Tournament Rewards and Risk Taking[¤]

Hans K. Hvide[y]

Norwegian School of Economics (LOS) and

Berglas School of Economics, Tel-Aviv University

October 14, 1999

Abstract

I consider two seemingly unrelated puzzles; 1.Why is relative performance evaluation (RPE) used less in CEO compensation than agency theory suggests? 2.Why is sometimes, e.g., for fund managers, a mediocre performance more highly rewarded than excellence? I consider a simple tournament model, where agents can in‡uence the spread of output in addition to its mean. I show that standard tournament rewards induce risky and lazy behavior from the agents. This …nding sheds light on Puzzle 1. Second, I consider a scheme that ranks agents according to their relative closeness to a benchmark k. I show that there exists intermediate values of k such that the risky-lazy problem of the standard tournament can be mitigated. This result sheds light on Puzzle 2.

[¤]Thanks to Eddie Dekel, Thore Johnsen, Eirik G. Kristiansen, Jim March, Isaac Meilijson, Trond Olsen, Yossi Spiegel, Harald Troye, Yoram Weiss, Knut Aase, and to seminar participants at NHH, Tel-Aviv University and Tromso University for valuable comments and discussions. The main bulk of this paper was written while the author was on a post-doc stay at the Berglas School of Economics in Tel-Aviv, in the academic year 98/99. I warmly thank the school for its kind hospitality. Thanks also to the Maritime Program at NHH for providing …nancial support.

[y]Mailing adress: LOS-Centre, Rosenberggt 39, 5015 Bergen, Norway. Email: hans.hvide@nhh.no.

1 Introduction

To the extent that real-world rewards are based on measures of performance, they often depend on relative performance. For example, promotion is awarded to the most productive member of a level in an organization; the CEO of the least profitable firm in an industry gets fired, and the mutual fund with the highest return one year gets a higher investor inflow the next year.

The main theoretical rationale for rewarding relative performance stems from the Informativeness Principle (Holmstrom, 1982), which, informally, states that an optimal compensation contract conditions rewards on any variable that is (incrementally) informative about work intensity (effort). Recently, a corollary of the Informativeness Principle known as the relative performance evaluation (RPE) hypothesis has been extensively tested in the rapidly growing empirical literature on CEO compensation.[1] The idea behind the RPE hypothesis is that if firms in the same industry face some common random shock, like changes in industry demand, an optimal compensation contract for a CEO makes his payment conditional on the profits of the other firms in the industry (in addition to being made conditional on the profit of the CEO's own firm, and possibly other variables). The higher the profit of the other firms, the lower the reward of the CEO.

Although the findings of the empirical literature are not conclusive, researchers in the field tend to be puzzled by the lack of evidence for RPE in the CEO compensation data. For example, Aggarwal & Samwick (1999a) 'suggest that relative performance evaluation considerations are not incorporated into executive compensation contracts' (p. 104, ibid.). And, Murphy (1999, page 40) states that: 'The paucity of RPE in options and other components of executive compensation remains a puzzle worth understanding'.

A seemingly unrelated puzzle is that rewards based on relative performance do not always go in favor of the agent with the highest performance; sometimes 'modest' performances are more highly rewarded than 'very high' performances. For example, at workplaces, due to envy or other effects, very high performances are sometimes discouraged by peers.[2] While the rewards from peers are of informal nature, there are also

[1] See Murphy (1999) or Prendergast (1998) for an overview of this literature, and for more on the Informativeness Principle.

[2] Interestingly, one of the most well-known pieces in the Norwegian satiric literature, 'The Law of

examples where formal rewards work in favor of mediocrity. For example, fund manager compensation schemes sometimes has an outlier e¤ect: a very low return and a very high return yields a lower reward from the principal than performances in the middle. Such incentives are puzzling, since they seemingly do not motivate hard work, e.g., in acquiring and analyzing …nancial data.

The purpose of this paper is to show that adding a notion of risk taking to the agents' choice set in a standard tournament model both can shed light on both why RPE is used less than agency theory suggests (Puzzle 1), and why rewards are sometimes non-monotonic in performance (Puzzle 2).[3]

In a tournament, a principal sets a prize, and several agents then compete to attain the highest observed output, and win the prize. This paper departs from the existing literature on tournaments by assuming that agents can in‡uence the spread of their distribution of output, in addition to the mean. To increase the spread of output is assumed to have no intrinsic cost. Thus neither the …rm's expected pro…ts nor the worker's utility depends directly on the agents' choice of risk. However, there is an indirect link: if the equilibrium risk taking is high, then the marginal increase in the probability of winning from increasing e¤ort is low, and hence equilibrium e¤ort is low. A low equilibrium e¤ort in turn implies a less lucrative prize structure (since expected total production is low). Notice, however, that it is not obvious that equilibrium risk taking will be high; if an agent decides to work hard, then he has an incentive to choose a low risk.

It turns out that there is a huge moral hazard problem in the model. In Proposition 1, it is shown that with no limits to possible risk taking, agents exert zero e¤ort and choose an in…nite risk in equilibrium. Since the expected production is zero in this case,

---

Jante' by Aksel Sandemose, describes a society where excellence is strongly discouraged. For example, two of the ten laws of Jante are 'Thou should not believe you are better than anyone else', 'Thou should not believe you are something'. Although the laws of Jante tend to focus on self-beliefs, rather than accomplishment, it seems fair to say that they strongly discourage excellence.

[3] It should be emphasized that since data on the (explicit) incentives facing fund managers is scarce, Puzzle 2 is more a speculation than the …rmly established Puzzle 1. However, an example of a non-monotonic reward structure was provided to me by the CEO of Skandia Fund Managment (SFM), Harald Troye (SFM manages approximately $50 billion in the Scandinavian Market). SFM …rst selects an initial pool of fund managers. Second, the relationship is terminated with those managers that get a return too low or too high compared to a benchmark return. SFM engages in a long(er) term relationship with the remaining managers.

the tournament breaks down as a reward scheme. This result is somewhat modi…ed in Proposition 2, where possible risk taking is limited, but still the moral hazard problem is grave. Proposition 1 and Proposition 2 together indicate that a reason why CEO compensation to a small extent depends on the relative performance of the …rm is that putting too much emphasis on relative performance in compensation contracts may induce risky and lazy behavior from the CEOs.

Given this negative result, I ask whether the tournament reward scheme can be modi…ed to avoid the risky-lazy 'trap' of the standard tournament. To this end, a scheme where agents are ranked according to the relative closeness of their output to a benchmark k is considered. The idea behind this scheme, labeled k-contracts, is that excessive risk can be avoided, which in turn can provide incentives for working hard.

The second main result states that there exists intermediate values of the benchmark k such that …rst best level of e¤ort can be implemented under risk neutrality. This positive result sheds light on why sometimes higher rewards are given to agents with a modest performance than to agents with a very high performance.

It is not hard to motivate why risk taking is a plausible additional choice variable to e¤ort in environments where rewards are made conditional, or potentially are made conditional, on relative performance. For example, employees aspiring for promotion can choose to work in a risky environment or a safe environment before choosing how hard to work. CEOs can choose whether the …rm should pursue a safe or a risky R&D pro…le, and a variety of other decisions that also a¤ects the risk pro…le of a …rm.[4] And, …nally, fund managers can choose the riskiness of their portfolio, in addition to choosing how much resources to spend on providing and analyzing relevant stock information.

I now discuss related literature. First the literature on tournaments is dealt with, and then I turn to discussing literature that provides alternative explanations to the two puzzles outlined. Several recent papers in the …nancial literature employ tournament models. For example, Brown et al (1996) and Chevalier & Ellison (1997) consider the portfolio choice of mutual fund managers, and '... suggest that viewing the mutual fund market as a tournament in which all funds having comparable investment objectives compete with

[4] For example, the CEO can in‡uence which type of workers the …rm employs, or whether the …rm should enter emerging markets or not.

one another provides a useful framework for a better understanding of the portfolio managment decision-making process' (Brown et al., p. 85). Importantly, the empirical results of those papers indicate that the in‡ow of new investors to a fund depends crucially on its relative performance, and moreover that risk taking is an important decision variable for mutual fund managers.

Tournaments were …rst studied by the classic Lazear & Rosen (1981), who showed that a …rst best provision of e¤ort can be implementable with a tournament reward scheme.[5] Tournament models with risk taking as the choice variable was …rst considered by Bronars (1987).[6] Importantly, the papers of the received tournament literature consider e¤ort or risk taking as choice variables for the agents, in contrast to the present paper that considers the interaction between e¤ort and risk taking.

On Puzzle 1, Aggarwal & Samwick (1999b) argue that imperfect competition in the product markets can neutralize the RPE e¤ect on compensation schemes. (However, if Cournot competition prevails, rather than Bertrand competition, the model of Aggarwal & Samwick (1999b) strengthens the prediction of RPE hypothesis.) In contrast to Aggarwal & Samwick (1999b), I point out harmful e¤ects of RPE in compensation schemes, even when product markets are competitive.[7]

With respect to Puzzle 2, Heinkel & Stoughton (1994) derives an optimal reward scheme for fund managers, where managers with a 'too' high return will be replaced. However, this result refers to the solution of an adverse selection problem, while the scheme proposed in the present paper solves an unrelated moral hazard problem. Moreover, there

---

[5] Later papers in the tournament literature include Nalebu¤ & Stiglitz (1983), Bhattacharya & Guasch (1988), Clark & Riis (1998), and Fullerton & McA¤ee (1999).

[6] Bronars (1987) showed that leaders in a sequential tournament have an incentive to 'lock in' their gains by playing safe, while followers choose a risky strategy. Recently, several papers have proved a variety of other properties of risk taking in tournaments. For example, Cabral (1997) proves a similar result to Bronars (1987). Dekel & Scotchmer (1999), using tools from evolutionary game theory, …nd an evolutionary pressure towards risk loving preferences if those who breed in a population is determined in a contest-like fashion (and where a child inherits the risk preferences of its parents). And Hvide & Kristiansen (1999), investigating the selection properties of tournaments, show that the (expected) quality of the winning agent may decrease in the (average) quality of all the competing agents.

[7] An older argument against RPE is that compensation schemes that put too much weight on relative performance are sensitive to collusion between the agents that are compared. For illustration, if the sum of compensation for two workers is constant, then both workers would be better o¤ if they could collude in slacking their e¤ort. However, since collusion typically requires a long-term relationship, such arguments seems more applicable to explain lack of intra-…rm RPE than lack of inter-…rm RPE.

is no notion of risk taking in the model of Heinkel & Stoughton (1994).[8]

Part 2 sets up the model and contains the analysis, while Part 3 concludes. Some of the proofs are relegated to the appendix.

2 Analysis

Section 2.1. sets up a standard tournament model with effort as the only choice variable; Section 2.2 adds a notion of risk taking to that model, and Section 2.3 introduces k-contracts.

2.1 The Tournament Model

There is one risk-neutral principal and several agents, for convenience assumed to be only two.[9] To focus on incentive effects, the agents are assumed to be risk-neutral. The value of agent i's output equals $Y_i = \mu_i + \varepsilon_i$, where μ_i is agent i's choice of effort, and where ε_i is an iid shock with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = \sigma^2$. Y_i and Y_j are the only contractible variables. The cost of effort for agent i, $V_i(\mu_i)$ is assumed to satisfy V_i', V_i'' > 0 and $V_i(0) = V_i'(0) = 0$. The first-best level of effort, denoted μ_i^*, is the μ_i that solves $V'(\mu_i) = 1$. For convenience, there are only two agents, with a symmetric cost of effort; $V_1(\cdot) = V_2(\cdot) = V(\cdot)$. Furthermore, ε_i is assumed to be normally distributed.[10]

Under a rank-order scheme, the principal fixes the prizes W_1 and W_2 [where $W_1 > W_2$], and the agents then compete in winning the first prize W_1, which is awarded to the agent with the highest Y_i. Expected utility for agent i, U_i, equals,

[8] In their seminal paper on the basic principal-agent model, Grossman & Hart (1982) construct an example where a non-monotonic reward scheme can solve a moral hazard problem, but due to assumptions that violate the monotone likelihood ratio property. [Briefly, a distribution of output that satisfies MLRP has the property that the higher observed output, the harder the agent probably worked.] In contrast to Grossman & Hart, I show that a non-monotonic scheme may perform well without assuming any violation of MLRP.

[9] All results can easily be generalized to hold for an arbitrary number of agents.

[10] With the exception of the normality assumption, the model in this section is identical to the standard tournament model of Lazear & Rosen (1981) [the normality case is encompassed by their model]. As will be clear below, the normality assumption is not required to obtain the results.

$$U_i = P_i W_1 + (1 - P_i) W_2 - V(\mu_i) = P_i \Delta W + W_2 - V(\mu_i) \qquad (1)$$

where $\Delta W = W_1 - W_2$, and $P_i = Prob(Y_i > Y_j) = Prob(\mu_i - \mu_j > \epsilon_j - \epsilon_i)$. For agent 1 we get, $P_1 = Prob(Y_1 > Y_2) = Prob(\mu_i - \mu_j > \epsilon) = G(\mu_1 - \mu_2)$, where $G(.,.)$ is the cdf of ϵ $[\epsilon \equiv \epsilon_2 - \epsilon_1]$. Clearly ϵ is normally distributed with $E(\epsilon) = 0$ and $E(\epsilon^2) = 2\sigma^2$. The first order condition for optimal provision of effort becomes,

$$\frac{\partial U_i}{\partial \mu_i} = \frac{\partial P_i}{\partial \mu_i} \Delta W - \frac{\partial V}{\partial \mu_i} = 0; \; i = 1, 2. \qquad (2)$$

Notice that due to the option-like structure of the prizes, only the difference between the first and the second prize, ΔW, enters the first order conditions. By symmetry, if there exists an equilibrium, then in equilibrium $\mu_1 = \mu_2$, and the outcome is purely random, i.e., $P = \frac{1}{2}$, since $G(0) = \frac{1}{2}$. By substituting $\mu_1 = \mu_2$ in (2), equilibrium effort, μ_i^*, can be characterized by,

$$\frac{\partial V}{\partial \mu_i} = \Delta W g(0); \, i = 1, 2. \qquad (3)$$

Inserting for the normal density,

$$\frac{\partial V}{\partial \mu_i} = \frac{\Delta W}{2\sqrt{\sigma^2 \pi}}, \; i = 1, 2 \qquad (4)$$

From inspecting (4), it can easily be seen that μ_i^* is implementable with an appropriate choice of ΔW. This observation leads to the well-known equivalence result: under risk neutrality, both individual schemes (that base compensation on individual output) and tournaments can implement first best level of effort. As we shall see in the next section, the equivalence between individual schemes and tournaments is no longer true when agents in addition to effort can choose risk.

Notice also that it follows from (4) that the equilibrium effort is decreasing in σ. Intuitively, a higher σ makes the tournament 'more random', which decreases the marginal gain of increasing effort (the increased probability of winning), and hence reduces equilibrium effort.

2.2 Risk Taking in the Tournament Model

I adhere to the assumptions of the previous section, with one exception. The shock ε_i is now assumed to have variance equal to η_i^2, where $\eta_i^2 = \sigma^2 + s_i^2$, with $\sigma > 0$ and $s_i \in \mathbb{R}_+$. The interpretation of σ is the level of non-diversifiable, background, noise, and s_i is the degree of voluntary spread in the output distribution. Thus s_i is a choice variable for agent i, while σ is, as before, a parameter. The cost of adjusting s_i is assumed to be uniformly zero, implying that the first best level of effort is not affected from introducing risk taking.

Notice that risk taking added to the agents' choice set makes no difference for the efficiency of individual schemes, they still implement the first best level of effort.[11] It is now showed that the equivalence between linear schemes and tournaments breaks down when risk taking is added to the agents' choice set.

Proposition 1 *The unique equilibrium in the tournament game induces infinite variance and zero effort from both agents.*

Proof. I first show that $X^* = \{s_i = s_j = \infty$ and $\mu_i = \mu_j = 0\}$ is a Nash Equilibrium, and then show uniqueness. Suppose $\mu_i = 0$ and $s_i = \infty$. Then agent j wins with probability $\frac{1}{2}$ irrespectively of his choice of μ_j and s_j. Therefore, $\mu_j = 0$ and $s_j = \infty$ is a best reply to $\mu_i = 0$ and $s_i = \infty$, and hence X^* is a NE. To prove that X^* is a unique NE, first consider tuples with (i) $\mu_i < \mu_j$. For (i) to be a Nash equilibrium, clearly $s_i = \infty$, since that choice of s_i maximizes P_i. That implies $\mu_i = 0$. But, in that case $\mu_j = 0$ is a best reply from agent j, which contradicts (i). So in any Nash equilibrium we have

[11] Consider a linear scheme with reward to agent i equal to, $a + b_i Y_i$, where a and b_i are parameters set by the principal. The principal sets $b_i = 1$, and expected utility of an agent equals $\mu_i - V(\mu_i)$, independently of the choice of risk. The first order condition for optimal effort thus becomes, $1 - V'(\mu_i) = 0$, which is identical to the first order condition for first best. Hence individual schemes can implement first best even when risk taking is an additional choice variable.

that $\mu_i = \mu_j$. Tuples with (ii) $\mu_i = \mu_j > 0$ are now excluded. If $\mu_i = \mu_j$ then $P = 1/2$. But since both players have positive cost of effort, player i can gain by changing μ_i (one obvious improvement is to set $\mu_i = 0$ and $s_i = \infty$). But then we are in case (i). Hence neither (i) nor (ii) is consistent with Nash behavior, and X^* is a unique NE. ■

Thus if agents can choose their level of risk taking, in addition to their effort, a tournament induces extremely risky and lazy behavior from workers. Hence the equivalence between individual schemes and tournaments is no longer true when risk taking is added to the agents' choice set. Proposition 1 contradicts the intuition of Lazear & Rosen (1981), which state: "In this paper the worker has no choice over [the variance of individual output]. This does not affect the risk neutral solution but does have an effect if workers are risk averse, since they tend to favor overly cautious strategies ... ." (footnote 1, page 843).[12]

Since Proposition 1 is obtained under rather special assumptions, let me comment on its robustness. First notice that exactly the same argument, and the same negative result, goes through if risk-averse agents play the tournament. Hence agents choose infinite variance and zero effort in equilibrium even if they are risk averse. Second, independence and normality of the shocks is not necessary to obtain the result; all that is required by $G(\cdot)$ for the result to go through is that $g(0)$ is decreasing in σ_i. Since lack of independence in the sense of a positively correlated shocks is one of the main justifications for applying tournaments (see e.g., Nalebuff and Stiglitz 1983), it is worth noticing that the negative result also holds for any degree of correlation between the shocks, provided that an equilibrium exists. Hence, Proposition 1 is rather robust in these respects.

However, since the meaning of 'infinite variance' is somewhat unclear, it seems useful to consider a case where there are limits to risk taking. It is now assumed that $s_i \in [s^{min}, s^{max}]$, $\forall i$, where $0 < s^{min} < s^{max}$, with s^{max} finite. Hence risk taking is bounded by a lower limit s^{min} and an upper limit s^{max}. To avoid non-existence problems, I consider the game where the agents first choose level of risk taking and then, after observing each

[12] Also Murphy (1999) seems to be overly optimistic with respect to the optimality properties of RPE when the agent has additional choice variables to effort (page 41, ibid.): 'RPE remains a strong prediction of the model after expanding the managerial action set, since paying based on relative performance provides essentially the same incentives as paying based on absolute performance, while insulating risk-averse managers from common shocks.'

others choice, decide how hard to work.[13] The following result indicates that the basic moral of Proposition 1 also holds with more general assumptions on possible risk taking.

Proposition 2 In the unique subgame perfect equilibrium, both agents choose $s_i = s^{max}$. in the first stage, and the corresponding low effort in the second stage.

Proof. In equilibrium at stage 2, $P = \frac{1}{2}$ independently of level of risk taken at stage 1. Since the equilibrium effort is a decreasing function in the sum of s_1 and s_2, both agents choose $s_i = s^{max}$ at stage 1, in dominant strategies. Since the equilibrium risk taking at stage 1 is high, the equilibrium effort at stage 2 is consequently low. ■

Proposition 2 shows that even when there are limits to risk taking, the moral hazard problem induced by a tournament reward structure is serious: equilibrium behavior by the agents is risky and lazy. Notice that a comparative statics exercise on s^{max} yields a simple result; the equilibrium effort is monotonically decreasing in s^{max}. This can be interpreted as the greater opportunity of taking risk, the less efficient is a tournament reward structure.[14]

Since real life rewards typically are conditioned on a mixture of relative and absolute performance measures, it is important to see whether the result is robust to letting the incentive scheme also depend on absolute measures. Making the compensation to agent i depend on his absolute output, in addition to relative output, through a linear component, would make the agent care about expected output, in addition to the probability of winning. But the linear component in the scheme does not affect the incentives for risk taking; the agents will still choose s^{max} in the first stage to decrease the equilibrium effort in the second stage. Hence the risky-lazy problem is present also when the compensation scheme combines relative and absolute factors.[15]

[13] For example, sales persons choose an area to work in before deciding how many hours to work, a CEO determines the R&D profile of the firm before deciding how much effort to put into the job, and a fund manager specializes in low or high-risk shares before deciding how much information to collect and process.

[14] Thus if the principal can affect s^{max} cheaply, he would set it as low as possible. Notice also that Proposition 1 is not a corollary of Proposition 2, due to the sequential structure imposed in Proposition 2.

[15] However, if a term that is linear in output is added to the payment scheme and agents are risk averse, then there would be a risk-return trade-off that is worth studying further.

To conclude, Proposition 1 and Proposition 2 together indicate that when agents can choose both level of effort and level of risk taking, rewarding relative performance induces risky and lazy behavior in equilibrium. A direct empirical value of this finding is that it sheds light on Puzzle 1, why relative performance evaluation is used less in CEO compensation than what standard agency theory suggests. Specifically, if risk taking is a choice variable for a CEO then the principal (e.g., the board) should be careful in conditioning rewards on the performance of other CEO's, since such schemes induce risky and lazy behavior from CEOs.

Since a tournament insures agents against common shocks, it is interesting to see whether a tournament can be modified to avoid the risky-lazy 'trap' of standard tournaments. I turn to that task in the next section. The positive result obtained will shed light on Puzzle 2, why a modest performance is sometimes more highly rewarded than a very high performance.

2.3 A Modified Tournament: k-contracts

The idea behind the contract form proposed in this section is that if agents are motivated to achieve a moderately high output, instead of a very high output, they might get an incentive to choose a moderate level of risk taking, which, in the next turn, can create incentives to work hard.[16]

Consider a modified tournament reward structure, where the winner of the tournament is the agent with output closest to a finite benchmark k. To avoid confusion with standard tournaments, this modified tournament structure is labeled k-contracts.

The distance between k and agent i's observed output, D_i, equals,

$$D_i = |Y_i - k| \tag{5}$$

Denote by $Q_i(.)$ agent i's probability of having an observed output closer to k than

[16] An often-voiced criticism of non-monotonic schemes is that they give incentives to dispose with parts of the output (if output falls in the non-monotonic range). However, since disposal is equivalent to theft, this criticism applies to all compensation schemes with marginal reward less than marginal output. Contracts in practice prescribe punishment for theft (or 'disposal'), if detected. Here, I simply assume that disposal is not a choice variable for the agents.

agent j, and hence win the tournament. Formally, $Q_i(\ldots) = Prob(D_i < D_j)$. The expected utility for agent i under a k-contract then equals,

$$U_i = Q_iW_1 + (1 - Q_i)W_2 - V(\mu_i) = Q_i \cdot W + W_2 - V(\mu_i) \tag{6}$$

The following remark clarifies the relation between k-contracts and standard tournaments.

Remark 2.1 If $k = \infty$ the agents play a standard tournament game.

Proof. Recall that $P_i(\ldots)$ is the probability of agent i winning in the standard tournament case, where $P_i = Prob(Y_i - Y_j > 0)$. I show that $Q_i(\ldots)$ and $P_i(\ldots)$ converge when k goes to infinity. By definition, $Q_i(\ldots) = Prob(D_i < D_j)$. Since $D_i > 0$; we have that,

$$Q_i(\ldots) = Prob(D_i < D_j) = Prob(D_i^2 < D_j^2) = Prob[(Y_i - Y_j)(Y_i + Y_j - 2k) < 0]. \tag{7}$$

When k tends to infinity, $(Y_i - Y_j)(Y_i + Y_j - 2k) < 0$ occurs if and only if $(Y_i - Y_j) > 0$. Hence, from (7), $Q_i(\ldots) = Prob[(Y_i - Y_j)(Y_i + Y_j - 2k) < 0]$ converges to $Prob(Y_i - Y_j > 0) = P_i(\ldots)$ when k tends to infinity. ■

Remark 2.1 shows that standard tournament reward structure is a special case of k-contracts; when k tends to infinity, a k-contract and a standard tournament, as studied in the previous sections, are identical.

It is intuitively clear that a finite k will induce finite levels of risk in equilibrium, when equilibria exist. However, it is not clear exactly which level of risk taking that will occur, as a function of k. In providing a link between k and risk taking, the following lemma will be useful. First a standard definition.

Definition 2.1 (FOSD). Let $G_i(d; \ldots)$ and $H_i(d; \ldots)$ be cdf's of D_i. $G_i(d; \ldots)$ first order stochastic dominates $H_i(d; \ldots)$ if $G_i(d; \ldots) \geq H_i(d; \ldots)$ for all d, with $G_i(d; \ldots) > H_i(d; \ldots)$ for some d.

Let $F(d; \sigma_i)$ be the cdf of D_i, as a function of σ_i, holding μ_i and k constant at $\hat{\mu}_i$ and

$\hat{k}$, respectively, where $\hat{k} > \hat{\mu}_i$. Furthermore, define $\eta_i^* = \hat{k} - \hat{\mu}_i$. Now choose two values of η_i, denoted η_i^1 and η_i^2, where $\eta_i^1 < \eta_i^2$. Then we have the following.

Lemma 1 $F(d; \eta_i^1)$ first order stochastic dominates $F(d; \eta_i^2)$, for $\eta_i^* \leq \eta_i^1 < \eta_i^2$.

Proof. See the appendix. ■

Lemma 1 puts an upper bound on the risk taking of agent i in that any choice of standard deviation η_i larger than η_i^* generates a distribution of D_i that is dominated. The intuition for Lemma 1 is that η_i^* is the choice of standard deviation that maximizes the probability of hitting very close to the benchmark k. If η_i is set larger than η_i^* then the distribution generated will perform worse with respect to the probability of hitting very close to k, and the potential gains from an increased probability of hitting farther from k does not offset this effect.

Corollary 1 Suppose $\sigma > k$. Then $s_i = 0$ is a (strictly) dominating choice for agent i.

Proof. First notice that regardless of η_i, it is dominated for agent i to choose $\mu_i > k$. Now fix μ_i at $\hat{\mu}_i$ and k at $\hat{k}$, where $\hat{\mu}_i \leq \hat{k}$, and recall that $\eta_i^* = \hat{k} - \hat{\mu}_i$. By a simple transformation, it follows that a choice of s_i^2 larger than s_i^{*2} is dominated, where $s_i^{*2} = (\hat{k} - \hat{\mu}_i)^2 - \sigma^2 = (k^2 - \sigma^2) + \mu_i(\mu_i - 2k)$, which is negative for $\sigma > k$. It follows from Lemma 1 that $s_i = 0$ is a dominating choice for agent i. ■

The corollary shows that $\sigma > k$ is a sufficient condition for agents to choose $s_i = 0$ in equilibrium.

Equipped with these results, we have the following.

Proposition 3 For a sufficiently large σ, the first best provision of effort is implementable with a k-contract.

Proof. See the appendix. ■

Hence in contrast to the standard tournament scheme, k-contracts and individual schemes are equivalent: they both implement first best. The intuition behind Proposition 3 is that to avoid excessive risk taking, and hence a low level of effort, the principal rewards the agent with output closest to a positive constant k rather than rewarding the highest output. Reduced risk taking in turn makes it possible to give incentives for effort by increasing the prize spread, ΔW.

Notice that first best can also be implemented if agents in addition to being rewarded for relative performance, are also rewarded according to their absolute performance. So Proposition 3 is robust to adding compensation based on individual performance to the scheme. Second, it is sufficient for Proposition 3 that the distribution of the shocks has the FOSD property described in Lemma 1. In addition to the normal, a simple distribution as the uniform also has this property.[17]

The empirical value of Proposition 3 is that it gives an explanation for the second puzzle, why mediocrity is sometimes more highly valued than excellence, e.g., in fund manager compensation schemes.

Since linear schemes can also implement first best in the case where agents choose both effort and risk, it is not obvious why k-contracts should be preferred to linear schemes. The downside with individual schemes compared to k-contracts, however, is that they do not exploit commonality of the shocks, which may important e.g., in the market for fund managers. Hence k-contracts can insure risk averse agents as well as linear schemes, and provide stronger incentives. I have computed examples with risk averse agents and found that k-contracts can dominate linear schemes, provided that agents are not too risk averse and that shocks are sufficiently correlated.[18]

3 Conclusion

This paper has considered an agency problem where agents choose level of risk in addition to effort. Two main results were obtained. First, opening up for risk taking seriously increases the moral hazard problem in a tournament; the equilibrium level of risk taking becomes high, and the equilibrium level of effort becomes low. This result, it was argued,

[17] It is unknown whether more general distributions have the FOSD property of Lemma 1. However, even if a distribution of errors does not satisfy the FOSD property, it is conjectured that a modified technique can be applied to make Proposition 3 hold also for distributions of the shocks that do not satisfy the FOSD property.

[18] These examples have been obtained with numerical tecniques for utility functions with constant absolute risk aversion, and can be provided upon request.

can shed light on Puzzle 1, that RPE is used less in practice than indicated by the Informativeness Principle. Second, by modifying the tournament reward structure to give the prize to the agent with output closest to a …nite benchmark k, rather to the one with the highest output, …rst best can be implemented. This positive results seems useful in understanding why compensation schemes, e.g., for fund managers, are sometimes non-monotonic.

4 References

Aggarwal, R. K. and A. A. Samwick (1999a). The Other Side of the Trade-o¤: The Impact of Risk on Executive Compensation. Journal of Political Economy, 107, 65-105.

Aggarwal, R. K. and A. A. Samwick (1999b). Executive Compensation, Strategic Competition, and Relative Performance Evaluation: Theory and Evidence. Journal of Finance, forthcoming, December issue.

Bronars, S. (1987). Risk Taking in Tournaments. Working paper, University of Texas.

Brown, K. C., W. V. Harlow, and L. T. Starks (1996). Of Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry. Journal of Finance, 51, 85-110.

Cabral, L. (1997). Sailing, Tennis and R&D: The Choice of Variance and Covariance in Sequential Tournaments. Working paper, London Business School.

Chevalier, J. and G. Ellison. (1997). Risk taking by mutual funds as a response to incentives. Journal of Political Economy, 105, 1167-1200.

Clark, D. and C. Riis. (1998). Contests With More than One Prize. American Economic Review, 88, 276-89.

Dekel, E. and S. Scotchmer. (1999). On the Evolution of Preferences in Winner-Takes-All Games. Working paper 4-99, Berglas School of Economics, Tel-Aviv University. Forthcoming, Journal of Economic Theory.

Fullerton, R. L. and R. P. McAfee (1999). Auctioning Entry Into Tournaments. Journal of Political Economy, 107, 573-606.

Grossman, S. J. and O. Hart. (1983). An Analysis of the Principal-Agent Problem. Econometrica, 51, 7-35.

Heinkel, R. and N. M. Stoughton (1994). The Dynamics of Portfolio Management Contracts. The Review of Financial Studies, 7, 351-87.

Holmstrom, B. (1982). Moral Hazard in Teams. Bell Journal of Economics, 13, 324-40.

Hvide, H. K. and E. G. Kristiansen (1999). Risk Taking in Selection Contests. Working Paper 5-99, Berglas School of Economics, Tel-Aviv University

Lazear, E. and S. Rosen (1981). Rank-Order Tournaments as Optimum Labor Contracts. Journal of Political Economy, 89, 841-64.

Murphy, K. J. (1999). Executive Compensation. Forthcoming, Handbook of Labor Economics, Vol. 3, North Holland (eds. O. Ashenfelter and D. Card). Available at http://www-rcf.usc.edu/~kjmurphy/working.htm.

Nalebuff, B. and J. Stiglitz. (1983). Prizes and Incentives: Towards a General Theory of Compensation and Competition. Bell Journal of Economics, 14, 21-43.

Prendergast, C. (1998). The Provision of Incentives in Firms. Journal of Economic Literature, 37, 7-63.

5 Appendix on k-contracts

I start out with a remark establishing some distributional properties of the stochastic variable D_i, the distance between agent i's output Y_i and the benchmark k. Throughout the appendix, I skip subscripts when possible.

Remark 5.1 D has cdf equal to $F(d;\ldots) = \frac{1}{\sqrt{\pi}}\int_{\alpha}^{\beta} e^{-t^2}dt$, where $\alpha = \frac{\sqrt{2}(\mu - k - d)}{2\sigma}$, and $\beta = \frac{\sqrt{2}(\mu - k + d)}{2\sigma}$.

Proof. Recall that $D = |k - Y|$, where Y is normally distributed with mean $k - \mu$ and variance σ^2. Hence the cdf of D equals,

$$F(d;\ldots) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{k-d}^{k+d} e^{-\frac{(d-\mu)^2}{2\sigma^2}}dd \tag{8}$$

where $d \geq 0$. This is just the probability that a single realization of normally distributed variable with expectation μ and variance σ^2 falls within a distance d of a benchmark k.

By standard procedures, the integral simplifies to,

$$F(d;\cdot) = \frac{1}{\sqrt{\pi}}\int_{\alpha}^{\beta} e^{-t^2}dt, \text{ where } \alpha = \frac{\sqrt{2}(\mu - k - d)}{2\eta}, \text{ and } \beta = \frac{\sqrt{2}(\mu - k + d)}{2\eta} \tag{9}$$

It is easily checked that $F(d;\cdot)$ indeed induces a probability distribution, i.e., that $\lim_{d\to\infty} F(d;\cdot) = \lim_{d\to\infty} \frac{1}{\sqrt{\pi}}\int_{\alpha}^{\beta} e^{-t^2}dt = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-t^2}dt = 1$. Notice that since D_i^2 is χ^2-distributed, D_i is distributed as the square root of a χ^2 variable.

Differentiating $F(d;\cdot)$ with respect to d, we obtain the density $f(d;\cdot)$,

$$f(d;\cdot) = \frac{\partial F(d;\cdot)}{\partial d} = \frac{e^{-\frac{(\mu - k - d)^2}{2\eta^2}} + e^{-\frac{(\mu - k + d)^2}{2\eta^2}}}{\sqrt{2\pi\eta^2}} \tag{10}$$

■

Proof. of Lemma 1.

Recall that, by definition, $\eta^2 = \sigma^2 + s^2$, and $\eta^* = \hat{k} - \hat{\mu}$. I wish to show that any choice of η greater than η^* is dominated in the sense of FOSD.

Substitute $\mu = \hat{\mu}$ and $k = \hat{k}$ into $F(d;\cdot)$ from Remark 1, substitute for η^*, and differentiate with respect to η, to obtain,

$$\frac{\partial F(d;\cdot)}{\partial \eta} = \frac{1}{\sqrt{2\pi\eta^2}}\left[(\eta^* - d)e^{-\frac{(\eta^* - d)^2}{2\eta^2}} - (\eta^* + d)e^{-\frac{(\eta^* + d)^2}{2\eta^2}}\right] \tag{11}$$

I proceed to show that this expression is negative for $\eta > \eta^*$, and hence Lemma 1 follows. Denote the first term of the right side of (11) by A_1, and the second term by A_2. Moreover, substitute in $\eta^* + \alpha$ for η, where $\alpha > 0$. Hence $A_1 = (\eta^* - d)e^{-\frac{(\eta^* - d)^2}{2(\eta^* + \alpha)^2}}$ and $A_2 = (\eta^* + d)e^{-\frac{(\eta^* + d)^2}{2(\eta^* + \alpha)^2}}$. Since $A_2 > 0$, $\frac{\partial F(d;\cdot)}{\partial s} < 0$ is equivalent to $\frac{A_1}{A_2} < 1$, for $d > 0$. I finish the proof by showing that $\frac{A_1}{A_2} < 1$, for $d > 0$.

$$\frac{A_1}{A_2} = \frac{(\eta^* - d)e^{-\frac{(\eta^* - d)^2}{2(\eta^* + \alpha)^2}}}{(\eta^* + d)e^{-\frac{(\eta^* + d)^2}{2(\eta^* + \alpha)^2}}} = \frac{\eta^* - d}{\eta^* + d} e^{\frac{2d\eta^*}{(\eta^* + \alpha)^2}} = \frac{\eta^* - d}{\eta^* + d} e^{\frac{2d\eta^*}{(\eta^* + \alpha)^2}} \tag{12}$$

Notice that from (12) it follows that $\frac{A_1}{A_2} = 1$ when $d = 0$. I show that $\frac{A_1}{A_2} < 1$ for any $d > 0$. Differentiating (12) with respect to d yields,

$$\frac{\partial(\frac{A_1}{A_2})}{\partial d} = -2 \frac{e^{\frac{2d}{(\eta^* + \alpha)^2}} \eta^*(2\eta^*\alpha + \alpha^2 + d^2)}{(\eta^* + d)^2(\eta^* + \alpha)^2} \tag{13}$$

which is negative for $d > 0$. Hence $\frac{A_1}{A_2} < 1$, for $d, \alpha > 0$, and consequently $\frac{\partial F(d, \ldots)}{\partial \eta} < 0$ for $\eta > \eta^*$, and $d > 0$, and Lemma 1 follows. ■

Proof. of Proposition 3.

Suppose σ is larger than the first best level of effort, μ_i^*. I show that this condition is sufficient for first best to be implementable. First notice that, for a given k, to choose effort level μ_i larger than k is a dominated choice for agent i. Hence we can restrict attention to $\mu_i \in [0, k]$, $i = 1, 2$. Moreover, choose k such that $\mu_i^* < k < \sigma$. Then, by Corollary 1, $s_i = 0$ is a dominating strategy for agent i, and we can restrict attention to solve for equilibrium in choice of effort. The first order conditions are,

$$\frac{\partial U_i}{\partial \mu_i} = \frac{\partial Q_i}{\partial \mu_i} \cdot W - \frac{\partial V_i}{\partial \mu_i} = 0; \ i = 1, 2. \tag{14}$$

The probability of agent i winning under a k-scheme, $Q_i(.\,.)$, equals,

$$Q_i = \int_0^{\infty} F_i(d) f_j(d)\,dd \tag{15}$$

$$= \int_0^{\infty} \frac{e^{-\frac{(\mu_j - k - d)^2}{2\sigma^2}} + e^{-\frac{(\mu_j - k + d)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\alpha} e^{-t^2} dt\right] dd \tag{16}$$

Define $erf(x) = \int_{-\infty}^{x} e^{-t^2} dt$, and $erfc(x) = 1 - erf(x)$. Differentiate (..) by μ_1 and normalize by setting $\sigma = 1$ to obtain,

$$\left.\frac{\partial Q_1}{\partial \mu_1}\right|_{\mu_1<\mu_2} = -\frac{1}{2\sqrt{\pi}}\{[(e^{\frac{1}{4}(\mu_1+\mu_2-2k)^2})(erfc(k - \frac{1}{2}\mu_1 - \frac{1}{2}\mu_2) - e^{\frac{1}{4}(\mu_1-\mu_2)^2} + \tag{17}$$

$$e^{\frac{1}{4}(\mu_1-\mu_2)^2}(erfc(\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2)]e^{\mu_1+\mu_2-k-\frac{1}{2}\mu_1^2-\frac{1}{2}\mu_2^2}\} \tag{18}$$

while,

$$\left.\frac{\partial Q_1}{\partial \mu_1}\right|_{\mu_1>\mu_2} = -\frac{1}{2\sqrt{\pi}}\{[(e^{\frac{1}{4}(\mu_1+\mu_2-2k)^2})(erfc(k - \frac{1}{2}\mu_1 - \frac{1}{2}\mu_2) + e^{\frac{1}{4}(\mu_1-\mu_2)^2} - \tag{19}$$

$$e^{\frac{1}{4}(\mu_1-\mu_2)^2}(erfc(\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2)]e^{\mu_1+\mu_2-k-\frac{1}{2}\mu_1^2-\frac{1}{2}\mu_2^2}\} \tag{20}$$

Substitute for $\mu_1 = \mu_2$ to obtain,

$$\left.\frac{\partial Q_i}{\partial \mu_i}\right|_{\mu_i=\mu_j} = \frac{erf(k - \mu_i)}{2\sqrt{\pi}} \tag{21}$$

which is continuous and increasing in k. Therefore, since $V(.\,.)$ is convex, the symmetric equilibrium is increasing in k. From equation (2) and equation (21) it is evident that the symmetric equilibrium is increasing (continuously) in ΔW, where equilibrium effort equals k, in the limit. Hence there exist a ΔW such that the efficient provision of effort is implemented. ■