Total Probability of Collision as a Metric for Finite Conjunction Assessment and Collision Risk Management

Ryan C. Frigm
Omitron, Inc.

Matthew D. Hejduk
Astrorum Consulting, LLC

Lauren C. Johnson and Dragan Plakalovic
Omitron, Inc.

ABSTRACT

On-orbit collision risk is becoming an increasing mission risk to all operational satellites in Earth orbit. Managing this risk can be disruptive to mission and operations, present challenges for decision-makers, and is time-consuming for all parties involved. With the planned capability improvements to detecting and tracking smaller orbital debris and capacity improvements to routinely predict on-orbit conjunctions, this mission risk will continue to grow in terms of likelihood and effort. It is very real possibility that the future space environment will not allow collision risk management and mission operations to be conducted in the same manner as it is today. This paper presents the concept of a finite conjunction assessment—one where each discrete conjunction is not treated separately but, rather, as a continuous event that must be managed concurrently. The paper also introduces the Total Probability of Collision as an analogous metric for finite conjunction assessment operations and provides several options for its usage in a Concept of Operations.

1. INTRODUCTION & MOTIVATION

The Probability of Collision (Pc) has become a universal metric and statement of on-orbit collision risk. Although several flavors of the computation exist and are well-documented in the literature, the basic calculation requires the same input: estimates for the position, position uncertainty, and sizes of the two objects involved. The Pc is used operationally to make decisions on whether a given conjunction poses significant collision risk to the primary object (or space asset of concern). It is also used to determine necessity and degree of mitigative action (typically in the form of an orbital maneuver) to be performed. The predicted post-maneuver Pc also informs the maneuver planning process regarding the timing, direction, and magnitude of the maneuver needed to mitigate the collision risk. Although the data sources, techniques, decision calculus, and workflows vary for different agencies and organizations, they all have a common thread. The standard conjunction assessment and collision risk concept of operations[1, 2, 3, 4] (CONOPS) – such as the one implemented by the NASA Robotic Conjunction Assessment Risk Analysis (CARA) team – predicts conjunctions, assesses the collision risk (typically, via the Pc), and plans and executes avoidance activities for conjunctions as a discrete events.

As the space debris environment continues to increase and improvements are made to remote sensing capabilities and sensitivities to detect, track, and predict smaller debris objects, the number of conjunctions will in turn continue to increase, with some analysts estimating an order-of-magnitude increase in the number of predicted conjunctions. This increase will challenge the paradigm of treating each conjunction as a discrete event. The challenge will not be limited to workload issues, such as manpower and computing performance, but will also include the ability for satellite owner/operators to successfully execute their mission while also managing on-orbit collision risk. Executing a propulsive maneuver occasionally can easily be absorbed into the mission planning and operations tempo, whereas continuously planning evasive maneuvers for multiple conjunction events is time-consuming and would disrupt mission and science operations beyond what is tolerable. At the point when the number of conjunctions is so large that it is no longer possible to consider each individually, some sort of an amalgamation of events

and risk must be considered. This shift is to one where each conjunction cannot be treated individually and the effects of all conjunctions within a given period of time must be considered together. This new paradigm can be called finite Conjunction Assessment (CA) risk management.

This paper considers the use of the Total Probability of Collision (TPc) as an analogous collision risk metric in the finite CA paradigm. While the TPc computation is straightforward and its physical meaning is understandable, the implications of its usage operationally require a change in mindset and approach to collision risk management. This paper explores the necessary changes to evolve the basic CA and collision risk management CONOPS from discrete to non-discrete, or finite, CA, including aspects of collision risk assessment and collision risk mitigation. It proposes numerical and graphical decision aids to understand both the "risk outlook" for a given primary as well as mitigation options for the aggregate collision risk. Both concepts make use of the TPc as a metric for finite collision risk management. Several operational scenarios are used to demonstrate the proposed concepts in practice.

2. THE TOTAL Pc METRIC

If a set of n probabilistic events are independent, meaning that the outcome of at least one of these events is not dependent on the outcome of any of the others, then the probability of any one of these events' taking place is calculable through a straightforward formula if the individual probabilities of each event are known. In a "Total Pc" context, if a series of conjunction events with the same primary are statistically independent, then the situation is analogous: the risk assessment metric desired is the probability that at least one of these conjunctions could result in a collision. One could, of course, maintain that such a series is not truly composed of independent events because if the primary actually sustains a collision as part of the first event, it would no longer be an intact payload capable of causing a second collision in one of the subsequent conjunctions. While practically true, this is in fact a trivial objection; for the question is not whether collisions will actually take place but the frequency with which the primary and secondary trajectories will align so that for at least one of the conjunctions the objects will pass closer than a pre-defined miss distance.

Leaving the question of actual event independence aside for a moment, if such independence can in fact be presumed, the derivation of the compounding probability formula follows quickly from de Morgan's Laws of Complementarity. For simplicity, the following derivation addresses only three events, but it can easily be generalized to n events if desired. One wishes, as shown below, the probability of event A, B, or C taking place:

$$P(A \cup B \cup C)$$

$$= 1 - \overline{P(A \cup B \cup C)} \tag{1}$$

$$= 1 - (\overline{P_A} \cap \overline{P_B} \cap \overline{P_C}) \tag{2}$$

$$= 1 - (\overline{P_A})(\overline{P_B})(\overline{P_C}) \tag{3}$$

$$= 1 - (1 - P_A)(1 - P_B)(1 - P_C) \tag{4}$$

$$= P_A + P_B + P_C - P_A P_B - P_B P_C - P_A P_C + P_A P_B P_C \tag{5}$$

Eq. 1 rewrites this as the complement of the complement probability. In Eq. 2, De Morgan's Law allows the complementarity to be distributed to each of the three probabilities by inverting the conjunction. One invokes the independence assumption in proceeding to Eq. 3, in which the conjunction of probabilities (*e.g.,* $P_A \cap P_B$) are written out as direct products ($P_A \cdot P_B$). Finally, the complements are written out in a different form in Eq. 4, producing the familiar cumulative probability formula. For three events, this formula is multiplied out fully in Eq. 5. One can see that it is essentially the sum of the three individual probabilities with the intersections subtracted but yet the three-way intersection added. Examining a Venn diagram helps to understand this formula intuitively.
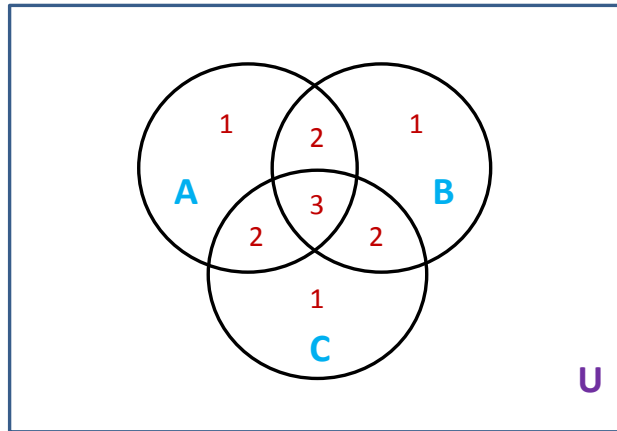
Fig. 1: Venn diagram illustrating physical meaning of probability compounding formula

In the diagram above, the three circles represent P_A, P_B, and P_C in the overall universe U. The red numbers represent the number of times each sub-region is counted if one simply sums the three probabilities ($P_A + P_B + P_C$). By subtracting off the regions $P_A P_B$, $P_B P_C$, and $P_A P_C$, one ends up subtracting off the middle section three times, which leaves it unrepresented; this is why $P_A P_B P_C$ needs to be re-added as the last term in the formula (Eq. 5).

Returning to the issue of event independence, it is not difficult to postulate independence in a situation in which one primary encounters a set of conjunctions but each against a different secondary; there is no reason to suppose that a close approach with a particular secondary is likely to promote a close approach with some other secondary. However, a situation in which a primary sustains multiple conjunctions with a single secondary—the repeating conjunction scenario—does suggest some level of correlation/dependence and should be investigated more thoroughly.

A repeating conjunction situation between a single primary and secondary that produced three high-Pc conjunctions (all three higher than 1E-04) was identified in the CARA archives and subjected to a Monte Carlo investigation. One million trials were run in which the primary and secondary objects' epoch positions were perturbed (in accordance with their accompanying epoch covariances) and propagated forward. Perturbations were performed both in equinoctial elements (and converted back to Cartesian elements, with only the position perturbations used) and directly in Cartesian space; results are tabulated separately for each approach. New times of closest approach for the three conjunctions were calculated for each pair of trajectories attempted, and the number of cases in which the two trajectories passed within the 20m hard-body radius (HBR) value were tabulated, both for each conjunction independently and as an aggregate (*i.e.,* the number of primary trajectories that violated the HBR value for any of the three conjunctions). The results are summarized in the table below:

Attribute		Equinoctial Perturbations	Cartesian Perturbations
From Monte Carlo	P_A	1.38E-03	1.36E-03
	P_B	8.20E-04	8.31E-04
	P_C	6.31E-04	7.16E-04
	Any compound terms ($P_A P_B$ &c.)	0	0
	P_A or P_B or P_C	2.84E-03	2.91E-03
Formula calculation (Eq. 4), using P_A, P_B, and P_C above		2.84E-03	2.91E-03
Formula calculation (Eq. 4, using 2-D Pc values for each event		2.29E-03	
Difference between MC and 2-D Pc (in orders of magnitude)		0.094	0.105

There are a number of interesting aspects in this set of results. First, it is somewhat surprising that no "compound" situations were observed as part of the Monte Carlo results—that is, single trajectories that violate the HBR for more than one of the conjunctions. One could argue that simply not enough Monte Carlo trials were run, and undoubtedly if a much larger number were then at least some compound situations would arise; but relatively large numbers of hits for each of the three conjunctions (~600-1500) were observed, so it would seem that at least a few compound cases should have been generated. Given that there were no such situations, the total Monte Carlo result is merely the sum of the three calculated probabilities (P_A, P_B, and P_C); this will of course introduce a difference between this simple sum and the calculation arising from the formula in Eq. 5; but as shown in the table, the results are identical to three significant figures. Comparing the Monte Carlo results to the total Pc calculated from the 2-D Pc values for each of the three conjunctions produces a less striking match but still quite nice alignment: the difference is only about a tenth of an order of magnitude, and differences in Pc of less than half an order of magnitude are rarely considered operationally significant. While it would be helpful to investigate a larger number of such empirical situations to assemble a more comprehensive gauge of the independence situation, it does seem that, for a case in which this assumption was questionable, the standard compound probability formula performed quite adequately.

3. FINITE CA RISK MITIGATION

It may take the reader by surprise to encounter a discussion of risk mitigation prior to that of risk assessment, for such is a reversal of the usual collision risk management process. However, the use of TPc as a metric for risk mitigation planning is not a new concept, and therefore it makes sense to outline TPc's current usage in operations before treatment of the new concept of operations. The CARA team at NASA GSFC has included and uses operationally the computation of TPc in their maneuver trade space tool, which allows the analyst to examine all of the options for selecting maneuver candidates; and Wysack[5] proposed optimization techniques that included this calculation to automatically select a candidate maneuver.

The reason the use of TPc first appeared in risk mitigation prior to risk assessment is both simplicity and convenience. For a discrete conjunction event, once a decision has been made to plan a maneuver, it follows logically that one ought to consider the effects of that maneuver on other known conjunctions. This concept even pre-dates inclusion of the TPc as a more closed-form consideration of these effects. Since the near beginnings of CA as an operational practice, after planning a potential collision avoidance maneuver the ephemeris containing the maneuver was re-screened prior to execution to ensure that the maneuver itself did not engender a subsequent dangerous conjunction situation. Many advances have occurred, including the first maneuver trade space developed by McKinley[6] and subsequent efforts at CARA to move his construct into a probability-based trade space, which have made the collision avoidance maneuver planning process more robust and executed on more compressed timelines; but the concept of considering the effects of a planned maneuvers on conjunctions other than the trigger event has been in practice for years. Incorporation of the TPc metric into this process was a logical incremental step for risk mitigation. Fig. 2 provides an example output from the latest version of CARA's maneuver trade space utility, where the contours represent the post-maneuver TPc for any maneuver size (x-axis) and maneuver timing (y-axis) pairing. There exists some challenges in selecting which post-maneuver conjunctions to consider in this analysis and maneuver planning but those will be discussed in more detail in the risk assessment section.
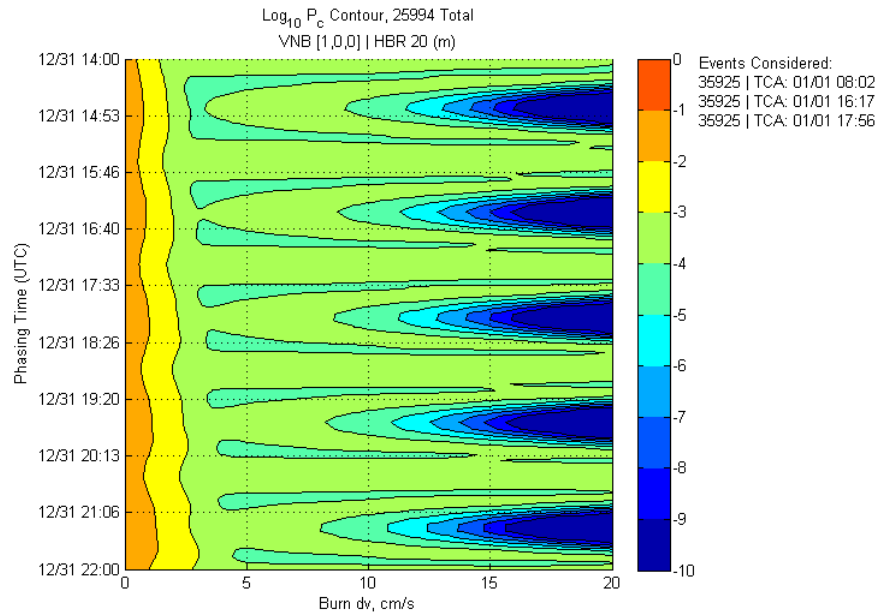
**Fig. 2: Example of a Probabilistic Maneuver Trade Space using TPc**

4. FINITE CA RISK ASSESSMENT

Plots like the one shown in Fig. 2 can be used to select a maneuver candidate, or maneuver candidates, that reduce the total collision probability because they have the following property:

TPc Property: The predicted collision probability, Pc, for any discrete post-maneuver conjunction will be equal or less than the total collision probability, TPc; meaning, if one decides to remediate the TPc to a given risk level, each discrete event will also be remediated to that level (or better).

Using Total Pc for Finite Collision Risk Management

For discrete conjunctions events, Pc has become commonplace as a metric to initiate planning of mitigation activities such as a collision avoidance maneuver. Typically a conjunction is predicted several days in advance of the time of closest approach (TCA) and is updated on a regular basis using new tracking and space environment information. With each update, the Pc can be re-calculated. Each new Pc provides a new estimate of the predicted risk at the TCA for that event; Fig. 3 shows an example of this Pc time history for a given discrete conjunction event, where the Pc (y-axis) for each prediction of the close approach is plotted versus time to TCA (x-axis) and the red line represents the risk threshold, above which maneuver planning is at least considered.
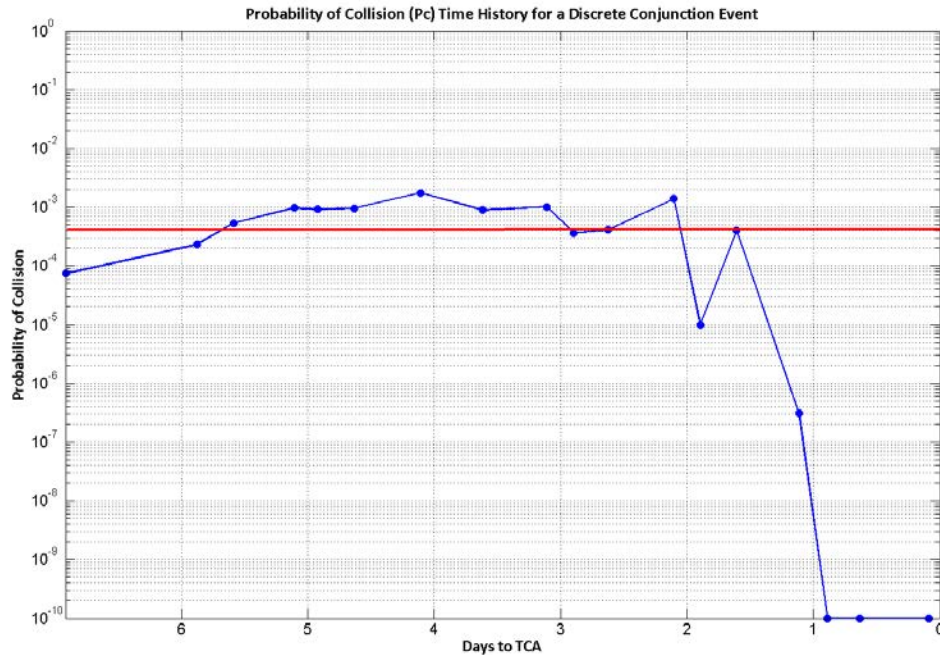
Fig. 3: Pc Time History and Canonical Pc Behavior for a Discrete Conjunction Event

For some events, this evolution of Pc is well behaved and follows the canonical pattern (as is the case in Fig. 3); for some, this evolution is more schizophrenic. Regardless, the Pc still informs the decision-making process of whether planning (and ultimately executing) a collision avoidance maneuver is prudent. For finite collision risk management, TPc can be used similarly to inform the decision-making process on whether mitigation actions should be considered. However, if use of Pc for discrete risk assessment seemed naively straightforward, fear not: the use of TPc for finite risk assessment will not be. These additional considerations will be discussed later in this section but, first, the basic concept will be laid out.

After conjunction assessment—the process of identifying conjunctions, sometimes referred to as a "screening"—is performed, the risk assessment process typically begins with the Pc calculation for each set of input data. The Pc, along with holistic evaluation of the conjunction, forms the basis for risk decisions. With the current size of the space object catalog, the number of conjunctions and number of high-risk conjunctions is quite manageable with a robust concept of operations in place, such as is with organizations like CARA or CNES's Conjunction Analysis and Evaluation Service: Alerts and Recommendations (CAESAR). Fig. 4a depicts the risk outlook for a typical protected asset with the current conjunction density by showing risk as a function of time, from present time to end of the prediction (or screening) period. The first data point on the x-axis represents the discrete conjunction event whose TCA is closest to present time; likewise, the last data point is the furthest. Here, five events are shown that range in event severity as measured by the Pc. This outlook is manageable. The paradigm shift from discrete to finite collision risk management is required when a conjunction assessments are executed against a substantially-larger catalog. Fig. 4b and c depict a much different situation; one in which the number of conjunctions is 5 and 10 times the current conjunction density, respectively. These outlooks are not at all unrealistic given estimates of what is anticipated in terms of tracked orbital debris with Initial Operating Capability (IOC) of Space Fence.
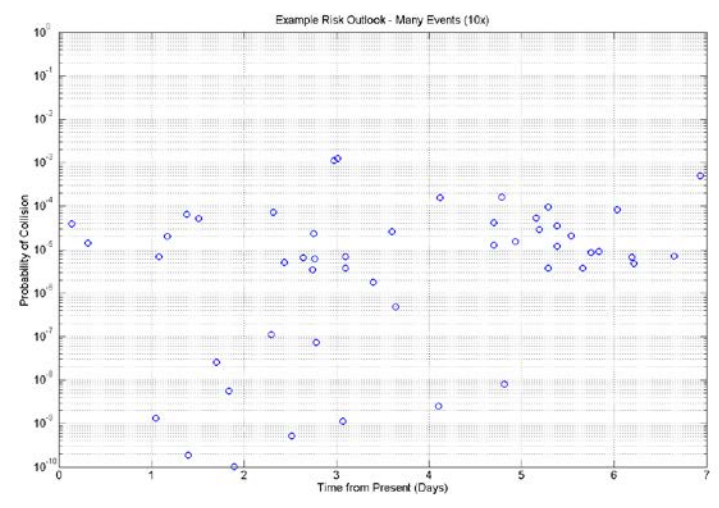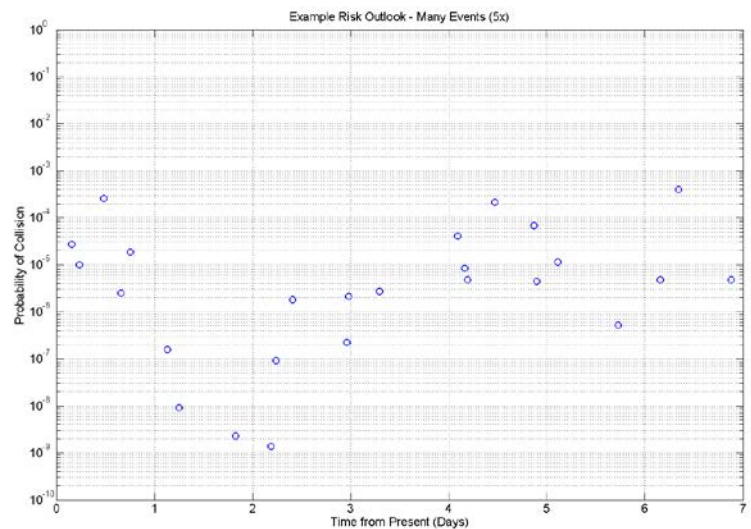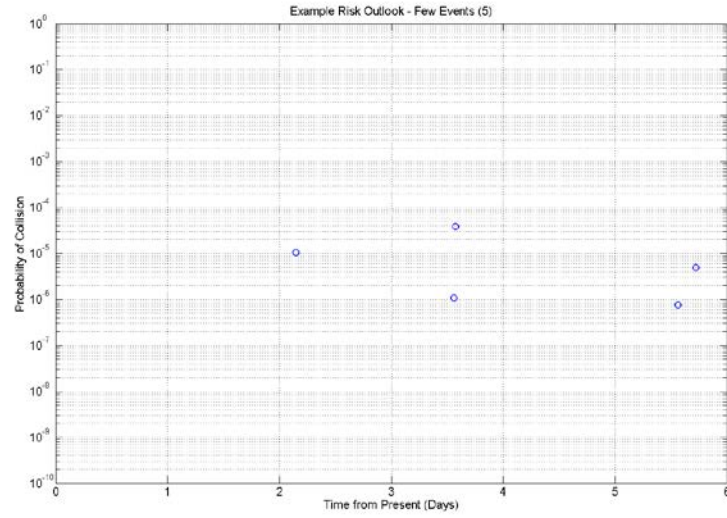
Fig. 4: Example Risk Outlook for (a) 1x, (b) 5x, (c) 10x Present-Day Catalog

In these scenarios, it is no longer feasible to react to discrete events; treating the "group" of conjunctions as an overall collision risk to asset is more tenable. In this situation of performing finite collision management, the discrete conjunctions can, and must, be processed in the same manner. These discrete collision probabilities are the only necessary input for the calculation of the TPc—the TPc for some number of the discrete events is a single numerical value. Fig. 5 shows an example graphical display of how the TPc value might be represented as it grows over time.
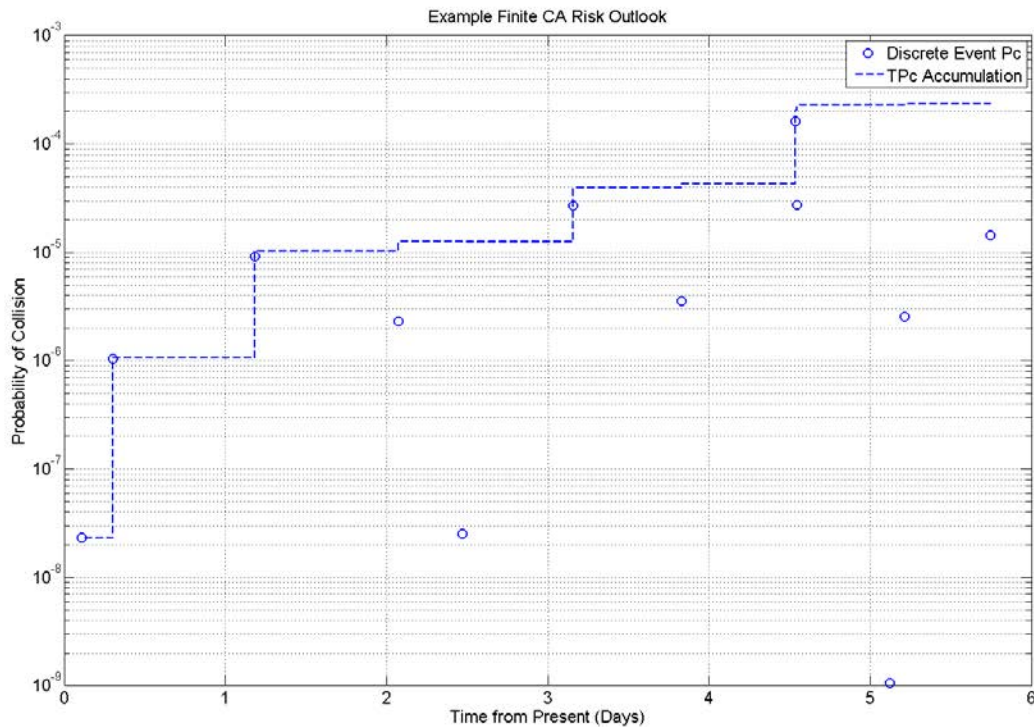
Fig. 5: Example Risk Outlook with Discrete Event Pc and Finite Event TPc

For this example, it is approximately 2.25×10^{-4} by the end of the 6-day look-forward period. Once a new screening is performed and a new prediction is processed for each conjunction, each Pc and the TPc can be re-calculated.

TPc Property: Total Pc as a single collision risk metric over a finite period of time still retains physical meaning like discrete collision probabilities—TPc is probability that any one of the conjunctions over that finite period of time may result in a collision.

What is more interesting than the single TPc value at the end of the entire look-forward period is how the TPc accumulates, as represented by the dashed line in Fig. 5. This line shows how the TPc increases with each discrete event that is convolved chronologically. From present time to the end of the consider period, this value will be a step function that is monotonically increasing. While Total Pc provides the single collision threat estimation over a finite period, the TPc accumulation also provides insight into how each contributes to that total risk. TPc accumulation versus time overlay identifies "gaps" in the Total Pc accumulation, over which there are few or no conjunctions or conjunctions with individual collision probabilities that are not significantly adding to the TPc.

TPc Property: TPc versus time identifies gaps in accumulation which may serve as natural breakpoints for the TPc calculations and target time periods for risk remediation.

Another interesting observation that arises from looking at many simulations of TPc accumulation is that a high TPc does not need to include any high risk discrete conjunctions. A high TPc may result from the convolving of many low risk conjunctions.

TPc Property: TPc prevents that "false sense of security" from not realizing that many low risk discrete conjunction event may result is higher overall risk of collision to the spacecraft

4. OPERATIONAL CONSIDERATIONS

For any practitioner of collision risk management, one aspect of every conjunction that is always discussed is the quality of the data used in the Pc calculation. For better or worse, it is typically folded into the decision calculus qualitatively rather than quantitatively. Evaluating data quality is difficult. Newman[4] provided some techniques to look at tracking adequacy, tracking distribution about the orbit, and OD residual analysis. Cerven[7] also provides a comprehensive approach to evaluating covariance realism in this application. There is an open debate within the community on the manner in which quality should be considered in this process. One position, typically embraced by those with a more theoretical and academic perspective, believe that the Pc already incorporates data quality as it uses relative position, uncertainty, and object size information. An opposing view, advanced usually by those who work daily conjunction assessment operations, holds that an additional overlay of data quality must be employed. The common thread is that understanding the quality is as important as the resultant probabilities.

Within the framework of finite risk management, risk assessment will rely on three important factors: likelihood (probability), timing, and quality.

This paper has focused on the TPc as the analogous metric to evaluate likelihood in finite CA, but it is important to mention the timing and quality aspects. If one refers back to Fig. 5, one can imagine different scenarios in which the TPc violates any chosen threshold at different times. It is known that the predicted Pc changes over time for a given discrete event. This dynamic nature will change the TPc and its accumulation over the considered period. Much like what is done in operations today, events that are several days away are often "de-weighted" (even if only by human perception) as it hoped that it will follow the canonical behavior (*i.e.,* with a shrinking covariance will manifest a substantial drop in Pc), and there is additional time to react should it remain a risk. This de-weighting exists in the TPc construct. Contributing events that are further out in time may be excluded from consideration by terminating the TPc accumulation earlier in the look-ahead period, such as over only four days instead of six.

Another option to account for the temporal consideration is to have a time-varying TPc threshold. This methodology would indirectly allocate more weight to nearer-term events that have had the benefit of the rest of the process, namely opportunities for tasking adjudication, more tracking, less propagation, and more operational planning. At the same time it also will not ignore significant TPc values throughout the entire consider period that may be high enough to warrant early maneuver planning and even execution. Fig. 6 provides some examples of what such a time-varying threshold might look like.
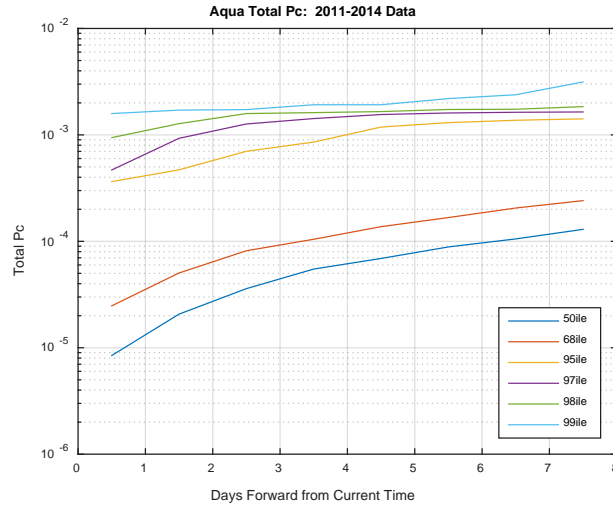
**Fig. 6: Empirical Time-Varying TPc Threshold Options**

In this case, the threshold is determined empirically by mining historical data from 2011 to 2014 for the Aqua satellite, calculating the TPc for different look-forward periods (from 0.5 to 7.5 days), and summarizing those results at different percentile levels. It is not clear that one would want to use this approach actually to set risk tolerance thresholds, but at least it does show the relationship between the choice of certain thresholds (or threshold curves) and the frequency that such a curve might be encountered with the current conjunction density.

A third option to more directly enfold timing and quality exists. If one truly would like a single encapsulating metric, folding in data quality and reaction time as a weighting function of sorts to the TPc can be done, but only if one is willing to sacrifice the physical meaning. The Pc is the probability that, given the uncertainty represented by the predicted covariance matrices of the two objects, the actual miss distance is less than the hard-body radius. It is possible to develop a single metric that includes all three considerations (probability, quality, and immediacy). A similar concept was proposed by Frigm[8] using fuzzy logic to combine probability and quality information into the F-value. The F-value was developed for discrete CA but one can easily envision the extension into finite CA.

6. ADVANCED CONCEPTS

Total Violation Time

One behavior to note when comparing the accumulated TPc against a time-varying TPc threshold is that it is possible for the TPc violations to be dynamic—that is to say, the TPc can be above the threshold at certain points and below at others. One variation of the TPc thresholds described previously to account for this dynamic nature is measuring the amount of the time that the cumulative TPc metric is above the threshold and defining a second threshold for this total violation time. Fig. 7 provides an example risk outlook illustrating the TPc Total Violation Time concept. In this example, the period of time over which the total risk is above the time-varying threshold is about 4 days, or about 57% of the entire considered period.
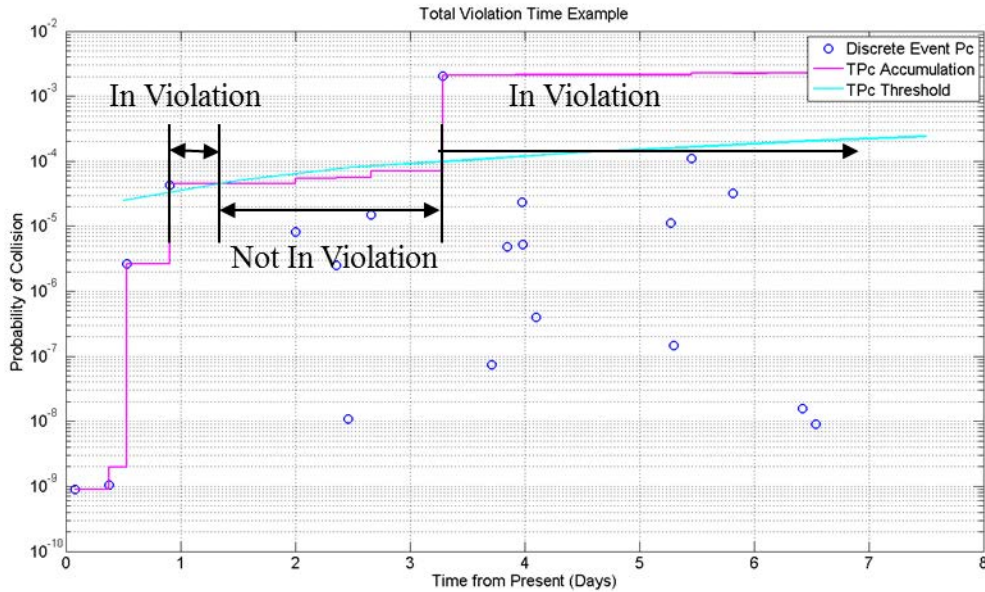
Fig. 7: Example Risk Outlook illustrating the TPc Total Violation Times

Collision Avoidance as an Orbit Maintenance Strategy

Given the frequency of collision avoidance maneuvers expected with a substantially larger catalog, coupled with the fact that most operational LEO satellites—particularly Earth-Observing satellites in sun-synchronous orbits—perform regular station-keeping orbital maneuvers, one interesting concept is to use collision avoidance as the primary means for orbit maintenance. Adopting such a strategy would reverse the paradigm that regular orbit maintenance (*i.e.,* drag make-up maneuvers) would be the "long" lead-time maneuvers and collision avoidance maneuvers would be the quick-turn maneuvers. Moreover, many owner/operators are moving towards having "canned" orbital maneuvers prepared, whether on-board or stored command loads on the ground. In this concept, the canned maneuvers may only become necessary when long periods of time have elapsed without the need for a collision avoidance maneuver. This concept offers an advantage in terms of a conservative collision risk posture; that is to say owner/operators could be more risk adverse to plan and execute collision avoidance maneuvers that play into an overall strategy for orbital maintenance rather than against it. A non-collision avoidance orbital maneuver becomes the non-routine event and is only performed when absolutely necessary.

6. CONCLUSIONS & FUTURE WORK

On-orbit collision risk management is becoming a ubiquitous and critical part of space mission operations. Technological advancements brought about by the sociopolitical awareness of on-orbit collision as a real and present space mission risk will continue to force the space situational awareness community to challenge current concept of operations and seek improved ways of doing business. The sun is setting on the days of evaluating discrete conjunctions uniquely. This paper has presented the concept of finite collision risk management and the treating of all prediction conjunctions in a finite look-forward period as a single risk to be assessed and mitigated. Such a paradigm shift is likely to be only one step towards preparing for the future of space operations, but some modification along these lines of the current paradigm is rapidly becoming necessary.

7. ACKNOWLEDGEMENTS

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