Tone Injection with Hexagonal Constellation for Peak-to-Average Power Ratio Reduction in OFDM

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***Abstract*— One of the main drawbacks of orthogonal frequency division multiplexing (OFDM) is the high peak-to-average power ratio (PAPR) of the OFDM signal. Tone injection (TI) technique is a promising candidate for the reduction of the PAPR of an OFDM signal. But this technique requires the increase of the transmit signal power because of the injected signal. In this paper, we propose the use of hexagonal constellation as a method to achieve PAPR reduction without increasing signal power in the TI technique.**

***Index Terms*— OFDM, PAPR, hexagonal constellation, tone injection (TI).**

I. Introduction

RECENTLY, orthogonal frequency division multiplexing (OFDM) has been receiving considerable attention for high-speed wireless communication systems. One of the major drawbacks of OFDM is, however, the high peak-to-average power ratio (PAPR) of the transmit signal. A number of approaches have been proposed to deal with the PAPR problem. These techniques include amplitude clipping, clipping and filtering, coding, partial transmit sequence, selected mapping, interleaving, tone reservation, tone injection (TI) [1], and active constellation extension. For an overview of these techniques, see [2]. Among them the TI technique has many advantages: it is a distortionless technique, it does not require the exchange of side information between transmitter and receiver, and there is no data rate loss. However, there is an increase in transmit signal power due to the injected signal in the TI technique.

In this paper, we propose the use of hexagonal constellation in the TI technique. We can have more signal points in a given area by using a hexagonal constellation instead of quadrature amplitude modulation (QAM) and this extra degree of freedom is utilized for PAPR reduction without increasing signal power in the TI technique.

II. System Model

Let us denote the data block of length N as a vector $\mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]^T$ where N is the number of subcarriers. Each symbol in $\mathbf{X}$ modulates one of a set of subcarriers, $\{f_n, n = 0, 1, \cdots, N-1\}$. The N subcarriers are chosen to be orthogonal, that is, $f_n = n\Delta f$, where $\Delta f = 1/NT$ and T is the duration of the data symbol X_n. The duration of an OFDM data block is *NT*. The complex envelope of the transmit OFDM signal is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \ 0 \le t < NT. \tag{1}$$

The PAPR of the transmit signal $x(t)$ is defined as

$$\text{PAPR} = \frac{\max\limits_{0 \le t < NT} |x(t)|^2}{1/NT \cdot \int_0^{NT} |x(t)|^2 \, dt}. \tag{2}$$

Signal samples of $x(t)$ are usually used instead of $x(t)$ itself for PAPR calculation. The signal samples are obtained by oversampling (1) by a factor of L to approximate the true PAPR better. The oversampled time-domain samples are obtained by an *LN*-point inverse discrete Fourier transform (IDFT) of the data block with $(L-1)N$ zero-padding.

III. Hexagonal Constellation and PAPR Reduction

The densest packing of regularly spaced points in two dimensions is the hexagonal lattice shown in Fig. 1 [3]. The volume or area of the decision region for each point is $\nu_H = \sqrt{3}d^2/2$ where d is the minimum distance between points. When compared with QAM constellation, we can pack more signal points for hexagonal constellation in a given area. Specifically, the ratio between the numbers of signal points is $(1/\nu_H)/(1/\nu_S) = d^2/(\sqrt{3}d^2/2) = 2/\sqrt{3}$ where $\nu_S = d^2$ is the volume or area of the decision region for each point in QAM constellation.

We can have extra degrees of freedom by using hexagonal constellation with appropriate number of points instead of QAM constellation. A rough idea can be illustrated with the hexagonal constellation with 7 signal points (7-HEX) shown in Fig. 1. Assume that 7-HEX is used instead of 4-QAM which has 4 symbols ‘1’, ‘2’, ‘3’, and ‘4’. In this case, we have 3 excess signal points in 7-HEX. So some of the points in 4-QAM may be associated with more than one point in 7-HEX. In Fig. 1, symbol ‘1’ in 4-QAM has 1 representation in 7-HEX and symbols ‘2’, ‘3’, and ‘4’ in 4-QAM have 2 representations in 7-HEX. At the receiver, either ‘2A’ or ‘2B’ can be considered as symbol ‘2’. We can freely choose between the two points so that the PAPR is reduced in the transmit signal. This idea can be directly related to the TI technique.

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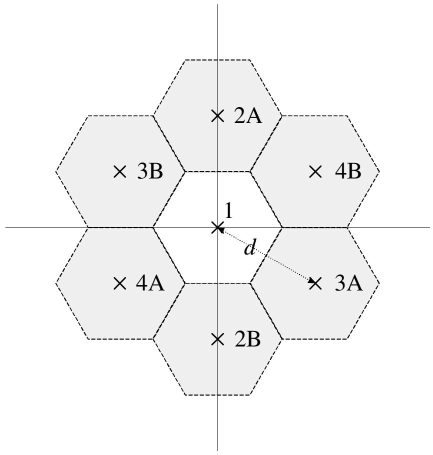

Fig. 1. Hexagonal constellation with 7 signal points (7-HEX).

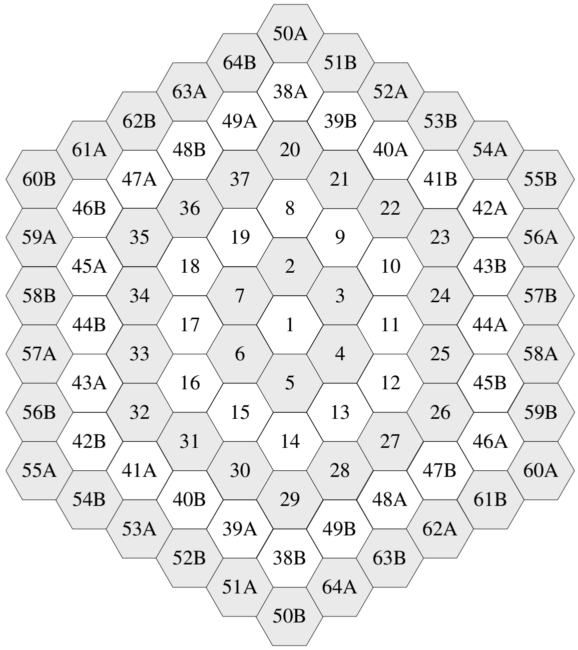

Fig. 2. Hexagonal constellation with 91 signal points (91-HEX).

IV. Tone Injection with Hexagonal Constellation

The TI technique [1] is based on adding a data-block-dependent time-domain signal to the original OFDM signal to reduce its peaks. The basic idea is to increase the constellation size so that each of the points in the original basic constellation can be mapped into several equivalent points in the expanded constellation. Since each symbol in a data block can be mapped into one of several equivalent points, these extra degrees of freedom can be exploited for PAPR reduction. This method is called tone injection because substituting a point in the basic constellation for a new point in the larger constellation is equivalent to injecting a tone of the appropriate frequency and phase in the OFDM signal. The problem of the ordinary TI technique is the increase of signal power due to the injected signal because all equivalent constellation points have larger amplitude than the original one.

In contrast, we can design the hexagonal constellation such that the amplitude of the equivalent points is the same and the average power of the signal points is less than or equal to that of the square QAM constellation with same data rate. Since the amplitude of all equivalent signal points is the same, there is no power increase due to the injected signal. For example, hexagonal constellation with 91 signal points (91-HEX) is shown in Fig. 2. Symbol '1', '2', $\cdots$, '37' in 64-QAM have 1 representation in 91-HEX and symbol '38', '39', $\cdots$, '64' in 64-QAM have 2 representations in 91-HEX. In this case, 2 representations of a single point have same amplitude and opposite signs. So we can find another representation by simply multiplying -1 to one representation. When the symbol in a certain subcarrier is between '38' and '64', we can reduce PAPR by choosing appropriate representation in 91-HEX.

The PAPR reduction capability of the TI technique with hexagonal constellation is based on the number of representations for the symbols with more than one representation, R, and the number of subcarriers with data symbols having more than one representation, N_{TI}. We can achieve the most PAPR reduction by searching all $(N_{TI})^R$ possible transmit signals. But this is too complex for practical realization when N_{TI} is large. So we can apply the reduced complexity algorithms which were proposed for the PTS technique in [4], [5]. Details of the TI technique with the simple iterative flipping algorithm [4] are as follows:

1) Identify the subcarrier positions with more than one representation and denote the subcarrier indices as $I = \{i_1, i_2, \cdots, i_{N_{TI}}\}$.
2) Set iteration count $j = 0$, choose the first representation for all positions in I, and compute PAPR of the resulting signal.
3) Increase j by 1. For the subcarrier i_j, select the representation which achieves the lowest PAPR among R possibilities with all other subcarriers being fixed.
4) If $j < N_{TI}$, go to step 3); otherwise terminate.

We can also use gradient guided search algorithm proposed in [5] to get better PAPR results with higher complexity.

It is possible to generalize the proposed scheme to general QAM constellations. We can think the hexagonal constellation as a layered structure. For example, the layer 2 is made up of points '2', '3', '4', '5', '6', and '7' in Fig. 2. For general QAM, we can choose the number of layers in hexagonal constellation so that the number of points in those layers exceeds that of QAM constellation under consideration. For instance, there are 7 points in layers up to 2 and 19 points in layers up to 3. So it is required to use more than 3 layers for 16-QAM. If we choose to use layers up to 3, 13 points in 16-QAM have 1 representation in hexagonal constellation with 19 points (19-HEX) and 3 points in 16-QAM have 2 representations in 19-HEX. If we use more layers than minimum required, it is possible to increase the number of points in QAM that have more than one representation in hexagonal constellation. It is also possible to partially use the outermost layer.

V. Numerical Results and Discussions

We assume OFDM systems with 64, 128, and 256 subcarriers ($N = 64, 128, 256$) with 64-QAM or 91-HEX constellation. The transmit signal is oversampled by a factor of 4

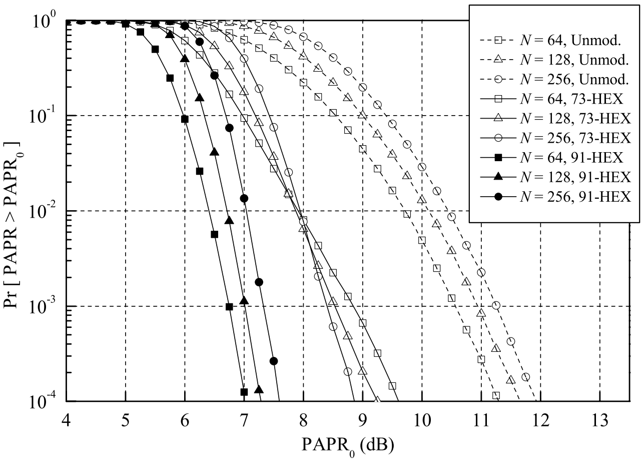

Fig. 3. CCDFs of the PAPR of the TI technique with the 91-HEX for OFDM systems with 64, 128, 256 subcarriers.

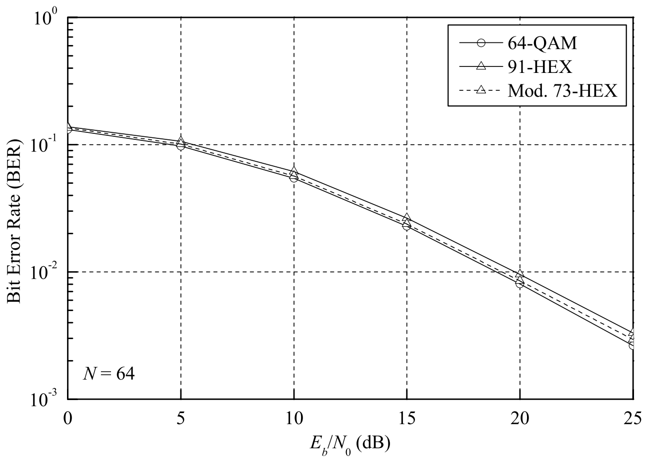

Fig. 4. BER of the TI technique with 91-HEX constellation for OFDM systems with 64 subcarriers in a Rayleigh fading channel.

($L = 4$) and 200,000 random OFDM blocks were generated to obtain the complementary cumulative density functions (CCDFs) of PAPR. When the minimum distance is d, the average power of the signal points of the 91-HEX is $10.36d^2$ while that of the 64-QAM is $10.50d^2$. So there is no average power increase from using the 91-HEX. The average power can be reduced even further if we reduce the number of signal points with more than 1 representation. It is found that the average power is $9.94d^2$ and $9.19d^2$ when we use hexagonal constellation with 85 points (85-HEX) and with 73 points (73-HEX), respectively. 43 points in 64-QAM have 1 representation in 85-HEX and 21 points in 64-QAM have 2 representations in 85-HEX. Similarly, 55 points in 64-QAM have 1 representation in 73-HEX and 9 points in 64-QAM have 2 representations in 73-HEX.

Figure 3 shows the CCDFs of PAPR of the TI technique with 91-HEX or 73-HEX. The CCDFs of PAPR of the unmodified OFDM signal with 64-QAM are also shown for comparison. It is shown that the unmodified OFDM signal has a PAPR which exceeds 10.6 dB for less than 0.1 percent of the blocks for $N = 64$. The 0.1 percent PAPRs of the unmodified OFDM signal are 10.9 dB and 11.2 dB for $N = 128$ and $N = 256$, respectively. We can lower the 0.1 percent PAPRs by 3.9 dB, 3.9 dB, and 3.9 dB with the proposed scheme with 91-HEX when $N = 64, 128, 256$, respectively. Also, we can lower the 0.1 percent PAPRs by 1.8 dB, 2.1 dB, and 2.8 dB with the proposed scheme with 73-HEX when $N = 64, 128, 256$, respectively. It is found that the gain of the proposed scheme is bigger for an OFDM system with larger number of subcarriers when we use 73-HEX. It might be because that there are smaller number of points with more than one representation in 73-HEX than in 91-HEX. As mentioned before, we can lower the 0.1 percent PAPRs of the proposed scheme by using gradient guided search [5] with additional computational complexity.

Figure 4 shows the BER of the TI technique with 91-HEX for an OFDM system with 64 subcarriers in a Rayleigh fading channel under the assumption of perfect channel estimation. BER of the 64-QAM is also shown for comparison. It is shown that the BER of the 91-HEX is slightly worse than that of the 64-QAM because the minimum distance is the same in both cases and the number of the nearest neighbors is larger for the 91-HEX. To get better BER, we can use hexagonal constellation with smaller average signal power such as 73-HEX and increase the minimum distance so that the average power of the hexagonal constellation is the same as that of 64-QAM. It is shown in Fig. 4 that the BER of this modified 73-HEX (denoted 'Mod. 73-HEX' in the legend) is very close to that of the 64-QAM. Note that we can achieve same PAPR reduction with the modified hexagonal constellation because PAPR does not depend on the minimum distance.

VI. Conclusions

In this paper, we proposed the use of hexagonal constellation in the TI technique. It is shown that we can achieve PAPR reduction from the TI technique without power increase. The proposed scheme can be easily applied even when the constellation size is large.

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