To Hedge or Not to Hedge:

Managing Demographic Risk in Life Insurance Companies

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Abstract

Demographic risk, i.e., the risk that life tables change in a nondeterministic way, is a serious threat to the financial stability of an insurance company having underwritten life insurance and annuity business. The inverse influence of changes in mortality laws on the market value of life insurance and annuity liabilities creates natural hedging opportunities. Within a realistically calibrated shareholder value maximization framework, we analyze the implications of demographic risk on the optimal risk management mix (equity capital, asset allocation, and product policy) for a limited liability insurance company operating in a market with insolvency-averse insurance buyers. Our results show that the utilization of natural hedging is optimal only if equity is scarce. Otherwise, hedging can even destroy shareholder value. A sensitivity analysis shows that a misspecification of demographic risk has severe consequences for both the insurer and the insured. This result highlights the importance of further research in the field of demographic risk.

Introduction

The financial performance of insurance companies writing life insurance and annuity policies is heavily dependent on possible deviations from the mortality assumptions made at the time the contracts were underwritten. Random deviations from mortality assumptions are therefore an important aspect of life insurance company risk management. Life tables used for pricing life insurance and annuity products typically incorporate an assumed future development (trend) of life expectancy improvement. Deviations from this development, on the one hand, can arise if the insurer's liability portfolio is too small to get the law of large numbers working fully. On the other hand—and this is the focus of our contribution—there is *demographic risk*, which we define as the risk that mortality laws and life tables themselves change in a nondeterministic way (see Olivieri, 2001). For example, imagine that a path-breaking technological or medical innovation leads to a sudden decrease in mortality at all ages. This would result in a severe deterioration of an annuity provider's solvency situation and a drop in its shareholder value, whereas a life insurance provider would benefit. Alternatively, an increase in the prevalence of obesity, which increases mortality at all ages (Swiss Re, 2004), would be beneficial for an annuity provider, but would result in a severe decline of shareholder value for a life insurance provider.[1] In either case, the inverse performance of annuity and term life insurance liabilities creates natural hedging opportunities (see Blake and Burrows, 2001).

That such issues of demographic risk are highly relevant is shown by the ongoing controversial scientific discussion on the future development of human life expectancy and the maximum possible lifespan.[2] Random deviations from presently assumed life tables are considered significantly probable.[3] However, despite its obvious importance, the risk management implications of demographic risk for a life insurer have received little attention

[1] As another example take the increased occurrence of very hot summers—comparable to the 2003 summer in Europe—as a consequence of a global climate change. They might diminish life expectancy considerably (see Valleron and Boumendil, 2004; Conti et al., 2005). Or, consider the uncertainty about the future degree of medical services rationing (see Schmidt, 2004), which will have an important impact on mortality, especially in countries with underfinanced governmental health sectors.

[2] Olshansky et al. (2002), e.g., claim that there is a biological limit to the maximum lifespan, which a few persons already reach. Robine and Vaupel (2002), however, argue that we will experience continuing lengthening of the maximum lifespan. Oeppen and Vaupel (2002) document that past prognoses of future life expectancy improvements almost always turned out too low.

[3] An example of the consequences of demographic risk is the recent introduction of new annuity tables, which forced German life insurers to make heavy adjustments to their reserves for the annuity contracts they had already sold (see Deutsche Aktuarvereinigung e.V., 2004). Similar experience for the United Kingdom is summarized in Willets et al. (2005).

in the scientific literature.[4] In our contribution, we analyze these policy implications in a shareholder value maximization framework. We derive the optimal risk management mix, i.e., the amount of equity to be inserted by shareholders, asset allocation (risky/risk-free), and product policy (term life insurance/life annuities) for a publicly held life insurance company with limited liability and access to a perfect capital market. Potential insurance buyers are assumed to be risk averse and not able to trade or diversify their risks perfectly. This implies that they are willing to pay insurance premiums beyond the expected value of insurance benefits. Thus, our model allows for premium loadings, which we assume to be given exogenously.[5] Empirical evidence on the dependence of insurance demand on an insurer's solvency situation (Cummins and Sommer, 1996; Sommer, 1996; Cummins and Danzon, 1997; Phillips et al., 1998) and the experimental evidence on insolvency aversion (Wakker et al., 1997) are incorporated into our model via a demand-reaction function for insurance contracts that decreases for a given insurance premium as the insurer's solvency situation worsens.[6] As a measure of the solvency situation we use the ruin probability, a measure also used by insurance regulators (European Commission, 2004) and rating agencies (Brand and Bahr, 2001).

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[4] See Blake and Burrows (2001), Dowd (2003), and Blake (2003), who discuss aspects of managing demographic risk by hedging life insurance with annuity contracts and the issuance of mortality bonds on a qualitative basis. Willets et al. (2005) also qualitatively discuss capital adequacy and reinsurance implications stemming from demographic risk. Olivieri (2001) computes moments of liability distributions of term life insurance and annuity portfolios under demographic risk and derives implications for solvency issues.

[5] As usual when loaded premiums are used, we assume an imperfectly competitive insurance market, which is necessary for the insurer's creation of shareholder value. Similarly, if the insurance buyers' willingness to pay was derived from the same shareholder value maximization calculus used by the insurer, i.e., from an arbitrage-free pricing model, the buyers would not accept loaded premiums (i.e., here, premiums above the arbitrage-free value), with the result that the insurer could not create shareholder value from the insurance business (Gründl and Schmeiser, 2002).

[6] In some countries, devices such as guarantee funds protect policyholders from bearing huge financial losses in the event of an insurer's insolvency. These devices can alleviate a buyer's fear of the insurer going insolvent. Nevertheless, there is empirical evidence that buyers are still wary of an insurer's default because, in such a case, the guarantee protection may cover only a part of the loss or payments may be substantially delayed (Phillips et al., 1998).

In accordance with Olivieri (2001), we model demographic risk by assuming a probability model for future changes to the life table, giving rise to natural hedging opportunities. We calibrate the model using German empirical data on stock market returns, the risk-free rate of return, the expected mortality of insurance buyers, and experimental results on insolvency aversion.

Our model framework is, in principle, comparable to the models used by Cummins and Sommer (1996) and Cummins and Danzon (1997), who from rather general micro-economic models derive hypotheses for empirical tests. Because of their abstractness, these models give no specific instructions for an optimal firm policy. Doherty and Tinic (1981) show that—in a shareholder value calculus—optimal reinsurance decisions of insurance companies may be driven by the willingness to pay of risk-averse insurance buyers. Our model is also in line with the model proposed by Froot and Stein (1998), who derive an optimal firm policy and risk management strategy for a bank subject to a convex cost-of-capital function for capital necessary to finance occurred losses. In contrast to Froot and Stein (1998), however, we focus on an explicit insurance context in which shareholder value is driven by collecting “customer liabilities” (Merton and Perold, 1993) with positive net present value, incorporating limited liability of shareholders.

The article is organized as follows. After setting up and calibrating the model, we analyze a situation without demographic risk. We show that equity capital is the preferred risk management measure. Then, demographic risk is introduced. Our results show that the insurer does not utilize any natural hedging opportunities in the product portfolio. Instead, adjustment of the equity position remains the preferred risk management measure.

Next, we study a situation where the insurer has a less than optimal and fixed amount of equity[7] and analyze the complex interplay of product policy and asset allocation as a function of the amount of equity available. We find that now there is a certain range of equity inserted, within which the insurer uses natural hedging opportunities. We show that within this range, hedging

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[7] This can be interpreted as a case with infinite transaction costs for adjusting equity capital. Thus our analysis includes the two polar cases of infinite and zero transaction costs of equity provision.

becomes more pronounced the less equity the insurer has. As general effects, we find that the insurer hedges more the higher premium loadings are and the more sensitively insurance buyers react to the insurer's solvency situation. After this, we examine the impact of a misspecification of the demographic probability model on shareholder value. We find that an error in this specification can create significant economic risk for both the insurer and the insured.

The last section concludes and suggests directions for future research. In particular, the sensitivity of our results with respect to alternative assumptions on premium loadings, insurance demand structure, and the demographic probability model—especially for suboptimal equity—points to the necessity for further empirical and experimental research into concretizing input parameters for insurance demand and demographic risk.

The Model

Formalization

We examine an insurance company founded at $t = 0$, offering term life insurance (*TL*) and annuity (*A*) contracts. At the time of founding, the insurer irreversibly decides on product policy, the amount of equity capital inserted, and asset allocation, and collects insurance premiums. Immediately after this, say at $t = 0^+$, information about the true life table becomes known. At the end of the year, at $t = 1$, the insurer receives asset returns and must settle liabilities at their current market value if the value of the assets is greater than or equal to liabilities. Otherwise, only a part of the liabilities, i.e., the current market value of assets, is paid out. Therefore, although the contract periods of the life and annuity business may go on for decades, the planning horizon focused on here is one year.

Let $l_{TL} \geq 0$ and $l_A \geq 0$ be the number of term life insurance and annuity contracts written by the insurer. Let $\pi_{TL} \geq 1$ and $\pi_A \geq 1$ be the insurance premiums received per contract of each type, which we interpret as a 1 € premium per contract multiplied by loading factors greater or equal to 1. Thus, π_{TL} and π_A can be used to denote the loading factors themselves.

Total premium income $l_{TL} \cdot \pi_{TL} + l_A \cdot \pi_A$ and the insurer's equity E_0 provide the company's total capital, of which the amount α_f is invested at the risk-free rate of return r_f and the rest, α_R, is invested at the risky rate of return R. Thus, at $t = 1$, the insurer will receive $\alpha_f \cdot \exp(r_f) + \alpha_R \cdot \exp(R)$ from its capital market investments. We restrict α_f and α_R to be nonnegative.

Let $S \in \mathbb{N}$ be the number of different life table scenarios,[8] $\hat{S}$ a random variable that realizes at $t = 0^+$, indicating the actual scenario, and $p_s \geq 0$ for every $s = 1, \ldots, S$ the probability that $\hat{S}$ realizes as s. We assume the random variable $\hat{S}$ to be stochastically independent from the risky rate of return R (see Cairns et al., 2004).

Let $\mathbf{z}_{TL}$ and $\mathbf{z}_A$ be two vectors of dimension S. We define $s = 1, \ldots, S$, $z_{TL,s}$ and $z_{A,s}$ to be the s^{th} component of $\mathbf{z}_{TL}$ and $\mathbf{z}_A$, respectively, representing the present value at $t = 1$ (i.e., the default-free market value) of the future liabilities per 1 € premium received for each contract of the corresponding type in scenario s. Then $l_{TL} \cdot z_{TL,\hat{S}} + l_A \cdot z_{A,\hat{S}}$ is the insurer's random liability at $t = 1$ resulting from its underwriting decisions l_{TL} and l_A realizing as $l_{TL} \cdot z_{TL,s} + l_A \cdot z_{A,s}$ with probability $p_s = \Pr[\hat{S} = s]$.[9]

Accounting for its limited liability, the insurer's stochastic equity at $t = 1$ is given by:

$$\hat{E}_1 = \max\{\alpha_f \cdot \exp(r_f) + \alpha_R \cdot \exp(R) - (l_{TL} \cdot z_{TL,\hat{S}} + l_A \cdot z_{A,\hat{S}}), 0\}. \tag{1}$$

Assuming a perfect capital market, let $E^*(\cdot)$ denote the expected value operator with respect to the thereby-induced risk-neutralized martingale measure on the one hand, and to our supposed life scenario probability measure on the other

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8 By choosing a finite point distribution for life table scenarios we follow Olivieri (2001). Cairns et al. (2004) give an overview of other models for stochastic mortality, several of which are similar to interest-rate models. Such models of stochastic mortality, however, all have a weak empirical foundation because stochastic mortality is a young field of research. Therefore, at present, model choice seems, at least to some extent, arbitrary.

9 As done by Jensen et al. (2001, p. 80), we thus assume that the insurer writes a portfolio of insurance policies large enough that—beyond the question of which scenario will actually occur—there is no other mortality risk.

hand.[10] Then, the insurer maximizes the excess shareholder value (in the following, "excess" will be left out) at $t = 0$, given as:

$$SHV_0^* = \max_{\substack{\alpha_f, \alpha_R, \\ l_{TL}, l_A, E_0}} \left\{ \exp\left(-r_f\right) \cdot \mathrm{E}^*(\hat{E}_1) - E_0 \right\}. \tag{2}$$

SHV_0^* measures the maximum additional shareholder value beyond E_0 (i.e., the net present value) created by entrepreneurial activities. Let us denote the resulting shareholder value maximizing asset and liability structure and the optimal equity at $t = 0$ by α_f^*, α_R^*, l_{TL}^*, l_A^*, and E_0^*, respectively. Aside from the nonnegativity restrictions

$$\alpha_f \geq 0,\ \alpha_R \geq 0,\ l_{TL} \geq 0,\ l_A \geq 0 \text{ and } E_0 \geq 0, \tag{3}$$

maximization will be subject to two types of constraints: a balance constraint and, for each contract type, the demand-reaction function, determining the maximum size of the market for insurance contracts. The balance constraint

$$\alpha_f + \alpha_R = E_0 + l_{TL} \cdot \pi_{TL} + l_A \cdot \pi_A \tag{4}$$

guarantees that at $t = 0$, total investment will be equal to the company's capital. As demand-reaction functions, we consider

$$l_{TL} \leq n_{TL} \cdot [1 - q_{TL} \cdot D(\alpha_f, \alpha_R, l_{TL}, l_A, E_0)] \tag{5}$$

and

$$l_A \leq n_A \cdot [1 - q_A \cdot D(\alpha_f, \alpha_R, l_{TL}, l_A, E_0)], \tag{6}$$

where $n_{TL} \geq 0$ and $n_A \geq 0$ for each contract type represent the maximum size of

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[10] Because of the independence of $\hat{S}$ and R, their probability measures are separable from each other. For the solution of our empirical data-based model, the transformation of the empirical probability measure to the equivalent martingale measure is done via CAPM risk adjustment (see Ingersoll, 1987, chapter 4). For demographic risk, the empirical probability measure is used (see Carriere, 1999).

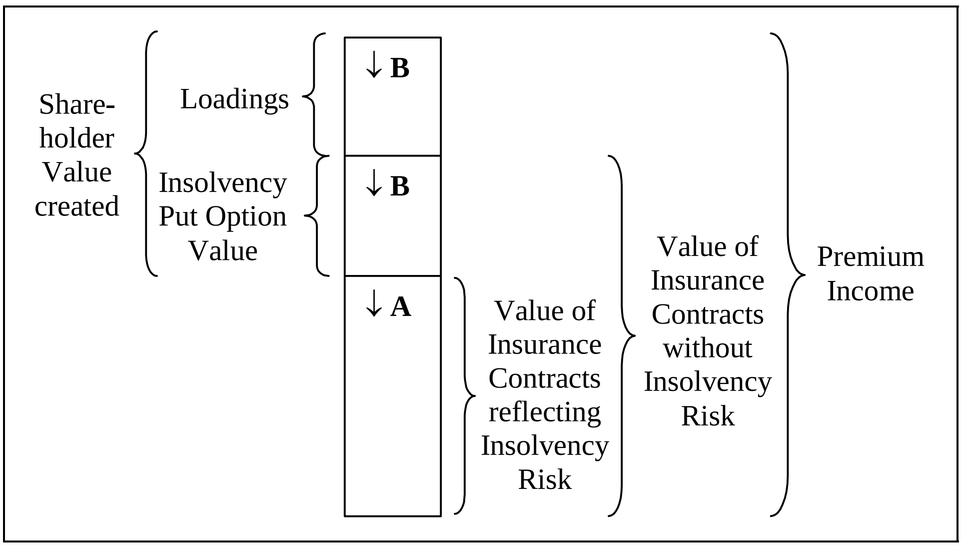
the corresponding insurance market if the insurer's ruin probability,

$$D(\alpha_f, \alpha_R, l_{TL}, l_A, E_0) = \Pr[\alpha_f \cdot \exp(r_f) + \alpha_R \cdot \exp(R) - (l_{TL} \cdot z_{TL,\hat{S}} + l_A \cdot z_{A,\hat{S}}) < 0], \tag{7}$$

is zero. For $D(\alpha_f, \alpha_R, l_{TL}, l_A, E_0) > 0$, the reaction parameters $q_{TL} \geq 0$ and $q_A \geq 0$ determine the percentage drop in maximum demand for the respective contract per percentage point of the insurer's ruin probability. Thus, we assume a linear relationship between ruin probability and percentage drop in insurance demand. Furthermore, as in Cummins and Sommer (1996), the ruin probability is perfectly observable by both the insurer and potential insurance buyers (e.g., by using rating information). Note that now not only the choice of the underwriting decision variables l_{TL} and l_A influences the distribution of the insurer's equity at $t = 1$, but, via the insurer's ruin probability $D(\alpha_f, \alpha_R, l_{TL}, l_A, E_0)$, the distribution of $\hat{E}_1$ also determines the upper bound of l_{TL} and l_A. This strong interdependence makes the optimization problem highly endogenous.

In this setting, the insurer has two conflicting ways of creating shareholder value: increasing the value of the insolvency put option or skimming premium loadings (see Figure 1).

Figure 1
Sources of Shareholder Value Creation for the Insurer



The value of the insolvency put option can be increased through a riskier firm policy, e.g., by inserting low amounts of equity or implementing a risky asset allocation and product policy (see Arrow A in Figure 1). However, as potential insurance buyers are insolvency averse, a risky solvency policy reduces revenues and collected loadings (Arrow B). A trade-off situation emerges. Note that because of the existence of a perfect capital market investing capital within the insurance company per se is not a source of shareholder value creation.

Calibration

Neither of the insurance products considered in our model, i.e., the term life insurance (*TL*) or the annuity (*A*) product, contain any options or guarantees (e.g., for *A*, payments to heirs in case of the annuitant's death), nor do they have a deferment time. Insurance premiums are paid at $t = 0$; insurance benefits are paid at the end of each year. Payments are fixed in nominal terms and all insured persons are male. Table 1 gives further contractual characteristics.

Table 1

Insurance Contracts

product name	type	premium payment	contract duration in years[11]	age of policyholder
TL	term life insurance	single premium	≤ 35	30
A	life annuity	single premium	≤ 56	65

Insurance policies are priced by usual actuarial methods, i.e., the price without an explicit loading equals the (at the risk-free rate) discounted expected liability at $t = 0$, accounting for the supposed probability model for demographic risk that we will describe below.[12] We consider loading factors of 1 (i.e., no loading), 1.005, 1.01, and 1.05 (i.e., loadings of 0.5%, 1%, and

[11] The life table used (DAV 2004 R) allows for a maximum age of the insured of 121.

[12] Because of the assumed independence of mortality risk and financial risk, this price is the lowest price a shareholder value maximizing insurer with no default risk would accept (see Carriere, 1999). For an insurer with default risk, we have an implicit loading that is equal to the insolvency put option value per contract (see Doherty and Garven, 1986).

5% of the discounted expected liability). We set $\pi_{TL} = \pi_A$ and henceforth denote the loading factor by π.

The stock market returns are assumed to have a lognormal distribution (see, e.g., Hull, 2003). The distribution parameters are estimated from a time series of the German stock market index DAX, based on annual data from 1950 to 2003.[13] The estimation gives a mean of 0.1102 and a standard deviation of 0.2678 for the normally distributed log-return R. For the risk-free rate of return $\exp(r_f)$, we use the sample mean of the annualized return time series of the German money market from 1950 to 2003, which gives a value of 1.05 (i.e., $r_f = 0.0488$).[14]

As in Olivieri (2001), we set the number of possible scenarios S at three. Scenario 1 (s = 1) is the base scenario, where the life tables used today by actuaries in Germany at $t = 0^+$ turn out to be correct. Therefore, for the term life insurance contract, the table DAV 1994 T (see Bundesaufsichtsamt für das Versicherungswesen, 1994) is used, and for the life annuity, the table DAV 2004 R (see Deutsche Aktuarvereinigung e.V., 2004) is used. In both Scenarios 2 and 3, the one-year death probabilities[15] change at all ages. Thus natural hedging opportunities arise. In Scenario 2 (3), the one-year death probabilities are multiplied by a constant that is chosen to increase (decrease) the implied life expectancy of a newborn by three years[16] compared to the original DAV tables.

Because of insufficient empirical evidence on the probability model underlying demographic risk, we investigate five alternative distributions $\left\{p_1^j, p_2^j, p_3^j\right\}$, j = 1, …, 5, for the random variable $\hat{S}$, which indicates the scenario actually occurred (see Table 2). In all distributions, the main weight

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[13] We are greatly indebted to Professor R. Stehle, Ph.D., Chair of Banking and Stock Exchanges, Humboldt-Universität zu Berlin (Germany), for providing us with the time series of the DAX.

[14] See the IMF *International Financial Statistics Online* database, http://ifs.apdi.net/imf.

[15] That is, the probability that a person will die at the end of the current year, conditional on having survived to the present date.

[16] This number is somewhat arbitrary because research in this area is still insufficient (see Cairns et al., 2004).

(p_1^j) is assigned to Scenario 1 where the DAV life tables are correct. The remaining probability mass, varying from 0% to 40%, is then equally shared between the two deviating Scenarios 2 and 3.

Table 2
Probability Distributions of $\hat{S}$

probability model	occurrence probability of scenario where life expectancy stays constant	increases	decreases
j	p_1^j	p_2^j	p_3^j
1	1.000	0.000	0.000
2	0.950	0.025	0.025
3	0.900	0.050	0.050
4	0.800	0.100	0.100
5	0.600	0.200	0.200

For these probability models, Table 3 shows the resulting liabilities at $t = 1$ for *TL* and *A* per 1 € (unloaded) premium income if Scenario s occurs.

Table 3
Liabilities at $t = 1$ for *TL* and *A* per 1 € (Unloaded) Premium Income for Alternative Probability Models $\{p_1^j, p_2^j, p_3^j\}$

probability model	probability for scenario 1	liability per 1 € unloaded premium at $t = 1$					
		term life insurance *TL* scenario s			annuity *A* scenario s		
j	p_1^j	1	2	3	1	2	3
1	1.00	1.05000	n/a	n/a	1.05000	n/a	n/a
2	0.95	1.04894	0.83644	1.30381	1.05001	1.09407	1.00556
3	0.90	1.04788	0.83560	1.30250	1.05002	1.09408	1.00557
4	0.80	1.04578	0.83392	1.29988	1.05004	1.09410	1.00559
5	0.60	1.04159	0.83057	1.29467	1.05008	1.09414	1.00562

To construct the liability vectors $\mathbf{z}_{TL}$ and $\mathbf{z}_A$, we collect for a given probability model j the scenario-dependent liabilities from Table 3.[17] Note that for every probability model and product, the expected liability at $t = 1$ is 1.05, i.e., the risk-free compounded net premium.

[17] For example, under probability model $j = 2$, we then have $\mathbf{z}_{TL} = (1.04894, 0.83644, 1.30381)^T$ and $\mathbf{z}_A = (1.05001, 1.09407, 1.00556)^T$.

As can be seen from Table 3, the liabilities of *TL* and *A* are—as expected—negatively correlated. Table 4 gives standard deviations, correlations of both products, and the hedge ratio (i.e., the ratio of the number of term life insurance contracts to the total number of contracts underwritten), which minimizes the variance of the product portfolio.

Table 4
Standard Deviations, Correlations, and Product Portfolio Variance Minimizing Hedge Ratio of Liabilities at $t = 1$

probability model	probability for scenario 1	standard deviation of liability at $t = 1$		correlation of liabilities at $t = 1$	portfolio variance minimizing hedge ratio of contracts
j	p_1^j	$\mathrm{Std}(z_{TL,\hat{s}})$	$\mathrm{Std}(z_{A,\hat{s}})$	$\mathrm{Corr}(z_{TL,\hat{s}}, z_{A,\hat{s}})$	$l_{TL} / (l_{TL}+l_{\mathrm{A}}) \cdot 100\%$
1	1.00	0	0	n/a	n/a
2	0.95	0.052	0.010	-0.994	15.82%
3	0.90	0.074	0.014	-0.993	15.82%
4	0.80	0.105	0.020	-0.999	15.91%
5	0.60	0.147	0.028	-0.998	15.97%

Table 4 shows that for all probability models, life insurance *TL* is more volatile than annuity *A*. This effect is mainly caused by the different impacts that the shifts in life expectancy have on the probability of the respective event insured.[18] To minimize product portfolio risks, for alternative demographic probability models, approximately 16% of the whole portfolio should consist of life insurance business.

To help determine an exact relationship between the solvency situation and insurance demand there is, unfortunately, only little experimental research

[18] In the life insurance business, the low probability event is insured. For example, the probability of dying during the first contract year (at age 35) is 0.0015. According to our model, a decline in mortality decreases this probability to 0.0012. That is, the probability of the event insured changes by –20%, which changes the market value of the contract significantly (compare Table 3). In the annuity business, it is the high probability event that is insured. For example, the probability of surviving the first contract year (at age 65) is 0.9924. Here, the decline in mortality changes the survival probability to 0.9937, which is only a change of +0.13% in the probability of the event insured, resulting in a rather low change in market value. (Note that although annuity payments in the very distant future are low probability events, too, these payments contribute only little to the market value because they are heavily discounted.)

(Wakker et al., 1997) and none with a focus on the demand for life insurance.[19] Because of this lack of quantitative foundation, we set the demand-reaction parameters $q_{TL} = q_A = q$, and vary q from 0 to 9 to track the consequences for the optimal risk management mix when we move from an entirely inelastic ($q = 0$) to a very elastic market that reacts to every percent of ruin probability by a 9% reduction in demand.[20] The maximum market sizes n_{TL} and n_A are both set to 100 contracts.[21]

Solving technique

The high endogeneity and complexity of our model makes it necessary to solve it numerically. As a solution technique we use a mixed integer linear programming approach. To apply this technique, we had to approximate the continuous distribution of the stock market return by some finite discrete probability distribution and adjust the risk-neutral measure accordingly.

Results

The situation without demographic risk

In a situation without demographic risk, the liability per contract at $t = 1$ is deterministic and equals 1.05 € for both product types. Product policy does not influence the volatility of the insurer's equity at $t = 1$ and, therefore, neither does it influence the insolvency put option (IPO) value or the insurer's solvency situation. Hedging between product types is not possible. In such a situation, it is always optimal to maximize the number of products sold (i.e.,

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[19] Empirical studies (Cummins and Sommer, 1996; Sommer, 1996; Cummins and Danzon, 1997; Phillips et al., 1998) reveal that there is a negative relationship between the solvency situation and potential insurance buyers' willingness to pay, but provide no explicit functional relationship.

[20] Compared to the value of an about 30% reduction in willingness to pay if there is a 1% probability that the insurer will not pay, as reported for fire insurance by Wakker et al. (1997), demand elasticities between 0 and 9 may seem rather low. However, it must be taken into consideration that, in contrast to Wakker et al. (1997), we do not assume that an insurer's insolvency is equivalent to total default of liabilities. Therefore, the Wakker et al.'s values cannot be used as input parameters. In our model, potential insurance buyers react to the probability that there is any positive default at all, which also includes partial defaults. Our calculations showed that the expected default per 1 € of liability is almost always smaller than 1/6. We thus chose the range for the reaction parameter at around 1/6 of the value given by Wakker et al. (1997).

[21] The model can be scaled arbitrarily in this number.

revenues) for any ruin probability given (as far as admitted by the demand reaction). Consequently, this also holds for the shareholder value (SHV) optimal ruin probability given at the optimal trade-off point (maximizing the IPO vs. maximizing loadings). This leads to a liability portfolio composed of 50% term life insurance and 50% annuity business, which results from the assumed symmetric market structure. The optimal risk management mix and the resulting optimal values of dependent variables are given in Figure 2 as a function in the demand-reaction parameter q and the loading factor π. Figure 2 illustrates the basic functioning of our model and the interrelations of various variables. First, in Figure 2a, the obvious negative relationship between SHV and the demand-reaction parameter q can be seen. The more strongly potential insurance buyers react to the solvency situation by refusing to buy insurance (i.e., the higher q), the less opportunities exist for the insurer to create SHV by exploiting insurance buyers. This effect is also evident in Figure 2e, where the percentage share of SHV that stems from the IPO is plotted. Higher loadings also increase SHV, as can be seen from Figure 2a.[22] Second, Figure 2d shows that the more strongly insurance buyers react to the solvency situation, the less risky the insurer's firm policy (measured by the ruin probability) is.

From the way the insurer controls its ruin probability the consequences of the discrepancy between its and the insureds' behavior become apparent. Insurance buyers care only about the insurer's ruin probability. For any ruin probability, the insurer who maximizes SHV then uses its risk management mix to optimize the IPO value. It turns out that on the continuum of possible risk management measures resulting in the same strictly positive ruin probability, it is optimal to invest 100% of the capital riskily and to insert the necessary amount of equity (see Figures 2b and 2c), instead of, e.g., inserting less equity and investing a proportion of the capital risk-free. Any substitution of equity in favor of risk-free investment is at the expense of the IPO value and, thus, the insurer's SHV.[23] Thus, equity capital is the preferred risk management measure.

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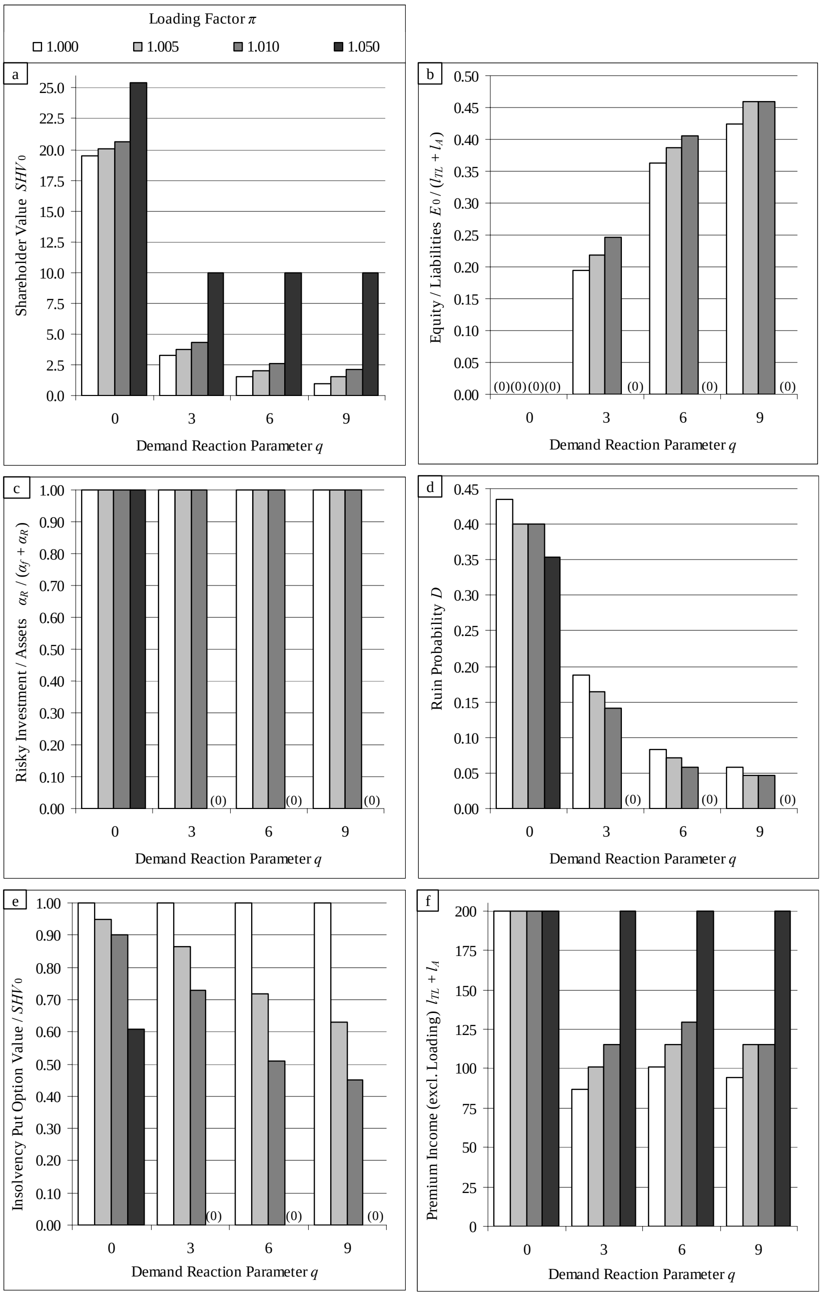
[22] Note that this holds only if insurance buyers do not react to the size of the load, which, in reality, will not generally be the case. Therefore, loading size must not be interpreted as a decision variable; rather, it is determined by a specific market condition.

[23] For example, for q = 3 and π = 1.005, the optimal equity to liability ratio is 0.219, which gives a SHV of 3.75 € when investing 100% of capital riskily (compare Figures 2b and

Figure 2

The Situation Without Demographic Risk, p_1 = 1.0—Optimal Risk Management Mix and Resulting Optimal Values of Dependent Variables



Note that for the cases where the ruin probability is zero, the amount of equity is optimal at and above the value plotted.

2c). Holding the ruin probability constant at 16.5% (compare Figure 2d) while reducing the equity to liability ratio to, e.g., 0.014 and investing instead only 10% of capital riskily gives a SHV of 0.78 €, i.e., a drop of 79.2% (in both cases the insurer sells about 50.6 insurance contracts of each type; compare Figure 2f).

The ruin probability (Figure 2d) is negatively related to the loading factor, since higher loadings increase the incentive to create SHV from loadings rather than from the IPO. This effect is also seen in Figure 2e, where the IPO value to SHV ratio is shown, and in Figure 2b, where the insurer (except for the case where $\pi = 1.05$) increases safety by inserting more equity the higher the load is in order to sell more contracts (see also Figure 2f).

If loading factors are high enough ($\pi = 1.05$), the insurer chooses a ruin probability of 0% (compare Figure 2d), sells the maximum possible number of contracts ($= n_{TL} + n_A$), and thus maximizes loadings collected. SHV creation via the IPO value becomes a dominated strategy. The insurer does not insert any equity capital at all and invests 100% risk-free. Note that in these cases, investing 100% risk-free is only a degenerate solution of our optimization problem: There are also lower fractions of risk-free investment that lead to the same optimal SHV (leaving the ruin probability unchanged at 0%).[24] This demonstrates that the insurer cannot create SHV by merely allocating assets. Because of the risk-neutral evaluation of capital market returns by E^*, asset allocation does not influence SHV if the ruin probability is 0%.

The situation with demographic risk

Introducing demographic risk makes liabilities stochastic and natural hedge opportunities arise (see Table 4). Surprisingly, however, hedging in the product portfolio does not occur. Again, the insurer sells as many contracts as possible at the chosen ruin probability and the product portfolio composition is 50:50 instead of, e.g., the liability portfolio standard deviation minimizing term life insurance to annuity ratio of about 16:84.

In a frictionless neoclassical model framework it is well known that hedging is irrelevant (see, e.g., Doherty, 2000, chapters 5 and 7). Our model introduces the friction that insurance buyers explicitly reward a risk-averse firm policy, i.e., a firm policy that leads to a low ruin probability, by buying insurance contracts with (for the insurer) positive net present values. In this case, it turns out that hedging is far from being irrelevant: it even destroys shareholder

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[24] For example, for $q = 9$ and $\pi = 1.05$, sensitivity analysis shows that risk-free investment can be lowered to 90% of total capital.

value. This is because product hedging, i.e., controlling the ruin probability via product policy, is an expensive risk management measure compared to providing equity capital (and allocating 100% of assets to risky investments). Product hedging implies a deliberate reduction of revenues, which means forgoing positive net present value opportunities. Certainly, hedging leads to a lower ruin probability, which increases market demand, but in order to reach the desired hedge ratio, the insurer will not sell as many contracts as possible for each product type. Therefore, to obtain a desired ruin probability, inserting more equity is always the optimal risk management measure because at this reduced ruin probability, the insurer can sell as many contracts as possible for each product type.[25/26]

The effects of loadings and the demand-reaction parameter on the optimal risk management mix and SHV are very similar for all probability models of demographic risk. Figure 3 gives the results for an example case of $p_1 = 0.6$.

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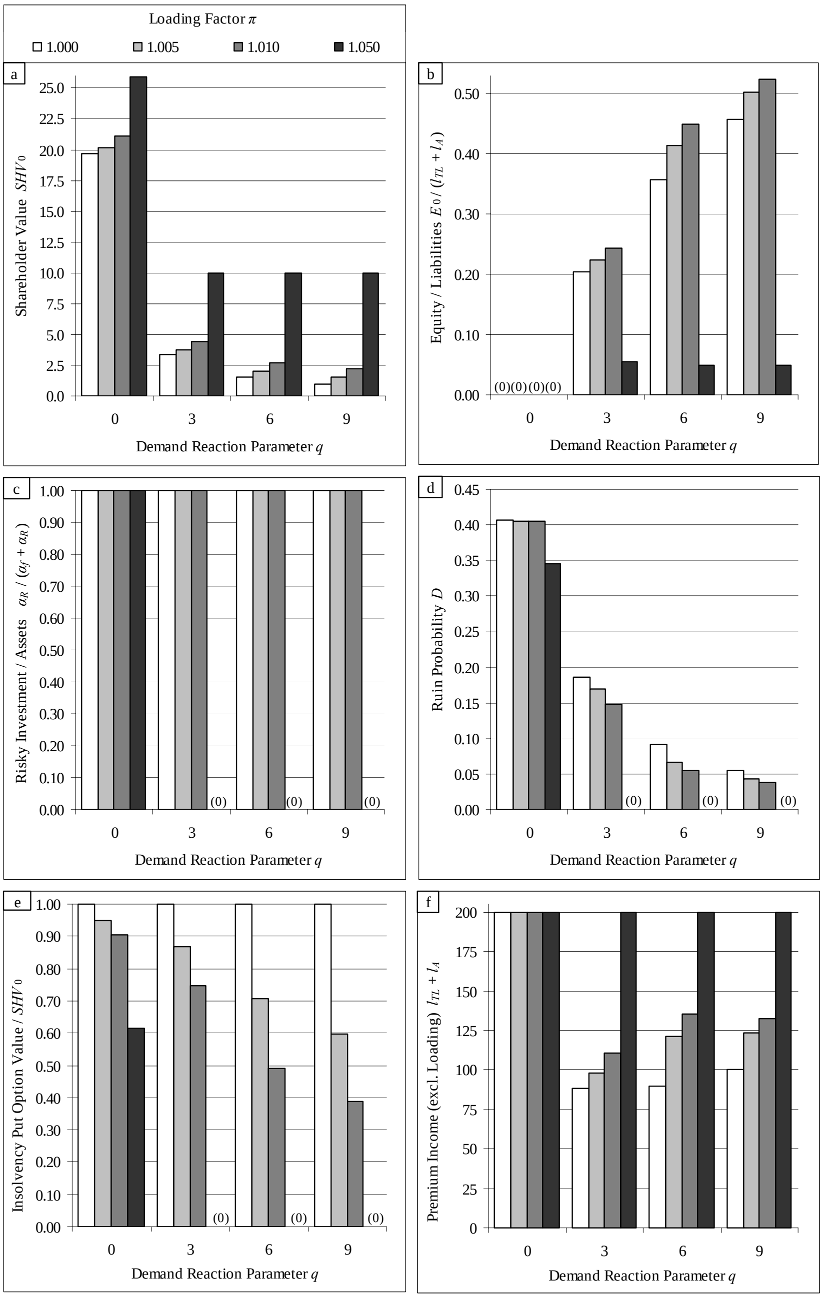
[25] A simple example for the case $q = 9$ and $\pi = 1.05$ and $p_1 = 0.6$ illustrates this. Assume that the insurer has a fixed equity capital of 2 €. As will be shown in the next subsection, in this situation the insurer optimally chooses a ruin probability of 0%, asset allocation of 100% risk-free investment, and sells 61.3 term life insurance and 100 annuity contracts (compare Figure 4l). The hedge ratio is 61.3 / 161.3 · 100% = 38%. This creates a SHV of about 8.07 €. The same ruin probability can be reached by increasing equity to 20 € and allocating 100% of assets risk-free. Then the insurer sells the maximum possible 100 contracts for each product type, which creates a SHV of 10 € (compare Figure 3).

[26] The no-hedging result is stable with respect to the assumed market structure (here, the same maximum demand of 100 for both products). In our calibration, not selling the maximal number of contracts for a given ruin probability is always suboptimal compared to inserting equity. Therefore, given, e.g., a market structure of maximal demands for life insurance of 16 and for annuity business of 84 (replicating the standard deviation minimal hedge ratio), the insurer would sell products in a proportion of 16:84. Therefore, what may look like hedging actually is not. This no-hedging result should also be stable with respect to changes in the life table, i.e., the exact kind of demographic risk we are examining in this article, since the insurer does not even use the relatively good hedging opportunities given by our calibration (hedging opportunities would be worse if changes in the one-period survival probabilities occur only within a certain age range, e.g., are only relevant for the subpopulation older than 65).

Figure 3

The Situation with Demographic Risk, p_1 = 0.6—Optimal Risk Management Mix and Resulting Optimal Values of Dependent Variables



Note that for the cases where the ruin probability is zero, the amount of equity is optimal at and above the value plotted.

When comparing the alternative assumptions for the demographic probability model, the insurer's optimal decisions vary slightly, but—due to the complexity of the stochastic environment and the insurer's high level of flexibility in adapting the risk management mix—no clear tendencies can be identified. The next subsection, where the insurer has fewer options, i.e., no flexibility in the amount of equity, will reveal the influence of alternative demographic probability models.

The situation with demographic risk and fixed equity capital

The preceding two subsections demonstrated that if equity is unrestricted, inserting equity capital and investing capital—depending on the loading factor—either 100% or 0% riskily are the insurer's preferred risk management measures. With equity fixed below the optimal value, only the asset allocation and the size and composition of the liability portfolio remain as risk management measures. Now there are parameter constellations where an explicit product policy, i.e., the utilization of natural hedging opportunities, is optimal. Figure 4 shows the liability portfolio composition, represented by the percentage invested in the term life insurance product *TL*, as a function in the fixed equity given, the loading factor, and the demographic probability model. The demand-reaction parameter is set to q = 9. Since *TL* is more volatile (compare Table 4), numbers below 50% indicate hedging.

Figure 4 shows that if the insurer is restricted in the amount of equity, it uses a risk-reducing product policy, i.e., hedging as a risk management measure.[27] In Figures 4f–4l it can be seen that the less equity the insurer has, the more it is induced to hedge, because equity is insufficient to reach an optimal solvency situation. Given that demographic risk is comparatively high (Figures 4g–4l), hedging becomes less pronounced the higher the loading factor is. There are two reasons for this result. First, high loading factors are a source of additional equity. Second, high loading factors create stronger incentives to increase revenues instead of using product policy as a risk management measure. In the case of relatively low demographic risk (Figures 4a–4f), the situation is more difficult to explain because of the complex interplay of the risk management

[27] For the loading factor of 1, results are omitted because in this situation hedging destroys the only source of SHV creation, the IPO, and thus is always a dominated risk management measure.

measures available. Here, hedging occurs only within a certain range of the equity. This range becomes wider the higher the loading factor and the higher the demographic risk. To understand this "range" effect, one has to look at the insurer's whole set of optimal decisions simultaneously. This is done in Figure 5 with the parameter constellation used for Figure 4c (i.e., Figure 4c is the same as Figure 5e).

Figure 4

The Situation with Demographic Risk, q = 9—Optimal Liability Portfolio Composition (% Invested in TL)

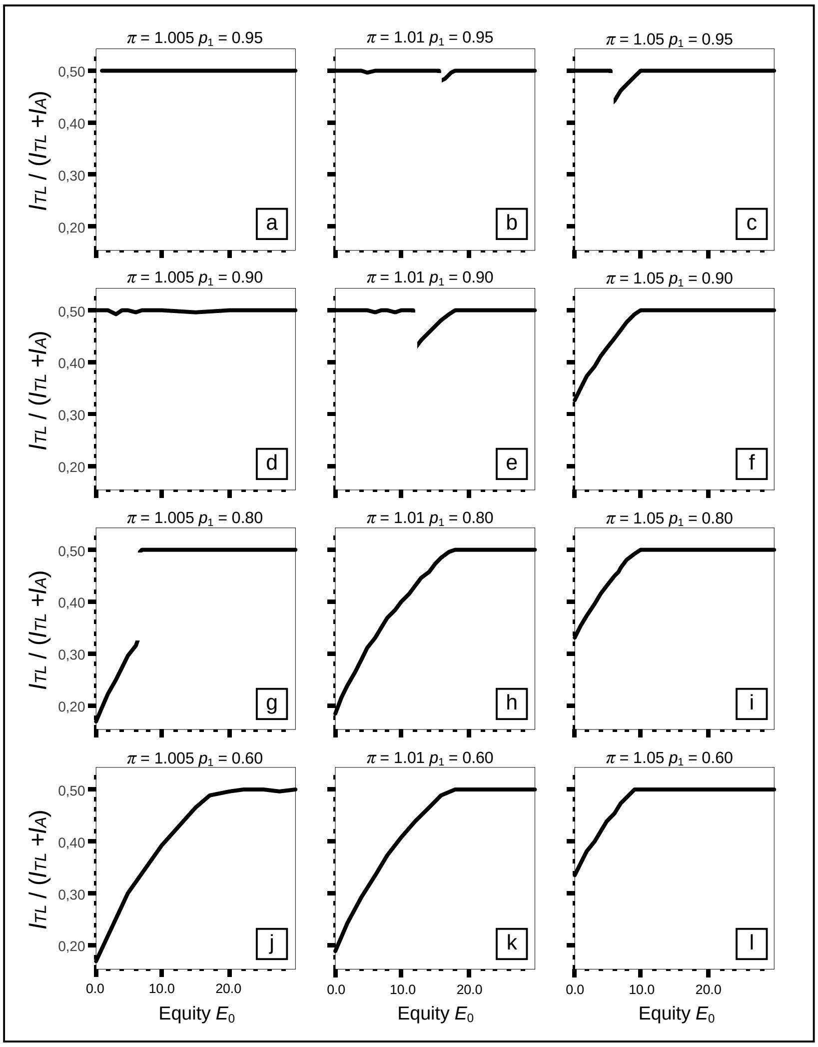
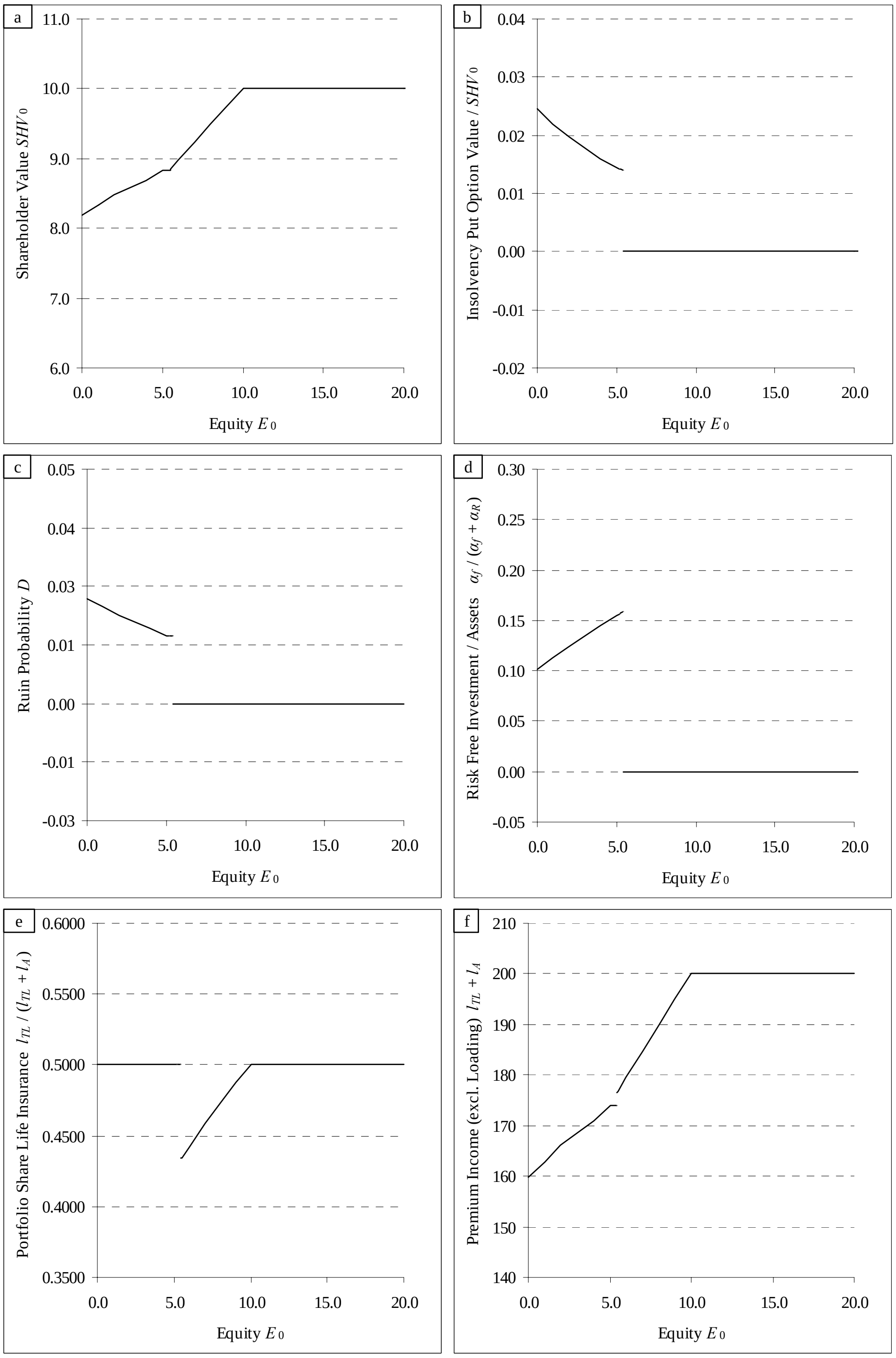


Figure 5

The Situation with Demographic Risk and Fixed Equity Capital, q = 9, π = 1.05, p_1 = 0.95—Optimal Risk Management Mix and Resulting Optimal Values of Dependent Variables

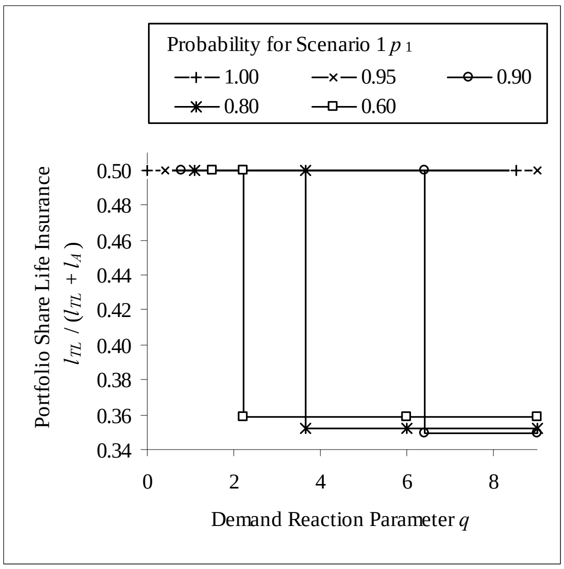


As can be seen from Figure 5a, the optimal equity in this parameter constellation is given at and above E_0 = 10 €. Below that value, the insurer's suboptimal solvency situation leads to hedging (Figure 5e), which reduces revenues (Figure 5f). The asset allocation stays at 100% risk-free investment. Reducing the equity further enhances hedging until equity falls below E_0 = 5.4 €. From there on, the entire risk management mix changes. The insurer stops hedging and invests some part of its capital riskily (Figure 5d). Here, the trade-off situation between creating SHV by skimming loadings and maximizing the IPO value alters the decision in favor of the IPO value (Figure 5b) by choosing a ruin probability greater than zero (Figure 5c).

The optimal decisions this trade-off situation leads to also depend on the demand-reaction parameter q. With an absolute inelastic demand (q = 0), it is clear that the insurer has no incentive to hedge. If demand becomes elastic (q > 0), insurance buyers reward a safe firm policy. Therefore, hedging incentives are the stronger the higher q is. This effect is shown in Figure 6. Here, for the alternative probability models for demographic risk the parameter q is varied while fixing the loading factor π at 1.05 and equity at E_0 = 1 € (for the hedge ratio at q = 9, compare Figures 4c, 4f, 4e, and 4i).

Figure 6

The Situation with Demographic Risk, π = 1.05, E_0 = 1 €—Optimal Liability Portfolio Composition (% Invested in *TL*)



For every alternative probability model for demographic risk there is one q at which a jump in the optimal product portfolio composition occurs and the insurer switches from no hedging to hedging. The higher the demographic risk (i.e., the lower p_1), the lower is the value of q at which the risk management mix changes. This is because the higher the volatility of the insurer's liabilities (compare Table 4), the higher is the influence of the product portfolio on the insurer's solvency situation, which cannot be compensated for by inserting equity (as equity is restricted). Thus, already at a smaller value of q it pays to switch to a less risky product policy, i.e., to hedge. The hedge ratio itself goes up with increasing demographic risk, i.e., for higher demographic risk the insurer hedges less. This finding is based on the fact that in our calibration with increasing demographic risk, the standard deviation minimal hedge ratio also increases, i.e., less hedging is necessary (compare Table 4). For low demographic risk, i.e., for p_1 = 0.95, no hedging is optimal in the given parameter constellation.

The consequences of falsity

As explained in the calibration subsection, there is very little research available on the probability distribution underlying demographic risk. Therefore, in the following we investigate the consequences on the insurer's SHV if—before the actual life table scenario realizes—it turns out (e.g., by publication of a new study) that the assumption on the scenario occurrence probabilities and, thus, the liability vectors $\boldsymbol{z}_{TL}$ and $\boldsymbol{z}_A$ were wrong, but it is too late for the insurer (and the insurance buyers) to react to this new information.[28] To do this, as before, we calculate the insurer's SHV and optimal policy (α_f^*, α_R^*, l_{TL}^*, l_A^*, E_0^*) under a certain assumption on the scenario occurrence probabilities. Then, we adopt an alternative probability model, adjust the vectors $\boldsymbol{z}_{TL}$ and $\boldsymbol{z}_A$, and—leaving the applied policy unchanged—calculate the new SHV using the maximand from the right hand side of Equation (2). Last, we compare the new SHV with the initial SHV.[29] To keep things simple, we assume that there are no other possible distributions of $\hat{S}$ than those we gave in Table 2. For the example case where π = 1.05

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[28] We thus are at a point in time somewhere between $t = 0$ and $t = 0^+$. However, to avoid notational inflation, we will say that we are at $t = 0$.

[29] We performed the same analysis for the situation with fixed equity. The results showed the same tendencies.

and q = 6, Table 5 contains the percentage changes in SHV that occur at t = 0 if we use distribution $\{p_1^{j_1}, p_2^{j_1}, p_3^{j_1}\}$ instead of the true distribution $\{p_1^{j_2}, p_2^{j_2}, p_3^{j_2}\}$ for our calculations.

Table 5
The Situation with Demographic Risk, π = 1.05, q = 6—Percentage Changes in the Insurer's SHV If the Wrong Scenario Distribution $\{p_1^{j_1}, p_2^{j_1}, p_3^{j_1}\}$ Was Used Instead of True Distribution $\{p_1^{j_2}, p_2^{j_2}, p_3^{j_2}\}$

probability model used in calculations		true probability model					
		j_2	1	2	3	4	5
j_1	$p_1^{j_1}$	$p_1^{j_2}$	1.00	0.95	0.90	0.80	0.60
1	1.00		-	0.97%	2.64%	3.92%	6.66%
2	0.95		-0.93%	-	1.67%	2.93%	5.65%
3	0.90		-2.57%	-1.64%	-	1.22%	3.87%
4	0.80		-3.72%	-2.82%	-1.24%	-	2.75%
5	0.60		-6.19%	-5.31%	-4.25%	-2.61%	-

The general tendency is clear: the larger the error in the supposed probability model, the larger the change in SHV. The relative changes vary from a 6.19% drop to a 6.66% increase in the insurer's SHV. Drops will occur in those cases where the new information on the scenario distribution reports a downward correction in the deviation probability, i.e., demographic risk was overestimated ($p_1^{j_1} \leq p_1^{j_2}$). This effect is mainly caused by the lower volatility of the insurer's equity at t = 1 ($\hat{E}_1$), which stems, on the one hand, from a drop in volatility of $\hat{S}$ and, on the other hand, from a drop in the volatility of the liabilities themselves (see Table 4). Analogously to standard option pricing theory, this leads to a drop in the IPO value and therefore—as premium income remains constant—in the insurer's SHV. Thus the insurer suffers from a simultaneous overestimation of the volatility of its equity at t = 1. Contrarily, the volatility of the insurer's equity at t = 1 and, thus, SHV go up when it turns out that, at the time insurance contracts were underwritten, demographic risk was underestimated ($p_1^{j_1} \geq p_1^{j_2}$).[30] In this case, insurance buyers pay the bill

[30] The consequences of using a suboptimal risk management mix per se are not clear. As both insurer and insurance buyers were wrong, the resulting ruin probability and number of contracts to be sold after the new probability model becomes public may be outside the feasible region of the optimization problem so that the error in the occurrence probability model does not necessarily lead to a loss for the insurer.

because they underestimated the resulting volatility of the insurer's equity and, thus, interpreted its solvency situation too optimistically.

It is important to note that because of the lack of research on the appropriate probability model, any model chosen that assigns some strictly positive weight to deviations from the current life tables may turn out to be either an underestimation or an overestimation of demographic risk. Thus, a priori, in the case of model error, it is not clear who will bear the consequences, since the actual direction of a possible misspecification is known only ex post.

Conclusions and Directions for Future Research

Our analysis revealed that the presence of demographic risk has a fundamental impact on the optimal risk management mix of a shareholder value maximizing life insurer operating in a market with insolvency-averse insurance buyers. In the situation where inserting equity capital, asset allocation, and product policy are the available risk management measures, it was found that the insurer prefers adjusting the amount of equity inserted as a way of coping with demographic risk. This measure is generally found in combination with allocating 100% of capital to risky investments and underwriting as many contracts as possible in each product line, thus avoiding shareholder value destructive product hedging. The risk management mix changes if the amount of equity capital available is scarce. In that case, product policy, i.e., the utilization of natural hedging opportunities between term life insurance and annuity contracts, can be shareholder value maximizing. The exact hedging strategy was shown to depend simultaneously on the amount of equity available, the degree of insurance buyers' insolvency aversion, the loading factor, and the demographic probability model. A misspecification of the demographic probability model turned out to have a significant impact on either the insurer or the insured: either the insurer benefits financially from an underestimation of demographic risks and the insured suffers a financial loss, or vice versa.

In Germany, the empirically found equity to liability ratio of life insurers is about 1.4% (German Insurance Association, 2004). This fact indicates the practical relevance of our contribution. If demographic risk is found to be sufficiently high and/or insurance buyers react rather strongly to the insurer's solvency situation, within our model, a ratio of 1.4% probably would lie within the range where either inserting higher amounts of equity or, if this is not possible, product hedging will be appropriate. To answer the question of the actual magnitude of demographic risk, however, further research is necessary and should be of high priority for both the insurance industry and policyholders, as both parties can suffer significant losses in the case of a wrong perception of demographic risk.

Another direction for future research is the introduction of mortality bonds (Blake and Burrows, 2001; Dowd, 2003; Blake, 2003) as a further risk management measure into our model framework. We expect that if arbitrage-free priced mortality bonds without any basis risk (i.e., cost-free perfect hedging instruments) were available and the insurer's equity was restricted mortality bonds might indeed be useful instruments because revenue-reducing product hedging could be avoided. In case mortality bonds bring about some basis risk, a combination of product hedge and mortality bonds could emerge as an optimal risk management mix. However, introducing mortality bonds into our model as an additional decision variable would further increase the complexity of the underlying optimization problem.[31]

The model could be extended by considering regulatory requirements, e.g., on the equity capital, asset allocation, ruin probability, or expected policyholder deficit. Another extension could be the inclusion of transaction costs for structuring the liability portfolio, which will be especially important in cases where the insurer starts with an initial portfolio of insurance contracts. Finally, the observed sensitivity of the insurer's optimal risk management mix with respect to the degree of potential insurance buyers' insolvency aversion highlights the importance of further research in the area of consumer behavior in life insurance markets.

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[31] In particular, analysis of a case where the insurer can decide on the issuance of such bonds would be a nontrivial task. If the bonds were priced arbitrage-free by market participants, the price would also need to reflect the insurer's insolvency risk. However, since in our model this risk is subject to the insurer's decision, another endogeneity would be created.

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