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TORSION OF A COMPOSITE BEAM OF RECTANGULAR CROSS-SECTION CONSISTING OF n ISOTROPIC MEDIA WITH INTERFACES PARALLEL TO ONE OF THE SIDES

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INTRODUCTION

1. The torsion problem of beams of polygonal and other cros-sections has been studied by several authors as KÖTTER (1), TREFFTZ (2), SETH (3), ARUTYUNYAN (4), ABRAMIAN and BABLOIAN (5), DEUTSCH (6) and others. The methods used can be broadly classified into two categories; (a) use of conformal transformation which involves a great deal of manipulational complexity in approximating and (b) use of intuition in forming the solution which does not seem to be much effective in case of a composite medium of high order. In this paper we have presented an approach that involves use of Green's function and Fourier expansion and can give in a systematic way the exact solution of the torsion problem of a composite beam of rectangular cross-section consisting of n (any number) of isotropic media with interfaces parallel to one the sides.

ANALYSIS

2. The origin is chosen to be situated at the centre of the pth medium $(p < n)$ with the axes of x and y in the plane of the cross-section parallel to the edges as shown in fig. 1. The division lines between the different media are parallel to the x-axis. The axis of the twist is in the direction of OZ which is perpendicular to the section. The dimensions of the section are shown in fig. 1.

Our problem consists in determining a torsion function $\psi(x, y)$ that satisfies the equation

$$\nabla^2\psi = -2\,, \quad \left(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \tag{2.1}$$

where $\psi = 0$ on the boundary.

The torsional rigidity D is given by

(2.2) $$D = 2\sum_{k=1}^{n} \mu_k \iint_{S_k} P_k(x, y)\, dx\, dy$$

where μ_k is the modulus of rigidity of the material of region S_k; and $\psi(x, y) = P_k(x, y)$ in S_k.

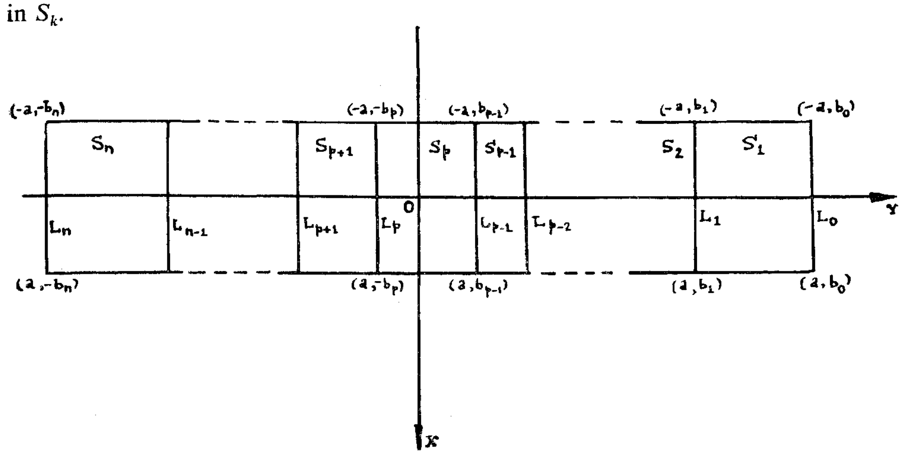

Fig. 1.

The regions S_k are given by

$$S_k = \left(-a \leqq x \leqq a,\ b_k \leqq y \leqq b_{k-1}\right), \quad k = 1, 2, 3, \ldots, p - 1\ ;$$

$$S_p = \left(-a \leqq x \leqq a,\ -b_p \leqq y \leqq b_{p-1}\right);$$

$$S_k = \left(-a \leqq x \leqq a,\ -b_k \leqq y \leqq -b_{k-1}\right), \quad k = p + 1, p + 2, \ldots, n\ .$$

The stress components are given by

$$\tau_{zx} = \alpha\mu_k \frac{\partial P_k}{\partial y}, \quad \tau_{yz} = -\alpha\mu_k \frac{\partial P_k}{\partial x} \quad \text{in} \quad S_k, \quad k = 1, 2, 3, \ldots, n$$

where α is the constant twist per unit length. Continuity of z-displacement component w and stress component τ_{yz} on the interface L_k yield the following boundary conditions

(2.3) $$P_k = K_{k+1} P_{k+1},\ K_{k+1} = \frac{\mu_{k+1}}{\mu_k}$$

and

$$\frac{\partial P_k}{\partial y} = \frac{\partial P_{k+1}}{\partial y}$$

on L_k, $k = 1, 2, 3, \ldots, n - 1$.

In (2.3) L_k is given by $y = b_k$ for $k = 1, 2, 3, \ldots, p - 1$; and by $y = -b_k$ for $k = p, p + 1, \ldots, n - 1$.

P_1 vanishes on the remaining boundaries of S_1, P_n vanishes on the remaining boundaries of S_n and P_k vanishes on $x = \pm a$ where $k = 2, 3, \ldots, n - 1$.

We apply the method of Green's function for built-up bodies. Let G_k be the Green's function for the medium S_k with the conditions

$$\frac{\partial G_k}{\partial n_k} = 0 \quad \text{on} \quad y = b_k \quad \text{where} \quad k = 1, 2, 3, \ldots, p - 1\,;$$

$$\frac{\partial G_k}{\partial n_k} = 0 \quad \text{on} \quad y = -b_k \quad \text{where} \quad k = p, p + 1, \ldots, n - 1\,;$$

$$\frac{\partial G_k}{\partial n_k} = 0 \quad \text{on} \quad y = b_{k-1} \quad \text{where} \quad k = 2, 3, \ldots, p\,;$$

$$\frac{\partial G_k}{\partial n_k} = 0 \quad \text{on} \quad y = -b_{k-1} \quad \text{where} \quad k = p + 1, p + 2, \ldots, n\,;$$

$$G_k = 0\,, \quad k = 2, 3, 4, \ldots, n - 1$$

on the boundaries $x = \pm a$;

$$G_1 = 0 \quad \text{on all boundaries of } S_1 \text{ except } L_1\,,$$

$$G_n = 0 \quad \text{on all boundaries of } S_n \text{ except } L_{n-1}\,.$$

Here $\overrightarrow{\delta n_k}$ is the element of normal to the bounding surface of S_k drawn away from the region.

The Green's function $G(x, y \mid u, v)$ due to a source at (u, v) satisfies

$$\nabla^2 G(x, y \mid u, v) = -4\pi\, \delta(x - u)\, \delta(y - v)\,.$$

By Green's identity we have

$$\text{(2.4)} \qquad \iint (G\, \nabla^2 \psi - \psi\, \nabla^2 G)\, \mathrm{d}u\, \mathrm{d}v = \int \left(G\, \frac{\partial \psi}{\partial n} - \psi\, \frac{\partial G}{\partial n} \right) \mathrm{d}s$$

$\mathrm{d}s$ being $\mathrm{d}u$ on v = constant and $\mathrm{d}v$ on u = constant.

In (2.4) the surface integral is to be taken over all the source points, the line integral over the entire boundary line enclosing the simply-connected region considered. Applying (2.4) to each of the regions S_k, $k = 1, 2, 3, \ldots, n$; and using

$$\frac{\partial}{\partial n_k} = - \frac{\partial}{\partial n_{k-1}} = \frac{\partial}{\partial y} \quad \text{on} \quad y = b_{k-1}, \quad k = 2, 3, \ldots, p$$

and on $y = -b_{k-1}$, $k = p + 1, p + 2, \ldots, n$ we get

$$(2.5) \quad -2\int_{b_1}^{b_0}\int_{-a}^{a} G_1^{(1)}\, du\, dv + 4\pi P_1(x, y) = - \int_{-a}^{a} (G_1^{(1)})_{v=b_1} \left(\frac{\partial P_1}{\partial v}\right)_{v=b_1} du\,,$$

$$(2.6) \quad -2\int_{b_k}^{b_{k-1}}\int_{-a}^{a} G_k^{(k)}\, du\, dv + 4\pi P_k(x, y) = - \int_{-a}^{a} (G_k^{(k)})_{v=b_k} \left(\frac{\partial P_k}{\partial v}\right)_{v=b_{k-1}} du\, +$$

$$+ \int_{-a}^{a} (G_k^{(k)})_{v=b_{k-1}} \left(\frac{\partial P_k}{\partial v}\right)_{v=b_{k-1}} du$$

where $k = 2, 3, 4, \ldots, p - 1$;

$$(2.7) \quad -2\int_{-b_p}^{b_{p-1}}\int_{-a}^{a} G_p^{(p)}\, du\, dv + 4\pi P_p(x, y) = \int_{-a}^{a} (G_p^{(p)})_{v=b_{p-1}} \left(\frac{\partial P_p}{\partial v}\right)_{v=b_{p-1}} du\, -$$

$$- \int_{-a}^{a} (G_p^{(p)})_{v=-b_p} \left(\frac{\partial P_p}{\partial v}\right)_{v=-b_p} du\,,$$

$$(2.8) \quad -2\int_{-b_{k+1}}^{-b_k}\int_{-a}^{a} G_{k+1}^{(k+1)}\, du\, dv + 4\pi P_{k+1}(x, y) =$$

$$= \int_{-a}^{a} (G_{k+1}^{(k+1)})_{v=-b_k} \left(\frac{\partial P_{k+1}}{\partial v}\right)_{v=-b_k} du - \int_{-a}^{a} (G_{k+1}^{(k+1)})_{v=-b_{k+1}} \left(\frac{\partial P_{k+1}}{\partial v}\right)_{v=-b_{k+1}} du\,,$$

where $k = p,\ p + 1, \ldots, n - 2$; and

$$(2.9) \quad -2\int_{-b_n}^{-b_{n-1}}\int_{-a}^{a} G_n^{(n)}\, du\, dv + 4\pi P_n(x, y) = \int_{-a}^{a} (G_n^{(n)})_{v=-b_{n-1}} \left(\frac{\partial P_n}{\partial v}\right)_{v=-b_{n-1}} du$$

where the superscript k indicates the observation point (x, y) is in S_k. The boundary conditions (2.3) with (2.5)–(2.9) give us the relations

$$(2.10) \quad -2\int_{b_1}^{b_0}\int_{-a}^{a} (G_1)_{y=b_1}\, du\, dv + 2K_2 \int_{b_2}^{b_1}\int_{-a}^{a} (G_2)_{y=b_1}\, du\, dv =$$

$$= -\int_{-a}^{a}\left[(G_1)_{\substack{v=b_1\\y=b_1}} + K_2(G_2)_{\substack{v=b_1\\y=b_1}}\right]\left(\frac{\partial P_2}{\partial v}\right)_{v=b_1} du + K_2\int_{-a}^{a}(G_2)_{\substack{v=b_2\\y=b_1}}\left(\frac{\partial P_2}{\partial v}\right)_{=bv_2} du\,,$$

(2.11) $$-2\int_{b_k}^{b_{k-1}}\int_{-a}^{a}(G_k)_{y=b_k}\,du\,dv + 2K_{k+1}\int_{b_{k+1}}^{b_k}\int_{-a}^{a}(G_{k+1})_{y=b_k}\,du\,dv =$$

$$= -\int_{-a}^{a}\left[K_{k+1}(G_{k+1})_{\substack{v=b_k\\y=b_k}} + (G_k)_{\substack{v=b_k\\y=b_k}}\right]\left(\frac{\partial P_{k+1}}{\partial v}\right)_{v=b_k} du +$$

$$+\int_{-a}^{a}(G_k)_{\substack{v=b_{k-1}\\y=b_k}}\left(\frac{\partial P_k}{\partial v}\right)_{v=b_{k-1}} du + K_{k+1}\int_{-a}^{a}(G_{k+1})_{\substack{v=b_{k+1}\\y=b_k}}\left(\frac{\partial P_{k+1}}{\partial v}\right)_{v=b_{k+1}} du$$

where $k = 2, 3, 4, \ldots, p-2$;

(2.12) $$2\int_{b_{p-1}}^{b_{p-2}}\int_{-a}^{a}(G_{p-1})_{y=b_{p-1}}\,du\,dv + 2K_p\int_{-b_p}^{b_{p-1}}\int_{-a}^{a}(G_p)_{y=b_{p-1}}\,du\,dv =$$

$$= -\int_{-a}^{a}\left[K_p(G_p)_{\substack{v=b_{p-1}\\y=b_{p-1}}} + (G_{p-1})_{\substack{v=b_{p-1}\\y=b_{p-1}}}\right]\left(\frac{\partial P_p}{\partial v}\right)_{v=b_{p-1}} du +$$

$$+\int_{-a}^{a}(G_{p-1})_{\substack{v=b_{p-2}\\y=b_{p-1}}}\left(\frac{\partial P_{p-1}}{\partial v}\right)_{v=b_{p-2}} du + K_p\int_{-a}^{a}(G_p)_{\substack{v=-b_p\\y=b_{p-1}}}\left(\frac{\partial P_p}{\partial v}\right)_{v=-b_p} du\,,$$

(2.13) $$-2\int_{-b_p}^{b_{p-1}}\int_{-a}^{a}(G_p)_{y=-b_p}\,du\,dv + 2K_{p+1}\int_{-b_{p+1}}^{-b_p}\int_{-a}^{a}(G_{p+1})_{y=-b_p}\,du\,dv =$$

$$= -\int_{-a}^{a}\left[K_{p+1}(G_{p+1})_{\substack{v=-b_p\\y=-b_p}} + (G_p)_{\substack{v=-b_p\\y=-b_p}}\right]\left(\frac{\partial P_{p+1}}{\partial v}\right)_{v=-b_p} du +$$

$$+\int_{-a}^{a}(G_p)_{\substack{v=b_{p-1}\\y=-b_p}}\left(\frac{\partial P_p}{\partial v}\right)_{v=b_{p-1}} du + K_{p-1}\int_{-a}^{a}(G_{p+1})_{\substack{v=-b_{p+1}\\y=-b_p}}\left(\frac{\partial P_{p+1}}{\partial v}\right)_{v=-b_{p+1}} du\,,$$

(2.14) $$-2\int_{-b_{k+1}}^{-b_k}\int_{-a}^{a}(G_{k+1})_{y=-b_{k+1}}\,du\,dv + 2K_{k+2}\int_{-b_{k+2}}^{-b_{k+1}}\int_{-a}^{a}(G_{k+2})_{y=-b_{k+1}}\,du\,dv =$$

$$= -\int_{-a}^{a}\left[K_{k+2}(G_{k+2})_{\substack{v=-b_{k+2}\\y=-b_{k+1}}} + (G_{k+1})_{\substack{v=-b_{k+1}\\y=-b_{k+1}}}\right]\left(\frac{\partial P_{k+2}}{\partial v}\right)_{v=-b_{k+1}} du +$$

$$+\int_{-a}^{a}(G_{k+1})_{\substack{v=-b_k\\y=-b_{k+1}}}\left(\frac{\partial P_{k+1}}{\partial v}\right)_{v=-b_k} du + K_{k+2}\int_{-a}^{a}(G_{k+2})_{\substack{v=-b_{k+2}\\y=-b_{k+1}}}\left(\frac{\partial P_{k+2}}{\partial v}\right)_{v=-b_{k+2}} du$$

where $k = p, p+1, \ldots, n-3$;

$$(2.15)\quad -2\int_{-b_{n-1}}^{-b_{n-2}}\int_{-a}^{a}(G_{n-1})_{y=-b_{n-1}}\,du\,dv + 2K_n\int_{-b_n}^{-b_{n-1}}\int_{-a}^{a}(G_n)_{y=-b_{n-1}}\,du\,dv =$$

$$= -\int_{-a}^{a}\left[K_n(G_n)_{\substack{v=-b_{n-1}\\ y=-b_{n-1}}} + (G_{n-1})_{\substack{v=-b_{n-1}\\ y=-b_{n-1}}}\right]\left(\frac{\partial P_{n-1}}{\partial v}\right)_{v=-b_{n-1}} du \, +$$

$$+\int_{-a}^{a}(G_{n-1})_{\substack{v=-b_{n-2}\\ y=-b_{n-2}}}\left(\frac{\partial P_{n-1}}{\partial v}\right)_{v=-b_{n-2}} du\,.$$

Following INCE (7) (also c.f. MORSE and FESHBACK (8)) we construct, m having positive integral values,

$$(2.16)\quad G_1(x, y \mid u, v) =$$

$$= \sum_m \frac{8}{m}\,\frac{\sin\dfrac{m\pi(a-x)}{2a}\sin\dfrac{m\pi(a-u)}{2a}}{\cosh\dfrac{m(b_0-b_1)\pi}{2a}} \times \begin{cases} \cosh\dfrac{m\pi(y-b_1)}{2a}\sinh\dfrac{m\pi(b_0-v)}{2a}, & (y<v) \\[2ex] \cosh\dfrac{m\pi(y-b_1)}{2a}\sinh\dfrac{m\pi(b_0-y)}{2a}, & (y>v) \end{cases}$$

$$(2.17)\quad G_k(x, y \mid u, v) =$$

$$= \sum_m \frac{8}{m}\,\frac{\sin\dfrac{m\pi(a-x)}{2a}\sin\dfrac{m\pi(a-u)}{2a}}{\sinh\dfrac{m\pi(b_{k-1}-b_k)}{2a}} \times \begin{cases} \cosh\dfrac{m\pi(y-b_k)}{2a}\cosh\dfrac{m\pi(b_{k-1}-v)}{2a}, & (y<v) \\[2ex] \cosh\dfrac{m\pi(v-b_k)}{2a}\cosh\dfrac{m\pi(b_{k-1}-y)}{2a}, & (y>v) \end{cases}$$

$k = 2, 3, 4, \ldots, p-1$;

$$(2.18)\quad G_p(x, y \mid u, v) =$$

$$= \sum_m \frac{8}{m}\,\frac{\sin\dfrac{m\pi(a-x)}{2a}\sin\dfrac{m\pi(a-u)}{2a}}{\sinh\dfrac{m\pi(b_{p-1}+b_p)}{2a}} \times \begin{cases} \cosh\dfrac{m\pi(y+b_p)}{2a}\cosh\dfrac{m\pi(b_{p-1}-v)}{2a}, & (y<v) \\[2ex] \cosh\dfrac{m\pi(v+b_p)}{2a}\cosh\dfrac{m\pi(b_{p-1}-y)}{2a}, & (y<v) \end{cases}$$

$$(2.19)\quad G_k(x, y \mid u, v) = \sum_m \frac{8}{m} \frac{\sin \dfrac{m\pi(a-x)}{2a} \sin \dfrac{m\pi(a-u)}{2a}}{\sinh \dfrac{m\pi(b_k - b_{k-1})}{2a}} \times \begin{cases} \cosh \dfrac{m\pi(y+b_k)}{2a} \cosh \dfrac{m\pi(b_{k-1}+v)}{2a}, & (y < v) \\ \cosh \dfrac{m\pi(v+b_k)}{2a} \cosh \dfrac{m\pi(b_{k-1}+y)}{2a}, & (y > v) \end{cases}$$

for $k = p+1, p+2, \ldots, n-1$;
$\times$ stands for "multiplied by";

$$(2.20)\quad G_n(x, y \mid u, v) = \sum_m \frac{8}{m} \frac{\sin \dfrac{m\pi(a-x)}{2a} \sin \dfrac{m\pi(a-u)}{2a}}{\cosh \dfrac{m\pi(b_n - b_{n-1})}{2a}} \begin{cases} \cosh \dfrac{m\pi(v+b_{n-1})}{2a} \sinh \dfrac{m\pi(y+b_n)}{2a}, & (y < v) \\ \cosh \dfrac{m\pi(y+b_{n-1})}{2a} \sinh \dfrac{m\pi(v+b_n)}{2a}. & (y > v) \end{cases}$$

Inserting (2.16)–(2.20) in (2.10)–(2.15) we get the following results on simplification: (this includes multiplication of both sides by $\sin[m\pi(a-x)/2a]$ and integration with respect to x between $x = -a$ to $x = a$):

$$(2.21)\quad \frac{8a^2}{m^2\pi^2}(1 - \cos m\pi) \operatorname{sech} \frac{m\pi(b_0 - b_1)}{2a} + L_m(2) = B_2'^m(0, 1, 2)\, F_m(P_2, b_1) + D^m_{k,2}(1, 2)\, F_m(P_2, b_2),$$

$$(2.22)\quad B^m_{k,t-2}(t-4, t-3, t-2)\, F_m(P_{t-2}, b_{t-3}) + C_m(t-4, t-3)\, F_m(P_{t-3}, b_{t-4}) + D^m_{k,t-2}(t-3, t-2)\, F_m(P_{t-2}, b_{t-2}) = L_m(t-2),$$

where $t = 5, 6, \ldots, p+1$,

$$(2.23)\quad B^m_{k,p}(p-2, p-1, -p)\, F_m(P_p, b_{p-1}) + C_m(p-2, p-1)\, F_m(P_{p-1}, b_{p-2}) + D^m_{k,p}(p-1, -p)\, F_m(P_p, -b_p) = L_m(p),$$

$$(2.24)\quad B^m_{k,p+1}\{p-1, -p, -(p+1)\}\, F_m(P_{p+1}, -b_p) + C_m(p-1, -p)\, F_m(P_p, b_{p-1}) + D^m_{k,p+1}\{-p, -(p+1)\}\, F_m(P_{p+1}, -b_{p+1}) = L_m(p+1),$$

$$(2.25)\quad B^m_{k,t+2}\{-t, -(t+1), -(t+2)\}\, F_m(P_{t+2}, -b_{t+1}) + C_m\{-t, -(t+1)\} F_m(P_{r+1}, b_{-t}) + D^m_{k,t+2}\{-(t+1), -(t+2)\}\, F_m(P_{t+2}, -b_{t+2}) = L_m(t+2),$$

where $t = p, p+1, \ldots, n-3$;

$$(2.26) \qquad -\frac{8a^2}{m^2\pi^2}(1-\cos m\pi)\operatorname{sech}\frac{m\pi(b_n-b_{n-1})}{2a} + L_m(n) =$$

$$= C_m(n-1, n-2)\, F_m(P_{n-1}, -b_{n-2}) + B''^m_{k,n} F_m(P_{n-1}, -b_{n-1}),$$

where,

$$F_m(P_p, b_{p-1}) = \int_{-a}^{a} \left(\frac{\partial P_p}{\partial v}\right)_{v=b_{p-1}} \sin\frac{m\pi(a-u)}{2a}\, du\,,$$

$$L_m(p) = \frac{8a^2}{m^2\pi^2}(1-\cos m\pi)(K_p - 1)\,,$$

$$C_m(p-2, p-1) = \operatorname{cosech}\frac{m\pi(b_{p-2}-b_{p-1})}{2a},$$

$$B^m_{k,p}(p-2, p-1, -p) = -\left[K_p \coth\frac{m\pi(b_{p-1}+b_p)}{2a} + \coth\frac{m\pi(b_{p-2}-b_{p-1})}{2a}\right],$$

$$B'^m_2(0, 1, 2) = -\left[K_2 \coth\frac{m\pi(b_1-b_2)}{2a} + \tanh\frac{m\pi(b_0-b_1)}{2a}\right],$$

$$B''^m_{k,n} = -\left[K_n \tanh\frac{m\pi(b_n-b_{n-1})}{2a} + \coth\frac{m\pi(b_{n-1}-b_{n-2})}{2a}\right],$$

$$D^m_{k,p}(p-1, p) = K_p \operatorname{cosech}\frac{m\pi(b_{p-1}+b_p)}{2a}.$$

To these are added the following relations obtained from (2.3)

$$(2.27) \qquad F_m(P_k, b_{k-1}) = F_m(P_{k-1}, b_{k-1}) \quad \text{where} \quad k = 2, 3, 4, \ldots, p\,;$$

$$F_m(P_k, -b_k) = F_m(P_{k+1}, -b_k) \quad \text{where} \quad k = p, p+1, \ldots, n-1\,.$$

From (2.5)–(2.9) we see that once the F_m's are obtained the P_k's are determined completely. As an illustration we can see from (2.7)

$$P_p(x, y) = \sum_m \frac{16a^2}{m^3\pi^3}(1-\cos m\pi)\sin\frac{m\pi(a-x)}{2a} \times$$

$$\times \left\{1 - \cosh\frac{m\pi(y-b_{p-1})}{2a} - \frac{\sinh\dfrac{m\pi(b_p+b_{p-1})}{2a}\cosh\dfrac{m\pi(b_{p-2}-y)}{2a}}{\sinh\dfrac{m\pi(b_{p-2}-b_{p-1})}{2a}}\right\} +$$

$$+ \sum_m \frac{2}{m\pi} \operatorname{cosech} \frac{m\pi(b_{p-1} + b_p)}{2a} \sin \frac{m\pi(a - x)}{2a} \times$$

$$\times \left\{ \cosh \frac{m\pi(y + b_p)}{2a} F_m(P_p, b_{p-1}) - \cosh \frac{m\pi(b_{p-1} - y)}{2a} F_m(P_p, -b_p) \right\}.$$

Now the number of F_m's is $2n - 2$. From (2.21)–(2.26) and (2.27) we get $(2n - 2)$ relations in total which determine the $(2n - 2)$ F_m's completely. It can easily be seen that all the F_m's are equal to zero for m an even positive integer, and accordingly all the P_k's are zeros then. So in the expressions for the P_k's the summation is to be taken over odd positive integral values of m only. That the infinite sums giving the P_k's are uniformly convergent can be easily seen. It is checked that the results deduced from the above in case of a single isotropic beam of rectangular cross-section agree with those obtained by SOKOLNIKOFF (9).

Example. For $n = 3, \mu_1 = \mu_3, k = k' = \mu_2/\mu_1, b_1 = b_2 = b, b_0 = b_3 = b + b'$, the above procedure yields

$$P_1(x, y) = \frac{32a^2}{\pi_3} \sum_{n=0}^{\infty} H_n(x) \left[1 - \frac{1}{t'_n} \cosh \frac{(2n + 1)\pi(y - b)}{2a} - \right.$$

$$\left. - \frac{A_n}{t'_n} \sinh \frac{(2n + 1)\pi(b + b' - y)}{2a} \right] \quad \text{for} \quad -a \leqq x \leqq a, b \leqq y \leqq b + b';$$

$$P_2(x, y) = \frac{32a^2}{\pi^3} \sum_{n=0}^{\infty} H_n(x) \left[1 + \frac{2A_n}{s_n} \cosh \frac{(2n + 1)\pi y}{2a} \cosh \frac{(2n + 1)\pi b}{2a} \right]$$

$$\text{for} \quad -a \leqq x \leqq a, -b \leqq y \leqq b;$$

$$P_3(x, y) = P_1(x, -y) \quad \text{for} \quad -a \leqq x \leqq a, -(b + b') \leqq y \leqq -b;$$

where

$$H_n(x) = \frac{(-1)^n}{(2n + 1)^3} \cos \frac{(2n + 1)\pi x}{2a}, \quad t_n = \cosh(2n + 1)\pi t, \quad s_n = \sinh(2n + 1)\pi t,$$

$$t = \frac{b}{a}, \quad t'_n = \cosh(2n + 1)\pi t', \quad s'_n = \sinh(2n + 1)\pi t', \quad t' = \frac{b'}{2a},$$

$$A_n = \frac{s_n[(1 - k)t'_n - 1]}{s'_n s_n + k t'_n(1 + t_n)}.$$

Let $t' = t\lambda$ so that $b' = 2b\lambda$. We observe then the following:

Case (a): $t = 1$, $k = 1{\cdot}2$. For $\lambda = 1$, the maximum stress occurs at $(\pm a, 0)$. It is seen that the maximum stress, in fact, occurs at $(\pm a, 0)$ for $\lambda > 0$.

For $t = 1$, $k = 1$, $\lambda = 1$ the maximum stress occurs at $(\pm a, 0)$.

Case (b): $t = 1$, $k = \frac{5}{6}$.
Stress is maximum at $\{0, \pm(b + b')\}$ for $\lambda \geqq 1$. But for $\lambda = 0{\cdot}2$ maximum stress occurs at $(\pm a, 0)$. Therefore there is a λ satisfying $0{\cdot}2 < \lambda < 1$ for which stress can be maximum at four points of the cross-section as in the case of a single isotropic beam of square cross-section.

Case (c): $t = 0{\cdot}5$, $k = 1{\cdot}2$.
For $\lambda = 1$ the maximum stress occurs at $\{0, \pm(b + b')\}$.

Case (d): $t = 0{\cdot}5$, $k = \frac{5}{6}$.
Stress is maximum at $\{0, \pm(b + b')\}$ for $\lambda \geqq 1$. For $\lambda = 0{\cdot}2$, the maximum stress occurs at $(\pm a, 0)$.

From Case (a) and Case (b) it can be concluded that the position of occurrence of maximum stress depends on the dimensions as well as on the ratio of rigidity moduli.

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Souhrn

TORSE SPŘAŽENÉHO NOSNÍKU OBDÉLNÍKOVÉHO PRŮŘEZU SLOŽENÉHO Z *n* RŮZNÝCH ISOTROPNÍCH MATERIÁLŮ SE STYČNÝMI PLOCHAMI ROVNOBĚŽNÝMI S JEDNOU STRANOU

BASUDEV GHOSH

V práci je řešen problém torse spřaženého nosníku obdélníkového průřezu složeného z n různých isotropních materiálů se styčnými plochami rovnoběžnými s jednou stranou. Postup řešení se zakládá na použití Greenovy funkce pro složené těleso a Fourierovy sinové transformace. Uvažuje se příklad spřaženého nosníku složeného ze tří materiálů a sleduje se závislost polohy bodu největšího namáhání na poměru měr tuhosti.

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