



Topological indices of rhombus type silicate and oxide networks

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Topological indices of rhombus type silicate and oxide networks

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Abstract

For a molecular graph, a numeric quantity which characterizes the whole structure of a graph is called a topological index. In the studies of quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR), topological indices are utilized to guess the bioactivity of chemical compounds. In this paper, we compute general Randić, first general Zagreb, generalized Zagreb, multiplicative Zagreb, atom-bond connectivity (ABC) and geometric-arithmetic (GA) indices for the rhombus silicate and rhombus oxide networks. In addition, we also compute the latest developed topological indices such as the fourth version of ABC (ABC_4), the fifth version of GA (GA_5), augmented Zagreb and Sanskruti indices for the foresaid networks. At the end, a comparison between all the indices is included and the result is shown with the help of a cartesian coordinate system.

Keywords: Topological indices, rhombus silicate network and rhombus oxide network.

1 Introduction

The combination of chemistry, mathematics and information science leads to a new subject called by cheminformatics. It studies the quantitative structure-activity relationship (QSAR) and the quantitative structure- property relationship (QSPR) that are used to predict the biological activities and properties of chemical compounds. The physico-chemical properties and the topological indices play an important role in the studies of QSAR and QSPR.

Graph theory has found a considerable use in a branch of mathematical chemistry which is known as chemical graph theory. It applies graph theory to the mathematical model of chemical phenomenon. Computing of the topological indices is a subtopic of chemical graph theory, which correlates certain physico-chemical properties of the underlying chemical compound. Simply, we can say that the topological indices are designed by transforming a molecular graph into a number and these are invariant under graph isomorphisms. There are some main classes of topological indices such as distance based topological indices, degree based topological indices and polynomials related indices of graphs. Among these classes degree based topological indices are of great importance and play a vital role in chemical graph theory and particularly in chemistry.

In 1947, Wiener first time made the use of a topological index in chemistry working on boiling point of paraffin [13]. He introduced the notion of path

number of a graph as the sum of distances between any two carbon atoms in the molecules, in terms of carbon-carbon bonds which later called by the name of Wiener index. In graph theoretical language, the Wiener index is equal to the count of all shortest distances in a graph. After the Wiener index the theory of topological indices is started. In the progressive studies of indices, a large number of topological indices has been introduced by different chemist, and many researchers have proved different computing and theoretical results related to the certain topological indices for different families of molecular graphs.

In particular, Rajan et al. [7] computed the Randić, Zagreb and atom-bond connectivity indices of silicate ($SL(n)$), honeycomb ($HC(n)$) and hexagonal ($HX(n)$) networks. Recently, Baig et al. [2] calculated Randić, atom-bond connectivity, geometric-arithmetic, general zagreb, fourth atom-bond connectivity and fifth geometric-arithmetic indices for dominating oxide ($DOX(n)$), dominating silicate ($DSL(n)$), and regular triangulene oxide ($RTOX(n)$) networks. In this paper, we compute general Randić, first general Zagreb, generalized Zagreb, multiplicative Zagreb, atom-bond connectivity (ABC) and geometric-arithmetic (GA), the fourth version of ABC (ABC_4), the fifth version of GA (GA_5), augmented Zagreb and Sanskruti indices for the rhombus silicate ($RHSL(n)$) and rhombus oxide ($RHOX(n)$) networks. For further studies of the topological indices, we refer, [1, 3, 4, 6, 19, 20, 25].

The rest of the paper is organised as, Section 2 includes the definitions

and formulas which are frequently used in the main results. In Section 3, we compute the main results related to the topological indices of rhombus silicate networks. Section 4 includes the main results on topological indices for the rhombus oxide networks, and in Section 5, we compare the results of all the indices and draw the conclusion.

2 Preliminaries

A molecular graph $\Gamma = (V(\Gamma), E(\Gamma))$ with the vertex set $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ and the edge set $E(\Gamma)$ is a graph whose vertices denote atoms and edges denote bonds between that atoms of any underlying chemical structure. The order and size of a graph is $|V(\Gamma)| = v$ and $|E(\Gamma)| = e$ respectively. A graph is connected if there exists a connection between any pair of vertices. A loop is an edge that connects a vertex to itself and two or more than two edges with same end points are multiple edges. A network is simply a finite connected graph having no multiple edges and loops. The distance between two vertices u and v denoted by $d(u, v)$ is the length of the shortest path between u and v in graph Γ . The length of the shortest path between u and v is also called a $u - v$ geodesic. The longest path between any two vertices is called a $u - v$ detour. The degree of a vertex v denoted by $d(v)$ is the number of vertices that are connected to v by the edges. Moreover, we define $S_u = \sum_{v \in N_\Gamma(u)} (d(v))$ and $S_v = \sum_{u \in N_\Gamma(v)} (d(u))$, where $N_\Gamma(u) = \{v \in V(\Gamma) | uv \in E(\Gamma)\}$ and $N_\Gamma(v) = \{u \in V(\Gamma) | uv \in E(\Gamma)\}$.

Now, we define certain topological indices which will be used frequently in

the main results. We begin from Wiener index which is defined in 1947.

Definition 2.1. [13] Let Γ be a graph and (u, v) be any ordered pair of its vertices with $d(u, v)$ as a $u - v$ geodesic. Then, the Wiener index of Γ is defined as

$$W(\Gamma) = \frac{1}{2} \sum_{(u,v)} d(u, v).$$

In 1972, Gutman and Trinajsti [14] derived a pair of molecular descriptors known as the first Zagreb index and the second Zagreb index for the total π -energy of conjugated molecules. Soon after these indices have been used as branching indices [16, 24]. Later the Zagreb indices found applications in QSPR and QSAR studies [15, 17].

Definition 2.2. Let Γ be a molecular graph. Then, the first Zagreb and the second Zagreb indices denoted by $M_1(\Gamma)$ and $M_2(\Gamma)$ respectively are defined as

$$M_1(\Gamma) = \sum_{uv \in E(\Gamma)} [d(u) + d(v)],$$

$$M_2(\Gamma) = \sum_{uv \in E(\Gamma)} [d(u) \times d(v)].$$

In 1975, Milan Randić [23] defined the following degree based topological index which is called Randić index.

Definition 2.3. Let Γ be a molecular graph. Then, the Randić index

denoted by $R_{-\frac{1}{2}}(\Gamma)$ is defined as

$$R_{-\frac{1}{2}}(\Gamma) = \sum_{uv \in E(\Gamma)} \frac{1}{\sqrt{d(u) \times d(v)}}.$$

In 1998, Bollobás and Erdős [5], and Amic et al. [8] independently generalized the concept of Randić index and established the generalized Randić index. It has been extensively studied by both theoretical chemists and mathematicians. By the way, many important properties and results are established. To know in details, we refer [12, 27, 28].

Definition 2.4. Let Γ be a molecular graph. Then, for a real number α , the general Randić index denoted by $R_\alpha(\Gamma)$ is defined as

$$R_\alpha(\Gamma) = \sum_{uv \in E(\Gamma)} [d(u) \times d(v)]^\alpha.$$

Thus, for $\alpha = -\frac{1}{2}$, $\alpha = \frac{1}{2}$ and $\alpha = 1$, then the general Randić index is a Randić index, reciprocal Randić index and the second Zagreb index respectively. For further study of the general Randić index, we refer [12, 28, 27].

Li and Zheng (2005)[26] defined the first general Zagreb index as follows.

Definition 2.5. Let Γ be a molecular graph. Then, for a real number α the first general Zagreb index denoted by M_1^α is defined as

$$M_1^\alpha = \sum_{u \in V(\Gamma)} [d(u)]^\alpha.$$

In 2011, Iranmanesh and Azari [18] introduced the generalized Zagreb index based on degree of vertices of a connected graph Γ .

Definition 2.6. Let Γ be a molecular graph. Then, for $r, s \in Z^+$, the generalized Zagreb index is denoted by $M_{r,s}(\Gamma)$ and defined as

$$M_{r,s}(\Gamma) = \sum_{uv \in E(\Gamma)} [d(u)^r d(v)^s + d(v)^r d(u)^s].$$

In 2012, Ghorbani and Azimi [21] defined the new multiplicative versions of $M_1(\Gamma)$ and $M_2(\Gamma)$ as follows.

Definition 2.7. Let Γ be a molecular graph. Then, the first multiplicative Zagreb index and the second multiplicative Zagreb index denoted by $PM_1(\Gamma)$ and $PM_2(\Gamma)$ are defined as

$$PM_1(\Gamma) = \prod_{uv \in E(\Gamma)} (d(u) + d(v)),$$

$$PM_2(\Gamma) = \prod_{uv \in E(\Gamma)} (d(u) \times d(v)).$$

In 1998, Estrada et al.[10] introduced the atom bond connectivity index which has been applied to study the stability of alkanes and the strain energy of cycloalkanes. Later on, Ghorbani and Hosseinzadeh (2010) [22] defined the forth version of atom bond connectivity index.

Definition 2.8. Let Γ be a molecular graph. Then, the atom bond connectivity index and the forth version of atom bond connectivity index denoted

by $ABC(\Gamma)$ and $ABC_4(\Gamma)$ respectively are defined as

$$ABC(\Gamma) = \sum_{uv \in E(\Gamma)} \sqrt{\frac{d(u) + d(v) - 2}{d(u) \times d(v)}},$$

$$ABC_4(\Gamma) = \sum_{uv \in E(\Gamma)} \sqrt{\frac{S(u) + S(v) - 2}{S(u) \times S(v)}}.$$

In 2009, Vukičević and Furtula [9] defined the geometric-arithmetic (GA) index. Later on the fifth version of the geometric-arithmetic (GA_5) index is defined by Graovac et al. (2011) [1].

Definition 2.9. Let Γ be a molecular graph. Then, the geometric-arithmetic index and the fifth version of the geometric-arithmetic index denoted by $GA(\Gamma)$ and $GA_5(\Gamma)$ respectively are defined as

$$GA(\Gamma) = \sum_{uv \in E(\Gamma)} \frac{2\sqrt{d(u) \times d(v)}}{d(u) + d(v)},$$

$$GA_5(\Gamma) = \sum_{uv \in E(\Gamma)} \frac{2\sqrt{S(u) \times S(v)}}{S(u) + S(v)}.$$

In 2010, Furtula et al. [6] proposed the modified version of the ABC index called by augmented Zagreb index and recently Sunilkumar M. Hosamani (2016) defined the Sanskruti index motivated by the augmented Zagreb index.

Definition 2.10. Let Γ be a molecular graph. Then, the augmented Zagreb index and the Sanskruti index denoted by $AZI(\Gamma)$ and $S(\Gamma)$ re-

spectively are defined as

$$AZI(\Gamma) = \sum_{uv \in E(\Gamma)} \left[\frac{d(u) \times d(v)}{d(u) + d(v) - 2} \right]^3,$$

$$S(\Gamma) = \sum_{uv \in E(\Gamma)} \left[\frac{S(u) \times S(v)}{S(u) + S(v) - 2} \right]^3.$$

Now, we discuss the construction of rhombus silicate and rhombus oxide networks. The most interesting class of minerals by far is silicate. These are obtained by fusing metal oxides or metal carbonates with sand. Essentially all the silicates contain SiO_4 tetrahedra as it is a basic unit. In chemistry, the corner vertices of SiO_4 tetrahedron represent oxygen ions and the center vertex represents the silicon ion. In graph theory, we call the corner vertices as oxygen nodes and the center vertex as silicon node. By the different arrangements of the tetrahedron silicate, we can obtain different silicate structures. Similarly, different silicate networks are constructed by different silicate structures. In Figure 1, the rhombus silicate network of dimension 3 is presented. In general, the vertices and the edges in a rhombus silicate of dimension n are $5n^2 + 2n$ and $12n^2$, respectively. If we delete all the silicon ions from the rhombus silicate network, then we obtain rhombus oxide network as shown in Figure 2 of dimension 3. For dimension n , the vertices and the edges of a rhombus oxide network are $3n^2 + 2n$ and $6n^2$ respectively, where dimension means the number of vertices existing in any side of the rhombus silicate or oxide network. In present study, for dimension n , the rhombus silicate and rhombus oxide networks are denoted by $(RHSL(n))$ and $(RHOX(n))$ respectively.

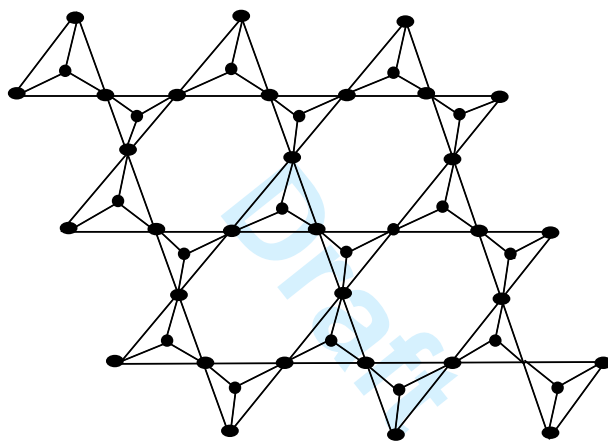


Figure 1: Rhombus silicate $RHSL(3)$

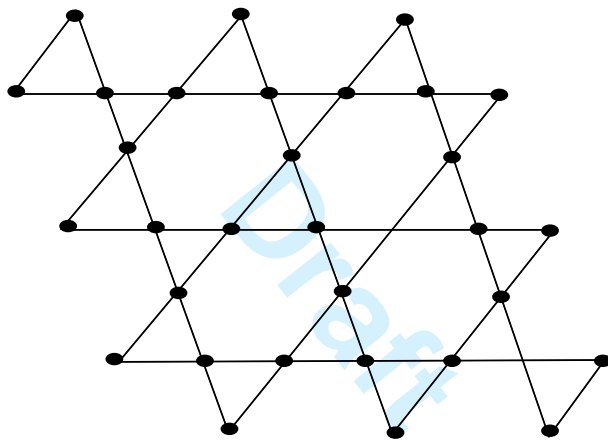


Figure 2: Rhombus oxide $RHOX(3)$

3 Topological indices of the rhombus silicate network

In this section, we compute the topological indices for rhombus silicate. In Theorem 3.1, Theorem 3.2 and Theorem 3.3, we find the topological indices which base on the degrees of the vertices such as general Randić, first general Zagreb, generalized Zagreb, multiplicative Zagreb, atom-bond connectivity (ABC), geometric-arithmetic (GA) and augmented Zagreb index (AZI). Moreover, Theorem 3.4 consists on the latest devolved topological indices which base on the sum of the degrees of neighbors of the vertices such as fourth version of ABC (ABC_4), the fifth version of GA (GA_5), and Sanskruti index (SI).

For this let us define the partitions of the vertex set $V(RHSL(n))$ and the edge set $E(RHSL(n))$ of $RHSL(n)$ with respect to degree of vertices. There are two types of vertices in $RHSL(n)$ namely of degree 3 and of degree 6. Thus, we have

$$V_1 = \{v \in V(RHSL(n)) | d_v = 3\} \text{ and } V_2 = \{v \in V(RHSL(n)) | d_v = 6\},$$

where $|V_1| = 2n(n+2)$ and $|V_2| = n(3n-2)$. Consequently, $|V(RHSL(n))| = v = |V_1| + |V_2| = 5n^2 + 2n$. There are three types of edges with respect to degrees of end vertices in $RHSL(n)$ namely with degrees of end vertices

| $E_{\{d(u),d(v)\}}$ | $E_{\{3,3\}}$ | $E_{\{3,6\}}$ | $E_{\{6,6\}}$ |
|-----------------------|---------------|-----------------|-----------------|
| $ E_{\{d(u),d(v)\}} $ | $4n + 2$ | $6n^2 + 4n - 4$ | $6n^2 - 8n + 2$ |

Table 1: The edge partition of the edges of $RHSL(n)$ based on degrees of end vertices.

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| $E_{\{s_u, s_v\}}$ | $ E_{\{s_u, s_v\}} $ |
|--------------------|----------------------|
| (12,12) | 6 |
| (12,24) | 6 |
| (15,24) | 8 |
| (18,24) | 2 |
| (24,27) | 8 |
| (15,15) | $4(n - 1)$ |
| (18,27) | $4(2n - 3)$ |
| (27,27) | $8(n - 2) + 2$ |
| (15,27) | $8(2n - 3)$ |
| (27,30) | $8(n - 2)$ |
| (18,30) | $2(n - 2)(3n - 4)$ |
| (30,30) | $6(n - 2)^2$ |

Table 2: The edge partitions of $RHSL(n)$ based on degree sum of the neighbors of end vertices.

$\{3, 3\}$, degrees of end vertices $\{3, 6\}$ and degrees of end vertices $\{6, 6\}$.

Thus, we have

$$E_{\{3,3\}} = \{uv \in E(RHSL(n)) | d_u = 3, d_v = 3\},$$

$$E_{\{3,6\}} = \{uv \in E(RHSL(n)) | d_u = 3, d_v = 6\},$$

and
$$E_{\{6,6\}} = \{uv \in E(RHSL(n)) | d_u = 6, d_v = 6\},$$

where $|E_{\{3,3\}}| = 4n + 2$, $|E_{\{3,6\}}| = 6n^2 + 4n - 4$ and $|E_{\{6,6\}}| = 6n^2 - 8n + 2$. Consequently, $|E(RHSL(n))| = e = |E_1| + |E_2| + |E_3| = 12n^2$. Similarly, we can define the partitions of the vertex set $V(RHSL(n))$ and the edge set $E(RHSL(n))$ of $RHSL(n)$ with respect to the sum of the degrees of neighbors of the vertices of $RHSL(n)$. Thus, for $uv \in E(RHSL(n))$, we obtain the Table 3.1 and Table 3.2.

Theorem 3.1. Let $\Gamma \cong RHSL(n)$ be a rhombus silicate network with dimension n . Then, for $n \geq 2$, its Randić index ($R_{-\frac{1}{2}}(\Gamma)$), reciprocal Randić index ($R_{\frac{1}{2}}(\Gamma)$), second Zagreb index ($R_1(\Gamma)$) and the general Randić index $R_\alpha(\Gamma)$ are given as

$$R_\alpha(\Gamma) = \begin{cases} (1 + n^2) + \frac{6n^2 + 4n - 4}{3\sqrt{2}}, & \text{for } \alpha = -\frac{1}{2}, \\ 36n^2 - 36n + 3\sqrt{2}(6n^2 + 4n - 4) + 18, & \text{for } \alpha = \frac{1}{2}, \\ 324n^2 - 180n + 18, & \text{for } \alpha = 1, \\ 6n^2(18^\alpha + 36^\alpha) + 4n(9^\alpha + 18^\alpha - 36^\alpha \times 2) + 2(9^\alpha - 2 \times 18^\alpha + 36^\alpha), & \text{for } \alpha \in \mathbb{R}. \end{cases}$$

Proof. Using Table 1 and the formulae defined in Section 2, we compute the required results as follows.

$$\begin{aligned} R_{-\frac{1}{2}}(\Gamma) &= \sum_{uv \in E(\Gamma)} \frac{1}{\sqrt{d_u d_v}} \\ &= \sum_{uv \in E_{\{3,3\}}} \frac{1}{\sqrt{d_u d_v}} + \sum_{uv \in E_{\{3,6\}}} \frac{1}{\sqrt{d_u d_v}} + \sum_{uv \in E_{\{6,6\}}} \frac{1}{\sqrt{d_u d_v}} \\ &= \frac{1}{\sqrt{9}}(4n + 2) + (6n^2 + 4n - 4) \frac{1}{\sqrt{18}} + (6n^2 - 8n + 2) \frac{1}{\sqrt{36}} \\ &= \frac{4n + 2}{3} + \frac{(6n^2 + 4n - 4)}{3\sqrt{2}} + \frac{(6n^2 - 8n + 2)}{6} \\ &= \frac{8n + 4 + 6n^2 - 8n + 2}{6} + \frac{6n^2 + 4n - 4}{3\sqrt{2}} \\ &= (1 + n^2) + \frac{6n^2 + 4n - 4}{3\sqrt{2}}. \end{aligned}$$

$$\begin{aligned}
R_{\frac{1}{2}}(\Gamma) &= \sum_{uv \in E(\Gamma)} \sqrt{d_u d_v} \\
&= \sum_{uv \in E_{\{3,3\}}} \sqrt{d_u d_v} + \sum_{uv \in E_{\{3,6\}}} \sqrt{d_u d_v} + \sum_{uv \in E_{\{6,6\}}} \sqrt{d_u d_v} \\
&= (4n+2)\sqrt{9} + (6n^2+4n-4)\sqrt{18} + (6n^2-8n+2)\sqrt{36} \\
&= 12n+6+3\sqrt{2}(6n^2+4n-4)+36n^2-48n+12 \\
&= 36n^2-36n+3\sqrt{2}(6n^2+4n-4)+18.
\end{aligned}$$

$$\begin{aligned}
R_1(\Gamma) &= \sum_{uv \in E(\Gamma)} (d_u d_v) \\
&= \sum_{uv \in E_{\{3,3\}}} (d_u d_v) + \sum_{uv \in E_{\{3,6\}}} (d_u d_v) + \sum_{uv \in E_{\{6,6\}}} (d_u d_v) \\
&= (4n+2)(9) + (6n^2+4n-4)(18) + (6n^2-8n+2)(36) \\
&= 36n+18+108n^2+72n-72+216n^2-288n+72 \\
&= 324n^2-180n+18.
\end{aligned}$$

$$\begin{aligned}
R_\alpha(\Gamma) &= \sum_{uv \in E(\Gamma)} (d_u d_v)^\alpha \\
&= \sum_{uv \in E_{3,3}} (d_u d_v)^\alpha + \sum_{uv \in E_{3,6}} (d_u d_v)^\alpha + \sum_{uv \in E_{6,6}} (d_u d_v)^\alpha \\
&= (4n+2)(9)^\alpha + (6n^2+4n-4)(18)^\alpha + (6n^2-8n+2)(36)^\alpha \\
&= 4n \times 9^\alpha + (6n^2+4n-4)(18)^\alpha + (6n^2-8n+2)(36)^\alpha \\
&= 6n^2(18^\alpha+36^\alpha) + 4n(9^\alpha+18^\alpha-36^\alpha \times 2) + 2(9^\alpha-2 \times 18^\alpha+36^\alpha).
\end{aligned}$$

Theorem 3.2. Let $\Gamma \cong RHSL(n)$ be a rhombus silicate network with

dimension n . Then, for $n \geq 2$, first general Zagreb ($M_1^\alpha(\Gamma)$), generalized Zagreb ($M_{r,s}(\Gamma)$), first multiplicative Zagreb ($PM_1(\Gamma)$) and the second multiplicative Zagreb ($PM_2(\Gamma)$) indices are given by

$$\begin{aligned} \text{(i)} M_1^\alpha(\Gamma) &= n^2(2 \times 3^\alpha + 3 \times 6^\alpha) + 2n(2 \times 3^\alpha - 6^\alpha), \\ \text{(ii)} M_{r,s}(\Gamma) &= (2 \times 3^{r+s})[3n^2(2^r + 2^s + 2(2^r 2^s)) + 2n(2 + 2^r + 2^s - 2^2(2^r)(2^s) + \\ & 2(1 - 2^r - 2^s + (2^r)(2^s))], \\ \text{(iii)} PM_1(\Gamma) &= 5184(1 + 2n)(1 - 4n + 3n^2)(-2 + 2n + 3n^2), \\ \text{(iv)} PM_2(\Gamma) &= 46656(1 + 2n)(1 - 4n + 3n^2)(-2 + 2n + 3n^2). \end{aligned}$$

Proof. Using Table 1 and the formulae defined in Section 2, we compute the required results as follows.

(i)

$$\begin{aligned} M_1^\alpha(\Gamma) &= \sum_{u \in V(\Gamma)} (d_u)^\alpha \\ &= (2n^2 + 4n)(3)^\alpha + (3n^2 - 2n)(6)^\alpha \\ &= n^2(2 \times 3^\alpha + 3 \times 6^\alpha) + 2n(2 \times 3^\alpha - 6^\alpha), \end{aligned}$$

(ii)

$$\begin{aligned}
M_{r,s}(T) &= \sum_{uv \in E(T)} (d_u^r d_v^s + d_v^r d_u^s) \\
&= \sum_{uv \in E_{\{3,3\}}} (d_u^r d_v^s + d_v^r d_u^s) + \sum_{uv \in E_{\{3,6\}}} (d_u^r d_v^s + d_v^r d_u^s) \\
&\quad + \sum_{uv \in E_{\{6,6\}}} (d_u^r d_v^s + d_v^r d_u^s) \\
&= (4n + 2)(3^r 3^s + 3^s 3^r) + (6n^2 + 4n - 4)(3^r 6^s + 6^r 3^s) \\
&\quad + (6n^2 - 8n + 2)(6^r 6^s + 6^s 6^r) \\
&= (4n + 2)(2 \times 3^{r+s}) + 2(3n^2 + 2n - 2)(3^{r+s})(2^r + 2^s) \\
&\quad + 2(3n^2 - 4n + 1)((2)(2^r)(2^s))(3^{r+s}) \\
&= (2 \times 3^{r+s})[(4n + 2) + (3n^2 + 2n - 2)(2^r + 2^s) \\
&\quad + (3n^2 - 4n + 1)((2)(2^r)(2^s))] \\
&= (2 \times 3^{r+s})[3n^2(2^r + 2^s + 2(2^r)(2^s)) + 2n(2 + 2^r + 2^s) \\
&\quad - (2^2)(2^r)(2^s)] + 2(1 - 2^r - 2^s + (2^r)(2^s)),
\end{aligned}$$

(iii)

$$\begin{aligned}
PM_1(\Gamma) &= \prod_{uv \in E(T)} (d_v + d_u) \\
&= \prod_{uv \in E_{\{3,3\}}} (d_v + d_u) \times \prod_{uv \in E_{\{3,6\}}} (d_v + d_u) \times \prod_{uv \in E_{\{6,6\}}} (d_v + d_u) \\
&= (4n + 2)(6)(6n^2 + 4n - 4)(9)(6n^2 - 8n + 2)(12) \\
&= (4n + 2)(54n^2 + 36n - 36)(72n^2 - 96n + 24) \\
&= 5184(1 + 2n)(1 - 4n + 3n^2)(-2 + 2n + 3n^2),
\end{aligned}$$

(iv)

$$\begin{aligned}
PM_2(\Gamma) &= \prod_{uv \in E(T)} (d_v \times d_u) \\
&= \prod_{uv \in E_{\{3,3\}}} (d_v d_u) \times \prod_{uv \in E_{\{3,6\}}} (d_v d_u) \times \prod_{uv \in E_{\{6,6\}}} (d_v d_u) \\
&= 9(4n+2)(6n^2+4n-4)(18)(6n^2-8n+2)(36) \\
&= (36n+18)(108n^2+72n-72)(216n^2-288n+72) \\
&= 46656(1+2n)(1-4n+3n^2)(-2+2n+3n^2).
\end{aligned}$$

Theorem 3.3. Let $\Gamma \cong RHSL(n)$ be a rhombus silicate network with dimension n . Then, for $n \geq 2$, atom-bond connectivity ($ABC(\Gamma)$), geometric-arithmetic ($GA(\Gamma)$) and augmented Zagreb index (AZI) are given by

- (i) $ABC(\Gamma) = 6.9039n^2 + 0.9447n - 0.1070$,
- (ii) $GA(\Gamma) = 11.6568n^2 - 0.2287n + 0.2287$,
- (iii) $AZI(\Gamma) = 381.9534n^2 - 259.674n + 48.0816$.

Proof. Using Table 1 and the formulae defined in Section 2, we compute the required results as follows.

(i)

$$\begin{aligned}
ABC(\Gamma) &= \sum_{uv \in E(\Gamma)} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} = \sum_{uv \in E_{\{3,3\}}} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} \\
&+ \sum_{uv \in E_{\{3,6\}}} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} + \sum_{uv \in E_{\{6,6\}}} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} \\
&= (4n+2)\sqrt{\frac{3+3-2}{3 \times 3}} + (6n^2+4n-4)\sqrt{\frac{3+6-2}{3 \times 6}}
\end{aligned}$$

$$\begin{aligned}
& + (6n^2 - 8n + 2)\sqrt{\frac{6+6-2}{6 \times 6}} \\
& = \frac{2}{3}(4n+2) + (6n^2 + 4n - 4)\frac{\sqrt{7}}{3\sqrt{2}} + (6n^2 - 8n + 2)\frac{\sqrt{10}}{6} \\
& = \frac{2}{3}(4n+2) + (3n^2 + 2n - 2)\frac{\sqrt{14}}{3} + (3n^2 - 4n + 1)\frac{\sqrt{10}}{3} \\
& = \frac{1}{3}[2(4n+2) + \sqrt{14}(3n^2 + 2n - 2) + \sqrt{10}(3n^2 - 4n + 1)] \\
& = 6.9039n^2 + 0.9447n - 0.1070.
\end{aligned}$$

(ii)

$$\begin{aligned}
GA(\Gamma) & = \sum_{uv \in E(\Gamma)} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} = \sum_{uv \in E\{3,3\}} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} \\
& + \sum_{uv \in E\{3,6\}} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} + \sum_{uv \in E\{6,6\}} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} \\
& = \frac{2 \times (4n+2)\sqrt{3 \times 3}}{3+3} + \frac{2(6n^2 + 4n - 4)\sqrt{3 \times 6}}{3+6} \\
& + \frac{(6n^2 - 8n + 2) \times 2\sqrt{6 \times 6}}{6+6} \\
& = (4n+2) + \frac{2 \times 3\sqrt{2}}{9}(6n^2 + 4n - 4) + (6n^2 - 8n + 2) \\
& = (6n^2 - 4n + 4) + \frac{2\sqrt{2}}{3}(6n^2 + 4n - 4) \\
& = 11.6568n^2 - 0.2287n + 0.2287.
\end{aligned}$$

(iii)

$$\begin{aligned}
AZI(T) &= \sum_{uv \in E(T)} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 = \sum_{uv \in E_{\{3,3\}}} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 \\
&+ \sum_{uv \in E_{\{3,6\}}} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 + \sum_{uv \in E_{\{6,6\}}} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 \\
&= (4n + 2) \left(\frac{3 \times 3}{3 + 3 - 2} \right)^3 + (6n^2 + 4n - 4) \left(\frac{3 \times 6}{3 + 6 - 2} \right)^3 \\
&+ (6n^2 - 8n + 2) \left(\frac{6 \times 6}{6 + 6 - 2} \right)^3 \\
&= (4n + 2)(11.3906) + (6n^2 + 4n - 4)(17.0029) \\
&+ (6n^2 - 8n + 2)(46.656) \\
&= 45.5624n + 22.7812 + 102.0174n^2 + 68.0116n - 68.0116 \\
&+ 279.936n^2 - 373.248n + 93.312 \\
&= 381.9534n^2 - 259.674n + 48.0816.
\end{aligned}$$

Theorem 3.4. Let $\Gamma \cong RHSL(n)$ be a rhombus silicate network with dimension n . Then, for $n \geq 2$, the fourth version of ABC (ABC_4), the fifth version of GA (GA_5), and Sanskruti indices are given as

$$(i) \quad ABC_4(\Gamma) = 3.2743n^2 + 1.1107n + 0.1107,$$

$$(ii) \quad GA_5(\Gamma) = 11.8095n^2 - 0.2047n + 0.1636,$$

$$(iii) \quad S(\Gamma) = 32124n^2 - 44196.4n + 15280.4$$

Proof. Using Table 2 and the formulae defined in Section 2, we compute the required results as follow.

(i)

$$\begin{aligned}
ABC_4 &= \sum_{uv \in E(\Gamma)} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} = 6\sqrt{\frac{12 + 12 - 2}{12 \times 12}} \\
&+ 6\sqrt{\frac{12 + 24 - 2}{12 \times 24}} + 8\sqrt{\frac{15 + 24 - 2}{15 \times 24}} + 2\sqrt{\frac{18 + 24 - 2}{18 \times 24}} \\
&+ 8\sqrt{\frac{24 + 27 - 2}{24 \times 27}} + 4(n-1)\sqrt{\frac{15 + 15 - 2}{15 \times 15}} \\
&+ 4(2n-3)\sqrt{\frac{18 + 27 - 2}{18 \times 27}} + [8(n-2) + 2]\sqrt{\frac{27 + 27 - 2}{27 \times 27}} \\
&+ 8(2n-3)\sqrt{\frac{15 + 27 - 2}{15 \times 27}} + 8(n-2)\sqrt{\frac{27 + 30 - 2}{27 \times 30}} \\
&+ 2(n-2)(3n-4)\sqrt{\frac{18 + 30 - 2}{18 \times 30}} + 6(n-2)^2\sqrt{\frac{30 + 30 - 2}{30 \times 30}} \\
&= 2.3452 + 2.0616 + 2.5647 + 0.6086 + 2.1999 + 1.4111(n-1) \\
&+ 1.1898(2n-3) + 2.1368(n-2) + 0.5342 + 2.5142(2n-3) \\
&+ 2.0846(n-2) + 0.5837(n-2)(3n-4) + 1.5232(n-2)^2 \\
&= 10.3142 + 1.4111(n-1) + 1.1898(2n-3) \\
&+ 2.1368(n-2) + 2.5142(2n-3) + 2.0846(n-2) \\
&+ 0.5837(n-2)(3n-4) + 1.5232(n-2)^2 \\
&= 3.2743n^2 + 1.1107n + 0.1107
\end{aligned}$$

(ii)

$$\begin{aligned}
 GA_5(\Gamma) &= \sum_{uv \in E(\Gamma)} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} \\
 &= \frac{12\sqrt{12 \times 12}}{12 + 12} + \frac{12\sqrt{12 \times 24}}{12 + 24} + \frac{16\sqrt{15 \times 24}}{15 + 24} \\
 &+ \frac{4\sqrt{18 \times 24}}{18 + 24} + \frac{16\sqrt{24 \times 27}}{24 + 27} + \frac{8(n-1)\sqrt{15 \times 15}}{15 + 15} \\
 &+ \frac{8(2n-3)\sqrt{18 \times 27}}{18 + 27} + \frac{[16(n-2) + 4]\sqrt{27 \times 27}}{27 + 27} \\
 &+ \frac{16(2n-3)\sqrt{15 \times 27}}{15 + 27} + \frac{16(n-2)\sqrt{27 \times 30}}{27 + 30} + \\
 &\frac{4(n-2)(3n-4)\sqrt{18 \times 30}}{18 + 30} + \frac{12(n-2)^2\sqrt{30 \times 30}}{30 + 30} \\
 &= 6 + 5.6569 + 7.7841 + 1.9794 + 7.9861 + 4(n-1) \\
 &+ 3.9192(2n-3) + 8(n-2) + 2 + 7.6665(2n-3) \\
 &+ 7.9889(n-2) + 1.9365(n-2)(3n-4) + 6(n-2)^2 \\
 &= 11.8095n^2 - 0.2047n + 0.1636.
 \end{aligned}$$

(iii)

$$\begin{aligned}
 S(\Gamma) &= \sum_{uv \in E(\Gamma)} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 \\
 &= 6\left(\frac{12 \times 12}{12 + 12 - 2}\right)^3 + 6\left(\frac{12 \times 24}{12 + 24 - 2}\right)^3 + 8\left(\frac{15 \times 24}{15 + 24 - 2}\right)^3 \\
 &+ 2\left(\frac{18 \times 24}{18 + 24 - 2}\right)^3 + 8\left(\frac{24 \times 27}{24 + 27 - 2}\right)^3 + 4(n-1)\left(\frac{15 \times 15}{15 + 15 - 2}\right)^3 \\
 &+ 4(2n-3)\left(\frac{18 \times 27}{18 + 27 - 2}\right)^3 + [8(n-2) + 2]\left(\frac{27 \times 27}{27 + 27 - 2}\right)^3 \\
 &+ 8(2n-3)\left(\frac{15 \times 27}{15 + 27 - 2}\right)^3 + 8(n-2)\left(\frac{27 \times 30}{27 + 30 - 2}\right)^3 \\
 &+ 2(n-2)(3n-4)\left(\frac{18 \times 30}{18 + 30 - 2}\right)^3 + 6(n-2)^2\left(\frac{30 \times 30}{30 + 30 - 2}\right)^3
 \end{aligned}$$

$$\begin{aligned}
&= 1682.5605 + 3646.6322 + 7368.7245 + 2519.424 + 18502.3446 \\
&+ 2075.5512(n-1) + 5775.1522(2n-3) + 22042.5859(n-2) + 5510.6464 \\
&+ 8303.7656(2n-3) + 25553.8873(n-2) \\
&+ 3235.4730(n-2)(3n-4) + 22417.8933(n-2)^2 \\
&= 39230.3320 + 2075.5512(n-1) + 5775.1522(2n-3) + 22042.5859(n-2) \\
&+ 8303.7656(2n-3) + 25553.8873(n-2) + 3235.4730(n-2)(3n-4) \\
&+ 22417.8933(n-2)^2 \\
&= 32124n^2 - 44196.4n + 15280.4.
\end{aligned}$$

4 Topological indices of the rhombus oxide network

In this section, we compute the topological indices such as general Randić, first general Zagreb, generalized Zagreb, multiplicative Zagreb, atom-bond connectivity (ABC), geometric-arithmetic (GA), augmented Zagreb, fourth version of ABC (ABC_4), the fifth version of GA (GA_5), and Sanskruti index (SI) for the rhombus oxide ($RHOX(n)$) network.

Before to the main theorems, we define the partitions of the vertex set $V(RHOX(n))$ and the edge set $E(RHOX(n))$ of $RHOX(n)$ with respect to degree of vertices. We note that each vertex $RHOX(n)$ is of degree 2

| | | | |
|-----------------------|---------------|---------------|-----------------|
| $E_{\{d(u),d(v)\}}$ | $E_{\{2,2\}}$ | $E_{\{2,4\}}$ | $E_{\{4,4\}}$ |
| $ E_{\{d(u),d(v)\}} $ | 2 | $4(2n - 1)$ | $6n^2 - 8n + 2$ |

Table 3: The edge partition based on the degrees of the end vertices of $RHOX(n)$.

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| $E_{\{s_u, s_v\}}$ | $ E_{\{s_u, s_v\}} $ |
|--------------------|----------------------|
| (6,6) | 2 |
| (6,12) | 4 |
| (8,12) | 4 |
| (8,14) | $4(2n - 3)$ |
| (12,14) | 8 |
| (14,14) | $2(4n - 7)$ |
| (14,16) | $8(n - 2)$ |
| (16,16) | $6(n - 2)^2$ |

Table 4: The edge partition based on the degree sum of neighbors of the end vertices of $RHOX(n)$.

or of degree 4. Thus, we have

$$V_1 = \{v \in V(RHOX(n)) | d_v = 2\} \text{ and } V_2 = \{v \in V(RHOX(n)) | d_v = 4\},$$

where $|V_1| = 4n$ and $|V_2| = 3n^2 - 2n$. Consequently, $|V(RHOX(n))| = v = |V_1| + |V_2| = 3n^2 + 2n$. Moreover, there are three types of edges with respect to degrees of end vertices in $RHOX(n)$ namely with degrees of end vertices $\{2, 2\}$, degrees of end vertices $\{2, 4\}$ and degrees of end vertices $\{4, 4\}$. Thus, we have

$$E_{\{2,2\}} = \{uv \in E(RHOX(n)) | d_u = 2, d_v = 2\},$$

$$E_{\{2,4\}} = \{uv \in E(RHOX(n)) | d_u = 2, d_v = 4\},$$

and
$$E_{\{4,4\}} = \{uv \in E(RHOX(n)) | d_u = 4, d_v = 4\},$$

where $|E_{\{2,2\}}| = 2$, $|E_{\{2,4\}}| = 4(2n - 1)$ and $|E_{\{4,4\}}| = 6n^2 - 8n + 2$. Consequently, $|E(RHOX(n))| = e = |E_{\{2,2\}}| + |E_{\{2,4\}}| + |E_{\{4,4\}}| = 6n^2$. Similarly, we can define the partitions of the vertex set $V(RHOX(n))$ and the edge set $E(RHOX(n))$ of $RHOX(n)$ with respect to the sum of the degrees of neighbors of the vertices of $RHOX(n)$. Thus, for $uv \in E(RHOX(n))$, we have the Table 4.1 and Table 4.2.

Theorem 4.1. Let $\Gamma \cong RHOX(n)$ be a rhombus oxide network with dimension n . Then, for $n \geq 2$, its Randić index ($R_{-\frac{1}{2}}(\Gamma)$), reciprocal Randić index ($R_{\frac{1}{2}}(\Gamma)$), second Zagreb index ($R_1(\Gamma)$) and the general Randić index $R_\alpha(\Gamma)$ are given as

$$R_\alpha(\Gamma) = \begin{cases} \frac{1}{2}(3n^2 - 4n + 3 + 2\sqrt{2}(2n - 1)), & \text{for } \alpha = -\frac{1}{2}, \\ 24n^2 - 32n + 8\sqrt{2}(2n - 1) + 12, & \text{for } \alpha = \frac{1}{2}, \\ 96n^2 - 64n + 8, & \text{for } \alpha = 1, \\ n^2(6 \times 16^\alpha) + 8n(8^\alpha - 16^\alpha) + (2 \times 4^\alpha)[1 + 2^\alpha(2^\alpha - 2)], & \text{for } \alpha \in \mathbb{R}. \end{cases}$$

Proof. Using Table 3 and the formulae defined in Section 2, we compute the required results as follow.

$$\begin{aligned} R_{-\frac{1}{2}}(\Gamma) &= \sum_{uv \in E(\Gamma)} \frac{1}{\sqrt{d_u \times d_v}} \\ &= \sum_{uv \in E_{\{2,2\}}} \frac{1}{\sqrt{d_u \times d_v}} + \sum_{uv \in E_{\{2,4\}}} \frac{1}{\sqrt{d_u \times d_v}} + \sum_{uv \in E_{\{4,4\}}} \frac{1}{\sqrt{d_u \times d_v}} \\ &= \frac{2}{\sqrt{2 \times 2}} + \frac{4(2n-1)}{\sqrt{2 \times 4}} + \frac{6n^2 - 8n + 2}{\sqrt{4 \times 4}} \\ &= \frac{2}{2} + \frac{4(2n-1)}{\sqrt{2 \times 4}} + \frac{(6n^2 - 8n + 2)}{\sqrt{4 \times 4}} \\ &= \frac{2}{2} + \frac{6n^2 - 8n + 2}{4} + \frac{4(2n-1)}{2\sqrt{2}} \\ &= \frac{4 + 6n^2 - 8n + 2}{4} + \frac{4(2n-1)}{2\sqrt{2}} \\ &= \frac{6n^2 - 8n + 2}{4} + \sqrt{2}(2n-1) \\ &= \frac{1}{2}(3n^2 - 4n + 3 + 2\sqrt{2}(2n-1)), \end{aligned}$$

$$\begin{aligned}
R_{\frac{1}{2}}(\Gamma) &= \sum_{uv \in E(\Gamma)} \sqrt{d_u \times d_v} \\
&= \sum_{uv \in E_{\{2,2\}}} \sqrt{d_u \times d_v} + \sum_{uv \in E_{\{2,4\}}} \sqrt{d_u \times d_v} + \sum_{uv \in E_{\{4,4\}}} \sqrt{d_u \times d_v} \\
&= 2\sqrt{2 \times 2} + 4(2n - 1)\sqrt{2 \times 4} + (6n^2 - 8n + 2)\sqrt{4 \times 4} \\
&= 4 + 8\sqrt{2}(2n - 1) + 4(6n^2 - 8n + 2) \\
&= 4 + 8\sqrt{2}(2n - 1) + 24n^2 - 32n + 8 \\
&= 24n^2 - 32n + 8\sqrt{2}(2n - 1) + 12,
\end{aligned}$$

$$\begin{aligned}
R_1(\Gamma) &= \sum_{uv \in E(\Gamma)} (d_v \times d_u) \\
&= \sum_{uv \in E_{\{2,2\}}} (d_v \times d_u) + \sum_{uv \in E_{\{2,4\}}} (d_v \times d_u) + \sum_{uv \in E_{\{4,4\}}} (d_v \times d_u) \\
&= 2(2 \times 2) + 4(2n - 1)(2 \times 4) + (6n^2 - 8n + 2)(4 \times 4) \\
&= 8 + 32(2n - 1) + 16(6n^2 - 8n + 2) \\
&= 96n^2 - 64n + 8,
\end{aligned}$$

$$\begin{aligned}
R_\alpha(\Gamma) &= \sum_{uv \in E(\Gamma)} (d_u \times d_v)^\alpha \\
&= \sum_{uv \in E_{2,2}} (d_u d_v)^\alpha + \sum_{uv \in E_{2,4}} (d_u d_v)^\alpha + \sum_{uv \in E_{4,4}} (d_u d_v)^\alpha \\
&= 2(2 \times 2)^\alpha + 4(2n - 1)(2 \times 4)^\alpha + (6n^2 - 8n + 2)(4 \times 4)^\alpha \\
&= 2 \times 4^\alpha + 4(2n - 1)(8)^\alpha + (6n^2 - 8n + 2)(16)^\alpha \\
&= (6n^2 \times 16^\alpha) + (8n \times 8^\alpha) - (8n \times 16^\alpha) + (2 \times 4^\alpha) + (2 \times 16^\alpha) \\
&\quad - (4 \times 8^\alpha) \\
&= n^2(6 \times 16^\alpha) + 8n(8^\alpha - 16^\alpha) + (2 \times 4^\alpha)[1 + 2^\alpha(2^\alpha - 2)].
\end{aligned}$$

Theorem 4.2. Let $\Gamma \cong RHOX(n)$ be a rhombus oxide with dimension n . Then, for $n \geq 2$, first general Zagreb ($M_1^\alpha(\Gamma)$), generalized Zagreb ($M_{r,s}(\Gamma)$), first multiplicative Zagreb ($PM_1(\Gamma)$) and the second multiplicative Zagreb ($PM_2(\Gamma)$) indices are given by

$$\begin{aligned}
\text{(i)} \quad M_1^\alpha(\Gamma) &= 2^\alpha \times 4n + 4^\alpha(3n^2 - 2n), \\
\text{(ii)} \quad M_{r,s}(\Gamma) &= 2^{r+s+2}[3n^2(2^{r+s}) + 2n(2^r + 2^s - 2^{r+s+1}) + (1 - 2^r - 2^s + 2^{r+s+1})], \\
\text{(iii)} \quad PM_1(\Gamma) &= 18432n^3 - 33792n^2 + 18432n - 3072, \\
\text{(iv)} \quad PM_2(\Gamma) &= 49152n^3 - 90112n^2 + 49152n - 8192.
\end{aligned}$$

Proof. Using Table 3 and the formulae defined in Section 2, we compute the required results as follows. (i)

$$\begin{aligned}
M_1^\alpha(\Gamma) &= \sum_{u \in V(\Gamma)} (d_u)^\alpha \\
&= 4n(2)^\alpha + (3n^2 - 2n)(4)^\alpha \\
&= 2^\alpha \times 4n + 4^\alpha(3n^2 - 2n).
\end{aligned}$$

(ii)

$$\begin{aligned}
M_{r,s}(T) &= \sum_{uv \in E(T)} (d_u^r d_v^s + d_v^r d_u^s) = \sum_{uv \in E_{\{2,2\}}} (d_u^r d_v^s + d_v^r d_u^s) \\
&+ \sum_{uv \in E_{\{2,4\}}} (d_u^r d_v^s + d_v^r d_u^s) + \sum_{uv \in E_{\{4,4\}}} (d_u^r d_v^s + d_v^r d_u^s) \\
&= 2(2^r \times 2^s + 2^s \times 2^r) + 4(2n-1)(2^r \times 4^s + 2^s \times 4^r) \\
&+ (6n^2 - 8n + 2)(4^r \times 4^s + 4^s \times 4^r) \\
&= (4 \times 2^{r+s})[1 + (2n-1)(2^r + 2^s) + (3n^2 - 4n + 1)2^{r+s}] \\
&= 2^{r+s+2}[3n^2(2^{r+s}) + 2n(2^r + 2^s - 2^{r+s+1}) \\
&+ (1 - 2^r - 2^s + 2^{r+s+1})],
\end{aligned}$$

(iii)

$$\begin{aligned}
PM_1(\Gamma) &= \prod_{uv \in E(T)} (d_v + d_u) \\
&= \prod_{uv \in E_{\{2,2\}}} (d_v + d_u) \times \prod_{uv \in E_{\{2,4\}}} (d_v + d_u) \times \prod_{uv \in E_{\{4,4\}}} (d_v + d_u) \\
&= 2(2+2) \times 4(2n-1)(2+4) \times (6n^2 - 8n + 2)(4+4) \\
&= 8 \times 24(2n-1) \times 8(6n^2 - 8n + 2) \\
&= 18432n^3 - 33792n^2 + 18432n - 3072,
\end{aligned}$$

(iv)

$$\begin{aligned}
PM_2(\Gamma) &= \prod_{uv \in E(T)} (d_v \times d_u) \\
&= \prod_{uv \in E_{\{2,2\}}} (d_v \times d_u) \times \prod_{uv \in E_{\{2,4\}}} (d_v \times d_u) \times \prod_{uv \in E_{\{4,4\}}} (d_v \times d_u) \\
&= 2(2 \times 2) \times 4(2n - 1)(2 \times 4) \times (6n^2 - 8n + 2)(4 \times 4) \\
&= 4096(12n^3 - 22n^2 + 4n - 2) \\
&= 49152n^3 - 90112n^2 + 49152n - 8192.
\end{aligned}$$

Theorem 4.3. Let $\Gamma \cong RHOX(n)$ be a rhombus oxide network with dimension n . Then, for $n \geq 2$, atom-bond connectivity ($ABC(\Gamma)$) index, geometric-arithmetic ($GA(\Gamma)$) index and augmented Zagreb index (AZI) are given by

$$\begin{aligned}
\text{(i)} \quad ABC(\Gamma) &= 3.6742n^2 + 0.7578n - 0.1894, \\
\text{(ii)} \quad GA(\Gamma) &= 6n^2 - 0.4575n + 0.2287, \\
\text{(iii)} \quad AZI(\Gamma) &= 113.7774n^2 - 87.7032n + 21.9258.
\end{aligned}$$

Proof. Using Table 3 and the formulae defined in Section 2, we compute the required results as follows.

(i)

$$\begin{aligned}
 ABC(\Gamma) &= \sum_{uv \in E(\Gamma)} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} = \sum_{uv \in E_{\{2,2\}}} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} \\
 &\quad + \sum_{uv \in E_{\{2,4\}}} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} + \sum_{uv \in E_{\{4,4\}}} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} \\
 &= 2\sqrt{\frac{2+2-2}{2 \times 2}} + 4(2n-1)\sqrt{\frac{2+4-2}{2 \times 4}} + (6n^2 - 8n + 2)2\sqrt{\frac{4+4-2}{4 \times 4}} \\
 &= \frac{2}{\sqrt{2}} + \frac{4(2n-1)}{\sqrt{2}} + (6n^2 - 8n + 2)\frac{\sqrt{6}}{4} \\
 &= \sqrt{2} + 2\sqrt{2}(2n-1) + (6n^2 - 8n + 2)\frac{\sqrt{6}}{4} \\
 &= \frac{4\sqrt{2} + 8\sqrt{2}(2n-1) + \sqrt{6}(6n^2 - 8n + 2)}{4} \\
 &= 3.6742n^2 + 0.7578n - 0.1894.
 \end{aligned}$$

(ii)

$$\begin{aligned}
 GA(\Gamma) &= \sum_{uv \in E(\Gamma)} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} \\
 &= \sum_{uv \in E\{2,2\}} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} + \sum_{uv \in E\{2,4\}} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} \\
 &\quad + \sum_{uv \in E\{4,4\}} \frac{2\sqrt{d_u \times d_v}}{d_u + d_v} \\
 &= \frac{2 \times 2\sqrt{2 \times 2}}{2 + 2} + \frac{4(2n - 1) \times 2\sqrt{2 \times 4}}{2 + 4} + \frac{(6n^2 - 8n + 2) \times 2\sqrt{4 \times 4}}{4 + 4} \\
 &= 2 + \frac{16(2n - 1)\sqrt{2}}{6} + \frac{8(6n^2 - 8n + 2)}{8} \\
 &= 2 + \frac{8}{3}(2n - 1)\sqrt{2} + (6n^2 - 8n + 2) \\
 &= 6n^2 - 8n + 4 + \frac{8\sqrt{2}}{3}(2n - 1) \\
 &= \frac{18n^2 - 24n + 12 + 8\sqrt{2}(2n - 1)}{3} \\
 &= \frac{18n^2 - 24n + 16\sqrt{2}n + 12 - 8\sqrt{2}}{3} \\
 &= 6n^2 - 0.4575n + 0.2287.
 \end{aligned}$$

(iii)

$$\begin{aligned}
AZI(T) &= \sum_{uv \in E(T)} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 \\
&= \sum_{uv \in E_{\{2,2\}}} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 + \sum_{uv \in E_{\{2,4\}}} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 \\
&\quad + \sum_{uv \in E_{\{4,4\}}} \left(\frac{d_u \times d_v}{d_u + d_v - 2} \right)^3 \\
&= 2 \left(\frac{2 \times 2}{2 + 2 - 2} \right)^3 + 4(2n - 1) \left(\frac{2 \times 4}{2 + 4 - 2} \right)^3 + (6n^2 - 8n + 2) \left(\frac{4 \times 4}{4 + 4 - 2} \right)^3 \\
&= 16 + 4(2n - 1) \times 8 + (6n^2 - 8n + 2) \left(\frac{8}{3} \right)^3 \\
&= 16 + 64n - 32 + (6n^2 - 8n + 2) \times 18.9629 \\
&= 113.7774n^2 - 87.7032n + 21.9258.
\end{aligned}$$

Theorem 4.4. Let $\Gamma \cong RHSL(n)$ be a rhombus oxide network with dimension n . Then, for $n \geq 2$, the fourth version of ABC (ABC_4), the fifth version of GA (GA_5), and Sanskruti index (SI) are given as

$$\begin{aligned}
\text{(i)} \quad ABC_4(\Gamma) &= \frac{\sqrt{10}}{3} + \frac{8}{3\sqrt{2}} + \sqrt{3} + 4(2n - 3)\sqrt{\frac{5}{28}} + \frac{8}{\sqrt{7}} + 2(4n - 7)\sqrt{\frac{13}{98}} + \\
&\quad \sqrt{8}(n - 2) + 6(n - 2)^2\sqrt{\frac{15}{128}}, \\
\text{(ii)} \quad GA(\Gamma) &= 2 + \frac{4}{9}\sqrt{72} + \frac{2\sqrt{96}}{5} + \frac{4\sqrt{112}}{11}(2n - 3) + \frac{8}{13}\sqrt{168} + 2(4n - 7) + \\
&\quad \frac{8\sqrt{224}}{15}(n - 2) + 6(n - 2)^2, \\
\text{(iii)} \quad S(\Gamma) &= 3728.2699n^2 - 5984.9639n + 2424.7359.
\end{aligned}$$

Proof. Using Table 4 and the formulae defined in Section 2, we compute the required results as follow.

(i)

$$\begin{aligned}
ABC_4 &= \sum_{uv \in E(\Gamma)} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} \\
&= \sum_{uv \in E_{\{6,6\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} + \sum_{uv \in E_{\{6,12\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} \\
&+ \sum_{uv \in E_{\{8,12\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} + \sum_{uv \in E_{\{8,14\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} \\
&+ \sum_{uv \in E_{\{12,14\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} + \sum_{uv \in E_{\{14,14\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} \\
&+ \sum_{uv \in E_{\{14,16\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} + \sum_{uv \in E_{\{16,16\}}} \sqrt{\frac{S_u + S_v - 2}{S_u \times S_v}} \\
&= 2\sqrt{\frac{6+6-2}{6 \times 6}} + 4\sqrt{\frac{6+12-2}{6 \times 12}} + 4\sqrt{\frac{8+12-2}{8 \times 12}} \\
&+ 4(2n-3)\sqrt{\frac{8+14-2}{8 \times 14}} + 8\sqrt{\frac{12+14-2}{12 \times 14}} + 2(4n-7)\sqrt{\frac{14+14-2}{14 \times 14}} \\
&+ 8(n-2)\sqrt{\frac{14+16-2}{14 \times 16}} + 6(n-2)^2\sqrt{\frac{16+16-2}{16 \times 16}} \\
&= \frac{2}{6}\sqrt{10} + \frac{16}{\sqrt{72}} + 4\sqrt{\frac{18}{96}} + 4(2n-3)\sqrt{\frac{20}{112}} \\
&+ 8\sqrt{\frac{24}{168}} + 2(4n-7)\sqrt{\frac{26}{196}} + 8(n-2)\sqrt{\frac{28}{224}} + 6(n-2)^2\sqrt{\frac{30}{256}} \\
&= \frac{\sqrt{10}}{3} + \frac{16}{6\sqrt{2}} + 4\sqrt{\frac{3}{16}} + 4(2n-3)\sqrt{\frac{5}{28}} + 8\sqrt{\frac{1}{7}} \\
&+ 2(4n-7)\sqrt{\frac{13}{98}} + 8(n-2)\sqrt{\frac{1}{8}} + 6(n-2)^2\sqrt{\frac{15}{128}} \\
&= \frac{\sqrt{10}}{3} + \frac{8}{3\sqrt{2}} + \sqrt{3} + 4(2n-3)\sqrt{\frac{5}{28}} \\
&+ \frac{8}{\sqrt{7}} + 2(4n-7)\sqrt{\frac{13}{98}} + \sqrt{8}(n-2) + 6(n-2)^2\sqrt{\frac{15}{128}}.
\end{aligned}$$

(ii)

$$\begin{aligned}
GA_5(\Gamma) &= \sum_{uv \in E(\Gamma)} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} \\
&= \sum_{uv \in E_{\{6,6\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} + \sum_{uv \in E_{\{6,12\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} \\
&+ \sum_{uv \in E_{\{8,12\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} + \sum_{uv \in E_{\{8,14\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} \\
&+ \sum_{uv \in E_{\{12,14\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} + \sum_{uv \in E_{\{14,14\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} \\
&+ \sum_{uv \in E_{\{14,16\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} + \sum_{uv \in E_{\{16,16\}}} \frac{2\sqrt{S_u \times S_v}}{S_u + S_v} \\
&= \frac{4\sqrt{6 \times 6}}{6+6} + \frac{8\sqrt{6 \times 12}}{6+12} + \frac{8\sqrt{8 \times 12}}{8+12} \\
&+ \frac{8(2n-3)\sqrt{8 \times 14}}{8+14} + \frac{16\sqrt{12 \times 14}}{12+14} + \frac{4(4n-7)\sqrt{14 \times 14}}{14+14} \\
&+ \frac{16(n-2)\sqrt{14 \times 16}}{14+16} + \frac{12(n-2)^2\sqrt{16 \times 16}}{16+16} \\
&= 2 + \frac{4}{9}\sqrt{72} + \frac{2\sqrt{96}}{5} + \frac{4}{11}(2n-3)\sqrt{112} + \frac{8}{13}\sqrt{168} \\
&+ \frac{14(4n-7)}{7} + \frac{8}{15}(n-2)\sqrt{224} + \frac{3}{8}(n-2)^2 16 \\
&= 2 + \frac{4}{9}\sqrt{72} + \frac{2\sqrt{96}}{5} + \frac{4}{11}(2n-3)\sqrt{112} + \frac{8}{13}\sqrt{168} \\
&+ 2(4n-7) + \frac{8}{15}(n-2)\sqrt{224} + 6(n-2)^2.
\end{aligned}$$

(iii)

$$\begin{aligned}
S(\Gamma) &= \sum_{uv \in E(\Gamma)} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 \\
&= \sum_{uv \in E_{\{6,6\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 + \sum_{uv \in E_{\{6,12\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 \\
&+ \sum_{uv \in E_{\{8,12\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 + \sum_{uv \in E_{\{8,14\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 \\
&+ \sum_{uv \in E_{\{12,14\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 + \sum_{uv \in E_{\{14,14\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 \\
&+ \sum_{uv \in E_{\{14,16\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 + \sum_{uv \in E_{\{16,16\}}} \left(\frac{S_u \times S_v}{S_u + S_v - 2} \right)^3 \\
&= 2 \left(\frac{6 \times 6}{6 + 6 - 2} \right)^3 + 4 \left(\frac{6 \times 12}{6 + 12 - 2} \right)^3 + 4 \left(\frac{8 \times 12}{8 + 12 - 2} \right)^3 \\
&+ 4(2n - 3) \left(\frac{8 \times 14}{8 + 14 - 2} \right)^3 + 8 \left(\frac{12 \times 14}{12 + 14 - 2} \right)^3 + 2(4n - 7) \left(\frac{14 \times 14}{14 + 14 - 2} \right)^3 \\
&+ 8(n - 2) \left(\frac{14 \times 16}{14 + 16 - 2} \right)^3 + 6(n - 2)^2 \left(\frac{16 \times 16}{16 + 16 - 2} \right)^3 \\
&= 93.312 + 364.5 + 606.815 + 702.464(2n - 3) + 2744 + 856.797(4n - 7) \\
&+ 4096(n - 2) + 3728.270(n - 2)^2 \\
&= 3728.270(n - 2)^2 + 702.464(2n - 3) + 856.797(4n - 7) \\
&+ 4096(n - 2) + 3808.627 \\
&= 3728.2699n^2 - 5984.9639n + 2424.7359.
\end{aligned}$$

5 Comparison and Conclusion

In this section, we compare the results of all the computed topological indices for the rhombus silicate and rhombus oxide networks with the help of cartesian coordinate systems separately.

In Section 3 and Section 4, the formulae of all the indices are found in term of n , where n is dimension (number of vertices in any side) of the rhombus silicate or rhombus oxide network. In Figure 3, along the horizontal line the values of n for silicate network are taken and along the vertical line the computed values of the indices are shown. We note that the values of the all the topological indices of the silicate network other than Sanskurti index (SI) and augmented Zagreb index (AZI) coincide. Similarly, in Figure 4, the horizontal line shows the values of n and the vertical line takes the computed values of the indices of the oxide network. The values of the all the topological indices of the oxide network other than Sanskurti index (SI) and augmented Zagreb index (AZI) coincide. Moreover, in both cases, all the topological indices remain constant approximately except Sanskurti index (SI) and augmented Zagreb index (AZI) with increasing values of n . The latest developed index SI shows a dominant change with the increasing value of n for both the networks. Now, we close our discussion with the following lines.

In this paper, certain degree-based topological indices, namely general Randić, first general Zagreb, generalized Zagreb, multiplicative Zagreb, atom-bond connectivity (ABC), geometric-arithmetic (GA), augmented Zagreb, the fourth version of ABC (ABC_4), the fifth version of GA (GA_5) and Sanskruti indices of the rhombus silicate and rhombus oxide networks are studied and analytical closed formulas for these networks are determined that will help to understand the underlying topologies of these networks.

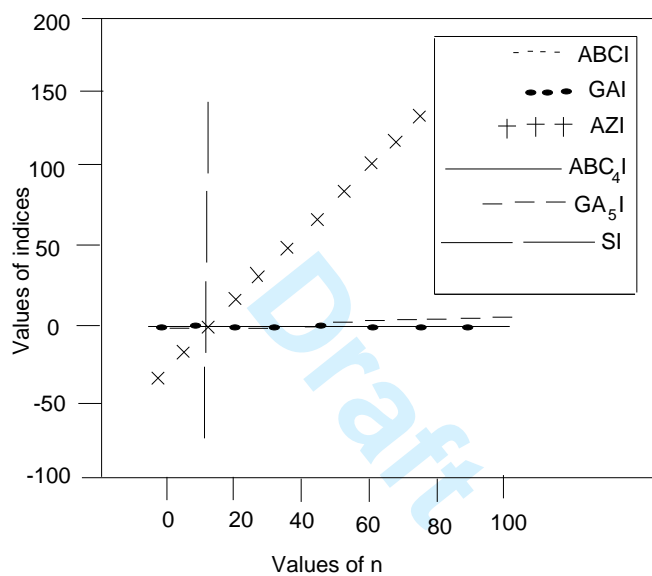


Figure 3: Comparison of the topological indices of the silicate network.

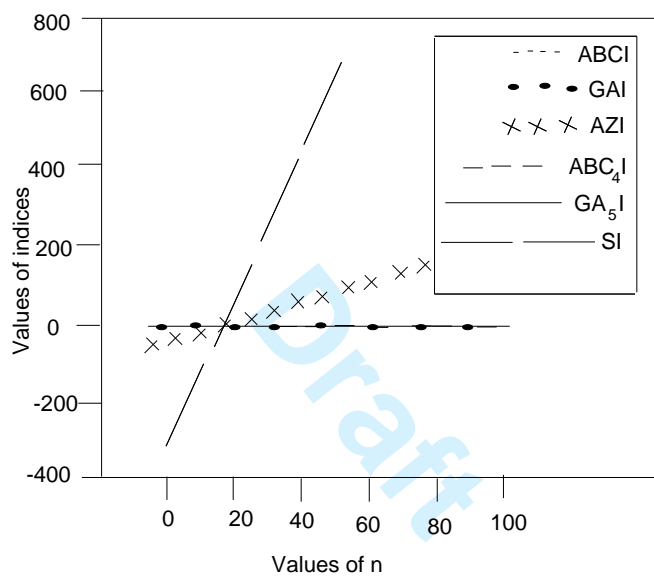


Figure 4: Comparison of the topological indices of the oxide network.

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