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TOWARD A BETTER UNDERSTANDING OF FRICTION AND HEAT/MASS TRANSFER IN MICROCHANNELS – A LITERATURE REVIEW*

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TOWARD A BETTER UNDERSTANDING OF FRICTION AND HEAT/MASS TRANSFER IN MICROCHANNELS – A LITERATURE REVIEW

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ABSTRACT

A critical review of published results is presented here to provide a better understanding of microchannel transport phenomena, together with the framework for future research. The main conclusions are (a) the onset of transition to turbulent flow in smooth microchannels does not occur if the Reynolds number is ≤1,000; (b) the Nusselt number varies as the square root of the Reynolds number in laminar flow; and (c) satisfactory estimates of transfer coefficients, within the accuracy of experimental errors, can be obtained by using either experimental results for smooth channels with large hydraulic diameter or conventional correlations.

1. INTRODUCTION

It is convenient to adopt an arbitrary definition of a microchannel as one in which the hydraulic diameter, D_h, ≤ 1000 μm (1 mm). Friction, heat and mass transfer in microchannnels are active research areas, especially within the past ten years. Potential applications include heat exchanger technology, cooling of electronic components, fuel processing for fuel-cell-powered vehicles, combustion and advanced propulsion systems, and reactors for a range of processes. A general review of the literature on microscale heat transfer was presented by Duncan and Peterson [1].

The unique feature of microchannels is the existence of mostly laminar flow; this is a primary reason for the interest in microreactors. Even for situations in which transitional and turbulent flows exist, the Reynolds numbers are usually <5,000. The theoretical significance of microchannel research is that it should provide clearer insight into laminar-flow heat/mass transfer than is possible with conventional flow systems. However, a study of the literature on single-phase friction and heat/mass transfer revealed that the present state of knowledge is unsatisfactory.

The objectives of this presentation, which focuses on critical analysis, comparisons, and discussion of published data, are two-fold: first, to provide a better understanding of microchannel transport phenomena by addressing and resolving some of the disagreement among various results, and second, to provide the framework for future research. The original intention was to present a review that covered single-phase friction and heat/mass transfer, adiabatic two-phase (gas/liquid) flow, boiling heat transfer, and microchannel fabrication. However, due to space limitations and the desire to maintain manageable length, only single-phase flows are considered here.

2. TRENDS FOR CONVENTIONAL CHANNELS

For microchannels, the published results for frictional pressure coefficients either agree with, or differ from, the well-established relationships. For heat transfer in laminar flow, nearly constant Nusselt numbers have been reported, and some results show that Nusselt number (or Sherwood number, Sh, in the case of mass transfer) increases with increasing Reynolds number. For heat transfer in turbulent flow, the conclusion from numerous studies is that the conventional correlations are not applicable to microchannels. Consequently, many empirical correlations have been proposed, but none has been verified. Moreover, an important observation that emerges from the literature is that critical Reynolds numbers, Re_c, for transition to turbulent flow can be significantly lower than those reported for larger-diameter channels. With background knowledge such as this, it is instructive to provide an overview of typical trends for friction factor and Nusselt number obtained with a larger-diameter smooth tube.

Figure 1 is a compact presentation of friction factor (f), $Nu/Re^{1/2}$, and C_h versus Re for a smooth tube with an inside diameter of 13.39 mm (Obot *et al.* [2]; Obot [3]; Esen *et al.* [4]). In laminar and turbulent flow, f follows the established relationships (solid lines). Another observation is that $Nu/Re^{1/2}$ or C_h is constant in laminar flow and increases with increasing Re after the onset of transition to turbulent flow. The Re_c values are 2,040 and 1,900 for Pr = 0.7 and 6.8, respectively. Of particular importance to the present consideration is the consistency in Re_c value determined from the friction factor and Nusselt number results.

Because Re_c is almost independent of the Prandtl number (Pr) in a smooth tube, Nusselt number in laminar or turbulent flow can be generalized by scaling with $Pr^{0.4}$ for wall cooling (or fluid heating). The expression established in [2, 3] for laminar flow and $0.7 \le Pr \le 125$ is

$$C_h = Nu/(Re^{1/2}\,Pr^{0.4}) = 0.14. \qquad (1)$$

Alternatively, in the most general form that considers f, Nu, Pr, and Re, the expression is

$$C_{h,f} = Nu/(Re^{3/2}\,f\,Pr^{0.4}) = 0.008. \qquad (2)$$

Equation (1) is valid for a smooth channel with the hydraulic diameter, D_h, as the length scale. Equation (2) holds for smooth and rough channels, and D_h must be used. The error band on Eq. (1) or (2) is ±20%. It is worth noting that when friction factor and heat transfer data are presented as a plot of $C_{h,f}$ versus Re there are two distinct trends: the horizontal line for laminar flow and the decreasing trend with increasing Reynolds number for both the transition and turbulent

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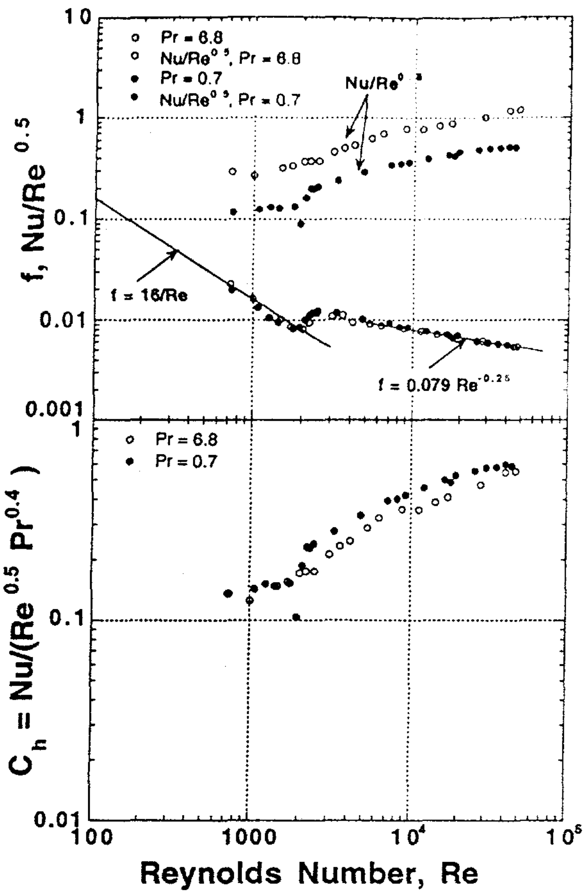

Fig. 1. Typical trends for a conventional channel

regimes. In other words, there is no transition region for any particular channel geometry. These generalized trends are indications that, in terms of the expended power, laminar flow affords the best heat transfer performance.

For turbulent flow, the simplest expression that provides adequate representation of the smooth-tube results obtained for $0.7 \le Pr \le 125$ is the Dittus-Boelter equation [2]. It must not be inferred from Fig. 1 that $Nu \propto Re^{1/2}$ in turbulent flow. The same Reynolds number exponent is used for laminar and turbulent flow to illustrate the consistent trends in going from laminar to turbulent flow. Another reason is that the same graphical format is used in the subsequent presentation of microchannel results, in order to provide clear picture of the Nu variation with Re in laminar flow and the transition process.

It is evident that close agreement with well-established relationships will result when Re_c is ≈2,000. When Re_c differs markedly from this threshold value, friction factor and Nusselt number results for smooth channels can lie above or below the solid lines on Fig. 1 depending on whether Re_c is higher or lower than 2,000. For instance, triangular-shaped channels are characterized by Re_c values that are well below 2,000, depending on the height-to-base ratio. As a result, f generally falls below the solid lines in Fig. 1. Similarly, relative to the Nu versus Re curves for a circular tube, those for triangular passages are the lower set. The trends are exactly the opposite for rectangular channels because Re_c is always greater than for a cylindrical geometry; its value increases with increasing aspect ratio to a limiting value for high aspect ratios [5].

In the presence of roughness, there are modifications to the trends outlined above. Of these, the relevant one that occurs frequently is the reduction in the transition Reynolds number. The friction factors are generally higher than those for the smooth channel for all flow regimes. Relative to the results for a smooth tube, the trends for the Nu versus Re curves parallel those depicted in Fig. 1 for Pr = 6.8; the slope of the line (log-log plot) in turbulent flow is roughly the same as that of the smooth tube. The shift in the transition Reynolds number introduces some ambiguity to the application in turbulent flow of the familiar Reynolds number similarity that asserts dynamically similar flows at the same Re.

In summary, the above observations are central to proper analysis, interpretation, and discussion of friction factor and heat/mass transfer data obtained with microchannels. It is worth noting that the dependence of Re_c on channel geometry for noncircular passages is due to the inseparably connected effects of the transition process and the length scale used to reduce the data to dimensionless form [5].

3. MICROCHANNEL STUDIES

This section consists of three parts: pressure drop studies; heat/mass transfer studies; and laminar, transitional and turbulent flow in microchannels. Consistent with the objectives, the focus is on experimental studies.

Before the presentation and discussion of published results, two general comments should be made. The first relates to the definition of aspect ratio (β) which, for horizontal rectangular channels, is usually taken as the width divided by the height (or spacing between the top and bottom plates). With the exception of Acosta *et al.* [6], who used the above definition, the height-to-width ratio (h/w) is widely referred to as the microchannel aspect ratio. In this paper, the traditional definition is used, that is, the inverse of h/w, because it is more appropriate in the comparisons and discussion of friction factor results. The second deals with the reliability of the absolute values of the original results, especially those for Nusselt numbers in laminar flow. Clearly, some of the results may be open to question. In this review, the emphasis is on trends because they should provide the framework for future research.

A summary of the geometric details of the channels used in selected studies is presented in Table 1. It is evident that the studies with noncircular geometry can be separated into two groups: one for aspect ratios in the $1 \le \beta < 10$ range and the other for $\beta > 10$. In the text, multichannel configurations are denoted by (X_n, N, h/w), where X_n is the relative channel-to-channel spacing, N is the number of channels, and h/w is as defined above.

3.1 Pressure Drop Studies

About ten full-scale studies on pressure drop have been conducted. Of the early studies, Wu and Little [7] obtained data for eight channels; one (S3) was smooth (h/w = 0.45) and the remaining seven were characterized by varying degrees of roughness. Note that the h/w values in Table 1 are based on the width of the top plate; the dimensions for the bottom plate were not given in the paper. A detailed discussion of their results is given later.

Acosta *et al.* [6] presented friction factors for three values of aspect ratio (20, 30, and 40) that are almost indistinguishable. This is consistent with the asymptotic behavior of the transition Reynolds number for large aspect ratios. Although these results give a value of Re_c (= 2,770) that is close to the estimated values from published results (Jones [8]; Obot [5]), friction factors in laminar flow are considerably lower than the expected values for large aspect ratios. Note that the laminar flow results referred to here (see Fig. 12 in Ref. 6) contrast sharply with the consistent trends given in Fig. 6 of the same paper. Pfahler *et al.* [9] covered a narrow Re range (50 to 300) in their experiments. Their results indicated that (f x Re) = C, where C is a function of β and Re. The dependence of C on Re is not the expected behavior.

The two studies by Peng *et al.* [10, 11] complement one another in that the geometric details of the channels and the Re range (50 to 3,000) were the same. The documented transition Reynolds numbers in these studies are significantly lower than those for smooth channels of large D_h, even lower than the values in the literature for very rough channels.

Table 1. Channel Geometric Details for Selected Studies

Author(s)	Cross-section	D_h (μm)[a]	h/w[b]
Acosta *et al.* [6]	rectangular	$368.9 \leq D_h \leq 990.4$	$0.019 \leq h/w \leq 0.05$
Wu & Little [7]	trapezoidal	$45.5 \leq D_h \leq 83.1$	$0.19 \leq h/w \leq 0.45$
Pfahler *et al.* [9]	rectangular	1.6 and 3.4	0.008 and 0.017
Peng *et al.* [10]	rectangular	$133 \leq D_h \leq 343$	$0.33 \leq h/w \leq 1$
Peng *et al.* [11]	rectangular	$133 \leq D_h \leq 343$	$0.33 \leq h/w \leq 1$
Choi *et al.* [12]	cylindrical	$3 \leq D_h \leq 81.2$	-
Yu *et al.* [13]	cylindrical	19.2, 52.1 and 102	-
Harley *et al.* [14]	trapezoidal	$1.0 \leq D_h \leq 35.9$	$0.0053 \leq h/w \leq 0.161$
Wang & Peng [16]	multiple rectangular	$311 \leq D_h \leq 747$	$0.88 \leq h/w \leq 3.5$
Peng & Peterson [17]	rectangular	$133 \leq D_h \leq 343$	$0.5 \leq h/w \leq 3.0$
Wu & Little [18]	trapezoidal	$134 \leq D_h \leq 164$	$0.17 \leq h/w \leq 0.29$
Peng *et al.* [19]	multiple rectangular	$311 \leq D_h \leq 646$	$1.17 \leq h/w \leq 3.5$
Adams *et al.* [20]	cylindrical	760 and 1090	-
Peng & Peterson [22]	rectangular	$311 \leq D_h \leq 747$	$0.88 \leq h/w \leq 3.5$

[a]Range is provided for D_h or h/w values more than 3.
[b]For trapezoidal geometry, values for h/w are based on the larger (top) dimension.

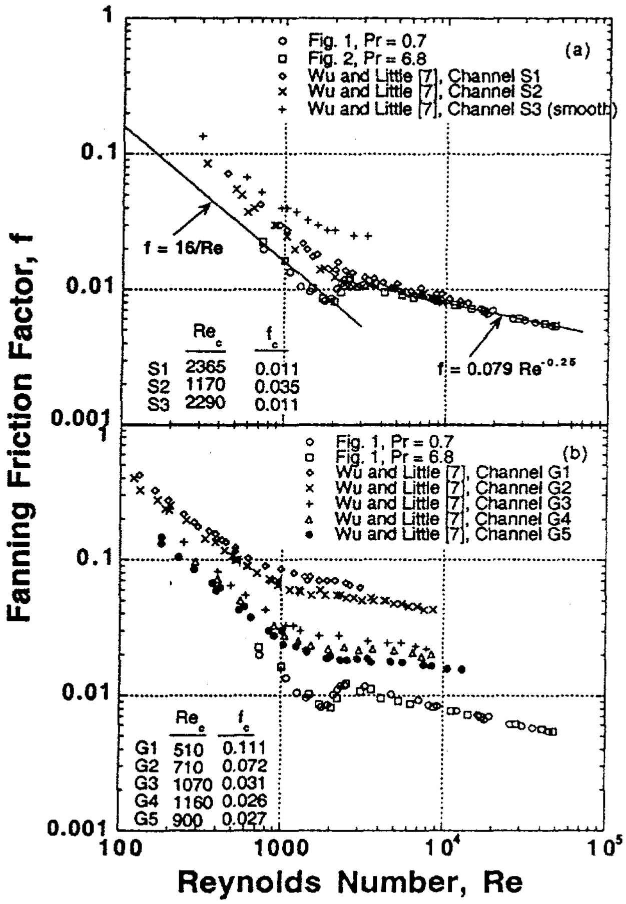

Fig. 2. Friction factor results of Wu and Little [7]

Further discussion of their results will be given in Sections 3.2 and 3.3.

Choi *et al.* [12] and Yu *et al.* [13] presented friction factor results obtained with microtubes for $20 < Re < 20{,}000$ and $200 < Re < 20{,}000$, respectively. The two sets of results exhibit a common trend, that is, a logarithmic regression line through the laminar friction factors falls somewhat below the theoretical circular tube line, and the corresponding values in turbulent flow are slightly the lower set. These trends are not demonstrative for a cylindrical geometry and can occur because the flow rates used in the data reduction are higher than the actual values.

Harley *et al.* [14] carried out comprehensive theoretical and experimental studies of laminar flow in microchannels. For the experimental aspect of their investigations, tests were performed with eight different channels; the working fluids were nitrogen, helium and argon. The highest Re covered in the study was about 1,200, with no evidence of transition to turbulence. Agreement of the experimentally determined friction constants with theoretical predictions was within 8%.

The figures in the original papers were scanned; the data were painstakingly extracted with digiMatic software and then analyzed. For friction factor, it was established that verifiable trends for laminar, transitional, and turbulent flow were provided only by Wu and Little [7]. Accordingly, their results form the basis for the following presentation.

Figure 2 shows the extracted friction factor results of Wu and Little [7]; the smooth-tube results of Fig. 1 are also included for comparison. Recall that channel S3 is smooth and has $\beta = 2.25$. Also, the remaining channels have varying levels of roughness and β ranges from 3.32 to 4.75. Based on the results of Jones [8] and Obot [5], it is estimated that smooth channels with β values of 2 and 5 should give friction factors in laminar flow that are higher than the smooth-tube values by ≈10% and ≈40%, respectively. The trends for the smooth channel S3 with $Re_c = 2{,}290$ are consistent with expectation; laminar friction factors are about 13% higher than the circular tube values.

For the other channels (Fig. 2), the higher values relative to those for the smooth tube were attributed to the roughness of the channel walls and uncertainties in the determination of the channel dimensions. On the contrary, it is evident that these differences are due to the combined effects of aspect ratio and roughness; the underlying physical phenomenon is the transition process. The conjecture is that the influence of roughness is of secondary importance, at least for channels S2, G3, G4, and G5. This will be evident from the subsequent presentation and discussion of heat transfer results.

By invoking the frictional law of corresponding states (Obot *et al.* [15]; Obot [3]), the reduced friction factor, f_m, and the similarity parameter, Re_m, were calculated by using the data in Fig. 2; the results are presented in Fig. 3. The method involves selecting a reference data set which, in the present case, is that in Fig. 1 for the smooth-tube with Pr = 0.7 ($Re_{c,r} = 2040$, $f_{c,r} = 0.008$). An arbitrary set of data is used to calculate the reduced parameters according to the following relationships:

$$Re_m = (Re_{c,r}/R_{c,a})Re_a \tag{3}$$

$$f_m = (f_{c,r}/f_{c,a})f_a\,, \tag{4}$$

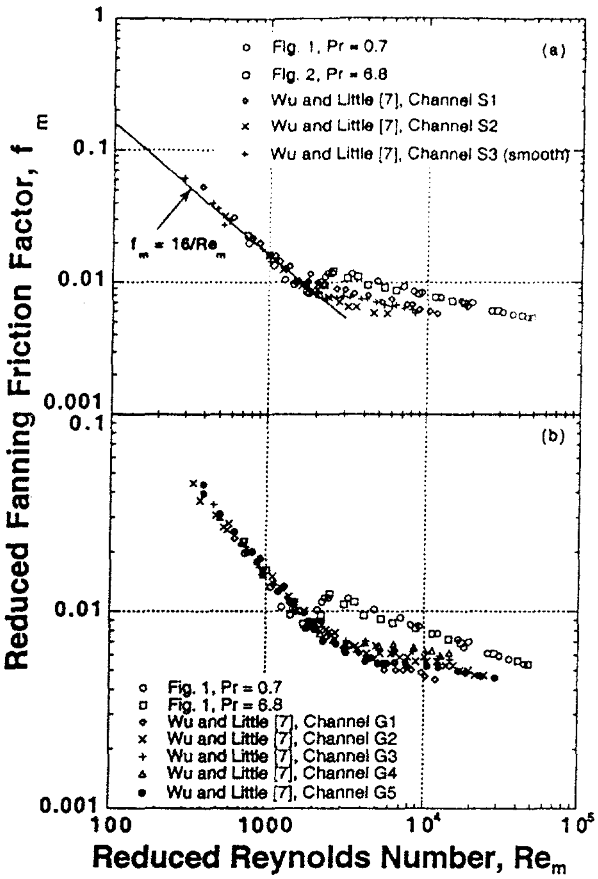

Fig. 3. Reduced friction factor results based on data in Fig. 2

where $Re_{c,r}$ and $f_{c,r}$ correspond to the transition Reynolds number and friction factor for the reference set, and $Re_{c,a}$ and $f_{c,a}$ are the corresponding transition parameters for any arbitrary set of results. The values of these transition parameters are given in Fig. 2. The values were determined by replotting the results on Fig. 2 as (f x Re) versus Re. The limiting point on the faired horizontal segment with increasing Re gives Re_c and (f_cRe_c); f_c is readily calculated by using the known Re_c.

Figure 3 establishes that the reduced friction factor data for microchannels are generalized. The microchannel results in Figs. 3(a) and (3b), taken together, are closely approximated by mean regression curves in laminar and turbulent flow with errors that are within the experimental uncertainty. This collapse of unrelated microchannel results is an indication that the differences between the data are due to the transition process, which manifests itself through variations in the transition parameters.

In laminar flow, the reduced microchannel data are almost indistinguishable from those for the circular tube. Deviations are observed in turbulent flow. This is due to the absence of a well-defined transition region in the original data; this is reflected in the shape of the curves (Fig. 2). The observed trends in the transitional flow region contrast sharply with those presented by Acosta *et al.* [6] and Yu *et al.* [13].

The results in Fig. 3 also imply several things. First, microchannel data can be generalized by the method developed for channels of large D_h. Second, estimates of microchannel pressure drop in laminar flow can be obtained by using conventional empirical correlations provided there are verifiable transition parameters.

Finally, several aspects of the experimental design and measurement might have contributed to some of the discrepancies and trends in published results for frictional pressure coefficients. First, for experiments with gases, it is extremely important to obtain very accurate measurements of the flow rates. Because the gas flow rates are very small and the inlet pressures are high, well-thought-out corrections must be applied to flow-meter calibrations to account for differences in pressure, temperature, and density between conditions during experimentation and calibration. In passing, note that the reported uncertainties in flow rate ranged from 2 to 5%.

The second aspect deals with the pressure differences from which friction factors are calculated. In most cases, the differential pressures were measured at locations external to the test sections, in contrast to measurements directly across test sections for larger-diameter channels. Clearly, microchannel friction factors are usually estimated values, based on pressures measured farther upstream and downstream of the test sections. Considering the peculiarities of each experimental design and that no two experiments are completely identical in every respect, it can be readily appreciated that the reliability of the estimates can vary from experiment to experiment. The above observations might shed some light on the rather high experimental uncertainties (10 to 15%) reported by various investigators.

3.2 Heat/Mass Transfer Studies

Figures 4-6 are plots of $Nu/Re^{1/2}$, $Nu/(Re^{0.5}\,Pr^{0.3})$, and $Nu/Re^{1/2}$ versus Re. The first figure was prepared by using the data of Wang and Peng [16] for water and methanol, the second from the results of Peng *et al.* [11], and the third is based on the work of Peng and Peterson [17]. The upper and lower plots in Fig. 6 correspond, respectively, to Figs. 4 and 5 in Peng and Peterson. However, unlike the original presentation, the same symbol is used for the eight different test sections (Fig. 6a) and nine values of concentration (Fig. 6b). The constants for the mean regression through all data and the standard deviations are given in the plots for those with moderate spread in the data.

In both Figs. 4 and 5, the trend for each channel is about the same. Specifically, the results can be approximated by a mean line with deviations that are, for the most part, under ±20% (that is, much better than the predictive equations provided in some of the original papers). There is hardly any evidence to support the existence of a flow type other than laminar flow for Re ≤ 1,000. The results in Fig. 6 do not modify the above observations. The trends in Fig. 6a cannot be attributed to transition to turbulent flow. The results for Re < 400 and Re > 500 are for different test sections, similar to the trends depicted in Fig. 4. It is of interest to note that the results in Fig. 6b were obtained with one test section. The spread in the data is a reflection of the residual effects of concentration for methanol/water mixtures. Even with this spread, the results are approximated by $Nu/Re^{1/2} = 0.125 \pm 0.032$, i.e., with a 26% variation about the mean value, an indication of lack of strong effect of concentration on Nusselt number.

Figure 5 shows that the effects of h/w or D_h are small. For instance, h/w = 0.667 with D_h = 240 μm gives essentially the same values as with h/w= 0.75 and D = 343 μm. Likewise, the two results for h/w = 1 with D_h = 200 and 300 μm are very close. Also, the results suggest that comparable values would be obtained by fixing h/w at 0.5 and doubling D_h. The only significant deviation is observed for h/w = 0.33 with D_h = 150 μm. Peng *et al.* [11] reported significant effects of h/w and D_h on Nu for laminar and turbulent flow; this is not confirmed by the present analysis of the same data. The lack of a strong effect of h/w or D_h on microchannel heat transfer is consistent with the well-established behavior for channels of large D_h.

In Fig. 4, the results for water can be placed in two groups: those for X_n > 10 [(13, 6, 3.5) and (20, 4, 3.5)] and those for X_n < 10 [(4.25, 4, 0.88), (6, 4, 1.17), (6, 6, 1.75) and (9.5, 4, 1.75)]. For the latter group, a careful study of the results reveals that the highest values of $Nu/Re^{1/2}$ are realized with (6, 6, 1.75). In other words, the dominant factors are the relative spacing, X_n, between the channels and the number of channels, N. Accordingly, the results for (X_n, N, h/w) =

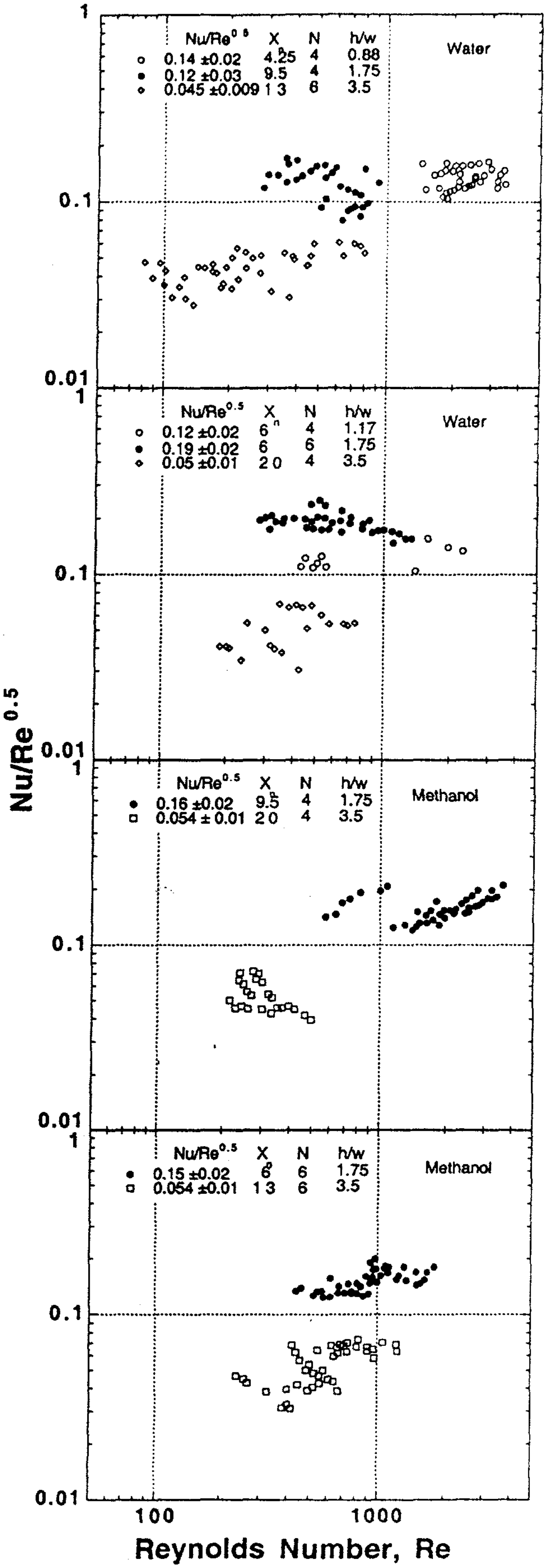

Fig. 4. Nu/Re$^{0.5}$ vs Re based on results of Wang and Peng [16]

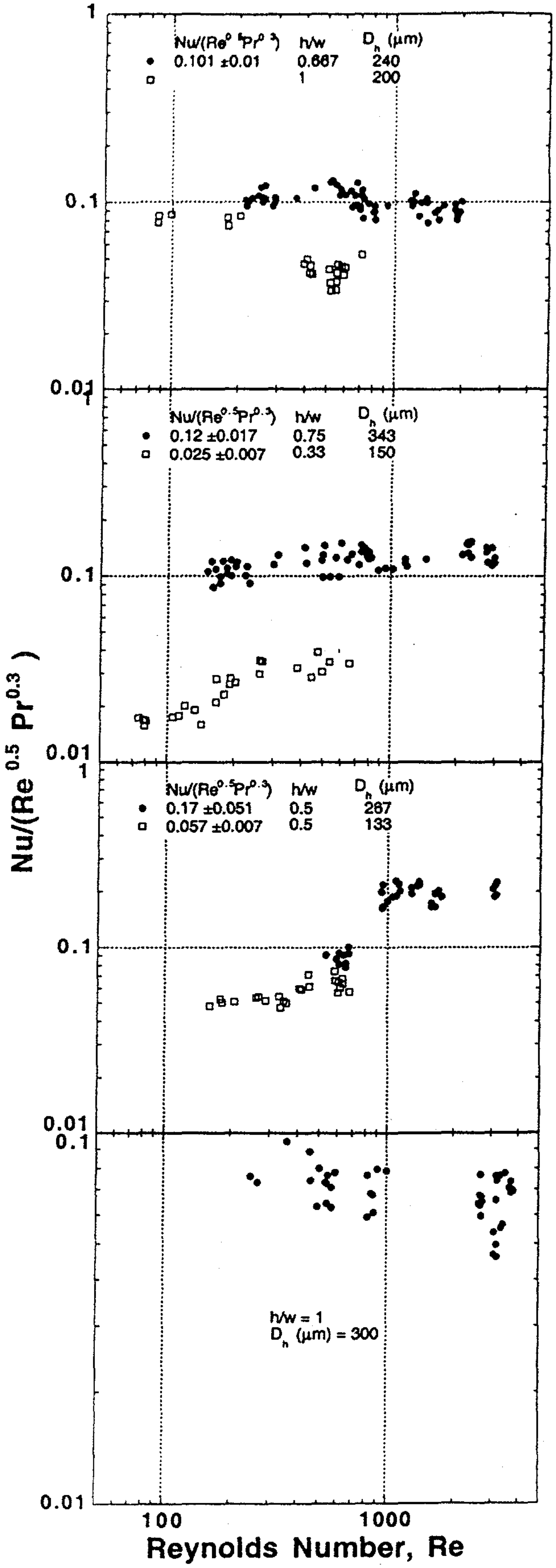

Fig. 5. Nu/(Re$^{0.5}$ Pr$^{0.3}$) vs Re based on results of Peng *et al.* [11]

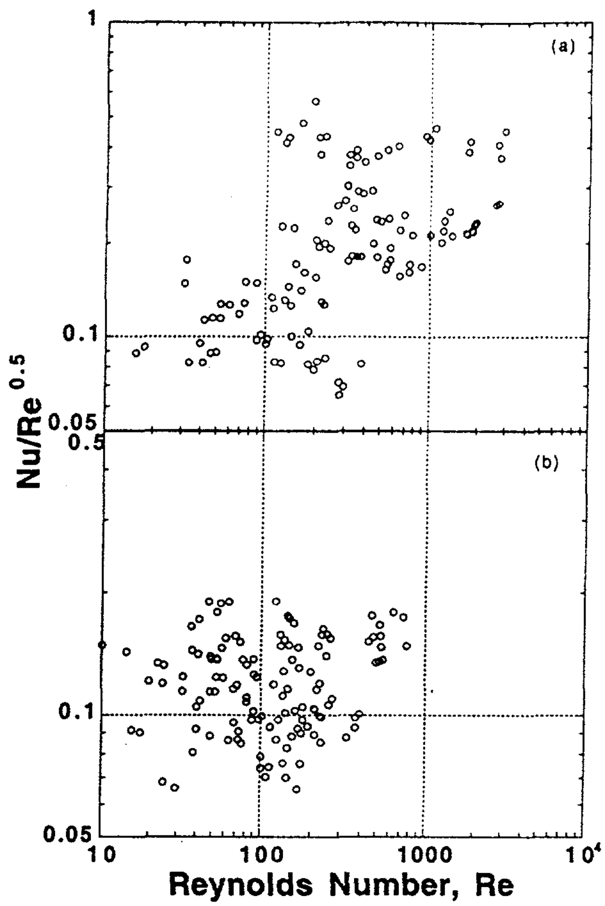

Fig. 6. Nu/Re 0.5 versus Re based on the results of Peng and Peterson [17]

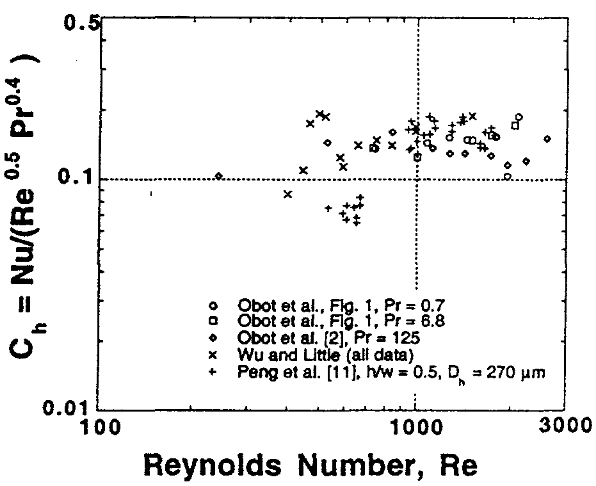

Fig. 7. Comparison of microchannel results with those in Fig. 1 for laminar flow

(13, 6, 3.5) and (20, 4, 3.5) lie well below those for (6, 6, 1.75) and (9.5, 4, 1.75); the two sets of results differ by a factor of ≈2.

The implication of the observed effects of X_n and N is that the smaller the value for X_n, or the greater the number of channels over a specified area, the larger the Nusselt number. There is inconclusive evidence to support the existence of an optimum value for X_n, although the presence of such a value could be expected. Clearly, there is a need for carefully designed experiments aimed at establishing the effects of X_n and N on heat/mass for multichannel configurations. It is worth noting that this situation is analogous to the well-established effect of relative rib-to-rib spacing on heat/mass transfer for transverse repeated-rib roughness.

Because Wu and Little [18] provided very limited results for each channel and Re ≤ 1,000, these are presented in the expanded scale plot in Fig. 7. For nitrogen, Pr is about 0.7; this value was used to scale the original results. The results for Pr = 0.7 and 6.8 (Fig. 1), ethylene glycol (Pr = 125) [2] and water [11] are also included for the purpose of comparison. For the latter, a Pr value of 6.8 was used in going from the original exponent of 0.3 to the present value of 0.4 for wall cooling or fluid heating. Beside the obvious conclusion that Nu ∝ $Re^{1/2}$, Fig. 7 establishes that the same Nu relationship may be applicable to both microchannels and larger-diameter tubes, at least for some range of conditions in laminar flow.

Choi *et al.* [12] presented results for 50 < Re < 20,000 with nitrogen as the working fluid. For Re ≤ 500, calculated values of $Nu/(Re^{0.5}\,Pr^{0.3})$ are significantly lower than those in Fig. 5. The values can be stated as $C_h = 0.033 \pm 0.011$ and 0.0997 ± 0.028 for Re ≤ 500 and 500 < Re ≤ 2,200, respectively. In the subsequent study by Yu *et al.* [13], the emphasis was mostly in turbulent flow as the Reynolds number was varied between 2,000 and 20,000.

Establishing the nature of the dependence of Nu on Re in laminar flow, consistent with boundary layer theory, has remained somewhat of a mystery for several reasons. First, interest in experimental studies has been minimal even for conventional channels. The second relates to the widely held view that Nu should be constant, as predicted by some of the models. The fact is that experimentally determined average fluid temperature does not vary linearly along the length of the test section, as dictated by the constant heat flux boundary condition. Also, the so-called constant wall temperature boundary condition is unrealistic because it is rarely realized experimentally.

For turbulent flow, there is consistency between boundary layer theory and channel flows in that Nu ∝ $Re^{0.8}$. There is no reason that Nu should not vary as the 1/2 power of Re for microchannels and channels of large D_h, in line with the well-established boundary layer theory. This is confirmed by the present analysis. With the exception of the results of Peng *et al.* [19], which indicated nearly constant heat transfer coefficients, the results of all microchannel studies indicate that Nu or Sh increases with Re in laminar flow. It is of interest to note that previously reported exponents on Re range from 0.3 to 0.62.

Studies of heat/mass transfer in turbulent flow were carried out by a number of researchers. For 1,000 ≤ Re ≤ 10,000, referred to by Wu and Little [18] as the transition zone, their experimental results for three of the four channels (# 1, 2, and 3) tested are, on average, ≈20% higher than the calculated values from an empirical equation. The mean line representing the results for channel # 4 is well above the other three sets of data.

Acosta *et al.* [6] used the electrochemical technique to obtain mass transfer coefficients for rectangular channels. Based on comparisons of their results with the Chilton-Colburn equation, they concluded that the smooth channel correlation of large D_h was valid for smooth microchannels. For rough channels, significant deviations from the Chilton-Colburn correlation were documented. The disagreement may occur because the Reynolds number similarity is no longer valid and corrections must be applied to account for the effect of transition to turbulence.

Adams *et al.* [20] tested two microtubes for Re values in the 3,200 to 23,000 range. The experimentally determined Nusselt numbers were somewhat higher than those calculated from the Gnielinski correlation. In the absence of laminar flow results, and given the fact that the error bands associated with empirical equations are no better than ±20%, a straightforward conclusion that conventional correlations are not valid for microchannels cannot be made.

A point that is often overlooked is that the outcome of comparisons of experimental Nusselt or Sherwood numbers with established correlations depends on the selected correlation, the Reynolds number range, and the Prandtl or Schmidt number. For instance, the Petkhov-Popov equation gives essentially the same Nu as the Dittus-Boelter correlation for Re = 10,000 with Pr = 0.7. The differences in the calculated Nu values increase with increasing Reynolds number to ≈15% when Re = 40,000. The latter consistently gives the higher values. By contrast, differences in the calculated Nu values between the two correlations are, on average, ≈10% for $10{,}000 \le Re \le 40{,}000$ with Pr = 6.8; the values deduced from the Petukhov-Popov equation are now the higher set. These differences must be considered, along with the fact that uncertainties on the order of ±25% are not uncommon with any particular correlation [21]. Thus, given the experimental uncertainties on measured Nu (usually no better than ±15%), the error bands on the correlations, and the working fluid, data deviations from well-established relationships of ≈25% could very well be expected. This degree of concurrence should be considered satisfactory.

To shed further light on the above discussion of heat/mass transfer in turbulent flow, comparisons of the results in Fig.1 with those of some of the investigators cited here are presented in Fig. 8. For the sake of completeness, laminar flow results are included in the figure. Adams *et al.* [20] covered the 3.7 to 6.43 Prandtl number range; the average value of 5.1 was used to scale their results for Nu. The anomalous results of Wu and Little for channel #4 are excluded in Fig. 8. The results of Choi *et al.* [12] and Yu *et al.* [13] are also included in the figure.

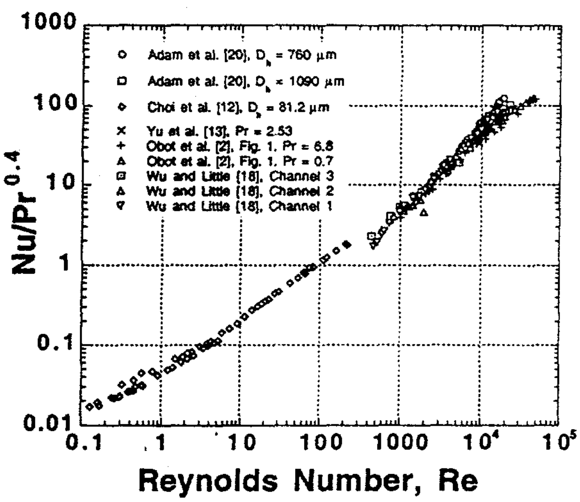

Fig. 8. Comparison of published microchannel results with those in Fig. 1

Figure 8 shows that the agreement among the various results is remarkable for both laminar and turbulent flow. In fact, a regression line through all turbulent flow results for $2{,}000 < Re \le 20{,}000$ would give deviations that are generally within 20%. This degree of concurrence provides indirect support for the absence of transition to turbulent flow with microchannels for $Re \le 1{,}000$. Also, it provides further confirmation that h/w and D_h are not important parameters, as concluded previously in the discussion of the results in Fig. 5. Further, the agreement is an indication that the widely speculated effect of roughness on Nu is not important for the studies considered here. It is important to note that the effect of roughness on pressure drop is more pronounced than that on heat transfer under identical flow conditions.

In conclusion, the observation that conventional relationships are not applicable to microchannels, as reported by numerous investigators, is invalid and must be rejected. The comparisons that led to this misleading observation were not thorough and failed to consider the factors raised in this paper. Also, in all cases, comparative evaluations with published experimental results for channels of large D_h or other microchannels were not made. The recommendation based on this exhaustive literature review amounts to this: satisfactory estimates of heat/mass transfer coefficients for smooth microchannels, within the accuracy of experimental errors, can be obtained by using either verifiable experimental data for channels of large D_h or conventional correlations. Recall that Acosta *et al.* [6] reached a similar conclusion.

3.3 Laminar, Transitional, and Turbulent Flow in Microchannels

The role of transition on friction and heat/mass transfer in microchannels is the least understood aspect of the literature on convection. To a lesser extent, this observation is also true for channels with high values of D_h. For microchannels, even the basic definitions of what constitutes laminar, transitional and turbulent flow are open to question, not to mention the criterion for transition to turbulent flow.

Wu and Little [7] reported that Re_c for glass channels without heat treatment was ≈350; the corresponding value for heat-treated glass channels was ≈900. These values differ markedly from those given in Fig. 2. Based on heat transfer data, Wu and Little [18] gave the flow zones as $Re < 1{,}000$ for laminar flow, $1{,}000 < Re < 3{,}000$ for transitional flow, and $Re > 3{,}000$ for fully turbulent flow. Note that the channels used for the heat transfer studies were not the same as those for the pressure drop experiments (Table 1).

Peng *et al.* [10, 11] indicated that the Re = 200-700 range represented the upper bound for laminar flow transition to turbulence, and that fully turbulent flow occurred in the 400 to 1500 Re range. Based on their heat transfer results, Peng and Peterson [22] gave $Re < 400$ for laminar flow, $400 < Re < 1{,}000$ for the transition region, and $Re > 1{,}000$ for fully turbulent flow. According to Wang and Peng [16], the initiation of fully turbulent flow occurred in the 1,000 to 1,500 Re range.

The results of Choi *et al.* [12], presented as a plot of (f x Re) versus Re (see Fig. 4 of Ref. 12), suggest that transition to turbulent flow occurs at Re of ≈2,000 with $D_h = 53$ μm or 81.2 μm and around Re = 500 with $D_h = 9.7$ μm or 6.9 μm. By contrast, the authors gave the extent of the laminar flow zone as $Re < 2{,}000$. Yu *et al.* [13] provided a friction factor correlation in laminar flow for $200 < Re < 2{,}000$. Because the results for D_h = 19.6, 52.1, and 102 μm were assigned the same symbol, the variability of the transition Reynolds number with D_h, as inferred from the results of Choi *et al.*, cannot be determined.

Stanley *et al.* [23] studied single-phase and two-phase flow in microchannels. For gases, the authors reported that transition to turbulent flow was completely suppressed for $D_h < 80$ μm; varying degrees of suppression were in evidence for D_h in the 80 to 150 μm range, but transition occurred for values of Re between 1500 and 2000 with $D_h > 150$ μm. Transition was not observed with water even for the largest D_h tested. The results that formed the basis for these observations were not presented in the paper.

There are misconceptions on the transition region. According to Reynolds [24], the transitional flow zone extends to about 1.3 Re_c. Unlike laminar or turbulent flow, the region is confined to a very narrow Re range, the upper limit of which is 1.4 Re_c, regardless of the channel geometry [25]. The lower and upper limits, respectively, correspond with remarkable consistency to the locations of the minimum and peak value on conventional f versus Re plot (Fig. 1). This well-established view of the transition region contrasts sharply with that inferred from the literature on microchannels.

The present analysis of the results of these studies leads to several conclusions. First, there is no strong evidence to support the existence of transitional or turbulent flow in smooth microchannels for $Re \le 1{,}000$. Second, in the study by Wu and Little [7], it is strikingly apparent from the values in Fig. 2 that Re_c varies from 510 to 2,365; the value for the smooth channel is 2290. Third, the results of Peng *et al.* [10, 11], Peng and Peterson [22], and Wang and Peng [16] do not exhibit any trend characteristic of *transitional flow* for Re ≤

1,000, and lack the detail to permit determination of Re_c for Re > 1,000. Finally, and strangely enough, the methods used to determine the transition Reynolds numbers and/or the extent of any particular flow regime were not provided in the papers.

4. CONCLUSIONS AND RECOMMENDATIONS

Critical analysis, comparisons, and discussion of published results on pressure drop and heat/mass transfer in microchannels are presented. The main conclusions are: (a) there is hardly any evidence to support the occurrence of transition to turbulence in smooth microchannels for Re $\leq$ 1,000; (b) Nu $\propto Re^{1/2}$ in laminar flow; and (c) estimates of transfer coefficients for smooth microchannels, accurate within experimental errors, can be obtained by using either conventional correlations or experimental results for channels of large D_h.

There is a need for carefully crafted experimentation aimed at determining pressure drop and heat transfer characteristics, as well as defining the role of transition on pressure drop and heat transfer for single isolated channel geometry. Similar studies are also needed for multichannel configurations to enable design and scaleup of microreactors and microheat exchangers.

NOTATION

C_h	=	$Nu/(Re^{1/2} Pr^{0.4})$, Eq. (1)
$C_{h,f}$	=	$Nu/(Re^{3/2} f Pr^{0.4})$, Eq. (2)
D_h	=	hydraulic diameter, μm
f	=	Fanning friction factor
f_m	=	reduced Fanning friction factor, Eq. (4)
h	=	depth of channel, μm
N	=	number of channels in multichannel configuration
Nu	=	average Nusselt number
Pr	=	Prandtl number
Re	=	Reynolds number
Re_c	=	critical Reynolds number at transition
Re_m	=	reduced Reynolds number, Eq. (3)
Sh	=	average Sherwood number
w	=	channel width, μm
x	=	channel center-to-center spacing, μm
X_n	=	relative channel-to-channel spacing, x/w
β	=	channel aspect ratio, w/h

Additional Subscripts

a	=	arbitrary data set
c,r	=	transition parameters for reference condition
c,a	=	transition parameters for arbitrary data set

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