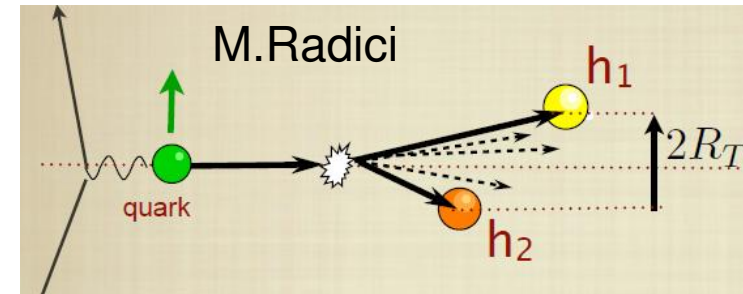


$$e H_1^\perp, h_L H_1^\perp \sin \phi_h$$

HT function related to force on the quark. M.Burkardt (2008)

Compare single hadron and dihadron SSAs

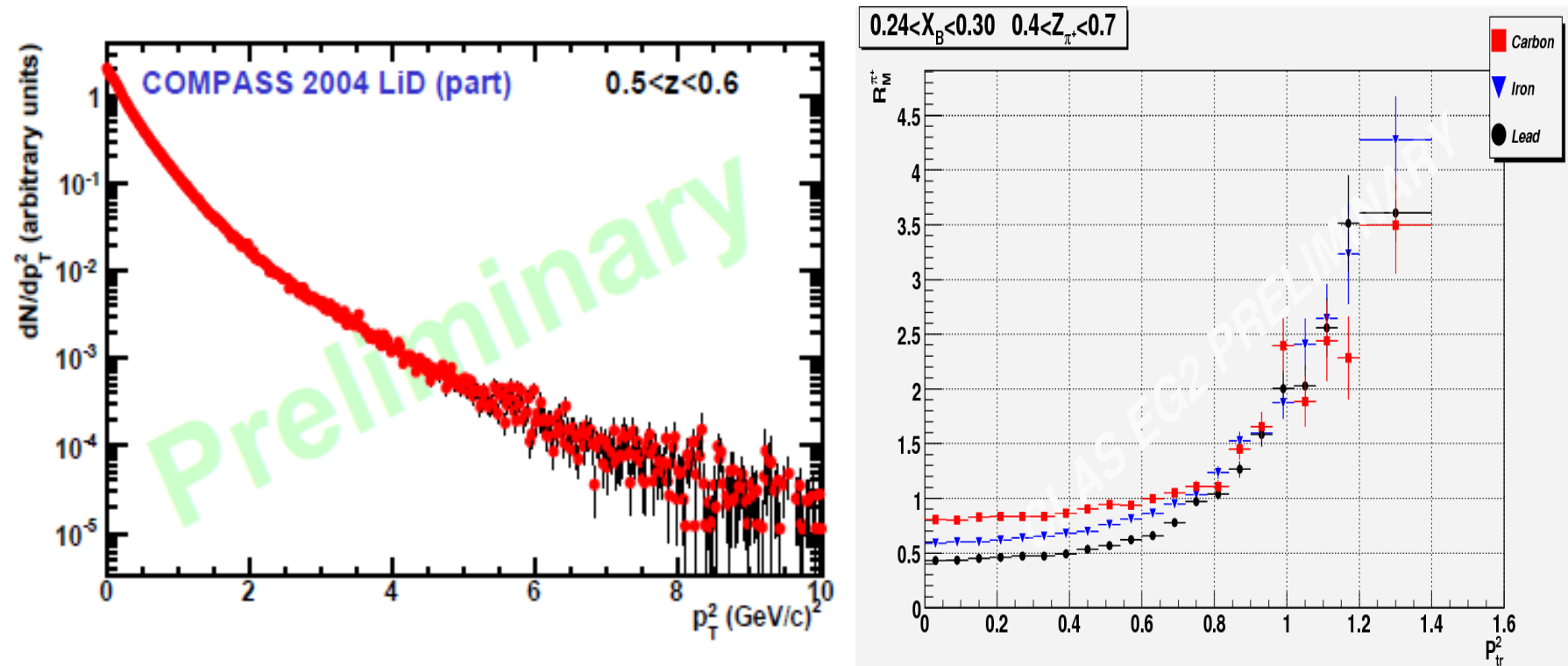
$$\frac{M}{M_h} x e(x) H_1^\triangleleft(z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^\triangleleft(z, \zeta, M_h^2)$$



Only 2 terms with common unknown HT \tilde{G} term!

$$\frac{M}{M_h} x h_L(x) H_1^\triangleleft(z, \zeta, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}^\triangleleft(z, \zeta, M_h^2)$$

Nuclear broadening Hadronic PT-distributions



Large PT may have significant nuclear contribution

Azimuthal moments with unpolarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 h_{1T}^\perp$

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{UU}^{\cos \phi} \propto \frac{M_h}{M} f_1 \frac{D^\perp}{z} - \frac{M}{M_h} x f^\perp D_1$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	$H_1 H_{1T}^\perp$

Azimuthal moments with unpolarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1^\perp, h_{1T}^\perp

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{UU}^{\cos\phi} \sim -h_1^\perp \frac{H}{z} + xhH_1^\perp$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	H_1, H_{1T}^\perp

SSA with unpolarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 h_{1T}^\perp$

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{LU}^{\sin \phi} \propto \frac{M_h}{M} f_1 \frac{G^\perp}{z} - \frac{M}{M_h} x g^\perp D_1$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	$H_1 H_{1T}^\perp$

SSA with unpolarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1^\perp, h_{1T}^\perp

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{LU}^{\sin \phi} \sim h_1^\perp \frac{E}{z} + xe H_1^\perp$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	H_1, H_{1T}^\perp

SSA with long. polarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 h_{1T}^\perp$

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{UL}^{\sin \phi} \propto \frac{M_h}{M} g_1 \frac{G^\perp}{z} + \frac{M}{M_h} x f_L^\perp D_1$$

q/h	U	L	T
U	D_1^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	$H_1 H_{1T}^\perp$

SSA with long. polarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 h_{1T}^\perp$

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{UL}^{\sin \phi} \sim h_{1L}^\perp \frac{H}{z} + x h_L H_1^\perp$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	$H_1 H_{1T}^\perp$

SSA with unpolarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 h_{1T}^\perp$

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{LL}^{\cos \phi} \sim \frac{M_h}{M} g_{1L} \frac{D^\perp}{z} + x e_L H_1^\perp$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	$H_1 H_{1T}^\perp$

SSA with unpolarized target

quark polarization

N/q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1^\perp, h_{1T}^\perp

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{LL}^{\cos \phi} \sim \frac{M_h}{M} h_{1L}^\perp \frac{E}{z} + x g_L^\perp D_1$$

q/h	U	L	T
U	D^\perp	D_L^\perp	D_T, D_T^\perp
L	G^\perp	G_L^\perp	G_T, G_T^\perp
T	H, E	H_L, E_L	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	D_1		D_{1T}^\perp
L		G_{1L}	G_{1T}^\perp
T	H_1^\perp	H_{1L}^\perp	H_1, H_{1T}^\perp

Twist-3 PDFs : “new testament”

N/q	U	L	T
U	f^\perp	g^\perp	h, e
L	f_L^\perp	g_L^\perp	h_L, e_L
T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

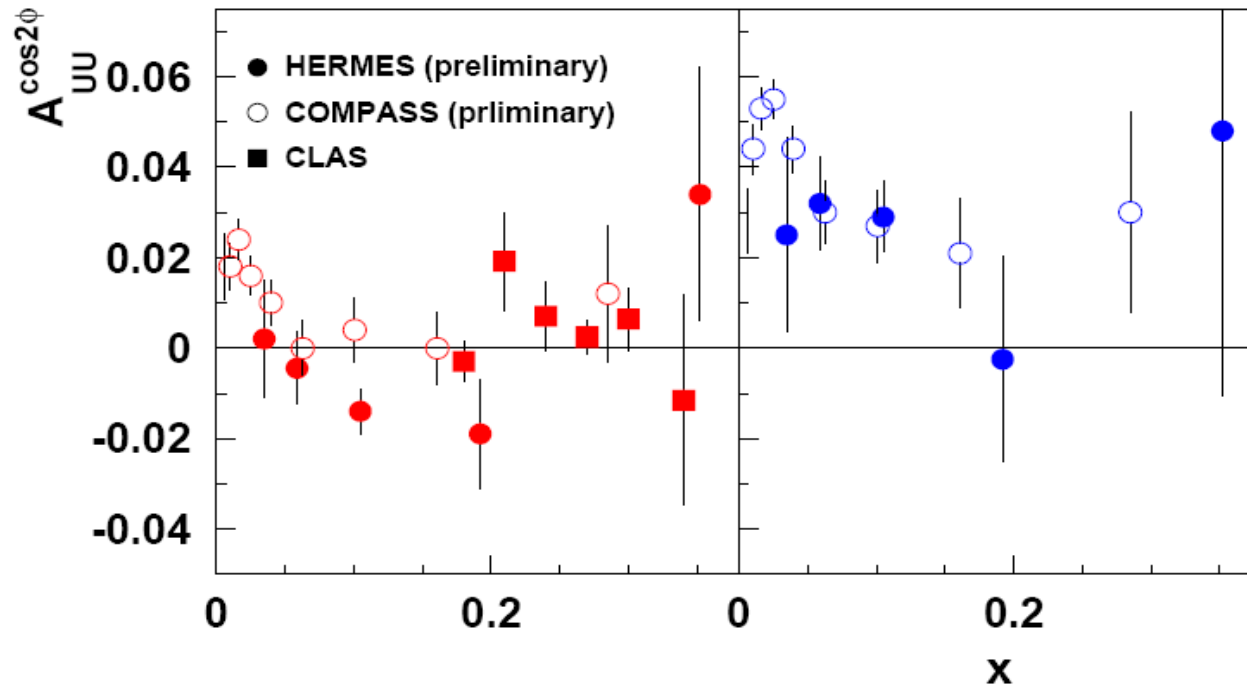
$$\begin{aligned} \frac{1}{2Mx} \text{Tr} [\tilde{\Phi}_{A\alpha} \sigma^{\alpha+}] &= \tilde{h} + i\tilde{e} + \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} (\tilde{h}_T^\perp - i\tilde{e}_T^\perp), \\ \frac{1}{2Mx} \text{Tr} [\tilde{\Phi}_{A\alpha} i\sigma^{\alpha+} \gamma_5] &= S_L (\tilde{h}_L + i\tilde{e}_L) - \frac{p_T \cdot S_T}{M} (\tilde{h}_T + i\tilde{e}_T), \\ \frac{1}{2Mx} \text{Tr} [\tilde{\Phi}_{A\rho} (g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho} \gamma_5) \gamma^+] &= \frac{p_T^\alpha}{M} (\tilde{f}^\perp - i\tilde{g}^\perp) - \epsilon_T^{\alpha\rho} S_{T\rho} (\tilde{f}_T + i\tilde{g}_T) \\ &\quad - S_L \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M} (\tilde{f}_L^\perp + i\tilde{g}_L^\perp) - \frac{p_T^\alpha p_T^\rho - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} \epsilon_{T\rho\sigma} S_T^\sigma (\tilde{f}_T^\perp + i\tilde{g}_T^\perp), \end{aligned}$$

SIDIS ($\gamma^*p \rightarrow \pi X$) x-section at leading twist

$$\frac{d\sigma}{dx dy dz d^2\vec{P}_h} = \frac{4\pi\alpha^2 s}{Q^4} [x(1-y + y^2/2)F_{UU} \xrightarrow{\text{TMD PDFs}} f_1 D_1$$

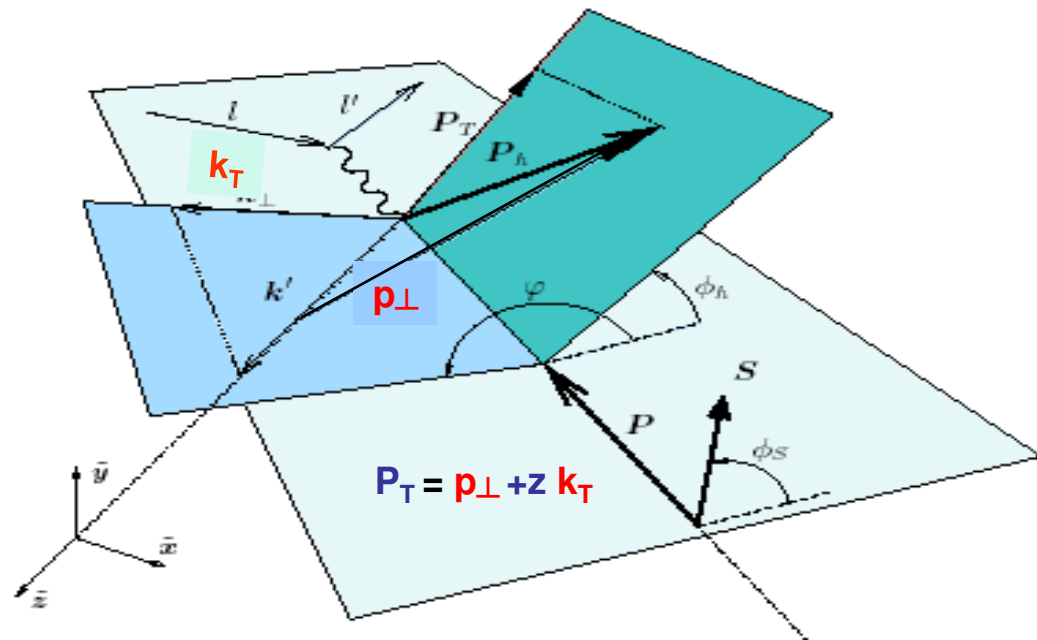
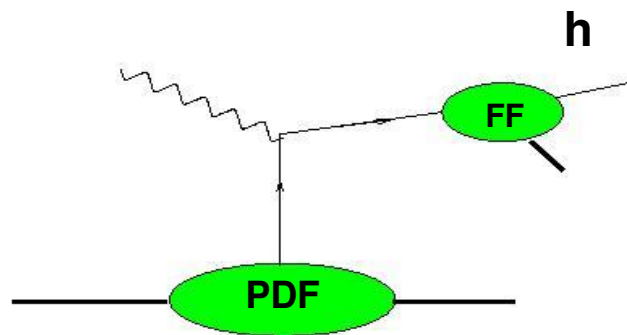
$$-x(1-y)\cos(2\phi)F_{UU}^{\cos 2\phi}] \xrightarrow{h^+} h_1^\perp H_1^\perp$$

h^+ h^-



- Measure Boer-Mulders distribution functions and probe the polarized fragmentation function
- Measurements from different experiments consistent

SIDIS: partonic cross sections



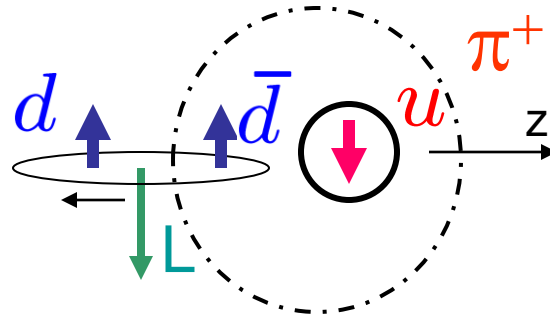
$$d\sigma^h \propto \sum f^{H \rightarrow q}(x, k_T) \otimes d\sigma_q(y) \otimes D^{q \rightarrow h}(z, p_\perp)$$



$$d\sigma^h \propto \sum f^{H \rightarrow q}(x) d\sigma_q(y) D^{q \rightarrow h}(z)$$

Collins effect

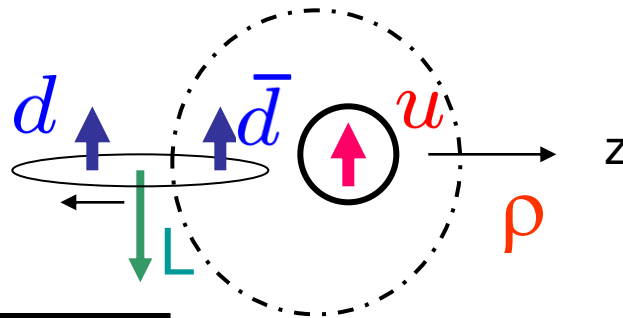
Simple string fragmentation for pions (Artru model)



leading pion out of page

$$h_1 H_1^\perp u \rightarrow \pi$$

ρ production may produce an opposite sign A_{UT}



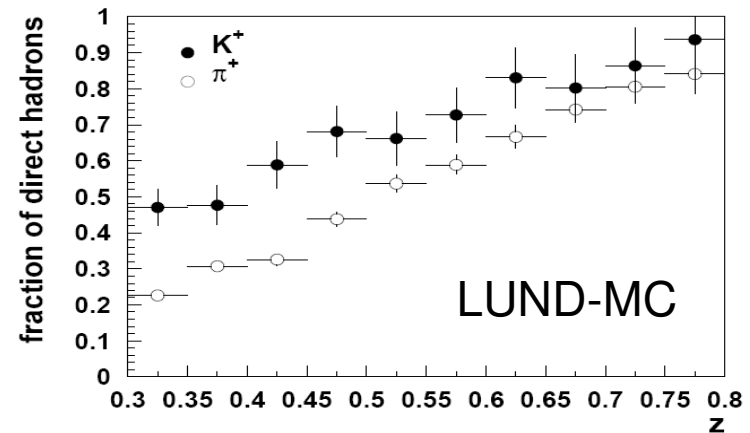
Leading ρ opposite to leading π (into page)

$$H_1^\perp u \rightarrow \rho \sim -\frac{1}{3} H_1^\perp u \rightarrow \pi$$

hep-ph/9606390

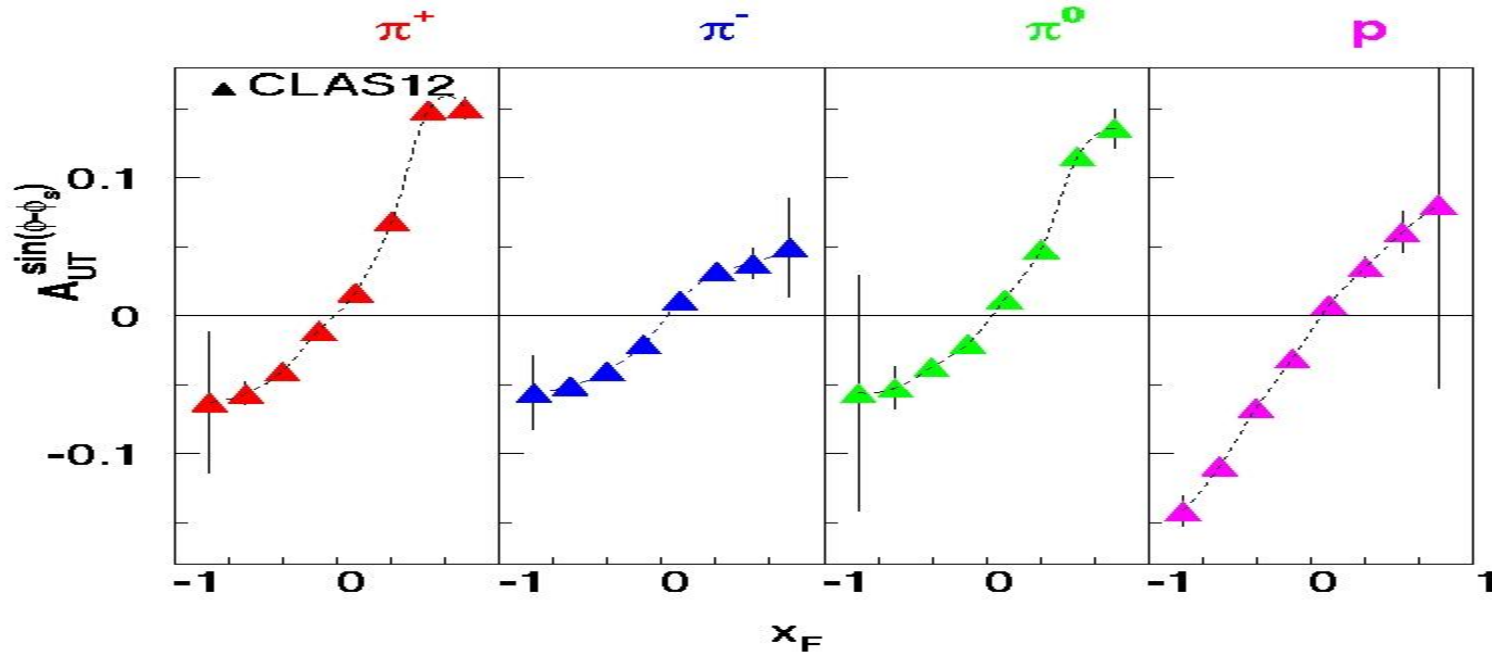
Fraction of ρ in $e\pi X$	% left from $e\pi X$ asm
20%	~75%
40%	~50%

Fraction of direct kaons may be significantly higher than the fraction of direct pions.



Sivers effect in the target fragmentation

A.Kotzinian



High statistics of **CLAS12** will allow studies of kinematic dependences of the Sivers effect in target fragmentation region

$$q(x, k_\perp)|_{k_\perp \gg \Lambda_{\text{QCD}}} = \frac{1}{(k_\perp^2)^n} \int \frac{dx'}{x'} f_i(x') \times \mathcal{H}_{q/i}(x; x'), \quad (23)$$

where $q(x, k_\perp)$ represents the TMD quark distribution we are interested, f_i represents the integrated quark distribution for the k_\perp -even TMDs, and higher twist quark-gluon correlation function for the k_\perp -odd TMDs. For the latter case, x' should be understood as two variable for the twist-three quark-gluon correlation functions as we discussed in the last section. The overall power behavior $1/(k_\perp^2)^n$ can be analyzed by the power counting rule [48]. The hard coefficient $\mathcal{H}_{q/i}(x; x')$ is calculated from perturbative QCD. In this paper, we will show the one-gluon radiation contribution to this hard coefficient.

The k_\perp -even TMD quark distribution functions, $f_1(x, k_\perp)$, $g_{1L}(x, k_\perp)$, and $h_1(x, k_\perp)$ be calculated from the associated integrated quark distributions [23]³. For the non-s contributions, they are expressed as [23],

$$\begin{aligned} f_1(x_B, k_\perp) &= \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} f_1(x) \left[\frac{1 + \xi^2}{(1 - \xi)_+} + \delta(1 - \xi) \left(\ln \frac{x_B^2 \zeta^2}{\vec{k}_\perp^2} - 1 \right) \right], \\ g_{1L}(x_B, k_\perp) &= \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} g_{1L}(x) \left[\frac{1 + \xi^2}{(1 - \xi)_+} + \delta(1 - \xi) \left(\ln \frac{x_B^2 \zeta^2}{\vec{k}_\perp^2} - 1 \right) \right], \\ h_1(x_B, k_\perp) &= \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} f_1(x) \left[\frac{2\xi}{(1 - \xi)_+} + \delta(1 - \xi) \left(\ln \frac{x_B^2 \zeta^2}{\vec{k}_\perp^2} - 1 \right) \right], \end{aligned}$$

where the color factor $C_F = (N_c^2 - 1)/2N_c$ with $N_c = 3$, $\xi = x_B/x$ and $\zeta^2 = (2v \cdot P)^2/v^2$.

TMDs: QCD based predictions

Large-x limit

$N \backslash q$	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Brodsky & Yuan (2006)

$$f_{1T}^\perp \sim (1-x)^4$$

$$g_{1T}^\perp \sim (1-x)^4$$

$$h_1 \sim (1-x)^3$$

$$h_{1T}^\perp \sim (1-x)^5$$

Burkardt (2007)

Large- N_c limit (Pobilitza)

$$f_1^{\perp u} > 0, f_1^{\perp d} > 0$$

$$h_1^{\perp u} < 0, h_1^{\perp d} < 0$$

Do not change sign (isoscalar)

$$f_{1T}^{\perp u} < 0, f_{1T}^{\perp d} > 0$$

All others change sign
 $u \rightarrow d$ (isovector)