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Torsional Fatigue and Bending Fatigue of Electropolished Low Carbon Steel Specimens*

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Rotating bending and torsional fatigue tests of plain and notched steel specimens with electropolished surfaces were carried out.

The main results obtained are as follows.

(1) The maximum shear stresses in the crack initiation limit of torsional fatigue are determined only by the stress gradient χ independently of the notch depth as in the case of bending fatigue.

(2) When the values of χ are equal, the maximum shear stress in the crack initiation limit of torsional fatigue is about 1.2 times that of bending fatigue.

(3) The surface states of notch roots after 10^7 cycles of the limit stress for crack initiation in torsional fatigue are similar to each other irrespective of the maximum stress repeated on the notch root, as in the case of bending fatigue.

The fatigue damages at the crack initiation limit under torsional stresses are severer than the ones under bending stresses.

(4) The mechanism of non-propagation of a crack in torsional notched specimens is different from that in bending notched specimens.

1. Introduction

Fatigue under combined stresses has been studied by many workers[1].

The aim of the present study is to investigate the effect of the normal stress on the plane on which the shear stress is maximum and to explain the mechanical factor which controls the fatigue damage of notched specimens under combined stresses. Taking the rotating bending and the torsional fatigue of notched and plain steel specimens as the typical example of the fatigue of ductile materials, the fatigue damages in both cases were compared using an optical microscope. All specimens were electropolished in order to observe the progress of fatigue damage.

Since the fatigue limit is the limit stress for fracture, it appears that the damage due to the stress of the fatigue limit might be similar in both cases of bending and torsion, but actually the damages in both cases at the limit of fracture are different from each other. This fact seems to have fundamental importance when the fatigue under combined stresses or mean stresses is treated.

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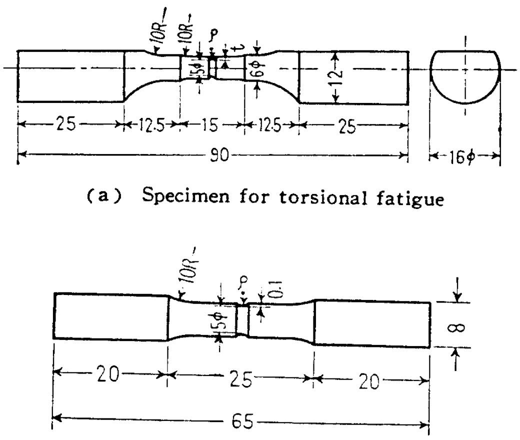
2. Materials, specimens and the testing method

The materials used are S 10 C rolled round bars (about 22 mm in diameter). The specimens were turned after annealing the bars for about 60 minutes at the temperature of 904°C. The chemical composition and the mechanical properties of these annealed bars are shown in Table 1.

The forms of the specimens are shown in Fig. 1. The combinations of the minimum diameter d, the notch depth t and the root radius of notch ρ and the stress concentration factor α (after Neuber's law) are shown in Table 2. The forms of the notch are 60° V-grooves (i.e. when ρ is large, the forms become circular arc). The plain specimens were polished by emery paper and the notched ones by emery powder and then the surface layers of those specimens were electropolished to the depth of about 10 microns. The electrolytic solution used consists of phosphoric acid (1 000 g), gelatine (20 g) and oxalic acid (20 g).

The machines used are the 4 kg-m Schenk-type torsional fatigue testing machine (being a constant deflection type, it was adjusted manually to keep constant stress. The rate of stress repetitions was nearly 3 600 cycles per minute) and the 1.5 kg-m Ono-type rotary bending fatigue testing machine (the rate of stress repetitions was 3 000 cycles per minute).

For the specimens which endured the stress repetitions of more than 10^7 cycles, the existence of the non-propagating crack was inspected by observing directly the surface of the root of the notch using an optical microscope (×400). The existence of the non-propagating crack in the electropolished specimens can be easily inspected by this method (Figs. 11, 12).



(a) Specimen for torsional fatigue

(b) Specimen for rotaing bending fatigue

Fig. 1 Specimens

3. Results and discussion

3·1 Fatigue strength and crack strength

The values of fatigue strength τ_{w1}, σ_{w1} (the limit stress for crack initiation) and of crack strength τ_{w2}, σ_{w2} (the limit stress for fracture) are shown in Table 2. The stresses are all nominal ones, calculated by the formulas $\tau=16M_{tor}/(\pi d^3)$ and $\sigma=32M_{bend}/(\pi d^3)$ (M_{tor}: torque, M_{bend}: bending moment, d: diameter of the minimum section). In

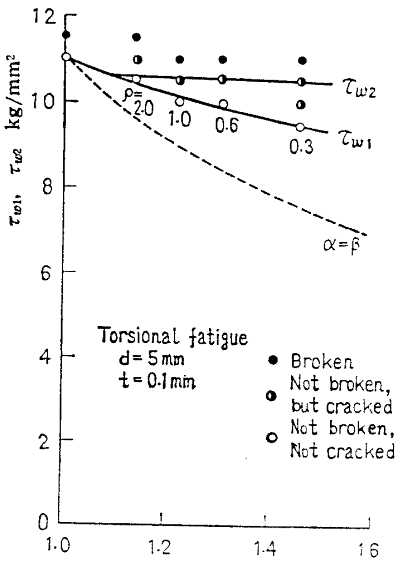


Fig. 2 Torsional fatigue

Table 1 Chemical composition and mechanical properties

Chemical composition %								Mechanical properties kg/mm², %			
C	Si	Mn	P	S	Cu	Al	Ni+Cr	σ_{sl}	σ_B	σ_T	ψ
0.13	0.22	0.39	0.013	0.022	0.09	0.010	0.10	20.7	38.0	78.7	67.7

σ_{sl}: lower yield point, σ_B: tensile strength,
σ_T: actual stress at fracture,
ψ: area contraction

Table 2 Dimensions of specimens, fatigue strengths (τ_{w1} or σ_{w1}) and crack strengths (τ_{w2} or σ_{w2})

d mm	t mm	ρ mm	Torsional fatigue				Bending fatigue			
			α_t	τ_{w1}	τ_{w2}	$\alpha_t\tau_{w1}$	α_b	σ_{w1}	σ_{w2}	$\alpha_b\sigma_{w1}$
5.0	—	∞ & 100	1.00	11.0	—	11.0	1.00	19.0	—	19.0
	1.0	2.0	1.14	10.5	11.0	12.0	1.26	16.0	—	20.2
		1.0	1.22	10.0	10.5	12.2	—	—	—	—
		0.6	1.30	10.0	10.5	13.0	1.59	14.0	14.5	22.3
		0.3	1.45	9.5	10.5	13.8	1.90	13.0	14.5	24.7
	0.5	2.0	1.16	10.5	—	12.2				
		1.0	1.28	10.0	10.5	12.8				
		0.6	1.40	9.5	10.5	13.3				
		0.3	1.64	8.5	10.5	13.9				
	3.0	2.0	1.17	10.0	—	11.7				
		1.0	1.30	9.5	10.0	12.3				
		0.3	1.72	8.0	9.0	13.8				

α: stress concentration factor
τ_{w1}, σ_{w1}: fatigue strength
τ_{w2}, σ_{w2}: crack strength
Unit of stress: kg/mm²

this paper, the fatigue strength is defined as the limit stress for crack initiation and the crack strength as the limit stress for fracture in the range of non-propagating crack existing. According to this definition, the so-called fatigue limit based on fracture is equal to fatigue strength in the range without non-propagating crack and is equal to crack strength in the range of non-propagating crack existing.

The values of τ_{w1}, τ_{w2} and σ_{w1}, σ_{w2} are shown respectively in Figs. 2~4 and in Fig. 5.

The relation between the maximum shear stress τ_{max} [the maximum shear stress repeated at the root of a notch in the fatigue limit based on crack initiation, that is $\alpha_t\tau_{w1}$ for torsion and $(1/2)\alpha_b\sigma_{w1}$ for bending] and χ [the stress gradient at the root of the notch, that is $(1/\tau_{max})\ (d\tau/dx)_{x=0}$] is shown in Fig. 6.

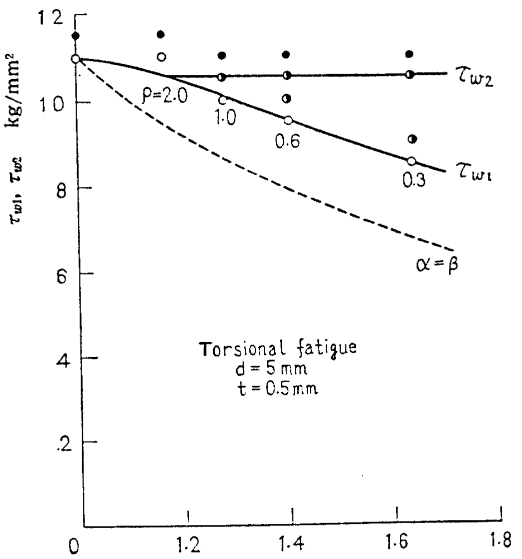


Fig. 3 Torsional fatigue

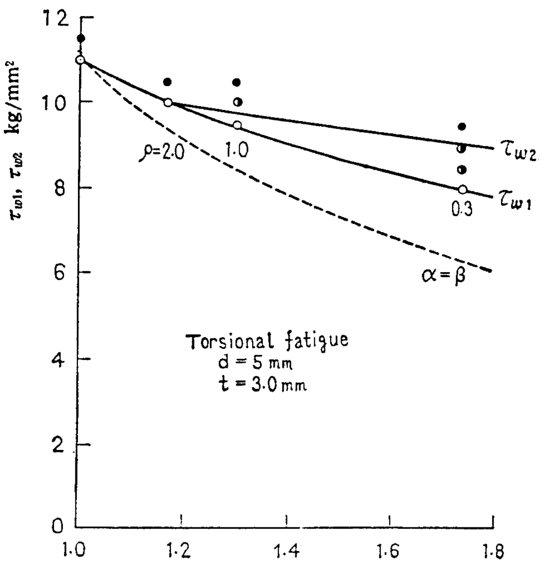


Fig. 4 Torsional fatigue

In this paper the following approximate expressions[2] for χ were used.

$\chi = 1/\rho + 2/d$ for torsion

$\chi = 2/\rho + 2/d$ for bending

From Figs. 2~6, the experimental results are summarized as follows:

(i) In the case of torsional fatigue, the maximum shear stress repeated at the root of a notch in the fatigue limit based on crack initiation becomes almost constant, irrespective of the depth of the

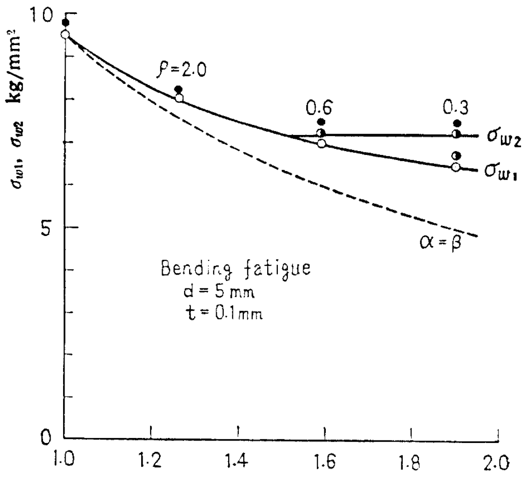


Fig. 5 Rotating bending fatigue

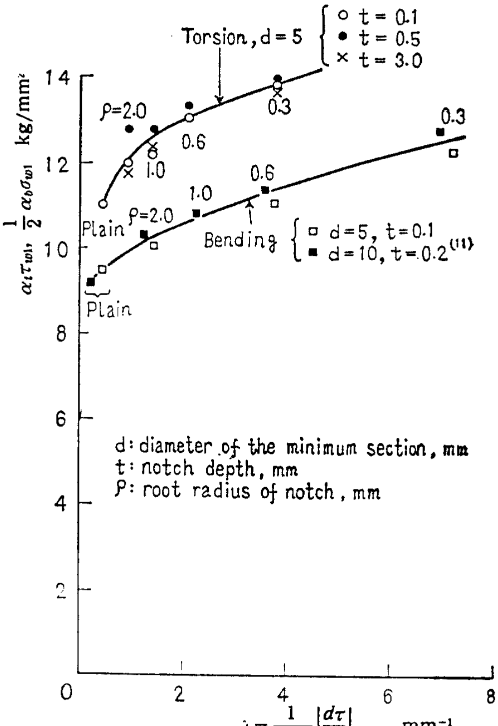


Fig. 6 Relations between the maximum shear stress repeated at the notch root in crack initiation limit and the stress gradient

notch t (and irrespective of the minimum diameter d, because the mechanical factor of size effect is the same as that of notch effect) if the stress gradient χ is constant.

The same fact has been confirmed for bending fatigue by the experiment of both electropolished and work-hardened specimens having various depths of the notch and diameters at the notch root[3].

This fact results from the fact that in torsion the stress distribution becomes approximately equal in the same manner as in bending if the stress gradient is constant (see Fig. 7. In most cases the stress distribution at the notch root is almost determined from the root radius of the notch ρ alone except for the case where ρ is large in comparison with d).

(ii) The maximum shear stress at the notch root in the crack initiation limit is larger in torsion than in bending, if the stress gradients are kept equal in both cases. The ratio of the maximum shear stress in torsion to the one in bending is about 1.2 irrespective of the stress gradient [that is, $\alpha_t\tau_{w1}/(\alpha_b\sigma_{w1})\fallingdotseq 0.6$. This can be explained by the viewpoint from which the fatigue process is divided into the following two stages (in this paper, the ductile materials are treated).

Process (I): The process in which the damage due to slips repeated in the crystal is accumulated in slip planes or grain boundaries and as a result the slip planes or the grain boundaries, the sizes of which are about one grain, approach to the free surface in the states of energy. In this process, the amount of slips repeated in the crystals on surface layer is of fundamental importance and it seems to be almost determined by the amplitude of τ_{max} and the stress distribution (mainly by the stress gradient) alone.

Process (II): The process in which a crack, the size of which is about one grain, propagates into the neighboring grains and enlarges in size until the specimen fractures. In this process, the severity of the damage repeated at the tip of the crack is of fundamental importance and it is determined by the amplitude of τ_{max}, the stress distribution and other stress components (i.e. mainly σ_{max}).

Namely, the fact (ii) results from the fact that at the limit of crack initiation the micro-crack has already initiated and yet the length is different in torsion and in bending. The micro-crack in the crack initiation limit should be called the limit crack length beyond which the crack propagates and extends to the whole circumference of the specimen. The limit crack length of the material used in this experiment was about one or two grain order in bending and about 1 mm in torsion. When the shear stress sligthly higher than the maximum shear stress repeated in the crack initiation limit of bending is repeated, keeping the stress gradient equal, the micro-cracks of one grain order initiate in both bending and torsion, and the micro-cracks always propagate to the whole circumference in bending while in torsion such cracks do not propagate at all at such a stress level.

(iii) The crack strength τ_{w2} in torsional fatigue is nearly equal to the fatigue strength of the plain specimen (that is, the notch factor in torsion is nearly equal to 1). This is due to the fact that in torsion cracks propagate not only along the circumference of the minimum section but also into the portion of larger section of the notch and yet the crack surface endures more or less the reversed shear stress and as a result the damage at the crack tip does not increase so much in spite of the increase of the crack length (in bending, the crack surface endures the compressive stress but does not the tensile stress at all).

3·2 Observation of slip bands

In Fig. 8 the surface states after 10^7 cycles of the same τ_{max} and χ in bending and torsion are shown. In Figs. 9 and 10, the surface states of plain and notched specimens after 10^7 cycles of the limit stress for crack initiation are shown. Figure 9 is a comparison of bending and torsion and Fig. 10 is that of various values of the depth and the root radius of the notch in torsion.

From Figs. 8~10 the following are recognized.

(i) The densities of slip bands after 10^7 cycles of the same τ_{max} and χ in bending and torsion are similar to each other. This can be understood if we consider that the fatigue damage before the initiation of a crack is a function of the amount of slip repeated in the surface grain and the amount of slip is uniquely determined by the amplitude of shear stress and the stress gradient provided the

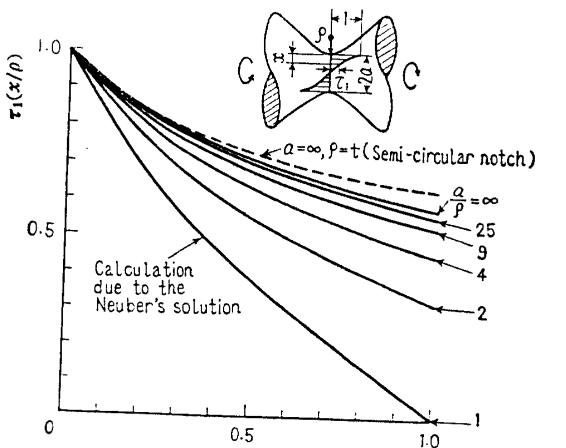
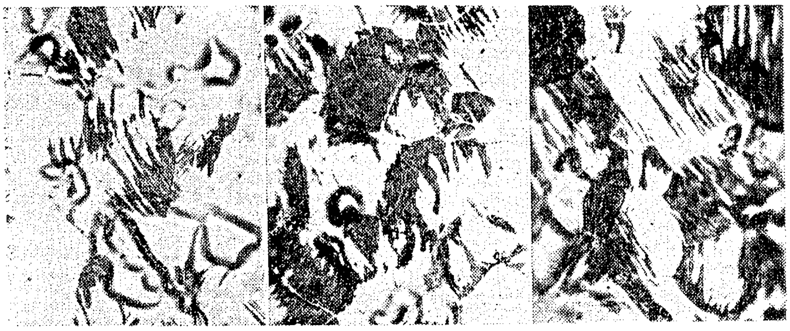


Fig. 7 Stress distribution near the notch root of a round bar subjected to torsional moment

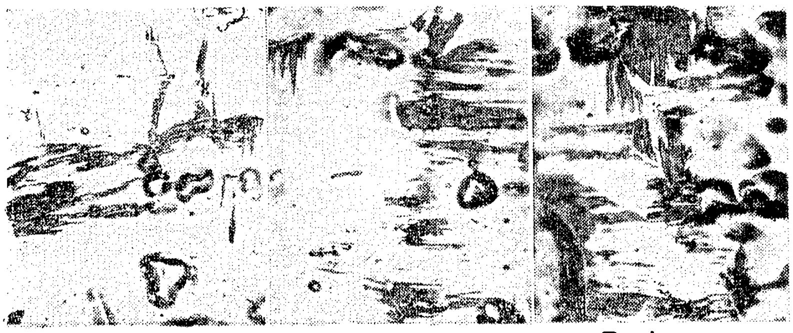
number of cycles is constant.

Since the density of the surface grains convenient to slip is higher in bending than in torsion as Peterson has pointed out[4], it is supposed that the density of the slip bands in bending will be higher than that in torsion, provided τ_{max} and χ are equal in both cases. But so long as the observation in Fig. 8 is concerned, the difference is small.

(ii) The surface states compared in torsion alone after 10^7 cycles of the stress in the crack initiation limit are similar to each other and the same thing can be said about bending. But the damage in torsion is severer (the density of the slip bands is higher) than in bending. This can be explained in the same manner as the term (ii) of section 3·1. The density of slip bands of plain specimens is slightly lower than that of notched specimens in both torsion and bending. This



Bending　　Bending　　Bending



Torsion　　Torsion　　Torsion

(a) τ_{max}=9.5 kg/mm² χ=0.4 mm⁻¹　(b) τ_{max}=10.1 kg/mm² χ=1.4 mm⁻¹　(c) τ_{max}=11.1 kg/mm² χ=3.7 mm⁻¹

Fig. 8 Comparison of the surface states after 10^7 cycles of stress repetitions, between bending and torsional fatigues in which both τ_{max} and χ are equal to each other (×240)

$\alpha\tau_{w1}$=11.0 kg/mm², $\rho=\infty$(plain)　$\alpha\tau_{w1}$=12.0 kg/mm², ρ=2.0 mm　$\alpha\tau_{w1}$=13.0 kg/mm², ρ=0.6 mm　$\alpha\tau_{w1}$=13.8 kg/mm², ρ=0.3 mm

(a) Torsional fatigue

$\alpha\sigma_{w1}$=19.0 kg/mm², $\rho=\infty$(plain)　$\alpha\sigma_{w1}$=20.2 kg/mm², ρ=2.0 mm　$\alpha\sigma_{w1}$=22.3 kg/mm², ρ=0.6 mm　$\alpha\sigma_{w1}$=24.7 kg/mm², ρ=0.3 mm

↔ Axial direction

(b) Bending fatigue

Fig. 9 Comparison of the surface states of the notch roots after 10^7 cycles of the stresses of crack initiation limits in bending and torsional fatigue tests (×240) is probably due to a statistical factor. Namely, in the case of notched specimens, Fig. 9 shows the limit of crack initiation in the grains within the narrow notched root, while in the case of plain specimens it shows that in the grains within comparatively broad surface.

3·3 Non-propagating crack and incipient crack

Figures 11, 12 show the non-propagating cracks of electropolished specimens. The one is the photograph of the surface of the notch root after 10^7 cycles of stress and the other is the photograph of the longitudinal section of the notched part etched with 5% picric acid alcohol solution after polishing by buff. Micro-cracks present themselves as a black line when the surface of a specimen is removed about 1~2μ by electropolishing after stress repetitions (see Fig. 13). We will call them the incipient crack in this paper. They correspond to the so-called Thompson's persistent slip bands[5]. They were found to initiate more often along grain boundaries than along slip bands, so far as the materials used in this experiment are concerned.

In Table 3 the numbers of incipient cracks observed by this method after 10^7 cycles of the stress in the crack initiation limit are given.

Figure 13 shows the incipient cracks which initiated along grain boundaries and slip bands.

From the facts mentioned above, the following conclusions are drawn.

(i) In torsion the root radius of notch ρ is not necessarily constant at the branch point contrary to the case of bending. And if the root radius of notch varies, the crack strength generally does not become constant even if the diameter of the

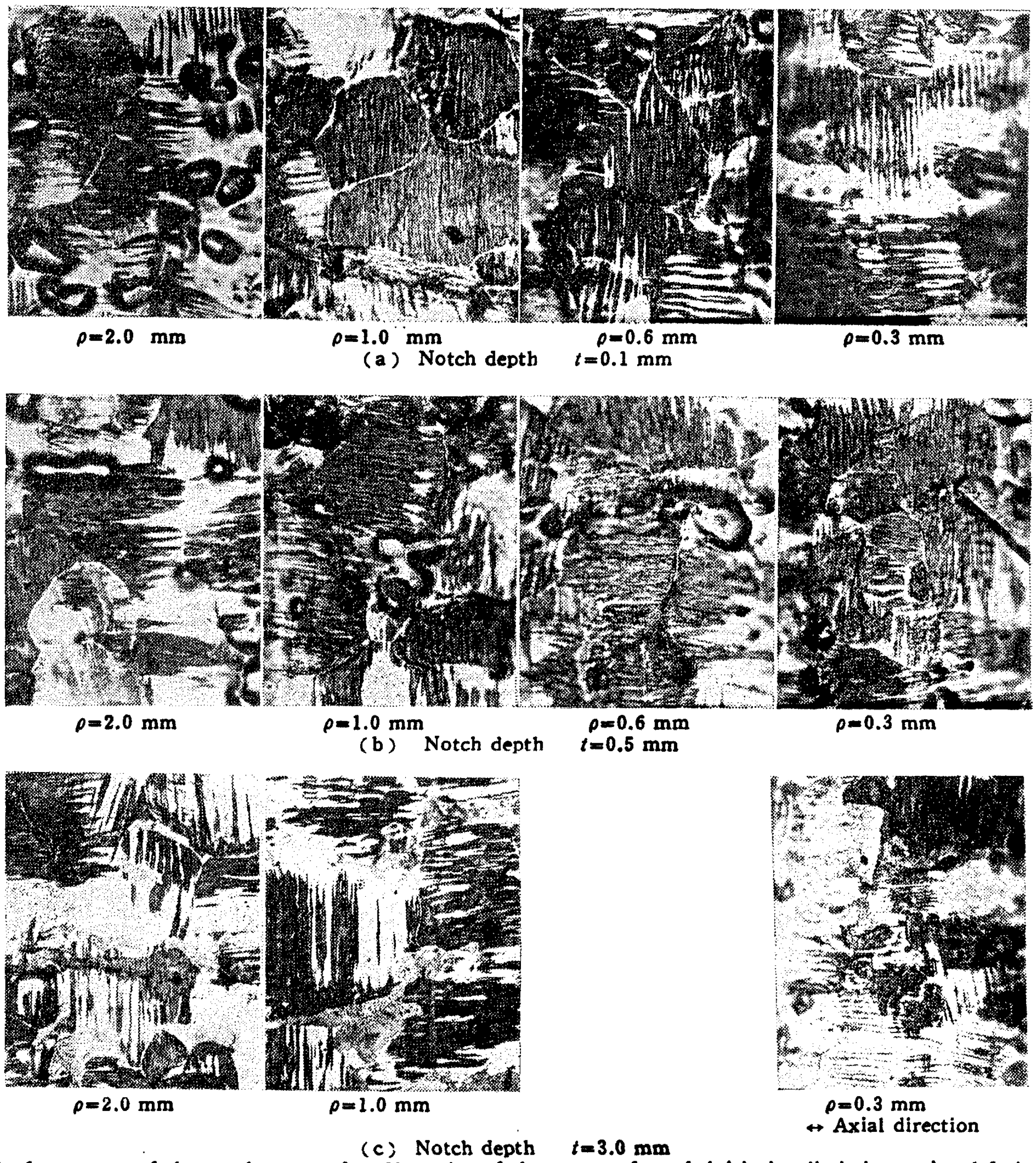
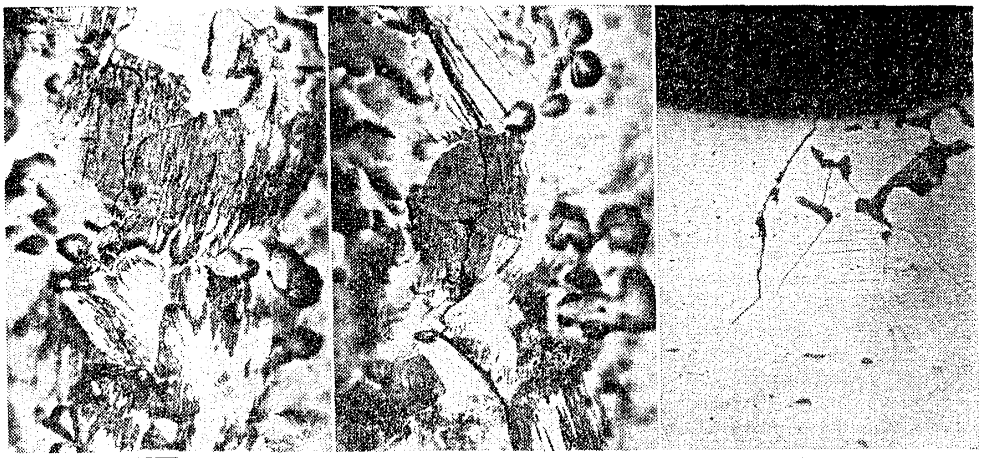


Fig. 10 Surface states of the notch roots after 10^7 cycles of the stress of crack initiation limit in torsional fatigue (×240)

minimum section and the notch depth are kept constant (Figs. 2~4).

This is due to the difference in the non-propagating mechanism of the crack of notched specimens between torsion and bending. In the case of bending fatigue, the fatigue damage in the tensile process differs from the one in the compressive process, and that is the main cause of the non-propagating crack.

In bending the crack propagates through the minimum section alone, and therefore whether the



$\sigma=14.5$ kg/mm^2, $t=0.1$ mm, $\rho=0.3$ mm; $\sigma=13.5$ kg/mm^2, $t=0.1$ mm, $\rho=0.3$ mm
(a) Non-propagating cracks viewed in the notch roots

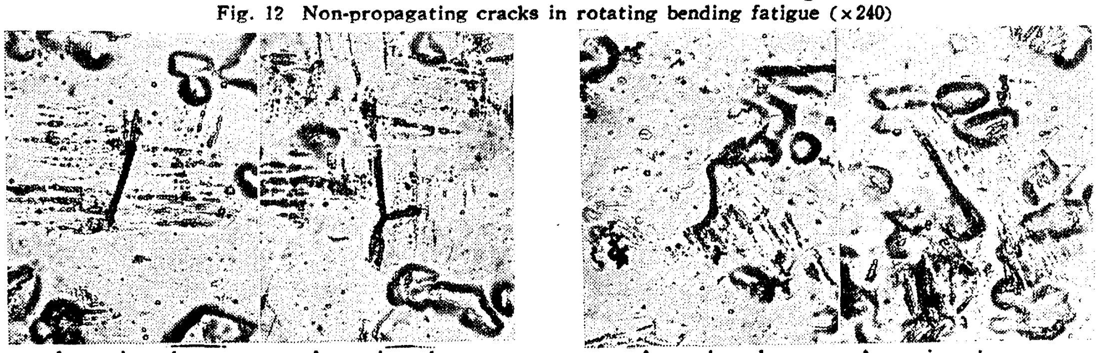
$\sigma=14.5$ kg/mm^2, $t=0.1$ mm, $\rho=0.3$ mm
(b) Non-propagating crack in the longitudinal section

Fig. 11 Non-propagating cracks in torsional fatigue (×240)

$\tau=11.0$ kg/mm^2, $t=0.1$ mm, $\rho=2.0$ mm; $\tau=10.5$ kg/mm^2, $t=0.1$ mm, $\rho=1.0$ mm
(a) Non-propagating cracks viewed in the notch roots

$\tau=10.5$ kg/mm^2, $t=0.1$ mm, $\rho=0.3$ mm
(b) Non-propagating crack in the longitudinal section

Fig. 12 Non-propagating cracks in rotating bending fatigue (×240)



Appearing along a grain boundary; Appearing along slip bands
(a) Torsional fatigue

Appearing along a grain boundary; Appearing along slip bands
(b) Bending fatigue

Fig. 13 Incipient cracks (×240)

crack propagates or not is determined by the stress distribution near the root of the notch alone. Since the stress distribution is controlled almost only by ρ except the case where ρ is extremely large (ρ at the branch point is about 0.5 mm for many ductile materials[3] [6]~[11]), ρ at the branch point becomes constant after all, irrespective of the diameter of the minimum section and the depth of the notch (ρ at the branch point of the material used in this experiment is about 0.6 mm[11]).

On the contrary, in torsion the non-propagation of crack in sharp notches becomes possible as a result of the fact that the crack surface can endure more or less the reversed shear stress. In torsion, since the cracks propagates, to a certain degree, to a larger portion of the section, whether the non-propagation of the crack occurs or not seems not to be determined by the stress distribution in the minimum cross section alone. Therefore the branch point is not determined by ρ alone as in the case of bending. The typical example can be seen in the experiments in which the torsional fatigues of the notched and the filleted specimens of S 35 C have been compared[12] (ρ at the branch point is about 2 mm for the notched specimens, and about 0.3 mm for the filleted specimens). From the above consideration, it can also be understood that the crack strength varies with ρ.

(ii) Along grain boundaries and slip bands, the incipient cracks were observed. In this material, the incipient cracks were found more along grain boundaries than along slip bands.

It is natural that the incipient crack initiates along slip bands where the damage by the repeated stress is serious. The incipient crack along a grain boundary seems to initiate as the result of joining of the points where the slip bands and a grain boundary cross each other and consequently the damage becomes serious.

Table 3 Comparison of the numbers of incipient cracks
(a) After 10^7 cycles of the stress of crack initiation limit in torsional and bending fatigue tests

ρ mm	Torsion	ρ mm	Bending
∞	354	∞	26
1.0	79	2.0	7
0.6	72	0.6	19
0.3	57	0.3	6

(b) After 10^7 cycles in torsional and bending fatigue tests in which τ_{max} and χ are equal respectively in both cases

τ_{max} kg/mm²	χ mm⁻¹	Torsion	Bending
20.2	0.4	17	26
22.3	1.4	2	7
24.7	3.7	0	19

If that is the case, the operation of a number of slip planes in a grain will be the necessary condition for the initiation of an incipient crack along a grain boundary (in the low cycle fatigue of electropolished 7:3 brass, cracks initiate along grain boundaries under large strain amplitude and almost along slip bands under small strain amplitude).

(iii) In torsional fatigue non-propagating cracks whose length were about 1 mm were observed even in plain specimens.

The non-propagating cracks in plain specimens have been observed already by Wadsworth[13], Watanabe et al.[14] and Ōkubo et al.[15]

There are two types of non-propagating cracks. They are the non-propagating crack due to (a) non-uniformity of the matrix and the one due to (b) non-uniformity of the stress distribution.

Namely, though the cracks in torsional fatigue initiate along the plane on which the shear stress becomes nearly the maximum, the cracked surface endures more or less the shear stress and the stress concentration does not increase so much in spite of the increase of the crack length. Therefore the crack can be arrested if it comes upon the barrier during the process of propagation across several crystals. Such a non-propagating crack observed in torsional fatigue is due to the cause (a). In bending fatigue, the normal stress acts on the plane along which the crack initiates and the severity of the damage repeated at the crack tip increases rapidly with the increase of the crack length, therefore the length of the non-propagating crack observed is about 1～2 times the grain diameter.

In the case of the crack initiation from inclusions, the mechanism of non-propagation seems to be based on (b) even in plain specimens. In fact, in the fatigue of nodular cast iron the non-propagating cracks of plain specimens from spherical graphites were observed in both cases of bending and torsion, and they were similar to those of notched specimens[16].

(iv) The number of incipient cracks observed after 10^7 cycles of the stress at the fatigue limit based on crack initiation is greater in torsion than in bending. This is the fact associated with the term (ii) of the section 3.2.

When the numbers were obtained keeping the maximum shear stress (we took it equal to the shear stress at the crack initiation limit of bending) and the stress gradient equal, they are nearly equal in bending and torsion.

This, as mentioned in the term (ii) of the section 3.1, is due to the fact that the fatigue process up to the crack initiation is determined mainly by the amplitude of the shear stress and the stress gradient.

4. Conclusions

The results of the torsional and bending fatigue tests on S 10 C electropolished specimens (plain and notched) are summarized as follows.

(1) In torsional fatigue, as in the case of bending the maximum shear stress repeated at the notch root at the fatigue limit based on crack initiation is determined only by the stress gradient $\chi=(1/\tau)(d\tau/dx)$ at the notch root and the larger is the stress gradient, the higher is the maximum shear stress.

(2) When the values of χ are equal, the maximum shear stress in the crack initiation limit of torsional fatigue is about 1.2 times that of bending fatigue.

(3) At the fatigue limit based on crack initiation, the fatigue damage, taking the density of slip bands and incipient cracks as a measure, is more serious in torsion than in bending (the amount of slip bands and the number of the incipient cracks are larger in torsion than in bending). But in the test keeping the maximum shear stress and stress gradient equal, the damages of the surface after 10^7 cycles are similar to each other.

(4) The facts of (1)~(3) imply that the amplitude of maximum shear stress and the stress gradient are fundamental factors which control the fatigue process up to crack initiation.

(5) Two types of non-propagating cracks which originate from two different causes, (a) metallurgical non-uniformity of matrix and (b) non-uniformity of stress distribution, were observed. In torsion the stress concentration at the crack tip does not increase so much in spite of the increase of the crack length and therefore pretty long non-propagating cracks based on (a) are apt to be found (in plain specimens of torsion, several non-propagating cracks whose length was about 1 mm, were observed).

(6) The mechanism of non-propagation of a crack in torsional notched specimens is different from that in bending notched specimens.

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