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Top-quark-mass prediction from supersymmetric grand unified theories

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We consider a supersymmetric grand-unified-theory (GUT) framework motivated by SO(10) or E_6 unification in which the parameter $\tan\beta(\equiv v_2/v_1)$ of the minimal supersymmetric standard model is constrained by the condition that the Yukawa couplings h_t, h_b, and h_τ are all equal at the GUT scale. With $\alpha_s(M_Z)=0.106\pm0.006$, the estimate for the b-quark mass, which depends on $\tan\beta$, lies in the "observed" range $m_b(m_b)=4.25\pm0.10$ GeV, provided that the top-quark mass is 142^{+26}_{-49} GeV.

The minimal supersymmetric extension of the standard model (MSSM) introduces an important new parameter $\tan\beta\equiv v_2/v_1$, the ratio of the vacuum expectation values that provide masses for u-type and d-type quarks (plus the charged leptons) [1]. Phenomenological considerations require that $1<\tan\beta<m_t/m_b$ [2]. Embedding the MSSM in supersymmetric (SUSY) SU(5) [3–5] leaves $\tan\beta$ undetermined, which means that the SU(5) prediction for m_b depends on an additional free parameter [6].

In this Brief Report we consider a supersymmetric grand unified framework, based on groups such as SO(10) and E_6, in which $\tan\beta$ is constrained by the condition that the Yukawa couplings h_t, h_b, and h_τ are all equal at the grand-unified-theory (GUT) breaking scale M_X. For $\mu<M_X$, $\tan\beta$ differs from m_t/m_b by a (small) calculable amount. With $\alpha_s(M_Z)=0.106\pm0.006$, the estimated b-quark mass lies within the "measured" range $[m_b(m_b)=4.25\pm0.10$ GeV)] [7] provided that the top-quark mass is 142^{+26}_{-49} GeV.

Our starting point is the assumption that the third-generation fermions acquire mass from the coupling **16×16×10**, where the **10**-plet contains the two Higgs doublets that develop vacuum expectation values (VEV's) v_1 and v_2 in an SO(10) theory, or from the coupling $\mathbf{27}^3$ in an E_6 theory. This implies that the Yukawa couplings h_t, h_b, and h_τ are all equal at M_X (see Table I for an estimate of M_X to one loop). For $M_S<\mu<M_X$ [$M_S=1$ TeV denotes the SUSY-breaking scale and $t\equiv\ln\mu(\mathrm{GeV})/16\pi^2$] the evolution equations for the gauge and Yukawa couplings to one loop are [6,8] (with $\alpha_i\equiv g_i^2/4\pi$, $i=1,2$ and $\alpha_s\equiv g_3^2/4\pi$)

$$\begin{aligned}
dg_1/dt&=(2n_g+\tfrac{3}{5})g_1^3\ ,\\
dg_2/dt&=(-6+2n_g+1)g_2^3\ ,\\
dg_3/dt&=(-9+2n_g)g_3^3\ ,\\
dh_t/dt&=h_t(6h_t^2+h_b^2-\tfrac{16}{3}g_3^2-3g_2^2-\tfrac{13}{15}g_1^2)\ ,\\
dh_b/dt&=h_b(h_t^2+6h_b^2-\tfrac{16}{3}g_3^2-3g_2^2-\tfrac{7}{15}g_1^2)\ ,\\
dh_\tau/dt&=h_\tau(3h_b^2-3g_2^2-\tfrac{9}{5}g_1^2)\ .
\end{aligned}\tag{1}$$

For $M_Z<\mu<M_S$, the equations are

$$\begin{aligned}
dg_1/dt&=(\tfrac{4}{3}n_g+\tfrac{2}{10})g_1^3\ ,\\
dg_2/dt&=(-\tfrac{22}{3}+\tfrac{4}{3}n_g+\tfrac{1}{3})g_2^3\ ,\\
dg_3/dt&=(-11+\tfrac{4}{3}n_g)g_3^3\ ,\\
dh_t/dt&=h_t(9h_t^2+h_b^2-8g_3^2-\tfrac{9}{4}g_2^2-\tfrac{17}{20}g_1^2)\ ,\\
dh_b/dt&=h_b(h_t^2+9h_b^2-8g_3^2-\tfrac{9}{4}g_2^2-\tfrac{5}{20}g_1^2)\ ,\\
dh_\tau/dt&=h_\tau(6h_b^2-\tfrac{9}{4}g_2^2-\tfrac{9}{4}g_1^2)\ .
\end{aligned}\tag{2}$$

At the tree level the Yukawa couplings are given by

$$\begin{aligned}
h_t&=\frac{m_t\sqrt{1+\tan^2\beta}}{174\tan\beta}\ ,\\
h_b&=\frac{m_b\sqrt{1+\tan^2\beta}}{174}\ ,\\
h_\tau&=\frac{m_\tau\sqrt{1+\tan^2\beta}}{174}\ ,
\end{aligned}\tag{3}$$

where $\sqrt{v_1^2+v_2^2}=174$ GeV.

In Fig. 1 we plot $\tan\beta$ vs $m_t(m_t)$, where $\tan\beta$ is deter-

TABLE I. One-loop predictions for $\sin^2\theta_W(M_Z)$ and M_X with SUSY SO(10) or E_6 GUT broken directly to the minimal supersymmetric extension of the standard model.

$\alpha_s(M_Z)$	M_S (TeV)	M_X (GeV)	$\sin^2\theta_W(M_Z)$
0.100	1.0	0.42×10^{16}	0.235
0.106	1.0	0.71×10^{16}	0.233
0.112	1.0	1.02×10^{16}	0.231

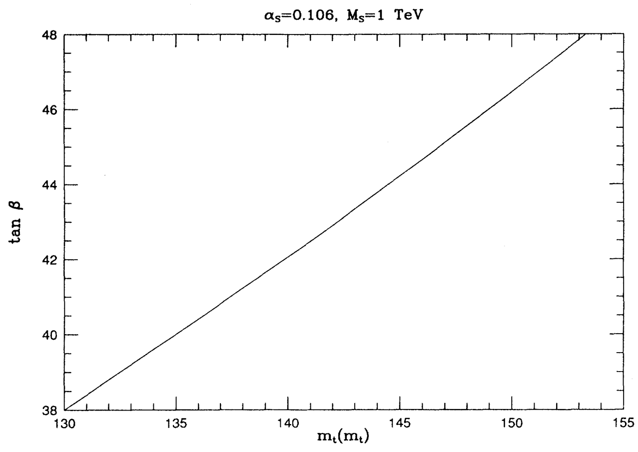

FIG. 1. Plot of $\tan\beta$ vs $m_t(m_t)$ with $\alpha_s=0.106$ and $M_S=1$ TeV.

mined by the requirement that for a given $m_t(m_t)$, the three Yukawa couplings h_t, h_b, and h_τ meet at the GUT scale M_X. In Fig. 2 an example of the evolution of the Yukawa couplings as functions of the momentum scale is shown. It may be noticed that h_t/h_b is of order 1 in the entire range and asymptotically reaches 1 from above. In Fig. 3 we plot $m_t(\text{physical})\approx m_t(m_t)[1+4\alpha_s(m_t)/\ 3\pi]$ vs $m_b(m_b)$. Note that between M_Z and m_b the QCD corrections are included to two loops. For $\alpha_s(M_Z)$, following the first paper in Ref. [9], we take the range 0.106 ± 0.006. Our conclusion from this is that the top-quark mass is 142^{+26}_{-49} GeV. A larger value for $\alpha_s(M_Z)$, say 0.12, leads to a top-quark mass in the range 171–182 GeV.

Independent of the constraint from $m_b(m_b)$, one can approximately bound h_t by setting the right-hand side of the evolution equation for its logarithm to zero. It turns out that, for $h_t\lesssim 1.05$, the system of equations lies in the perturbative domain [6,10]. In the first paper of Ref. [6], $\tan\beta$ was set to unity which gives an approximate bound on the top-quark mass of $(1.05)(1/\sqrt{2})(174\text{ GeV})\simeq 130$ GeV. Our study involves large values of $\tan\beta$ and as a consequence, we end up with an approximate upper

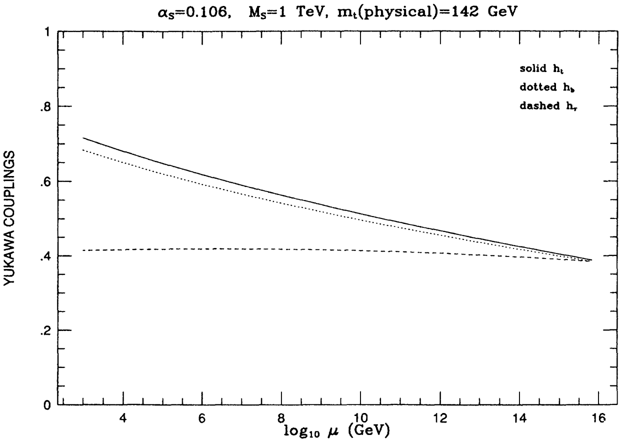

FIG. 2. Plot of Yukawa couplings vs $\log_{10}\mu$(GeV) for the case $\alpha_s=0.106$, $M_S=1$ TeV and $m_t(\text{physical})=142$ GeV.

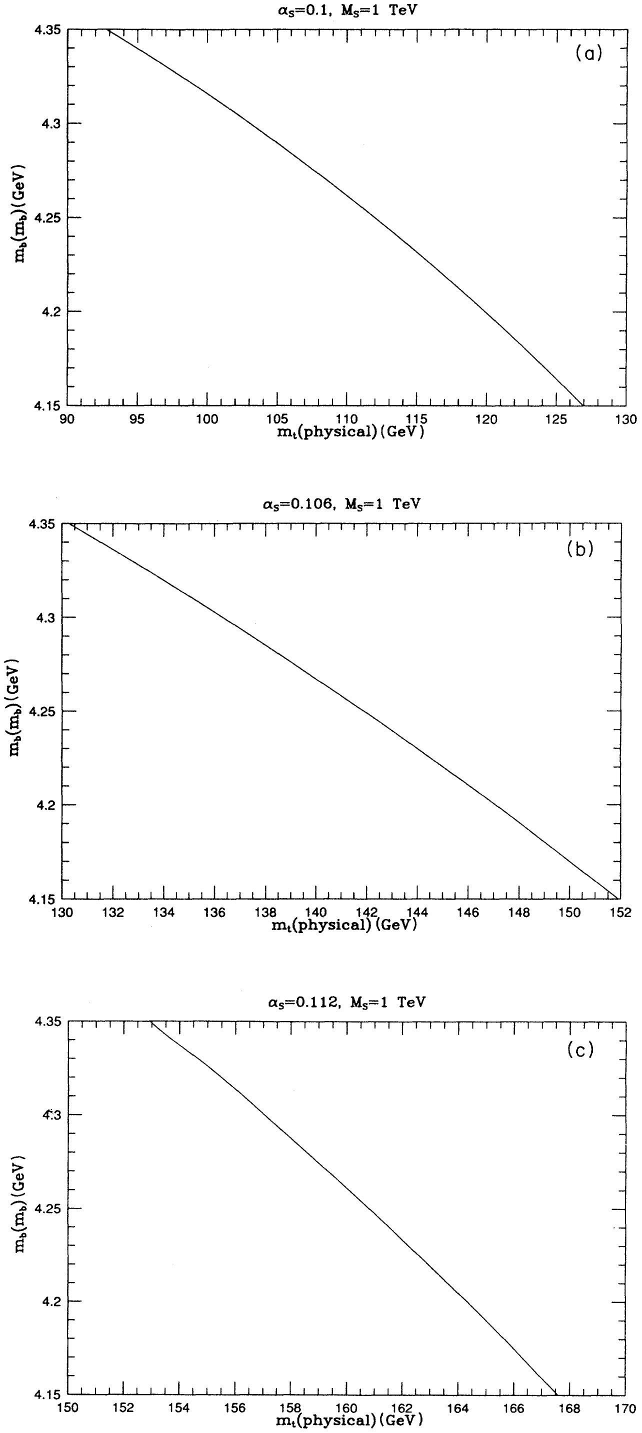

FIG. 3. Plots of $m_b(m_b)$ vs $m_t(\text{physical})$ for typical choices of parameters.

bound on m_t of $(1.05)(174\ \mathrm{GeV}) \simeq 183$ GeV, which is similar to the second paper of Ref. [6].

In conclusion, some recent investigations [9] suggest that supersymmetric grand unified theories directly broken to the MSSM are in striking agreement with data. For instance, the predicted value for $\sin^2\theta_W$ is in excellent agreement with recent results. Moreover, the observed gauge couplings when extrapolated to high energies appear to meet at a common scale close to 10^{16} GeV (with $M_S \simeq 1$ TeV). Our results on the top-quark mass take us a step further in this direction. We have shown that certain supersymmetric GUT's also predict a heavy top quark.

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