6-1993

Tolerance Specification of Robot Kinematic Parameters Using an Experimental Design Technique

Y.H. Andrew Liou
Cleveland State University

Paul P. Lin
Cleveland State University, p.lin@csuohio.edu

Richard R. Lindeke
University of Minnesota - Duluth

Hsiang-Dih Chiang
Cleveland State University

Publisher's Statement

NOTICE: this is the author's version of a work that was accepted for publication in Robotics and Computer-Integrated Manufacturing. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Robotics and Computer-Integrated Manufacturing, 10, 3, (06-01-1993); 10.1016/0736-5845(93)90055-O

Original Citation

Andrew Liou, Y., Lin, P., Lindeke, R., , & Chiang, H. (1993). Tolerance specification of robot kinematic parameters using an experimental design technique-the Taguchi method. Robotics and Computer Integrated Manufacturing, 10(3), 199-207. doi:10.1016/0736-5845(93)90055-O

TOLERANCE SPECIFICATION OF ROBOT KINEMATIC PARAMETERS USING AN EXPERIMENTAL DESIGN TECHNIQUE—THE TAGUCHI METHOD

Y. H. ANDREW LIOU,* PAUL P. LIN,† RICHARD R. LINDEKE‡ and H. D. CHIANG†

*Industrial Engineering, Cleveland State University, U.S.A., †Mechanical Engineering, Cleveland State University, U.S.A. and ‡Industrial Engineering, University of Minnesota at Duluth, U.S.A.

This paper presents the tolerance specification of robot kinematic parameters using the Taguchi method. The concept of employing inner and outer orthogonal arrays to identify the significant parameters and select the optimal tolerance range for each parameter is proposed. The performance measure based on signal-to-noise ratios (S/N) using the Taguchi method is validated by Monte Carlo simulations. Finally, a step-by-step tolerance specification methodology is developed and illustrated with a planar two-link manipulator and a five-degree-of-freedom Rhino robot.

1. INTRODUCTION

Although some investigators have considered probabilistic models of closed-loop mechanisms in the past,[1–6] minimal attention has been paid to the probabilistic analysis of open-loop mechanisms, especially robot manipulators. It is known that errors in robot joint variables are random in nature. The random nature of the errors seems to make it necessary to use a probabilistic approach to the solution of the kinematic model.

Some significant studies[7–13] relevant to the stochastic analysis of robot error have been performed. Among them Benhabib *et al.*[13] introduced direct and inverse robot error analyses. The inverse method led to the development of a feasible joint tolerance domain, but it was hypothetical in that joint tolerances were assumed either all equal or unequal. Besides, a method to identify the more significant joints in order to tighten the tolerances of those joints was not suggested.

Bhatti and Rao[14] introduced the concept of reliability through which a statistical measure of manipulator performance can be made. They defined "manipulator reliability" as the probability of end-effector position and/or orientation (pose) falling within a specified range from the desired pose. The manipulator reliability of a simple two-link planar manipulator was calculated analytically, and that of a six-degree-of-freedom manipulator was obtained by the Monte Carlo simulation method. Manipulator reliability was found to be pose dependent. In industrial robot applications, one can find, by a bubble sort approach, an optimal robot configuration to reach a given pose with least error in the work space. However, the reliability also depends on the size of the allowable region specified, which means that it cannot be used as a performance measure to identify the significant kinematic parameters of a robot manipulator. In this paper, a performance measure, signal-to-noise ratios (S/N) using the Taguchi method, is proposed. The Taguchi method, an experimental design technique,[15–17] provides a simple way to design an efficient and cost effective experiment. The Taguchi method employs systematic orthogonal arrays that contain the manipulator's parameters at their different nominal values, in order to investigate the effect of maximum positive and negative variations from nominal. As such, it provides a maximum and controlled "error" for each parameter in a single experimental set. The results of the designed experiment can be analyzed using the analysis of variance (ANOVA) technique, and the kinematic parameters that contribute most to the observed performance measure can be identified.

In the tolerance specification of robot kinematic parameters three important tasks can be accomplished. These are, one, to identify the significant parameters and their interactions, two, to evaluate the robot's performance as a function of tolerance range, and, three, to tighten the tolerances of significant parameters while considering improvement of performance measures. In this study, a step-by-step toler-

Acknowledgment—This Project was supported by the Manufacturing Engineering Education Foundation, Society of Manufacturing Engineers, Grant No. 589-1418 and the Fenn College of Engineering, Cleveland State University.

ance specification methodology using the Taguchi method is presented, and the applicability of this method is investigated and validated by Monte Carlo simulations. Two different types of robot manipulators, a two-link planar manipulator and a five-degree-of-freedom Rhino robot, are used to illustrate the developed methodology. The presented tolerance specification technique is based on an assumption that a robot slowly approaches a static pose. The dynamic effects on the tolerances are not taken into account.

2. MAJOR SOURCES OF ERRORS IN ROBOT MANIPULATORS

In robot control, a set of joint variables is calculated when the desired pose of a robot end-effector is given. However, a discrepancy always exists between the actual and the desired pose due to the following sources of error.

2.1. *Errors in manufacture and assembly*

The errors due to tolerances in manufacture and assembly cause variations in the robot kinematic parameters. These errors can be minimized by tightening the tolerances. This, however, may increase the production cost considerably. Another possible remedy to this problem is to identify the true kinematic parameters by employing parameter identification techniques.[18] However, the identification procedure is tedious and time consuming. It requires highly sophisticated measurement devices such as the theodolite.[19] These errors can be classified as deterministic errors which generally do not vary during a robot motion.

2.2. *Errors in actuators and controllers*

Another source of errors is the result of joint drive compliance between the angular encoder and the actual angular output, dynamic effects or other random disturbances. These variations can be minimized by placing sensors to act as feedback mechanisms at the joints. However, these errors cannot be completely eliminated due to the resolution of sensing devices and the accuracy of controller response. These errors can be decomposed into two components. One component is the deterministic error which always acts in the same direction, an example being the bias in joint position. The other component is indeterministic error which is random and is assumed to follow a normal (Gaussian) distribution.

3. TOLERANCE SPECIFICATION FOR ROBOT KINEMATIC PARAMETERS USING THE TAGUCHI METHOD

The Taguchi method has been widely and successfully used in the U.S. and Japan to optimize industrial designs and processes for years. This method identifies those factors (independent variables) that have a significant effect on the performance (dependent) variable by using designed experiments. Traditional design of experiment (DOE) approaches utilize factorial and Latin square[20] design techniques for a small number of factors, and employ fractional factorials, response surface methodology (RSM) and orthogonal arrays for large numbers of factors. Professor Taguchi adopted orthogonal arrays to simplify the experimental design procedure.

Like other DOE techniques, designing an experiment and analyzing data are the two main phases involved in conducting experimental studies using the Taguchi method. In the following sections, these two phases to perform tolerance specification for robot kinematic parameters, involving some special techniques, are presented.

3.1. *Designing an experiment using orthogonal arrays*

While designing an experiment, the factors, their levels and interactions need to be determined. For the aforementioned tolerance design, each kinematic parameter could be considered as an independent variable, but the factors actually studied are the tolerance ranges of the kinematic parameters, and these ranges are under the control of the robot designer, at a given manufacturing cost. The random error from the nominal value of each kinematic parameter within a given tolerance range can also be treated as an independent variable. However, this error is an uncontrollable factor, or noise. The independent variables selected are then assigned to a limited set of discrete values (levels) rather than randomly over the distribution space as is done in Monte Carlo simulation. In the Taguchi method, two levels (minimum and maximum) are usually recommended, but three levels (minimum, medium and maximum) are also used. In addition, the interaction effects between factors are also investigated.

While using the Taguchi method, the experiment is designed by following the column assignments specified by an orthogonal array (OA). The OA design employed is based on the number of factors, their levels and the number of selected interactions. The basic OA designs in the Taguchi method are intended for use in experiments employing two-level independent variables. They are $L_4(2^3)$, $L_8(2^7)$, $L_{16}(2^{15})$ and $L_{32}(2^{31})$ orthogonal arrays. The a, b and c for an orthogonal array $L_a(b^c)$ represent the number of runs, the number of levels of each factor and the number of columns in the array, respectively. A second set of OAs is used for three-level factors, the $L_9(3^4)$, $L_{27}(3^{13})$ and $L_{81}(3^{40})$ OAs. In addition, several OA designs are used to specifically allow "mixed level" factors. They are $L_{18}(2^2 \times 3^7)$ and $L_{36}(2^3 \times 3^{13})$ OAs. After an OA is selected, designing an experiment becomes a "column assignment" task. Some of the above OAs are shown in the Appendix.

In the Taguchi method special techniques using inner/outer OAs are employed to study the controllable factors and the noise of controllable factors in one experiment. Usually, the latter is not included due to its random nature. If needed, in traditional DOE techniques, the noise can be studied by experimental blocking.

In the tolerance specification of robot kinematic parameters presented in this paper, the inner OA is used to study the controllable factors. For each experimental run, consisting of specific tolerance ranges for each kinematic parameter in an inner OA, the outer OA provides noise to each factor. The noises are directions that the tolerance value deviates from a nominal position. They represent "worst-case" tolerance deviations and satisfy the 3σ (3 times standard deviation) limits of normal Monte Carlo simulation variability. Each outer OA noise combination is treated as repetitive data in the inner OA. Thus, if an outer OA L_o is used for each run of an inner OA L_i, the size of the experiment is i * o runs.

For instance, $(\Theta_1)_m$, $(\Theta_2)_m$, $(\Theta_3)_m$ and $(\Theta_4)_m$ are the nominal values of kinematic parameters (revolute joint angles), Θ_1, Θ_2, Θ_3, Θ_4. In designing the inner OA, two tolerance ranges (tight, T and loose, L) are used for each of the factors. The outer array, then, specifies a value, nominal $\pm$ tolerance range specified by the inner array that will be used to calculate the experimental result. The structure of the inner and outer arrays is shown as follows:

Inner OA L_8
(Controllable Factors)

Run No.	Θ_1	Θ_2		Θ_3			Θ_4
1	T	T	T	T	T	T	T
2	T	T	T	L	L	L	L
3	T	L	L	T	T	L	L
4	T	L	L	L	L	T	T
5	L	T	L	T	L	T	L
6	L	T	L	L	T	L	T
7	L	L	T	T	L	L	T
8	L	L	T	L	T	T	L

Outer Array L_8
(Uncontrollable Factors)

$(\Theta_1)_m - \mathrm{T}$	$(\Theta_2)_m - \mathrm{T}$	$(\Theta_3)_m - \mathrm{L}$	$(\Theta_4)_m - \mathrm{L}$	$\rightarrow y_1$
$(\Theta_1)_m - \mathrm{T}$	$(\Theta_2)_m - \mathrm{T}$	$(\Theta_3)_m + \mathrm{L}$	$(\Theta_4)_m + \mathrm{L}$	$\rightarrow y_2$
$(\Theta_1)_m - \mathrm{T}$	$(\Theta_2)_m + \mathrm{T}$	$(\Theta_3)_m - \mathrm{L}$	$(\Theta_4)_m + \mathrm{L}$	$\rightarrow y_3$
$(\Theta_1)_m - \mathrm{T}$	$(\Theta_2)_m + \mathrm{T}$	$(\Theta_3)_m + \mathrm{L}$	$(\Theta_4)_m - \mathrm{L}$	$\rightarrow y_4$
$(\Theta_1)_m + \mathrm{T}$	$(\Theta_2)_m - \mathrm{T}$	$(\Theta_3)_m - \mathrm{L}$	$(\Theta_4)_m + \mathrm{L}$	$\rightarrow y_5$
$(\Theta_1)_m + \mathrm{T}$	$(\Theta_2)_m - \mathrm{T}$	$(\Theta_3)_m + \mathrm{L}$	$(\Theta_4)_m - \mathrm{L}$	$\rightarrow y_6$
$(\Theta_1)_m + \mathrm{T}$	$(\Theta_2)_m + \mathrm{T}$	$(\Theta_3)_m - \mathrm{L}$	$(\Theta_4)_m - \mathrm{L}$	$\rightarrow y_7$
$(\Theta_1)_m + \mathrm{T}$	$(\Theta_2)_m + \mathrm{T}$	$(\Theta_3)_m + \mathrm{L}$	$(\Theta_4)_m + \mathrm{L}$	$\rightarrow y_8$

Note:
T —tight tolerance range
L —loose tolerance range
y_i —individual data
$(\Theta_i)_m$—nominal value of Θ_i.

The objective of the inner OAs is to determine the significance of controllable factors, and to select the levels of significant factors to optimize the performance measure. Using the outer OA, noise is introduced into the experiment in a systematic manner. Thus, the results of inner OA analysis should be more "robust" against the noise of controllable factors.

3.2. *Analyzing data using the S/N ratio*

In the Taguchi method, the signal to noise ratio (S/N)[17] is used as a data transformation method to consolidate the repetitive data into one value which reflects the mean value and the amount of variation present in the data. The equations for calculating S/N ratios are based on the characteristics of the response variables being evaluated. In robotic tolerance design, the response variable is position or orientation error with a target of the smaller the better. The S/N equation for the case of the-smaller-the-better in the Taguchi method is:

$$\mathrm{S/N} = -10 \log\left[\left(\sum_{i=1}^{n} y_i^2\right)\Big/ n\right]$$

where: n = number of repetitive data
y_i = individual data.

The S/N ratio is essentially a measure of both the mean value (signal) and the standard deviation (noise) of a set of data. Factors that reduce variability in the error measures (positional or orientational) or move the error measures closer to zero (the target) increase the S/N ratio. The use of the S/N ratio to maintain accuracy in tolerance specification, as in parameter design results, is recommended since the outer array in the design attempts to dictate the variability during the experimentation through the control of noise factors. This contrasts with traditional DOE where replications of experimental runs let noise "contributions" seek their own levels.

Higher S/N ratios are indicative of experimental conditions that will be more robust during robot operation. Clausing[21] stated that the S/N ratio is a good performance measure of robustness against noise. Thus, it can be expected that statistically significant results found in analysis using S/N ratios will be the "Best Set" of kinematic parameter tolerances to control to render the robot more reliable.

3.3 *The step-by-step design methodology*

The methodology for tolerance design via the Taguchi method is described as follows:

Step 1. Identify the number of control factors, k, the robot kinematic parameters, and assign a reasonable amount of tolerance, t, to each factor.
Step 2. Design the inner array.
Step 3. Design the outer array.
Step 4. Calculate the orientation vectors **n**, **o**, **a**, and position vector **P** for all combinations of the inner and outer arrays.
Step 5. Calculate the S/N ratios of the end-effector's position or orientation error.
Step 6. Calculate the percentage of performance improvement based on the S/N ratios.
Step 7. Identify the significant factors by comparing the percentage of improvement on a case-by-case basis. Alternatively, an ANOVA, a test of statistical significance, can be performed.
Step 8. Find an optimum set of tolerance ranges.
Step 9. Fine-tune the set of ranges based on the considerations of cost and technical difficulty.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS OF TOLERANCE SPECIFICATION

In tolerance specification of robot kinematic parameters, a planar two-link manipulator was initially studied. Based on these results, the work was extended to a five-degree-of-freedom Rhino robot.

4.1. *Two-link robot manipulator*

For a planar two-link manipulator, there are four control factors which are the tolerances of the two revolute joint positions and those of the two link lengths. During this testing phase, the link length was considered an indeterministic factor due to errors in manufacture and assembly. Note that link lengths, unlike joint positions, do not randomly vary as a function of time during a robot motion. The objective here was to minimize the end-effector's positional error as this planar manipulator has limited orientation capabilities. The positional error is expressed by a square root of $[(P'_x - P_x)^2 + (P'_y - P_y)^2]$ where the unprimed and primed quantities represent the desired (nominal) and actual components of the end-effector's position, respectively.

The nominal value of the first and second links lengths, $(L_1)_m$ and $(L_2)_m$, are 10 cm and 8 cm, respectively, and the mean value of joint angles, $(\Theta_1)_m$ and $(\Theta_2)_m$, are both zero degrees. An initial value of tolerance for each of the four factors is assigned by a reasonable number, 0.03 cm (3 times σ of 0.01 cm) for the link lengths and 0.3 degree (3 times σ of 0.1 degree) for the joint positions. Thus, the tolerance ranges are set either at loose (original) value or tight (half-original) value. For the planar two-link manipulator, there are 16 (2^4) possible cases forming the inner othogonal array in the Taguchi method. The outer array (noise array) uses Taguchi's $L_8(2^7)$ OA (shown in the Appendix) which accounts for the randomized variations about the nominal values. The S/N ratio of each case is used as a performance measure. The improvement in positional error can be observed by comparing the percentage change of the performance measure in the inner array.

Table 1 shows the S/N ratios for this experiment. These ratios were calculated using both Taguchi method (TM) and Monte Carlo (MC) simulation techniques. The Monte Carlo simulation technique is based on random numbers generated within a specified range of each factor for approximately 10,000 runs to estimate the performance measure. As can be seen, the difference in S/N ratios calculated using MC and TM is very little. Significant parameters (factors) can be identified by comparing the improvement of S/N ratios. For instance, the performance improves from Case 16 to Case 14 by:

$$(22.38 - 19.05)/19.05 = 17.48\%$$

while the S/N ratio improves from Case 16 to Case 15 by 2.57%. It is obvious that Θ_1 is more significant than Θ_2 in terms of positional error. In the Taguchi method, the significant factors can be simply identified through the analysis of variance (ANOVA) shown in

Table 1. Results of Monte Carlo and Taguchi methods for a planar two-link manipulator

Case	L_1 (cm)	L_2 (cm)	Θ_1 (deg.)	Θ_2 (deg.)	Monte Carlo S/N	Taguchi S/N
1	0.015	0.015	0.15	0.15	25.08	25.07
2	0.015	0.015	0.15	0.30	23.53	23.54
3	0.015	0.015	0.30	0.15	20.13	20.10
4	0.015	0.015	0.30	0.30	19.57	19.55
5	0.015	0.030	0.15	0.15	24.21	24.22
6	0.015	0.030	0.15	0.30	22.91	22.92
7	0.015	0.030	0.30	0.15	19.84	19.81
8	0.015	0.030	0.30	0.30	19.30	19.29
9	0.030	0.015	0.15	0.15	24.23	24.22
10	0.030	0.015	0.15	0.30	22.92	22.92
11	0.030	0.015	0.30	0.15	19.84	19.81
12	0.030	0.015	0.30	0.30	19.31	19.29
13	0.030	0.030	0.15	0.15	23.49	23.32
14	0.030	0.030	0.15	0.30	22.36	22.38
15	0.030	0.030	0.30	0.15	19.56	19.54
16	0.030	0.030	0.30	0.30	19.06	19.05

The data used to calculate the S/N ratios are the differences between the actual and desired end-effector's positions (positional error) in terms of length unit.

Table 2. This alternative, without comparing data case by case, is much more computationally efficient. The F values in Table 2 clearly indicate that the tolerances of Θ_1 and Θ_2 are the most and second most significant factors, respectively. Another advantage of ANOVA is that the significance of parameter interaction can also be identified.

The tolerance of each robotic joint or link parameter would initially have been assigned a reasonable tolerance range. The main objective here is to find an optimum set of tolerance ranges. The strategy is to tighten the tolerances of the significant factors and widen those of the insignificant factors. Once the ranges have been determined, the tolerances can be further "fine-tuned" to make the end-effector's performance better and reduce the overall cost if possible.

Robot performance (position accuracy of the robot end-effector) can be improved by tightening the tolerances of some joints. The choices are as follows:

Choice 1: Tightening the tolerance of one factor. The tolerance of Θ_1 is identified as the most significant factor. Using the data of Case 16 (worst) as reference, the percentage improvement of Case 14 is 17.48% based on TM.

Choice 2: Tightening the tolerances of two factors. The best way is to tighten the tolerances of Θ_1 and Θ_2 (Case 13). The percentage improvement is 22.41%.

Choice 3: Tightening the tolerances of three factors. The best way is to tighten the tolerances of Θ_1, Θ_2 and L_1 or L_2 (Case 5 or 9). The percentage improvement is 27.14%.

Choice 4: Tightening the tolerances of four factors. This corresponds to Case 1. The percentage improvement is 31.60%.

The above analysis indicates that tightening the tolerance of one factor (Θ_1) is the most cost effective way to improve the end-effector's performance. If necessary and feasible, the tolerance of joint 1 can be further reduced to increase the performance. Of equal importance, the tolerances of insignificant factors such as L_1 and L_2 can be widened without affecting robot performance significantly. This step can lead to overall cost savings for manipulator construction.

Table 2. ANOVA table (position error)

Source of variance	DF	SS	Mean square	F Value	P
L_1	1	0.8977	0.8977	9.5957	<0.025
L_2	1	0.8967	0.8967	9.5857	<0.025
Θ_1	1	65.3989	65.3989	699.0795	$\ll 0.010$
Θ_2	1	3.3509	3.3509	35.8194	<0.010
Error	11	1.0290	0.0935		

where: DF = degrees of freedom
SS = sum of squares
F value = statistics of F distribution
P = level of significance.

It may be that the S/N ratios calculated using MC are more accurate than those found using TM since the former requires approximately 10,000 runs per case, while the latter only performs eight experimental runs. Although the percentage improvement using the Taguchi method is slightly lower than that using the Monte Carlo method, both methods reveal the same trends for performance improvement.

4.2. *Five-degree-of-freedom Rhino robots*

Based on the results found during the evaluation of the simple two-link robot, the work was extended to a five-degree-of-freedom Rhino robot (Fig. 1).[22] The robot has no prismatic but five revolute joints. The positional error can be quantified by using a concept of error sphere. The radial distance from the sphere center indicates the deviation from the desired position. When measuring the performance of an end-effector, orientation accuracy, in addition to position accuracy, is also concerned in the Rhino robots. Usually the end-effector's orientation is expressed in terms of three unit vectors: normal (**n**), orientation (**o**), and approach (**a**) vectors as shown in Fig. 2, where the position vector (**p**) is from the origin of the base coordinates to the intersection of the above three vectors (**n**, **o** and **a**).[23] Any end-effector's orientation can be achieved by a series of rotations about the z, y and x axes through the so-called roll, pitch and yaw angles. To quantify the orientation error, only one quantity involving three consecutive rotations will be examined. This quantity shown below indicates the deviation from the desired orientation

$$\mathrm{RPY}(\phi, \Gamma, \psi) = \mathrm{Rot}(z, \phi)\mathrm{Rot}(y, \Gamma)\mathrm{Rot}(x, \psi)$$

where ϕ, Γ and ψ are the nominal roll, pitch and yaw rotational angles of the end-effector, respectively required to reach a desired orientation. These transformation angles can be calculated from joint positions and link lengths. The RPY function determines an unique orientation. The overall orientation error can be determined by:

$$[(\phi' - \phi)^2 + (\Gamma' - \Gamma)^2 + (\psi' - \psi)^2]^{0.5}$$

where unprimed and primed quantities represent desired (nominal) and actual angles, respectively. The actual angles deviate from the nominal values while considering the tolerance of each robot kinematic parameter such as joint and link length.

As the results of the two-link manipulator indicated, revolute joint parameters are much more significant to positional accuracy than link parameters (i.e. link length). In practice, the effects of biased link lengths can be minimized by robot calibration or parameter identification techniques. Thus, the link parameters can be considered deterministic for an industrial robot. In this section, only the effect of tolerances in joint parameters are investigated.

The tolerance for each of the five joints is initially set at a level of 0.10 degrees. The desired position and

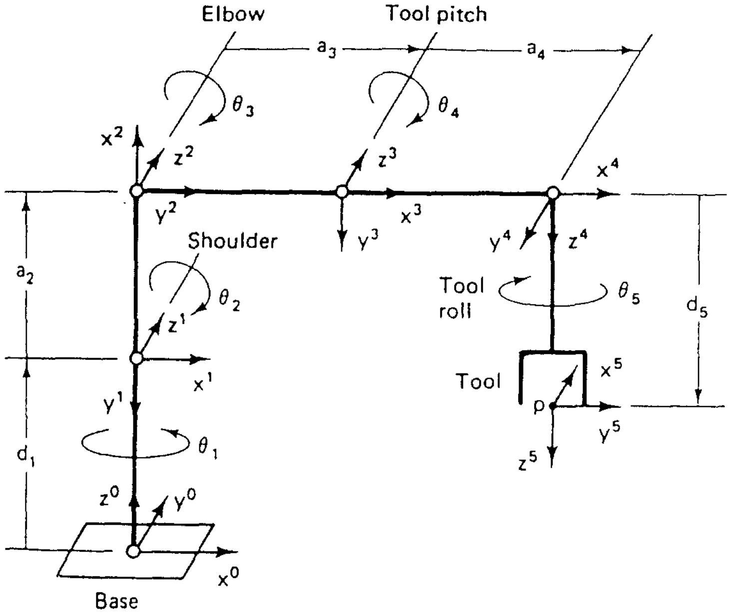

Fig. 1. Five-degree-of-freedom Rhino robot.

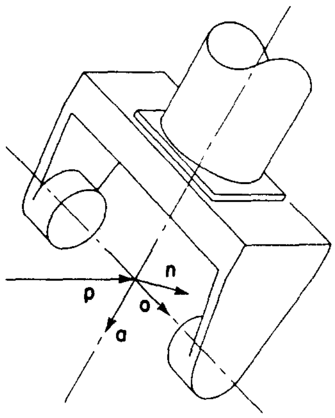

Fig. 2. **n**, **o**, **a** and **p** vectors.

orientation matrix (T_5) is as follows:

$$
\begin{vmatrix} \mathbf{n}_x & \mathbf{o}_x & \mathbf{a}_x & \mathbf{p}_x \\ \mathbf{n}_y & \mathbf{o}_y & \mathbf{a}_y & \mathbf{p}_y \\ \mathbf{n}_z & \mathbf{o}_z & \mathbf{a}_z & \mathbf{p}_z \\ 0 & 0 & 0 & 1 \end{vmatrix}
= \begin{vmatrix} 0.027 & -0.621 & 0.783 & 4.739 \\ -0.864 & 0.380 & 0.331 & 2.000 \\ -0.503 & -0.685 & -0.526 & 4.000 \\ 0 & 0 & 0 & 1 \end{vmatrix}.
$$

The inverse kinematic solution for the desired pose gives the nominal joint positions (in degrees) as follows:

$$
\begin{aligned}
\Theta_1 &= 22.881^\circ \\
\Theta_2 &= 70.350^\circ \\
\Theta_3 &= 100.075^\circ \\
\Theta_4 &= -228.676^\circ \\
\Theta_5 &= 126.276^\circ
\end{aligned}
$$

Tables 3 (position accuracy) and 4 (orientation accuracy) show the comparisons between the Monte Carlo and Taguchi methods.

For the performance measure, MC requires in total

Table 3. Comparison of position accuracy between two methods for Rhino robots

	Joint tolerances in degrees					Monte Carlo	Taguchi
Case	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	S/N	S/N
1	0.05	0.05	0.05	0.05	0.05	37.4663	37.4463
2	0.05	0.05	0.05	0.05	0.10	37.4663	37.4463
3	0.05	0.05	0.05	0.10	0.05	33.0682	33.0365
4	0.05	0.05	0.05	0.10	0.10	33.0682	33.0365
5	0.05	0.05	0.10	0.05	0.05	36.8484	36.8348
6	0.05	0.05	0.10	0.05	0.10	36.8484	36.8348
7	0.05	0.05	0.10	0.10	0.05	32.8284	32.8110
8	0.05	0.05	0.10	0.10	0.10	32.8284	32.8110
9	0.05	0.10	0.05	0.05	0.05	35.0049	34.9972
10	0.05	0.10	0.05	0.05	0.10	35.0049	34.9972
11	0.05	0.10	0.05	0.10	0.05	32.0110	31.9812
12	0.05	0.10	0.05	0.10	0.10	32.0110	31.9812
13	0.05	0.10	0.10	0.05	0.05	34.6443	34.6443
14	0.05	0.10	0.10	0.05	0.10	34.6443	34.6443
15	0.05	0.10	0.10	0.10	0.05	31.8222	31.7997
16	0.05	0.10	0.10	0.10	0.10	31.8222	31.7997
17	0.10	0.05	0.05	0.05	0.05	36.2207	36.1881
18	0.10	0.05	0.05	0.05	0.10	36.2207	36.1881
19	0.10	0.05	0.05	0.10	0.05	32.5735	32.5368
20	0.10	0.05	0.05	0.10	0.10	32.5735	32.5368
21	0.10	0.05	0.10	0.05	0.05	35.7489	35.7283
22	0.10	0.05	0.10	0.05	0.10	35.7489	35.7283
23	0.10	0.05	0.10	0.10	0.05	32.3589	32.3318
24	0.10	0.05	0.10	0.10	0.10	32.3589	32.3318
25	0.10	0.10	0.05	0.05	0.05	34.2551	34.2422
26	0.10	0.10	0.05	0.05	0.10	34.2551	34.2422
27	0.10	0.10	0.05	0.10	0.05	31.6185	31.5888
28	0.10	0.10	0.05	0.10	0.10	31.6185	31.5888
29	0.10	0.10	0.10	0.05	0.05	33.9497	33.9414
30	0.10	0.10	0.10	0.05	0.10	33.9497	33.9414
31	0.10	0.10	0.10	0.10	0.05	31.4457	31.4265
32	0.10	0.10	0.10	0.10	0.10	31.4457	31.4265

Each S/N ratio is based on the difference between the actual and desired end-effector's positions in terms of length unit.

Table 4. Comparison of orientation accuracy between two methods for Rhino robots

	Joint tolerances in degrees					Monte Carlo	Taguchi
Case	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	S/N	S/N
1	0.05	0.05	0.05	0.05	0.05	26.9170	26.7263
2	0.05	0.05	0.05	0.05	0.10	23.3902	23.1411
3	0.05	0.05	0.05	0.10	0.05	21.4527	21.2131
4	0.05	0.05	0.05	0.10	0.10	19.9180	19.6671
5	0.05	0.05	0.10	0.05	0.05	18.9388	18.6941
6	0.05	0.05	0.10	0.05	0.10	18.0117	17.7621
7	0.05	0.05	0.10	0.10	0.05	17.2533	17.0202
8	0.05	0.05	0.10	0.10	0.10	16.5242	16.2957
9	0.05	0.10	0.05	0.05	0.05	16.0481	15.8208
10	0.05	0.10	0.05	0.05	0.10	15.5471	15.3150
11	0.05	0.10	0.05	0.10	0.05	15.1021	14.8773
12	0.05	0.10	0.05	0.10	0.10	14.6446	14.4208
13	0.05	0.10	0.10	0.05	0.05	14.2785	14.0613
14	0.05	0.10	0.10	0.05	0.10	13.8954	13.6797
15	0.05	0.10	0.10	0.10	0.05	13.5483	13.3322
16	0.05	0.10	0.10	0.10	0.10	13.1906	12.9749
17	0.10	0.05	0.05	0.05	0.05	12.9743	12.7566
18	0.10	0.05	0.05	0.05	0.10	12.7314	12.5122
19	0.10	0.05	0.05	0.10	0.05	12.4979	12.2769
20	0.10	0.05	0.05	0.10	0.10	12.2467	12.0248
21	0.10	0.05	0.10	0.05	0.05	12.0373	11.8139
22	0.10	0.05	0.10	0.05	0.10	11.8106	11.5866
23	0.10	0.05	0.10	0.10	0.05	11.5950	11.3729
24	0.10	0.05	0.10	0.10	0.10	11.3670	11.1469
25	0.10	0.10	0.05	0.05	0.05	11.1951	10.9738
26	0.10	0.10	0.05	0.05	0.10	11.0075	10.7857
27	0.10	0.10	0.05	0.10	0.05	10.8279	10.6073
28	0.10	0.10	0.05	0.10	0.10	10.6364	10.4170
29	0.10	0.10	0.10	0.05	0.05	10.4709	10.2528
30	0.10	0.10	0.10	0.05	0.10	10.2938	10.0772
31	0.10	0.10	0.10	0.10	0.05	10.1247	9.9087
32	0.10	0.10	0.10	0.10	0.10	9.9466	9.7316

Each S/N ratio is based on the difference between the actual and desired end-effector's orientations in terms of degrees of rotation angle.

320,000 (32 × 10,000) evaluations, whereas TM requires only 512 (32 × 16) evaluations. The computational efficiency ratio between them is 625 to 1. The robot joint tolerances are designed according to the performance improvements which are summarized as follows:

(a) *Considering position accuracy*

The tolerance of joint 4 is identified as the most significant factor. Tightening the tolerance of Θ_4 by 50% (Case 30 in Table 3) will improve the end-effector's position accuracy by 8.00%. The tolerances of Θ_2 and Θ_1 are the second and third most significant factors, whereas those of Θ_3 and Θ_5 are very insignificant. When studying the ANOVA (Table 5) from TM, the interaction between Θ_2 and Θ_4 is found to be significant as well.

(b) *Considering orientation accuracy*

Θ_2, Θ_3 and Θ_4 are identified as the three significant factors by examining the S/N ratios in Cases 24, 28 and 30. Among these three factors, Θ_2 is the most significant. The ANOVA in Table 6 also identifies the same three significant factors.

(c) *Considering both position and orientation*

When both position and orientation (pose) accuracies are considered, Θ_4 is selected if only one joint is to be tightened. It is also found from the ANOVA that Θ_5 has no influence on position accuracy. Thus, the tolerance of Θ_5 is a candidate to be widened for possible cost reduction. However, this will worsen the orientation accuracy. If the designer wishes to increase pose accuracies, the tolerances of Θ_2, Θ_3 and Θ_4 can be simultaneously reduced by 50%. This will improve the position accuracy by 15.15%, and improve the orientation accuracy by 28.57%. Since the tolerance of Θ_3 is very insignificant to position accuracy, the most cost effective way to improve performance would be to tighten the tolerances of Θ_2 and Θ_4 only.

Table 5. ANOVA table (position error)

Source of variance	DF	SS	Mean square	F value	P
Θ_1	1	3.8738	3.8738	61.1878	$\ll 0.01$
Θ_2	1	18.8875	18.8875	298.3328	$\ll 0.01$
Θ_4	1	87.8498	87.8498	1387.6140	$\ll 0.01$
$\Theta_2 * \Theta_4$	1	2.4782	2.4782	35.8194	$\ll 0.01$
Error	27	1.7093	0.0633		

Table 6. ANOVA table (orientation error)

Source of variance	DF	SS	Mean Mean square	F value	P
Θ_2	1	29.0870	29.0870	178.0762	$\ll 0.01$
Θ_3	1	29.0870	29.0870	178.0762	$\ll 0.01$
Θ_4	1	29.0870	29.0870	178.0620	$\ll 0.01$
Error	28	4.5735	0.1633		

5. CONCLUSIONS

This paper presented a step-by-step methodology for systematic selection of tolerance ranges to use while performing the tolerance design of robot kinematic parameters. It is a new application of Taguchi's parameter design technique where the inner and outer orthogonal arrays contain tolerance ranges and deviations from nominal, respectively. Using the developed methodology, kinematic parameters or factors that have a significant effect on positional or orientational accuracy can be identified by comparing experimentally calculated S/N ratios on a case-by-case basis. Experimental results were compared to Monte Carlo simulation techniques and found to be nearly identical. The Taguchi method is found to be a much more computationally efficient alternative. When the Taguchi method was used, an analysis of variance (ANOVA), a test of statistical significance, was performed. The ANOVA identified the same significant factors as were found during exhaustive searches using a bubble sorting technique through case by case comparisons. ANOVA is a much more efficient identifier and not prone to interpretational error when robot kinematic parameters are interacting.

It is found that the performance measure based on the S/N ratio varies with the robot arm configuration.

This is in agreement with the finding of Bhatti and Rao's work[14] in which the manipulator reliability varies with the arm configuration. Therefore, in the robot tolerance specification one should first investigate the performance of important poses within the robot workspace. The parameter tolerances can then be set based on the average and the worst case within the workspace.

It should be noted that the Taguchi method presented is computational in nature. To experimentally verify the significant parameters identified by ANOVA, the Taguchi method or any DOE approaches can not be implemented to measure the robot end-effector's performance unless joint positions at extreme values (low and high) can be accurately and efficiently obtained. Furthermore, it was the authors' intention to develop a methodology that is most appropriate at the robot system design level, before initial construction begins. Its use will help a robot system designer to meet a specified pose tolerance level at a minimum cost.

REFERENCES

1. Rao, S. S., Given, S. S.: Analysis and synthesis of mechanical error in Geneva mechanism—a stochastic approach: Parts I and II. *ASME J. Mechanical Design* **104**: 63-77, 1982.
2. Schade, G. R., Mischke, C. R.: Deterministic vs. probabilistic models in slider-crank function generator design. *Proceedings of the 9th Reliability and Maintainability Conference*, Detroit, Michigan, Vol. 9, pp. 527-533, 20-22 July 1970.
3. Schade, G. R.: Probabilistic models in computer automated slider-crank function generator design. *ASME J. Mechanical Design* **103**(4): 835-841, Oct. 1981.
4. Schade, G. R.: A bivariate normal model of mechanism coupler-point position. *ASME J. Mechanisms, Transmissions Automation in Design* **105**: 599-605, 1983.
5. Baumgarten, J. R., Fixmer, J. V.: A note on a probabilistic study concerning linkage tolerances and coupler curves. ASME Paper 76-DET-3, 1976.
6. Dhande, S. G., Charkraborty, J.: Mechanical error analysis of spatial linkages. *ASME J. Mechanical Design* **100**(4): 732-738, 1978.
7. Waldron, K. J., Kumar, A.: Development of a theory of errors for manipulators. *ASME Proceedings of 5th World Congress of Theory of Machines and Mechanisms*, pp. 821-826, 1979.
8. Colson, J. C., Perrura, N. D.: Quasi-static performance of robots. *Robotics Computer-Integr. Manufacturing* **2**(3/4): 261-275, 1985.
9. Veitschegger, W., Wu, C. H.: Robot accuracy analysis based on kinematics. *IEEE J. Robotics Automation* RA-**2**(3): 171-179, Sept. 1986.
10. Azadivar, F.: The effect of joint position errors of industrial robots and their performance in manufacturing operations. *IEEE J. Robotics Automation* RA-**3**(2): 109-114, April 1987.
11. Baumgarten, J. R., Lynch, A. G.: Probabilistic estimate of hand position error for a six jointed robot. *Proceedings of SIFIR 89 International Workshop on Industrial Robots*, Zaragoza, Spain, pp. 181-185, 22-24 Nov. 1989.
12. Chen, J., Chao, L.: Positioning error analysis for robot manipulators with all rotary joints. *IEEE J. Robotics Automation* RA-**3**(6): 539-545, Dec. 1987.
13. Benhabib, B., Fenton, R. G., Goldenberg, A. A.: Computer-aided joint error analysis of robots. *IEEE J. Robotics Automation* RA-**3**(4): 317-322, Aug. 1987.
14. Bhatti, P. K., Rao, S. S.: Reliability analysis of robot manipulators. *ASME J. Mechanisms Transmissions Automation in Design* **110**: 175-181, 1988.
15. Taguchi, G.: *System of Experimental Design*, Vols 1 and 2, White Plains, UNIPUB/Kraus International Publication, 1987.
16. Ross, P. J.: *Taguchi Techniques for Quality Engineering*, McGraw-Hill, 1988.
17. Lindeke, R. R., Liou, Y. H. A.: Methods for optimization in the manufacturing system—the Taguchi method: an engineering approach to its implementation. *J. Mechanical Working Technol.* **20**: 205-218, 1989.
18. Hsu, T. W., Everett, L. J.: Identification of the kinematic parameters of a robot manipulator for positional accuracy improvement. *ASME Conference Proceedings of Computers in Engineering*, Vol. 2, pp. 263-267, 1985.
19. Mooring, B. W., Tang, G. R.: An improved method for identifying the kinematic parameters in a six-axis robot. *ASME Proceedings of the 1984 International Computers in Engineering Conference and Exhibition*, Vol. 1, pp. 79-84.
20. Montgomery, D. C.: *Design and Analysis of Experiments*, New York, John Wiley, 1991.
21. Clausing, D.: Taguchi methods to improve the development process, *IEEE International Conference on Communications*, 12-15 June, 1988.
22. Shahinpoor, M.: *A Robot Engineering Textbook*, New York, Harper & Row, 1987, 125-130.
23. Paul, R. P.: *Robot Manipulators: Mathematics, Programming, and Control*, Cambridge, MA, MIT Press, 1981.

APPENDIX: TAGUCHI'S ORTHOGONAL ARRAYS

L_8 (2^7) table

	Column No.						
Run No.	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

1 — low level
2 — high level.

L_9 (3^4) table

	Column No.			
Run No.	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

1 — low level
2 — medium level
3 — high level.

L_{16} (2^{15}) table

Run No.	Column No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1