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Tolerance and uncertainty

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Abstract

This paper describes the influence of the measurement uncertainty to the tolerance, as defined in international standards, and quotes documents on how to estimate measurement uncertainties. For industrial length measurements the principal contributors to the measurement uncertainty are summarised. They are detailed for machine tool acceptance tests, including the relevant contributor for the different types of measurements. Parameters for the volumetric accuracy of coordinate measuring machines (CMMs) and for machine tools are given, as well as the major influences on the overall performance of CMMs and machine tools.

1 The "golden rule"

The "golden rule" of metrology states, that the measurement uncertainty shall be less than 10% of the tolerance. If this requirement is fulfilled, there is practically no influence of the measurement uncertainty to the tolerance.

As tolerances become smaller, the "golden rule" cannot be fulfilled anymore – sometimes even the "golden rule" is changed to a margin of 20%, but that does not solve the problem.

ISO 14253-1, decision rules [1], and ISO 230-1, geometric testing of machine tools [2], clearly state, that any measurement uncertainty reduces the tolerance zone to the so called zone of conformance, if the conformance with the specification has to be proved. ISO 14253-1, decision rules, further states, that the tolerance zone is enlarged by the measurement uncertainty, if non-conformance with the specification has to be shown.

tolerance zone		
measurement uncertainty	zone of conformance	measurement uncertainty

Figure 1: Zone of conformance according to ISO 14253-1 for two-sided tolerance zone.

If the tolerance zone is two-sided, like the tolerance for a distance, e.g. 100 ± 0,020 mm, the measurement uncertainty is subtracted on both sides of the tolerance zone (see Fig. 1).

One-sided tolerances, like tolerances for roundness, roughness and parameters for machine acceptance, are reduced only on one side of the tolerance zone by the measurement uncertainty (see Fig. 2).

2 Estimation of measurement uncertainty

The basic document for the estimation of the measurement uncertainty is the Guide to the expression of uncertainty in measurement, GUM [3]. A practical application of the GUM for industrial length measurements is shown in an ISO specification, known as the PUMA method [4].

Special consideration to the bias and uncertainties arising from thermal effects, as a constant temperature other than the reference temperature of 20°C and as temperature variations, is given in an ISO specification under preparation [5].

As temperature effects are recognised as the major single source of errors on machine tools [6], the main thermal influences from [5] are repeated:

- correction of differential expansion between measurement equipment and object under test (i.e. workpiece or machine),
- uncertainty of the correction due to uncertainty in temperature measurements and uncertainty in thermal expansion coefficients,
- uncertainty due to temperature variation in space and time.

tolerance zone	
zone of conformance	measurement uncertainty

Figure 2: Zone of conformance according to ISO 14253-1 for one-sided tolerance zone (e.g. tolerance for roundness, roughness, machine accuracy parameters).

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The influence of temperature variations is checked by so called drift tests. For a drift test on a machine tool the relative movements between the tool side and the workpiece side are measured during a typical time period, e.g. typical machining time, with the machine axes and spindle(s) in hold position. For a drift test for a measuring equipment the changes in the readout are taken over a typical time period, e.g. the time between resetting the equipment, with the object under test (workpiece or machine) being not changed.

The calculation of the measurement uncertainty is simple in most cases, the more demanding part is the identification of the relevant contributors to the measurement uncertainty.

The result shall be given as the measurement result ± uncertainty, with a statement to the coverage factor k, that in general cases shall be equal to 2. An example: $L = 100{,}003 \pm 0{,}005$ (k=2).

3 Contributors to the measurement uncertainty

For the industrial length measurement, i.e. the measurement of workpieces, machine tools and the testing of measurement instruments, we can distinguish five groups of uncertainty contributors:

- the measurement equipment,
- the application of the equipment,
- the influence of the environment,
- the influence of the device under test, and
- the repeatability of the measurement set-up.

The uncertainty of the measurement equipment can either be taken from the data provided by the manufacturer of the measurement equipment or from the uncertainty of a calibration.

Uncertainty from the application of the equipment comes from the alignment of the workpiece in the equipment or the alignment of the equipment on a machine, from deformations due to fixtures on the workpiece or on the measurement equipment, from the selection of measuring points on the workpiece or along a machine axis, etc., always assuming that the measurement equipment is used according to the instructions of the manufacturer, and that the interpretation of the measurement task is correct.

The influence from a changing environment is estimated by a drift test during a time equivalent to the time of measurement, during the usual time between resetting the measurement equipment, during one shift, or during 24 hours, depending on the purpose of the measurement. Each length measurement has to be corrected to 20°C, thus introducing uncertainties due to the temperature measurement and due to the estimated thermal expansion coefficient, both for the object under test and for the measurement equipment.

Under "influence of the device under test" the following items are summarised: For a workpiece measurement it is the influence of the stiffness of the workpiece, the form error and roughness of the surface, and the shape (e.g. special uncertainties, if a radius of a partial arc should be measured). For a machine measurement it is the repeatability of the machine, especially if positioning accuracy parameters are evaluated.

Table 1:Uncertainty in machine tool measurement, assumptions.

characteristics of measuring instrument	
alignment telescope, maximum microscope error	13 µm on 0,350 mm
angle interferometer, maximum error	2 µm/m on 300 µm/m
dial gauge, electronic indicator, maximum error	2 µm on 1 mm
dial gauge, electronic indicator, maximum error	1 µm on 0,020 mm
dial gauge, reversal error	0 µm (compensated)
laser interferometer, positioning, maximum error	3 µm for 1000 mm, 25°C
laser interferometer, temperature measurement	0,7 °C
laser interferometer, straightness, maximum error	4 µm on 0,350 mm
laser interferometer, straightness, magnification of interference signal	36
precision level, maximum error	5 µm/m
precision level, foot distance	150 mm
straightedge, maximum out-of-straightness	12 µm according to [2]
test mandrel, maximum out-of-straightness	3 µm according to [2]
square, maximum deviation from 90°	5 µm/m
square, maximum out-of-straightness	12 µm according to [2]
wire, maximum straightness deviation	5 µm
application of measuring instruments	
alignment of alignment telescope	0,3 mm
alignment of laser beam to machine axis	2 mm
alignment of mechanical devices to machine axis	1 mm
alignment of straightness interferometer to axis	0,3 mm
alignment of wire to machine axis	0,3 mm
influence of air disturbance on	
- laser interferometer, straightness set up	0,1 µm on 1000 mm
- laser interferometer, angle set up	0,0 µm
- light beam	1 µm on 1000 mm
- wire	2 µm on 1000 mm
environmental characteristics	
air and machine temperature	25°C
uncertainty of thermal expansion coefficient	2 µm/m°C

Table 1, continued.

machine tool characteristics	
length of measured axis	1000 mm
maximum run out of spindle	20 μm acc. to [11]
maximum roll, pitch and yaw	60 μm/m acc. to [11]
maximum straightness error for any axis	50 μm acc. to [11]
nominal thermal expansion coefficient	12 μm/m°C
repeatability of machine tool (2 runs)	2 μm
thermal drift within 5 minutes	0 μm, 0 μm/m
thermal drift within 15 minutes	1 μm, 3 μm/m
thermal drift within 30 minutes	2 μm, 5 μm/m
thermal drift within 60 minutes	5 μm, 10 μm/m
evaluation of uncertainty	
assumed distribution within ranges	rectangular
coverage factor k	2

The repeatability of the measurement set-up shows the contribution of pitch, yaw and roll movements of the measuring equipment and/or the machine under test to any length measurement, if the Abbe-offsets during the measurement are not exactly known, or cannot exactly be repeated. An example of this contribution is given in [7] for a laser positioning measurement.

3.1 Example: machine tool acceptance, parameters

For the geometric machine tool testing different measurements have to be carried out. For a first estimate of the measurement uncertainties the following assumptions are made (see also [8]):

The characteristics of the measuring instruments in table 1 correspond to instruments available in the workshop; the data are according to the manufacturers' specifications. The measurement ranges are calculated from the task, the possible deviation of the machine tool, and the alignment of the instruments on the machine tool. Calibrated instruments and better aligned instruments will show smaller measurement errors (see also [7], including one uncertainty estimate for average, and one for improved industrial conditions).

The assumptions to the application of the measuring instruments of table 1 correspond to average industrial practice. The influence of air disturbance on light and laser are assumed according reports to the achievable repeatability of measurements and on discussions and reports of two ongoing European research projects [9, 10]. With more effort for the alignment of the instruments and with improved environmental conditions the given values may be reduced.

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The environmental characteristics are typical for industrial workshops, often they are worse. The repeatability value and the drift values for the machine tools are according to typical drift tests carried out in industrial workshops, where the values often are larger.

The values given in table 1 are assumed to be ranges, with a rectangular distribution of measurement values within the range.

3.2 Example: machine tool acceptance, estimated uncertainties

Table 2 shows the estimated uncertainties for machine tool acceptance measurements, using the parameters given in table 1, and compares the uncertainties with the tolerances given in the ISO standards for machining centres [11, 12]. The uncertainty related to the repeatability of the measurement set up is assumed to be zero.

According to table 2 just two measurements fulfil the "golden rule", for most measurements the measurement uncertainty reduced the tolerance zone significantly. In practice the situation is worse, because most machine tool manufacturers claim tolerances that are smaller than the ISO tolerances.

Table 2: Estimated uncertainties for machine tool acceptance measurements, measuring length = 1000 mm, measurement uncertainty U for k=2, rounded to µm, respectively µm/m.

measurement task	**U (k=2)**	**ISO tolerance**	**U as % of tolerance**	**reference for test & tolerance**
axial slip	1 µm	5 µm	20 %	[11]
parallelism spindle/axis	2 µm	15 µm	13 %	[11]
positioning in X, Y, Z				
- point at 1000 mm	8 µm			
- systematic deviation E	8 µm	23 µm	35 %	[12]
- accuracy A	10 µm	32 µm	31 %	[12]
roll/pitch/yaw				
- precision level	4 µm/m	60 µm/m	7 %	[11]
- laser, angular optics	2 µm/m	60 µm/m	3 %	[11]
run out spindle	1 µm	7 µm	14 %	[11]
squareness	10 µm/m	40 µm/m	25 %	[11]
straightness				
- alignment telescope	8 µm	20 µm	40 %	[11]
- laser, straightness optic	9 µm	20 µm	49 %	[11]
- straightedge	7 µm	20 µm	35 %	[11]
- straightedge, reversal	4 µm	20 µm	20 %	[11]
- taut wire, microscope	8 µm	20 µm	40 %	[11]

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Table 3: Magnitude of contributors to uncertainty of table 2, uncertainty U for k=2, relevant influence with grey, empty field: U < 0,1 µm.

measurement task	**U equipment**	**U applic.**	**U environm.**	**U repeat.**
axial slip	0,6 µm			
parallelism spindle/axis	2,2 µm	0,2 µm	0,6 µm	
positioning in X, Y, Z				
- point at 1000 mm	2,0µm	1,2 µm	7,6 µm	
roll/pitch/yaw				
- precision level	4,0 µm/m	0,2 µm/m	1,4 µm/m	
- laser, angular optics	0,4 µm/m	0,6 µm/m	1,4 µm/m	
run out spindle	1,0 µm			
straightness				
- alignment telescope	7,6 µm		1,4 µm	
- laser, straightness optic	2,2 µm		8,4 µm	
- straightedge	7,2 µm		0,6 µm	
- straightedge, reversal	1,6 µm		2,8 µm	1,2 µm
- taut wire, microscope	8,0 µm		1,6 µm	

In table 3 the principal sources of the measurement uncertainties are summarised, the principal source being the measurement equipment, the application of the equipment, the influence of the environment and the repeatability of the machine tool, as detailed at the begin of this clause.

For the so called classical measurements, i.e. the axial slip, the parallelism of the spindle to the axis, the angular measurements with the precision level, the run out, the straightness measurement with alignment telescope, mechanical straightedge or taut wire, the relevant contributor is the uncertainty of the measuring instrument.

For the advanced measurements, i.e. the positioning measurements, the laser interferometer measurements and the straightedge reversal, the environment is the most important contributor to the measurement uncertainty.

4 Volumetric accuracy

If the volumetric accuracy is calculated from the measurements of the geometric components, the uncertainty for a volumetric value becomes too large due to the measurement uncertainties. If we take a machining centre and the uncertainty values from table 2, and if we assume an Abbe offset on the workpiece side of 500 mm in Y and Z, and 100 mm in Z on the tool side, the

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uncertainty for the X position is 13 µm (k=2). As this value is mainly caused by the measurement uncertainties, and not by the machine tool, this uncertainty does not represent a machine performance parameter.

The situation for a coordinate measuring machine (CMM) is quite similar, as the measurement equipment for geometric deviations is the same. The advantage on CMMs is a better conditioned environment, unless the CMM is situated in the workshop, but the tolerances are in general smaller than for machine tools.

Therefore direct measurements in the working volume of the machine are applied to specify the volumetric performance of CMMs and machine tools. These are the measurement of precision spheres on CMMs [13] or circular paths on machine tools [14], the measurements of end bars arranged in space on CMMs [13] or diagonal positioning tests on machine tools [15], and the measurement of calibrated objects for determining the measurement uncertainty on CMMs [16] or the machining of test pieces [17].

The use of calibrated objects or machined test pieces is a powerful method for assessing the performance of CMMs and machine tools. But this method is limited to simple pieces, because as the calibrated objects or machined test pieces become more complex, the problem of the uncertainty of the calibration or measurement of these objects becomes relevant. Therefore the flexibility of theses modern machines cannot be covered with this method.

Moreover, the overall measurement uncertainty on a CMM and the accuracy of a machined workpiece depend on some more factors.

5 Measurement uncertainty on CMM, machining accuracy on machine tools

The influences to the overall measurement uncertainty and the machining accuracy may be grouped as follows:

- definition of the process,
- process parameters,
- environment,
- handling of the workpiece,
- workpiece characteristics, and
- machine performance.

Details to these groups are given in table 4. In general, the definition of the process has the largest influence on the result, the machine accuracy the smallest influence. However, the limit for the best result remains the machine accuracy, if time, money and knowledge is accumulated to minimise all the other influences, which - by the way - is seldom done.

Table 4: Influences on overall performance.

item	**CMM**	**machine tool**
definition of the process	def. of measurement task	machining sequence
process parameters	probe , number & distribution of points	tooling, cutting speed, feed
environment	absolute temperature, temperature changes, humidity, supply (air, electricity, etc.)	*same as for CMM*
handling of workpiece	cleaning, mounting	pre-machining, mounting
workpiece characteristics	roughness, form error, stiffness	hardness, homogeneity, stiffness
machine performance	geometry, probe system, probe calibration	geometry, control system, static & dynamic stiffness

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