PHYSICAL REVIEW A **84**, 020101(R) (2011)

Toward a resolution of the proton size puzzle

G. A. Miller,[1] A. W. Thomas,[2] J. D. Carroll,[2] and J. Rafelski[3]
[1]*Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA*
[2]*CSSM, School of Physics and Chemistry, University of Adelaide, Adelaide, South Australia 5005, Australia*
[3]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

(Received 21 January 2011; published 16 August 2011)

We show that off-mass-shell effects arising from the internal structure of the proton provide a new proton polarization mechanism in the Lamb shift, proportional to the lepton mass to the fourth power. This effect is capable of resolving the current puzzle regarding the difference in the proton radius extracted from muonic compared with electronic hydrogen experiments. These off-mass-shell effects could be probed in several other experiments. A significant ambiguity appearing in dispersion relation evaluations of the proton polarizability contribution to the Lamb shift is noted.

DOI: 10.1103/PhysRevA.84.020101 PACS number(s): 31.30.jr, 14.20.Dh, 21.10.Ft, 24.10.Cn

The recent, extremely precise extraction of the proton radius [1] from the measured energy difference between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ states of muonic hydrogen (H) has created considerable interest. Their analysis yields a proton radius that is smaller than the CODATA [2] value (extracted mainly from electronic H) by about 4 % or 5.0 standard deviations. This implies [1] that either the Rydberg constant has to be shifted by 4.9 standard deviations or that the QED calculations for hydrogen are insufficient. Since the Rydberg constant is extremely well measured, and the QED calculations seem to be very extensive and highly accurate, the muonic H finding presents a significant puzzle to the entire physics community.

Our analysis is motivated by the fact that muonic hydrogen is far smaller than electronic hydrogen and therefore more sensitive to corrections arising from hadron structure. In particular, we consider corrections associated with off-shell behavior of the photon-nucleon vertex, showing that it may account for the difference reported by Pohl *et al.*. Within our present knowledge of hadronic physics it is not possible to provide a precise value for this correction, so our result may be viewed as a phenomenological study of the sensitivity of muonic hydrogen to important aspects of proton structure. It should spur further study of processes which could be sensitive to off-shell proton structure. In alternate language, our explanation may be viewed as a new contribution from proton polarization, unconstrained by dispersion relations but accessible in systems other than the hydrogen atom.

We discuss the relevant phenomenology. Pohl *et al.* show that the energy difference between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ states, $\Delta\widetilde{E}$, is given by

$$\Delta\widetilde{E} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{meV}, \qquad (1)$$

where r_p is given in units of fm. Each of the three coefficients is obtained from extensive theoretical work [3–7], typically confirmed by several groups. Studies of the relevant atomic structure calculations and corresponding efforts to improve those have revealed no variations large enough to significantly affect the above equation [8,9]. Using this equation, we see that the difference between the Pohl and CODATA values of the proton radius would be entirely removed by an increase of the first term on the right-hand side of Eq. (1) by 0.31 meV $= 3.1 \times 10^{-10}$ MeV. Finding a new effect of about that value resolves the puzzle, provided the corresponding effect in electronic H is no more than the current difference between theory and experiment [3] (a few parts in a million).

The search to find such an effect has attracted considerable interest. New physics beyond the standard model must satisfy a variety of low-energy constraints and so far no explanation of the proton radius puzzle has been found that satisfies these constraints [10–13]. The third term of Eq. (1) [14] has been studied, with the result that its current uncertainties are far too small to resolve the proton radius puzzle [15,16].

We, therefore, seek an explanation based on the fact that the proton is not an elementary Dirac particle, with a significant anomalous magnetic moment. In particular, consider the electromagnetic vertex function which must depend on all of the relevant invariants. For a proton of initial four-momentum p, the most general expression must include a term, dependent on the proton virtuality, that is proportional to $p^2 - M^2$ or $\not{p}_N - M$, where the subscript N denotes acting on a nucleon and M is the nucleon mass. Such terms have been discussed for a very long time in atomic [6,7] and nuclear physics [17–29], especially in relation to the difference between free and bound deep inelastic structure functions [17–20], nucleon-nucleon scattering [21], and electromagnetic interactions involving nucleons [22,23], notably quasielastic scattering [24–29]. Such off-shell effects arise naturally in quantum electrodynamics [30].

Many possible forms [22,23] include the effects of proton virtuality; we consider three that could be significant here. The Dirac part of the vertex function for a proton of momentum p to absorb a photon of momentum $q = p' - p$ is expressed as:

$$\Gamma^\mu(p',p) = \gamma_N^\mu F_1(-q^2) + F_1(-q^2) F(-q^2) \mathcal{O}^\mu_{a,b,c}, \qquad (2)$$

with

$$\mathcal{O}_a^\mu = \frac{(p+p')^\mu}{2M}\left[\Lambda_+(p')\frac{(p\cdot\gamma_N - M)}{M} + \frac{(p'\cdot\gamma_N - M)}{M}\Lambda_+(p)\right]$$

$$\mathcal{O}_b^\mu = [(p^2 - M^2)/M^2 + (p'^2 - M^2)/M^2]\gamma_N^\mu$$
$$\mathcal{O}_c^\mu = \Lambda_+(p')\gamma_N^\mu \frac{(p\cdot\gamma_N - M)}{M} + \frac{(p'\cdot\gamma_N - M)}{M}\gamma_N^\mu \Lambda_+(p).$$

Other terms needed to satisfy the Ward-Takahashi identity do not contribute to the Lamb shift and are not shown. The proton Dirac form factor, $F_1(-q^2)$, is empirically well represented as a dipole $F_1(-q^2) = (1 - q^2/\Lambda^2)^{-2}$, ($\Lambda$ = 840 MeV) for the values of $-q^2 \equiv Q^2 > 0$ of up to about 1 GeV^2 needed here. $F(-q^2)$ is an off-shell form factor, and $\Lambda_+(p) = (p\cdot\gamma_N + M)/(2M)$ is an operator that projects on the on-mass-shell proton state.

We take the off-shell form factor $F(-q^2)$ to vanish at $q^2 = 0$ so the charge of the off-shell proton is the same as that of a free proton. This is also demanded by current conservation [22,23]. We assume $F(-q^2) = \frac{-\lambda q^2/b^2}{(1-q^2/\tilde{\Lambda}^2)^{1+\xi}}$. This is a simple, purely phenomenological nonunique form. At large values of $|q^2|$, FF_1 has the same falloff as F_1 if $\xi = 0$. We take $\widetilde{\Lambda} = \Lambda$ here.

We briefly discuss the influence of using Eq. (2). The ratio, R, of off-shell effects to on-shell effects, $R \sim \frac{(p\cdot\gamma_N - M)}{M}\lambda\frac{q^2}{b^2}$, ($|q^2| \ll \Lambda^2$), is constrained by nuclear phenomena such as the EMC effect (10–15%), uncertainties in quasielastic electron-nuclear scattering [24], and deviations from the Coulomb sum rule [25]. For a nucleon experiencing a 50-MeV central potential, $(p\cdot\gamma_N - M)/M \sim 0.05$, so $\lambda q^2/b^2$ could be of order 2. The nucleon wave functions of light-front quark-models [31] contain a propagator depending on M^2. Thus the effect of nucleon virtuality is proportional to the derivative of the propagator with respect to M or of the order of the wave function divided by difference between quark kinetic energy and M. This is about 3 times the average momentum of a quark (∼200 MeV/c) divided by the nucleon radius or roughly $M/2$. Thus $R \sim (p\cdot\gamma_N - M)2/M$, and $\lambda q^2/b^2$ is again estimated as of order 2.

The lowest order term in which the nucleon is sufficiently off-shell in a muonic atom for this correction to produce a significant effect is the two-photon exchange diagram of Fig. 1 and its crossed partner, an interference between one on-shell and one off-shell part of the vertex function.

The change in the invariant amplitude, $\mathcal{M}_{\rm Off}$, due to using Eq. (2) along with $\mathcal{O}_a^\mu$, to be evaluated between fermion spinors, is given in the rest frame by

$$\mathcal{M}_{\rm Off} = \frac{e^4}{M^2}\int \frac{d^4k}{(2\pi)^4}\frac{F_1^2(-k^2)F(-k^2)}{(k^2+i\epsilon)^2} \times (\gamma_N^\mu(2p+k)^\nu + \gamma_N^\nu(2p+k)^\mu) \times \left[\gamma_\mu \frac{(l\cdot\gamma - k\cdot\gamma + m)}{k^2 - 2l\cdot k + i\epsilon}\gamma_\nu + \gamma_\nu\frac{(l\cdot\gamma + k\cdot\gamma + m)}{k^2 + 2l\cdot k + i\epsilon}\gamma_\mu\right], \quad (3)$$

where the lepton momentum is $l = (m,0,0,0)$, the virtual photon momentum is k and the nucleon momentum $p = (M,0,0,0)$. The intermediate proton propagator is canceled by the off-mass-shell terms of Eq. (2). This graph can be thought of as involving a contact interaction and the amplitude in Eq. (3) as a new proton polarization correction corresponding to a subtraction term in the dispersion relation for the two-photon exchange diagram [32]. The resulting virtual-photon-proton Compton scattering amplitude corresponds to the T_2 term of conventional notation [33,34]. Equation (3) is gauge invariant and not changed by adding a term of the form $k^\mu k^\nu/k^4$ to the photon propagator.

Evaluation proceeds in a standard way by taking the sum over Dirac indices, performing the integral over k^0 by contour rotation and integrating over the angular variables. The matrix element $\mathcal{M}$ is well approximated by a constant in momentum space, for momenta typical of a muonic atom, and the corresponding potential $V = i\mathcal{M}$ has the form $V(\mathbf{r}) = V_0\delta(\mathbf{r})$ in coordinate space [3]. Then the relevant matrix elements have the form $V_0\,|\Psi_{2S}(0)|^2$, where Ψ_{2S} is the muonic hydrogen wave function of the state relevant to the experiment of Pohl *et al.* We use $|\Psi_{2S}(0)|^2 = (\alpha m_r)^3/(8\pi)$, with the lepton-proton reduced mass, m_r. The result,

$$\langle 2S|V|2S\rangle = \frac{-\alpha^5 m_r^3}{M^2}\frac{8}{\pi}\lambda\frac{mM}{b^2}F_L(m)$$

$$F_L(m) \equiv \frac{1}{2\beta^2} \times \int_0^\infty dx\frac{2x^2\left(\sqrt{x^2+\beta}-x\right)+\beta\left(2\sqrt{x^2+\beta}-3x\right)}{(1+x^2)^{5+\xi}}, \quad (4)$$

$\beta \equiv 4m^2/\Lambda^2$, shows a new contribution to the Lamb shift, proportional to m^4 and therefore negligible for electronic hydrogen. Using $\mathcal{O}_a^\mu$ leads to a vanishing hyperfine splitting (HFS) because the operator γ_N^μ is odd unless $\mu = 0$.

We next seek values of the model parameters λ,b,ξ of $F(-q^2)$. chosen to reproduce the value of the energy shift, 0.31 meV, to resolve the puzzle. With $\xi = 0$, $\widetilde{\Lambda} = \Lambda$, $\lambda/b^2 = 2.35/(79\ \text{MeV})^2$ is required. With this value, the corresponding change in the electronic H Lamb shift for the 2S state is about 9 Hz, significantly below the current uncertainty in both theory and experiment [3]. If ξ is changed substantially from 0 to 1 our value of λ would be increased by about 10%. Other tests of this effect could show sensitivity to the value of ξ or $\tilde{\Lambda}$.

The other operators appearing in Eq. (2) yield similar results. Using O_b gives a term of the T_2 form with a Lamb shift equal that of O_a and an HFS term that is about $-1/12$ of

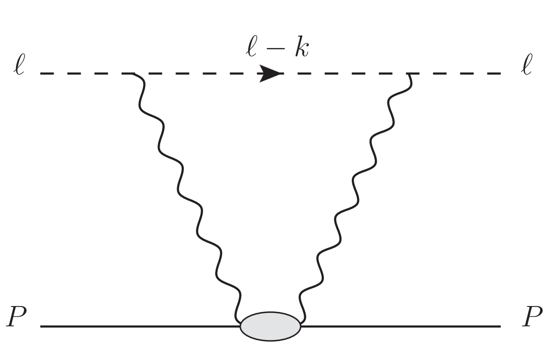

FIG. 1. Direct two-photon exchange graph corresponding to the hitherto neglected term. The dashed line denotes the lepton, the solid line the nucleon, the wavy lines denote photons, and the ellipse denotes the off-shell nucleon.

its Lamb shift. The use of O_c gives a term of the T_1 form and the same Lamb shift as O_a, as well as a HFS term that is -1.7 times its Lamb shift. In this case, the value of λ/b^2 would be about $-3/2$ times that stated above. The HFSs may be small enough to be well within current experimental and theoretical limits for electronic hydrogen.

It is necessary to comment on the difference between our approach, which yields a relevant proton polarization effect, and the dispersion relation approaches of others [34] (and also the very recent similar calculation [35]) which do not. We shall demonstrate that these approaches suffer from severe ambiguities when applied to the present problem. These works use a current-conserving representation of the virtual-photon proton scattering amplitude in terms of two unmeasurable scalar functions, $T_{1,2}$. Dispersion relations are used to relate $T_{1,2}$ to their measured imaginary parts. But one must introduce a subtraction to handle T_1. This is unconstrained by prior data [33] because the value of σ_L/σ_T at infinite photon energy is not determined [36]. Pachucki [34] [Eq. (31)], assumes a form proportional to q^2 times the very small proton magnetic polarizability. However, we are aware of no published derivation of this result, which has been recently criticized [37].

We shall now show that there are unknown terms in the dispersion relation which are not proportional to the magnetic polarizability. The problem with the dispersion relations is that terms with intermediate nucleon states are separated and evaluated using the Feynman diagrams involving an intermediate Dirac propagator. This allows the removal of an infrared divergence by subtracting the first iteration of the effective potential that appears in the wave function. But the Feynman diagrams involve intermediate off-shell nucleons, so their evaluation for composite particles must be ambiguous. On-shell form factors are used in evaluating these diagrams, and there is no fundamental reason for doing this [37]. Moreover, using the Dirac propagator to represent composite-fermion intermediate states, as done in Refs. [34,35], has been known to be incomplete for a long time [38]. This has significant consequences for physics [39].

We provide an example to explain. The proton Born term is typically evaluated by using the vertex function $\Gamma_1^\mu = \gamma^\mu F_1 + i\sigma^{\mu\nu} q_\nu F_{2/(2M)}$ (where F_2 is the Pauli form factor) in a Feynman diagram. This form is determined from measurements that involve evaluating the vertex function between on-shell nucleon spinors. For such measurements, there is an equivalent form $\Gamma_2^\mu = \gamma^\mu(F_1+F_2) - (P+P')^\mu/(2M)F_2$. However, using these to evaluate the Born diagrams gives different results. We isolate the resulting ambiguity by considering the propagators as evaluated in the rest frame

$$\frac{\not P \pm \not k + M}{(P\pm k)^2 - M^2 + i\epsilon} = \frac{\sum_s u(\pm\vec{k},s)\bar{u}(\pm\vec{k},s)}{(P\pm k)^2 - M^2 + i\epsilon} + \frac{\gamma^0}{M - i\epsilon + \sqrt{M^2+\vec{k}^2} \pm k^0}. \quad (5)$$

The first term of Eq. (5) corresponds to the nucleon pole term and using it with either of $\Gamma_{1,2}^\mu$ yields the same result. The second term corresponds to part of the left- and right-hand cut terms related to the production of antinucleons. This is also included in the contribution to the dispersion integral arising from inelastic states, so there is overcounting. Evaluations using the first term do not depend on choice of vertex function, but using the second term does.

Define the resulting difference in the virtual photon proton scattering amplitudes as

$$\Delta T^{\mu\nu} \sim \frac{\Gamma_1^\mu \gamma^0 \Gamma_1^\nu - \Gamma_2^\mu \gamma^0 \Gamma_2^\nu}{M - i\epsilon + \sqrt{M^2+\vec{k}^2} + k^0} + (\mu,\nu,k^0) \to (\nu,\mu,-k^0),$$

with $\Delta T^{\mu\nu} \propto F_2$. To gauge the size of such effects, we compute the contribution to the energy, ΔE, of the 2S state caused by $\Delta T^{\mu\nu}$. This is given by $\Delta E \propto \int d^4k L_{\mu\nu} \Delta T^{\mu\nu}/(k^2+i\epsilon)^2$, where $L_{\mu\nu}$ is lepton tensor including propagators. Explicit evaluation shows that this ΔE is about a substantial 0.4 meV. This is about the same value as needed to resolve the proton radius puzzle. This means that previous calculations of the proton polarizability effects are not well defined.

The ambiguities in the dispersion relation approach as applied to composite (non-Dirac) fermions indicate that additional (as yet unmeasured) proton structure properties need to be introduced. One constructive way to evaluate proton polarization effects is to use our postulated form of off-mass shell form factor, Eq. (2), and test its consequences in different physical environments.

In conclusion, we have shown that a simple off-shell correction to the photon-proton vertex, which arises naturally in quantum field theory and is consistent with gauge invariance, is capable of resolving the discrepancy between the extraction of the proton charge radius from Lamb shift measurements in muonic and electronic hydrogen. Off-shell effects of the proton form factor were an explicit concern of both Zemach [6] and Grotch and Yennie [7]. It is only with the remarkable improvement in experimental precision recently achieved [1] that it has become of practical importance. The effect postulated here can be investigated in lepton-nucleus scattering via the binding effects of the nucleon, as well as by lepton-proton scattering in arenas where two photon (or γ,Z) effects are relevant.

This research was supported by US DOE Energy Grant No. FG02-97ER41014 and LBNL Contract No. DE-AC02-05CH11231 (G.A.M.), Grant No. DE-FG02-04ER41318 (J.R.), Contract No. DE-AC05-06OR23177 (J.D.C., in part), and the Australian Research Council and the University of Adelaide (A.W.T. and J.D.C.). G.A.M. and J.R. gratefully acknowledge the support and gracious hospitality of the University of Adelaide while the project was undertaken. We thank M. C. Birse, J. A. McGovern, R. Pohl, T. Walcher, and A. Walker-Loud for useful discussions.

[1] R. Pohl *et al.*, Nature **466**, 213 (2010).
[2] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008).
[3] M. I. Eides, H. Grotch, and V. A. Shelyuto, Phys. Rep. **342**, 63 (2001).
[4] J. L. Friar, Ann. Phys. **122**, 151 (1979).

[5] S. G. Karshenboim, Phys. Rep. **422**, 1 (2005); E. Borie, Phys. Rev. A **71**, 032508 (2005); A. P. Martynenko, Phys. At. Nucl. **71**, 125 (2008); K. Pachucki and U. D. Jentschura, Phys. Rev. Lett. **91**, 113005 (2003); A. Veitia and K. Pachucki, Phys. Rev. A **69**, 042501 (2004); E. Borie and G. A. Rinker, *ibid.* **18**, 324 (1978); J. L. Friar, J. Martorell, and D. W. L. Sprung, *ibid.* **59**, 4061 (1999).

[6] A. C. Zemach, Phys. Rev. **104**, 1771 (1956).

[7] H. Grotch and D. R. Yennie, Rev. Mod. Phys. **41**, 350 (1969).

[8] U. D. Jentschura, Ann. Phys. **326**, 500 (2011).

[9] J. D. Carroll, A. W. Thomas, J. Rafelski, and G. A. Miller, Phys. Rev. A **84**, 012506 (2011).

[10] V. Barger, C. W. Chiang, W. Y. Keung, and D. Marfatia, Phys. Rev. Lett. **106**, 153001 (2011).

[11] J. Jaeckel and S. Roy, Phys. Rev. D **82**, 125020 (2010).

[12] P. Brax and C. Burrage, Phys. Rev. D **83**, 035020 (2011).

[13] U. D. Jentschura, Ann. Phys. **326**, 516 (2011)

[14] A. De Rújula, Phys. Lett. B **693**, 555 (2010).

[15] I. C. Cloet and G. A. Miller, Phys. Rev. C **83**, 012201 (2011)

[16] M. O. Distler, J. C. Bernauer, and T. Walcher, Phys. Lett. B **696**, 343 (2011)

[17] D. F. Geesaman, K. Saito, and A. W. Thomas, Annu. Rev. Nucl. Part. Sci. **45**, 337 (1995); M. M. Sargsian *et al.*, J. Phys. G **29**, R1 (2003).

[18] W. Melnitchouk, A. W. Schreiber, and A. W. Thomas, Phys. Rev. D **49**, 1183 (1994).

[19] F. Gross and S. Liuti, Phys. Rev. C **45**, 1374 (1992).

[20] S. A. Kulagin and R. Petti, Nucl. Phys. A **765**, 126 (2006); C. Ciofi degli Atti, L. L. Frankfurt, L. P. Kaptari, and M. I. Strikman, Phys. Rev. C **76**, 055206 (2007).

[21] F. Gross and A. Stadler, Phys. Rev. C **78**, 014005 (2008).

[22] A. M. Bincer, Phys. Rev. **118**, 855 (1960).

[23] H. W. L. Naus and J. H. Koch, Phys. Rev. C **36**, 2459 (1987).

[24] R. D. McKeown, Phys. Rev. Lett. **56**, 1452 (1986).

[25] Z. E. Meziani *et al.*, Phys. Rev. Lett. **52**, 2130 (1984).

[26] S. Strauch *et al.* (E93-049 Collaborations), Eur. Phys. J. A **19**, 153 (2004)

[27] M. Paolone, S. P. Malace, S. Strauch *et al.*, Phys. Rev. Lett. **105**, 072001 (2010).

[28] D. H. Lu, K. Tsushima, A. W. Thomas, A. G. Williams, and K. Saito, Phys. Rev. C **60**, 068201 (1999); J. R. Smith and G. A. Miller, *ibid.* **70**, 065205 (2004).

[29] I. C. Cloet, G. A. Miller, E. Piasetzky, and G. Ron, Phys. Rev. Lett. **103**, 082301 (2009).

[30] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2542 (1980).

[31] S. J. Brodsky, T. Huang, and G. P. Lepage, Springer Tracts Mod. Phys. **100**, 81 (1982).

[32] S. D. Drell and J. D. Sullivan, Phys. Lett. **19**, 516 (1965).

[33] J. Bernabeu and C. Jarlskog, Nucl. Phys. B **60**, 347 (1973).

[34] K. Pachucki, Phys. Rev. A **60**, 3593 (1999)

[35] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A **84**, 020102 (2011).

[36] K. Abe *et al.* (E143 Collaboration), Phys. Lett. B **452**, 194 (1999)

[37] R. J. Hill and G. Paz, e-print arXiv:1103.4617 [hep-ph].

[38] S. J. Brodsky and J. R. Primack, Ann. Phys. **52**, 315 (1969).

[39] S. J. Brodsky, F. J. Llanes-Estrada, and A. P. Szczepaniak, Phys. Rev. D **79**, 033012 (2009).