Topological Quantum Field Theory and Four-Manifolds

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Abstract. I review some recent results on four-manifold invariants which have been obtained in the context of topological quantum field theory. I focus on three different aspects: (a) the computation of correlation functions, which give explicit results for the Donaldson invariants of non-simply connected manifolds, and for generalizations of these invariants to the gauge group $SU(N)$; (b) compactifications to lower dimensions, and connections to three-manifold topology and to intersection theory on the moduli space of flat connections on Riemann surfaces; (c) four-dimensional theories with critical behaviour, which give some remarkable constraints on Seiberg-Witten invariants and new results on the geography of four-manifolds.

1. Introduction

One of the original motivations of Witten [19] to introduce topological quantum field theories (TQFT) was precisely to understand the Donaldson invariants of four-manifolds from a physical point of view. This approach proved its full power in 1994, when it was realized that all the information of Donaldson theory was contained in the Seiberg-Witten (SW) invariants. These new invariants led to a true revolution in four-dimensional topology, and they were introduced in [20] based on nonperturbative results in supersymmetric quantum field theory. The relation between Donaldson invariants and SW invariants was fully clarified in an important paper by G. Moore and E. Witten [15], where they introduced the so called u-plane integral.

In this note, I review some recent results on four-manifold invariants which have been obtained through the use of u-plane integral techniques. I emphasize how these results are related to the physics of four-dimensional quantum field theories. First, I discuss Donaldson invariants as correlation functions in TQFT. I present some new results for non simply connected manifolds (for product ruled surfaces, in particular), and for extensions of Donaldson theory to higher rank gauge groups. Second, I use compactifications of the field theory to make contact with results in three and two dimensions. In the two-dimensional case, I recover in fact Thaddeus' celebrated formula for the intersection pairings on the moduli space of flat connections on a Riemann surface. Finally, I consider qualitatively new physics (a field theory with critical behaviour) to obtain new relations between the SW invariants and classical invariants of four-manifolds. In the last section, I briefly consider some open problems.

2. Correlation Functions

2.1. General aspects

The Donaldson invariants of smooth, compact, oriented four-manifolds X [2] are defined by using intersection theory on the moduli space of anti-self-dual connections. The cohomology classes on this space are associated to homology classes of X through the slant product [2] or, in the context of topological field theory, by using the descent procedure [19]. Here we will restrict ourselves to the Donaldson invariants associated to zero, one and two-homology classes[1]. Define

$$\mathbf{A}(X) = \mathrm{Sym}(H_0(X) \oplus H_2(X)) \otimes \wedge^* H_1(X)\,. \tag{1}$$

Then, the Donaldson invariants can be regarded as functionals

$$D_X^{w_2(E)} \colon \mathbf{A}(X) \to \mathbf{Q}\,, \tag{2}$$

where $w_2(E) \in H^2(X, \mathbf{Z})$ is the second Stiefel-Whitney class of the gauge bundle. It is convenient to organize these invariants as follows. Let $\{\delta_i\}_{i=1,\dots,b_1}$ be a basis of one-cycles, $\{\beta_i\}_{i=1,\dots,b_1}$ the corresponding dual basis of harmonic one-forms, and $\{S_i\}_{i=1,\dots,b_2}$ a basis of two-cycles. We introduce the formal sums

$$\delta = \sum_{i=1}^{b_1} \zeta_i\, \delta_i, \qquad S = \sum_{j=1}^{b_2} v_i\, S_i\,, \tag{3}$$

where v_i are complex numbers, and ζ_i are Grassmann variables. The generator of the 0-class will be denoted by $x \in H_0(X, \mathbf{Z})$. We then define the Donaldson-Witten generating function:

$$Z_{DW}(p, \zeta_i, v_i) = D_X^{w_2(E)}(\mathrm{e}^{px+\delta+S})\,, \tag{4}$$

so that the Donaldson invariants can be read off from the expansion of the left-hand side in powers of p, ζ_i and v_i. The main result in [19] is that Z_{DW} can be understood as the generating functional of a twisted version of the $N=2$ supersymmetric gauge theory —with gauge group $SU(2)$— in four dimensions. In the twisted theory one can define observables $O(x)$, $I_1(\delta) = \int_\delta O_1$, $I_2(S) = \int_S O_2$ (where O_i are functionals of the fields of the theory) in one to one correspondence with the homology classes of X, and in such a way that the generating functional

$$\langle \mathrm{e}^{pO(x)+I_1(\delta)+I_2(S)} \rangle$$

is precisely $Z_{DW}(p, \zeta_i, v_i)$.

Based on the low-energy effective descriptions of $N=2$ gauge theories obtained in [17], Witten obtained a explicit formula for (4) in terms of SW invariants

[1] The inclusion of three-classes has been considered in [11].

for manifolds of $b_2^+ > 1$ and simple type [20]. The general framework to give a complete evaluation of (4) was established in [15]. The main result of Moore and Witten is an explicit expression for the generating function Z_{DW}:

$$Z_{DW} = Z_u + Z_{SW} \tag{5}$$

which consists of two pieces. Z_{SW} is the contribution from the moduli space M_{SW} of solutions of the SW monopole equations. Z_u (the u-plane integral henceforth) is the integral of a certain modular form over the fundamental domain of the group $\Gamma^0(4)$, that is, over the quotient $\Gamma^0(4)\setminus H$, where H is the upper half-plane. The explicit form of Z_u was derived in [15] for simply connected four-manifolds, and extended to the non-simply connected case in [11]. Z_u is non-vanishing only for manifolds with $b_2^+ = 1$, and provides a simple physical explanation of the failure of topological invariance of the Donaldson invariants on those manifolds [15].

2.2. Donaldson invariants in the non-simply connected case

Most of the computations of Donaldson invariants have focused on simply connected manifolds. The study of the nonsimply connected side was initiated in [15, 8], and finally a complete description of the invariants was given in [11]. Some additional results were obtained in [7]. The nonsimply connected case presents some new features, mostly when $b_2^+ = 1$. Of particular interest are the Donaldson invariants of product ruled surfaces $\mathbf{S}^2 \times \Sigma_g$, which as far as I know have not been completely determined from a mathematical point of view. Recall that the invariants depend on the chamber chosen in the Kähler cone. The result gets simpler in the limiting chambers of very small or large volumes for $\mathbf{S}^2$. We will take a symplectic basis of one cycles in Σ_g, δ_i, $i = 1, \cdots, 2g$, and consider the $\mathrm{Sp}(2g, \mathbf{Z})$-invariant element $\iota = -2\sum_{i=1}^{g} \delta_i \delta_{i+g}$. In the limit of small volume for $\mathbf{S}^2$, the generating functions $Z_g^{w_2(E)} = D^{w_2(E)}_{\mathbf{S}^2\times\Sigma_g}(\mathrm{e}^{px+r\iota+s\Sigma_g+t\mathbf{S}^2})$ are given by [11, 7]:

$$Z_g^{w_2(E)=0} = -\frac{i}{4}\left[(h_\infty^2 f_{2\infty})^{-1}\mathrm{e}^{2pu_\infty+2stT_\infty}\Big(2f_{1\infty}h_\infty^2 s + 2r\Big)^g \coth\Big(\frac{is}{2h_\infty}\Big)\right]_{q^0}, \tag{6}$$

$$Z_{g,\mathbf{S}^2}^{w_2(E)=[\mathbf{S}^2]} = -\frac{1}{4}\left[(h_\infty^2 f_{2\infty})^{-1}\mathrm{e}^{2pu_\infty+2stT_\infty}\Big(2f_{1\infty}h_\infty^2 s + 2r\Big)^g \csc\Big(\frac{s}{2h_\infty}\Big)\right]_{q^0}, \tag{7}$$

and they vanish for the other choices of Stiefel-Whitney class. For $g = 0$, one recovers the expressions for $\mathbf{S}^2 \times \mathbf{S}^2$ which were obtained in [15, 5]. The above equations involve the modular forms with q-expansions:

$$\begin{aligned}
u_\infty &= \frac{1}{2}\frac{\vartheta_2^4+\vartheta_3^4}{(\vartheta_2\vartheta_3)^2} = \frac{1}{8q^{1/4}}(1+20q^{1/2}-62q+\cdots)\,,\\
T_\infty &= -\frac{1}{24}\left(\frac{E_2}{h_\infty^2}-8u_\infty\right) = q^{1/4}(1-2q^{1/2}+6q+\cdots)\,,\\
h_\infty(\tau) &= \frac{1}{2}\vartheta_2\vartheta_3 = q^{1/8}(1+2q^{1/2}+q+\cdots)\,,
\end{aligned}$$

$$
\begin{aligned}
f_{1\infty}(q) &= \frac{2E_2+\vartheta_2^4+\vartheta_3^4}{3\vartheta_4^8}=1+24q^{1/2}+\cdots\,,\\
f_{2\infty}(q) &= \frac{\vartheta_2\vartheta_3}{2\vartheta_4^8}=q^{1/8}+18q^{5/8}+\cdots\,,
\end{aligned}
\tag{8}
$$

and the subscript q^0 means that one has to extract the q^0 power in the q-expansion. The expressions for the other limiting chamber can be found in two ways: since the wall-crossing formula in the nonsimply connected case was obtained in [11], one can sum to the above expression an infinite number of wall-crossing terms. Alternatively, one can perform a direct evaluation of the u-plane integral [7].

2.3. Extension to gauge group $SU(N)$

The Donaldson invariants are usually defined for the gauge group $SU(2)$. In principle, one can formally consider invariants of four-manifolds defined from anti-self dual $SU(N)$ gauge connections. Although this seems to be pretty difficult from a mathematical point of view, the evaluation of the would-be $SU(N)$ Donaldson invariants turns out to be tractable using quantum field theory [10]. The result is simpler for manifolds of simple type and with $b_2^+>1$. Not surprisingly, it can be expressed in terms of the cohomology ring of X and of SW invariants:

$$
\begin{aligned}
\langle \mathrm{e}^{pO(x)+I_2(S)}\rangle_{SU(N)} &= \alpha_N^{\chi}\beta_N^{\sigma}\sum_{k=0}^{N-1}\omega^{k(N^2-1)\chi_h}\sum_{\lambda^I}\prod_{I=1}^{N-1}SW(\lambda^I)\\
&\cdot\Big(\prod_{1\le I<J\le N-1} q_{IJ}^{-(\lambda^I,\lambda^J)}\Big)\exp\Big[p\omega^{2k}N+2\omega^{2k}S^2+2\omega^k\sum_{I=1}^{N-1}(S,\lambda^I)\sin\frac{\pi I}{N}\Big]\,,
\end{aligned}
\tag{9}
$$

where $\omega=\exp[i\pi/N]$, $\chi_h=(\chi+\sigma)/4$, and χ, σ are the Euler characteristic and the signature of X, respectively. The q_{IJ} are $\exp\pi i\tau_{IJ}$, where τ_{IJ}, $I\neq J$, are the leading terms of the offdiagonal effective couplings τ_{IJ}, which have been computed in [3]. The sum in (9) is over basic classes, and $(\,,\,)$ is the intersection form of X. Finally, α_N and β_N are universal constants. In the above expression we have only considered $SU(N)$ bundles with zero Stiefel-Whitney class. In addition, one can consider additional operators associated to higher Casimirs of the gauge group, that we have not included in (9). Notice that the above expression shows that the theory factorizes down to the "magnetic" Cartan torus $U(1)^{N-1}$, but there is an important mixing measured by the off-diagonal effective couplings.

3. Compactification

3.1. Down to three dimensions

To make contact with results in three-dimensional topology, one should consider four-manifolds of the form $X=\mathbf{S}^1\times Y$. Donaldson theory on these manifolds has been explored in [12]. Using results from supersymmetric gauge theory, we would expect the partition function of Donaldson-Witten theory on $Y\times\mathbf{S}^1$ for gauge group G to agree with the Rozansky-Witten invariant $Z_{RW}(Y,X_G)$ [16], where X_G is a hyperKähler manifold. When $G = SU(2)$, $X_{SU(2}$ is the Atiyah-Hitchin manifold and the Rozansky-Witten invariant is the Casson-Walker-Lescop invariant $\lambda_{\mathrm{CWL}}(Y)$. For $G = SU(N)$, $X_{SU(N)}$ is the reduced moduli space of N monopoles, which is a hyperKähler manifold of dimension $4(N-1)$.

This expectation can be partially checked. Using (9) and the Meng-Taubes theorem [14], one can prove that, for $b_1(Y) > 1$

$$Z_{DW}^{SU(N)}(Y \times \mathbf{S}^1) = N^2(\lambda_{\mathrm{CWL}}(Y))^{N-1}\,, \tag{10}$$

and the left hand side is in fact (up to an overall constant) $Z_{RW}(Y, X_{SU(N)})$, which has been recently computed by Habegger and Thompson [6]. This gives an interesting non-trivial check of (9). For $b_1(Y) = 1$ there are important subtleties in the correspondence with Rozansky-Witten theory, which have been discussed in [12] when the gauge group is $SU(2)$.

3.2. Down to two dimensions

The connection to two-dimensional moduli problems appears when one considers product ruled surfaces $X = \mathbf{S}^2 \times \Sigma_g$. Anti-self dual connections on $X = \mathbf{S}^2 \times \Sigma_g$ with zero instanton number and $w_2(E) = [\mathbf{S}_2]$ are in one-to-one correspondence with flat connections on Σ_g with odd degree, which form a moduli space M_g. Donaldson invariants correspond to intersection pairings on M_g, which were determined by Thaddeus in [18]. The $\mathrm{Sp}(2g, \mathbf{Z})$-invariant cohomology ring of M_g is generated by cohomology classes α, β and γ, of degrees 2, 4 and 6, respectively. The relation between the intersection pairings and the Donaldson invariants of product ruled surfaces is given by:

$$\langle \alpha^m \beta^n \gamma^p \rangle_{M_g} = -D^{w_2(E)=[\mathbf{S}^2]}_{\mathbf{S}^2\times\Sigma_g}((2\Sigma_g)^m(-4x)^n\iota^p)\,, \tag{11}$$

where the overall minus sign is due to a different choice of orientation. On the other hand, we know the explicit expression for the Donaldson invariants, which is given in (7), and we can then rederive some important results about the intersection pairings [7]. The first thing that we can prove is the recursive relation for insertions of γ. One easily sees that

$$\frac{\partial}{\partial r} Z_g^{w_2(E)=[\mathbf{S}^2]} = 2g Z_{g-1}^{w_2(E)=[\mathbf{S}^2]}\,, \tag{12}$$

and this implies, using (11), that $\langle \alpha^m \beta^n \gamma^p \rangle_{M_g} = 2g \langle \alpha^m \beta^n \gamma^{p-1} \rangle_{M_{g-1}}$, which is precisely Thaddeus' recursive relation.

We now compute the intersection pairings $\langle \alpha^m \beta^n \rangle$. To do this, we use the expansion:

$$\csc z = \sum_{k=0}^{\infty} (-1)^{k+1}(2^{2k}-2)B_{2k}\frac{z^{2k-1}}{(2k)!}\,, \tag{13}$$

where B_{2k} are the Bernoulli numbers. We have to extract now the powers $s^m p^n$ from the generating function (7). Fortunately, only the leading terms contribute in the q-expansion of the modular forms. Taking into account the comparison factors from (11), and the dimensional constraint $2m+4n=6g-6$, one finds

$$\langle \alpha^m \beta^n \rangle = (-1)^g \frac{m!}{(m-g+1)!} 2^{2g-2}(2^{m-g+1}-2)B_{m-g+1}\,, \tag{14}$$

which is exactly Thaddeus' formula for the intersection pairings.

The relation between topological Yang-Mills theory on $\mathbf{S}^2 \times \Sigma_g$ and two-dimensional moduli problems is in fact more interesting, since the Donaldson invariants in the limiting chamber of small volume for Σ_g correspond to the Gromov-Witten invariants of M_g. We refer the reader to [7] for results in this direction.

4. Critical Behaviour

4.1. Superconformal points

When one considers topological quantum field theories in four dimensions with qualitative new physics, one also finds a completely different kind of predictions from a mathematical point of view. In [13] we studied a quantum field theory with a critical behaviour on a four-manifold X of simple type and with $b_2^+ > 1$, namely twisted $N=2$ supersymmetric QCD with gauge group $SU(2)$ and one massive hypermultiplet with mass m. It is known [1] that the low-energy theory becomes superconformal for a certain critical value of the mass m_*, and that the quantities that characterize the theory (like the masses of the BPS particles) have a scaling behaviour near the critical point. The theory has the same BRST operators than topological Yang-Mills theory, although mathematically it describes equivariant intersection theory on the moduli space of $SU(2)$ monopoles (see [9] for a review). Using the results of [15] and some additional input, one can compute the analog of the generating function (4) for this theory, which now depends on the extra parameter m. To write the result, we need the family of Seiberg-Witten elliptic curves for the $N_f=1$ theory [17], parameterized by $(u,m) \in \mathbf{C}^2$ and given by:

$$y^2 = x^2(x-u) + 2mx - 1\,. \tag{15}$$

The curve is easily put into standard form $y^2 = 4x^3 - g_2 x - g_3$, with $g_2(u;m) = \frac{4}{3}(u^2-6m)$, $g_3(u;m) = \frac{1}{27}(8u^3 - 72mu + 108)$, and discriminant $\Delta(u;m) = g_2^3 - 27g_3^2$. This discriminant is a cubic in u and has three roots $u_j(m)$, $j=1,2,3$. For generic, but fixed, values of m one of the periods of (15) goes to infinity as $u \to u_j$ while the other period, $\varpi_j \equiv \varpi(u_j(m);m)$ remains finite, and in fact is given by $(\varpi_j)^2 = g_2/(36 g_3)$. The generating function of the critical theory is given by a sum over the singular fibers of the Weierstrass family (15) and over the basic classes of X:

$$Z(p,S;m) = k\sum_{j=1}^{3}\left(\frac{g_2^3(u_j(m);m)}{\Delta'(u_j(m);m)}\right)^{\chi_h}(\varpi_j(m))^{7\chi_h-c_1^2} \cdot\sum_x SW(x)(-1)^{(\upsilon^2+\upsilon\cdot x)/2}\exp\left[2pu_j+S^2T_j-i\frac{(S,x)}{2\varpi_j}\right] \quad (16)$$

Here $\Delta'=\frac{\partial}{\partial u}\Delta$, $T_j=-\frac{1}{24}\big((\varpi_j)^{-2}-8u_j\big)$, and k is a nonvanishing constant, independent of p,S,m. The topological data of the manifold X enter through υ, which is an integral lifting of $w_2(X)$, the basic classes x and their SW invariants, and the numerical invariants χ_h and $c_1^2=2\chi+3\sigma$.

The critical behaviour of this theory is associated to the cusp singularity of (15) when $m_*=\frac{3}{2}$, $u_*=3$. Indeed, when $z=m-m_*\to 0$, two of the roots of $\Delta(u;m)=0$, call them $u_\pm(m)$, coincide, and the period $\varpi_\pm$ diverges as $z^{-1/4}$, while $g_2(u_\pm(m);m)\sim z$ and $\Delta'(u_\pm(m);m)\sim\delta u_\pm\sim z^{3/2}$. At the third singularity all the various factors in (16) are given by nonvanishing analytic series in z, but, evidentally, the contributions from $u_\pm(m)$ contain factors which are diverging or vanishing as $z\to 0$. What can we say about the behaviour of the complete function $Z(p,S,m)$ as $z\to 0$? For physical reasons, we do not expect any divergence in the correlation functions: there are no infrared divergences in spacetime, since X is compact, and since the moduli space of vacua is also compact for $b_2^+>1$, we do not expect any divergence from the target geometry. In more physical terms, since we are working at finite volume, correlation functions should still be finite near the critical point. This implies that $Z(p,S,m)$ *must be a regular analytic function of* z *near* $z=0$.

4.2. Mathematical implications

Let's now see what are the mathematical implications of this fact. We first define the "twisted" Seiberg-Witten series as follows.

$$SW_X^{w_2(X)}(z):=\sum_x(-1)^{\frac{\upsilon^2+\upsilon\cdot x}{2}}SW(x)\mathrm{e}^{zx}\,. \quad (17)$$

This is a finite sum [20]. Notice that a change of lifting changes (17) only by a sign. We now make the key definition:

Definition 4.1. *Let X be a compact, oriented 4-manifold of simple type with $b_2^+>1$. We say that "X is SST" if $SW_X^{w_2(X)}(z)$ has a zero at $z=0$ of order $\geq\chi_h-c_1^2-3$.*

One has the following result, whose proof can be found in [13]:

Theorem 4.2. *If X is SST, then $Z(p,S,m)$ is regular at $m=m_*$.*

It is interesting to notice that, for most of the SST manifolds, the contributions to $Z(p,S,m)$ from the colliding singularities $u_\pm$ go to infinity separately as $z\to 0$, but when we sum the two contributions (and we are forced to do that because the manifold is compact) the infinities cancel and we get a finite result.

All the simple type, four-manifolds we are aware of are in fact SST. Using the definition, one can check that SST manifolds satisfy the following remarkable property, which gives a relation between SW invariants and the problem of geography for four-manifolds:

Theorem 4.3. (Generalized Noether inequality) *Let X be SST. If X has B distinct basic classes and $B > 0$, then*

$$B \geq \left[\frac{\chi_h - c_1^2}{2} \right].$$

In particular, $c_1^2 \geq \chi_h - 2B - 1$.

Although being SST is only a sufficient condition for $Z(p, S, m)$ to be finite, the analysis of [13] leads naturally to the following conjecture:

Conjecture 4.4. *All compact four-manifolds of simple type and with $b_2^+ > 1$ are SST.*

In fact, Feehan, Kronheimer, Lenness and Mrowka have proved in [4] that the above conjecture is true under some mild assumptions, by using the $PU(2)$ monopole equations.

5. Conclusions and Open Problems

I think it is fair to say that we have a rather complete understanding of the relation between Donaldson theory and TQFT in four dimensions. There are however a few open problems that deserve investigation, both in physics and in mathematics:

1) There are many predictions from TQFT that should still be checked from the mathematical side, and I think that this is interesting by itself. For example, the results (6) and (7), as well as the wall-crossing formula of [11] for nonsimply connected manifolds, may be obtained by generalizing [5]. The extension to $SU(N)$ seems still out of reach mathematically, but it would be extremely interesting to check (9) in some detail. One can invert the logic and say that (9) gives a good reason *not* to study the $SU(N)$ Donaldson invariants, since it shows that these generalizations have the same topological information than the SW invariants!

2) In a different direction, it would be interesting to study the theory for four-manifolds with $b_2^+ = 0$. A motivation to do that would be to shed some four-dimensional light (via compactification on a circle) on the relation between the Casson invariant and the three-dimensional Seiberg-Witten invariant for homology three-spheres.

3) Finally, the twisted counterparts of superconformal field theories in four dimensions certainly deserve closer scrutiny.

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