

TOPICS IN HARMONIC ANALYSIS

*Related to the
Littlewood-Paley
Theory*

BY
ELIAS M. STEIN

PRINCETON UNIVERSITY PRESS
AND THE
UNIVERSITY OF TOKYO PRESS

PRINCETON, NEW JERSEY
1970

CONTENTS

Preface	v
Introduction	1
Chapter I. Lie Groups (A Review)	
§1. Compact groups	5
§2. The Peter-Weyl theorem	12
§3. The Peter-Weyl theorem (Concluded)	15
§4. Lie groups; examples	20
§5. Lie algebras	23
§6. Universal enveloping algebra	28
§7. Laplacian	33
Chapter II. Littlewood-Paley Theory for a Compact Lie Group	
§1. The heat-diffusion semi-group	38
§2. The Poisson semi-group; the main theorem	46
§3. Proof of Theorem 2.	50
§4. Applications: Riesz transforms, etc.	57
Bibliographical remarks	64
Chapter III. General Symmetric Diffusion Semi-groups	
§1. General setting	65
§2. Analyticity of these semi-groups	67
§3. The maximal theorem	73
§4. A digression: L_2 theorems	82
Bibliographical remarks	88

Chapter IV. The General Littlewood-Paley Theory

§1. Conditional Expectation and Martingales	89
§2. The inequalities for martingales	94
§3. An additional “max” inequality	103
§4. The link between margingales and semi-groups	106
§5. The Littlewood-Paley inequalities in general	111
§6. Dénouement	120
Bibliographical remarks	122

Chapter V. Further Illustrations

§1. Lie groups	123
§2. Semi-simple case	128
§3. Sturm-Liouville	136
§4. Heuristics.....	137
Bibliographical remarks	141
References	143