TOTAL DOMINATION IN GRAPHS

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The total domination number $\gamma_t(G)$ of a graph G is the minimum cardinality of a set S of vertices so that every vertex of G is adjacent to a vertex in S. In this talk, we survey recent results on total domination in graphs. For example, we show that if G is a graph with no K_3-component and with all vertices of G contained in a traingle, then $\gamma_t(G) \leq \alpha'(G)$, where $\alpha'(G)$ denotes the matching number of G. We discuss recent results on total domination in graphs obtained using transversals in hypergraphs. For example, letting $f(3) = 0.7 - 1/130$, $f(4) = 0.65 + 1/300$ and $f(5) = (18f(4) + f(3))/20 \approx 0.6226154$, we show that if G is a graph of order n with $\delta(G) \geq 5$, then $\gamma_t(G) \leq (1 - f(5))n \leq 0.377385n$. Let $\mathcal{G}(n, p)$ be a random graph of n vertices where each edge is chosen with probability p. We show that for every $0 < \epsilon' < \epsilon$ and $p = (1 + \epsilon')\sqrt{\frac{1}{n}(2 \ln n)}$, almost every graph $G \in \mathcal{G}(n, p)$ has diameter two and $\left(\frac{1}{2\sqrt{2}} - \epsilon\right)\sqrt{n \ln(n)} < \gamma_t(G) \leq \left(\frac{1}{\sqrt{2}} + \epsilon\right)\sqrt{n \ln(n)}$. We also present results on total domination in hypergraphs.

Keywords: total domination, transversals.

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References

[1] M. A. Henning and A. Yeo, *Total domination in graphs (Springer Monographs in Mathematics)* 2013. ISBN: 978-1-4614-6524-9 (Print) 978-1-4614-6525-6 (Online).