Towards a better understanding of the potential of Interactive Whiteboards in stimulating mathematics learning.

Abstract

This research aims at exploring an effective set to stimulate mathematics understanding and learning in an Interactive Whiteboard (IWB) environment. IWB affordances appear to be best used when mathematical tasks engage students in mathematical reasoning and when all students are involved in the discussion. The intent of this research project was to design and implement, together with a small group of teachers, a series of lessons for the purpose of developing a useful framework for effective IWB use. In a first phase, the potential of the IWB in pursuing high-level mathematical tasks and promoting classroom interactivity was discussed in depth by the teachers and the researcher. Lessons were also planned in detail. In a second phase, the planned lessons were taught in the presence of the researcher, taped by him and subsequently analysed by the researcher and the teachers. The analyses highlighted the usefulness of the IWB in (a) improving high-level mathematical tasks and (b) creating a dialogic interactive discourse for better mathematical understanding and learning.

Two main patterns in productive IWB use emerge from the study. The first pattern is that the IWB promotes problem-solving activities through an intensive use of geometrical or other mathematical software. The second pattern is to use the IWB as a notepad with links to external sources, geometrical and other mathematical constructions, problems and activities, which the teacher, in collaboration with the students, 'tailors' following a thread. For both patterns, developing a strong synergy between the IWB affordances and students' interaction with it seems critical. The IWB appears as a powerful tool that allows students and teachers to alternate between different points of view and different visualisations of the same topic.

Keywords: Interactive Whiteboards, Mathematics Education, Learning Environments, Teaching/learning Strategies

Introduction

The research presented in this paper aims to find methods to utilise the distinctive role of the Interactive Whiteboard (IWB) in designing and enacting mathematics lessons that address high-level cognitive demands and improve intensive cooperation between teacher and students and between students themselves.

Interactive Whiteboard (IWB) systems provide a multimodality environment wherein images, texts, insertions from other software programmes (e.g. mathematical software) can be combined and manipulated directly on the screen by teachers and students. IWBs are equipped with dedicated software but they can also be considered a digital hub that allows teachers and students to integrate Internet or other hardware resources into lessons. Objects from other technologies, for instance geometric dynamic software, can easily be displayed on the IWB and can directly be manipulated by teachers and students to create an interactive experience accessible to all during lessons. Results from these manipulations can also be stored and retrieved for use in future lessons (Mercer, Hennessy & Warwick, 2010).

These affordances (preconditions for activity, as discussed by Gibson (1977)) make IWB a high-potential innovative tool in mathematical educational environments in which teachers' modelling processes, students' exploring activities and other instructional strategies have large opportunities. Nevertheless, these affordances are not self-evident; many teachers seem to use the IWB merely as a large-scale visual blackboard or a simple presentation tool. Teachers often fail to exploit the above-mentioned innovative pedagogical advantages of IWBs (Moss et al., 2007, Somekh et al., 2007). The use of IWBs is consequently potentially likely to have no significant impact on teachers' pedagogy. An in-depth study of the potential of IWBs in mathematical learning and teaching is consequently needed as well as a demonstration of situations in which IWB potential is beneficially and effectively exploited.

This study aims to present a series of environments in which learners engage in challenging mathematical tasks and intensive discourse, and in which extensive use is made of an IWB. IWB affordances appear to be used to maximum effect when mathematical tasks engage students in mathematical reasoning and when all students are involved in the discussion. The objective of this study was to design and implement, together with a small group of teachers, a series of lessons in which IWB promoted solving of high-level mathematical tasks and fostered intensive dialogic discourse.

Literature review and theoretical framework

Three elements were identified in a previous study (De Vita, Verschaffel & Elen, 2014) as essential ingredients in the development of an effective IWB-supported interactive environment for mathematics teaching and learning:

- the level of the mathematical content, i.e. the cognitively demanding mathematical tasks in which students are engaged;
- the quality of the discourse interaction between the teacher and the students, and amongst the students themselves;
- the support that IWB can lend to the previous two elements, i.e. mathematical content and discourse interaction.

For the level of mathematical content, most mathematics educators have argued that mathematics consists of more than knowledge of mathematical concepts, principles, techniques, procedures (e.g., Collins, Brown & Newman, 1989; Schoenfeld, 1992). For them, mathematics not only consists in applying standard procedures generally explained in school textbooks, but also in engaging in the processes of mathematical thinking, in reasoning about key mathematical concepts and in solving and managing mathematical problems.

Hiebert and Grouws (2007) highlight the concept of teaching for conceptual understanding in particular, i.e. "the mental connection among mathematical facts, procedures and ideas" (Hiebert & Grouws, 2007, p. 380). They individuate two critical approaches in the pattern of teaching for conceptual understanding: (1) teachers and students attending explicitly to concepts, i.e. treating mathematical connections in an explicit way, discussing the mathematical meaning underlying procedures, attending to the relationships between mathematical ideas, etc.; and (2) students struggling with important mathematics, i.e. expending efforts to make sense of mathematics, figuring something out that is not immediately apparent, solving problems "that are within reach and grappling with key mathematical ideas that are comprehensible but not yet well formed" (Hiebert & Grouws, 2007, p.387). These principles were integrated into the guidelines for mathematics teaching in several countries, for instance, in the Principles and Standards for School Mathematics (2000) and the Common Core State Standards Initiative (2009) in the US, in the national curriculum for mathematics in the UK (UK Department of education, 2014) and in the national guidelines for mathematics teaching in Italy (MIUR, 2012). Among other things, these guidelines for mathematics teaching focus on building new mathematical knowledge through problem-solving, on conjecturing relationships and generalisations, and on making rich connections between mathematical ideas to develop fluency, mathematical reasoning skills and competence in solving increasingly sophisticated problems. Relating to the importance of mathematical tasks, Doyle (1988) and Hiebert and Wearne (1993) observe that students' learning is mainly defined by the tasks they are given. Doyle (1988) argues that "the focus for tasks involving higher cognitive processes is on comprehension, interpretation, flexible application of knowledge and skills, and assembly of information from several different sources to accomplish work" (p. 170). Henningsen and Stein (1997) observe that "not only must the teacher select and appropriately set up worthwhile mathematical tasks, but the teacher must also proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task" (p. 546).

In addition to the level of the mathematical tasks, the quality of discourse interaction between teacher and students and amongst the students themselves is another essential element of an effective instructional set. Instruction consists of interactions involving teachers, students and content, i.e. how teachers and students interpreted and interacted with one another and with the task at hand. Effective instruction is dynamic, including teachers' and students' interaction, the mathematical tasks in which they engage, and the environment in which they act (Ball & Forzani, 2009; Cohen & Ball, 2001).

In their review of research in mathematics classrooms, Walshaw and Anthony (2008) highlight how

> "the opportunity for learning is influenced by what students are helped to coproduce through dialogue. The effective use of classroom discourse makes students' mathematical reasoning visible and open for reflection. In an environment where ideas are shared, students' own ideas become resources for their own learning. But more than that: their explanations stimulate, challenge, and extend other students' thinking. The most productive discourse is that which allows students to access important mathematical concepts and relationships, to investigate mathematical structure" (Walshaw & Anthony, 2008, p. 539).

Considering and discussing students' mathematical contributions allows students to see mathematics as a collective construction; it sustains students' learning by involving them in the formulation and verification of concepts, and helps students conceptualize mathematical activities. According to Walshaw and Anthony (2008), "a context that supports the growth of students' mathematical identities and competencies builds on students' responses, shapes the reasoning and thinking to an appropriate level, and moves ideas and solutions toward a satisfactory conclusion" (p. 539). An important result of this type of classroom discourse is a clear articulation of students' thinking, supported by careful listening and responsive scaffolding by the teacher.

A key challenge with respect to the present study is how IWB use can support engagement in high-level mathematical tasks and rich discourse interactions, the third element that we considered. Research in learning with technological tools has shown that technology can help produce learning environments in which students have ampler opportunities to construct mathematical meanings, to explore and experiment with mathematical ideas and to express these using a wide range of representations (Ruthven, 2007).

Ruthven (2008) highlights that integrating new mathematical technologies such as dynamic geometry and computer algebra into mathematics lessons is a complex challenge. Introduction of new mathematical technologies challenges well-established curricula as it requires smooth integration of new technology. An important theoretical development (dubbed *the instrumental approach*) offered in response to this challenge has been the analysis of the process through which use of a material resource, object or artefact is elaborated as it becomes a functional instrument for the user, the *instrumental genesis* (Artigue, 2002; Guin, Ruthven & Trouche, 2005). Similarly, Ruthven (2008) refers to the cultural process through which new tool-mediated forms of mathematical thinking emerge within a mathematical community as one of *instrumental evolution*.

Trouche (2004) and Drijvers, Doorman, Boon, Reed and Gravemeijer (2010) extend the instrumental approach to the notion of *instrumental orchestration*, defined as the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment in a given task situation with the aim of guiding students' instrumental genesis.

In the present study, the first two elements – the level of mathematical tasks and the quality of discourse interaction – were considered as preconditions and discussed by the teachers and the researcher in a first discussion phase before the actual enactment of the lessons. The design and improvement of the lessons was focused on IWB's potential contribution to mathematical learning and teaching.

In collaboration with the teachers, the modality and quality of the mathematical tasks covered in the classroom were categorised according to the classification developed by Stein, Grover and Henningsen (1996). Stein et al. (1996) define a *mathematical task* as a classroom activity the purpose of which is to focus students' attention on a particular mathematical idea. The kind of mathematical tasks with which students engage determines not only the contents they absorb but also how they come to think about, develop, use and make sense of mathematics. According to

Stein et al. (1996), "the tasks used in mathematics classrooms highly influence the kinds of thinking processes in which students engage" (p.462*).* Henningsen and Stein (1997) observe that high level demanding tasks are often more difficult and take more time than ordinary classroom activities, and thus more subject to factors that could cause a drop off in students' involvement to less demanding processes. For this reason, teachers should be especially attentive to the extent to which meaning is emphasized and the extent to which students are explicitly expected to demonstrate understanding of the mathematics underlying the activities in which they are engaged.

Stein and Smith (1998) distinguish between mathematical tasks at two levels and between four categories of cognitive demand:

- *Low-level tasks*:
  - *Memorisation*: committing facts, rules, formulas or definitions to memory;
  - *Procedures without connections to concepts or meaning*: the use of formulas, algorithms or procedures *without connection* to concepts, understanding or meaning;
- *High-level tasks*:
  - *Procedures with connections to concepts and meaning*: the use of formulas, algorithms or procedures *with connection* to concepts, understanding or meaning;

- *Doing mathematics*: including complex mathematical thinking and reasoning activities such as making and testing conjectures, framing problems, looking for patterns and so on.

The first precondition was that the teachers would work only on high-level tasks, which is not self-evident, i. e., demanding tasks wherein students are required to connect procedures to concepts and meaning or to do mathematics.

The quality of discourse interaction between teacher and students and amongst students themselves was the second element considered as a precondition.

Discourse interaction is rooted in the concept of ”dialogic teaching” (Alexander, 2008; Mercer, 2004) as a means to promote and enhance students' learning. According to Wolfe and Alexander, (2008), a dialogic approach is collective (teachers and students address learning tasks together), reciprocal (teachers and students listen to each other, share ideas and consider alternative viewpoints) and cumulative (teachers and students build on their own and each other's ideas to develop them into coherent lines of thinking and enquiry. Smith, Hardman and Higgins (2006), who conducted a study that specifically focused on the different types of discourse moves in an IWB environment, first developed the framework used to investigate this precondition. Their analysis scheme focused on the three-part IRF structure (Sinclair & Coulthard, 1975): an *Initiation*, usually in the form of a question submitted to students, a *Response* in which students attempt to answer the question and a F*eedback* that has the teacher provide some form of feedback to students' responses.

Each of the three parts is further divided into a detailed, multi-part frame. *Initiation* includes *explain*, *closed questions, open questions* and *students' questions. Response* includes *single student answers, choral responses, spontaneous contributions* and g*eneral discussion. Feedback* includes *repeat questions* (questions that shape the course of implementation provide teachers with specific ways of focusing students on the key mathematical ideas, leaving less room for "in the moment" decision-making vis-à-vis subsequent instructional moves)*, uptake questions* (teachers incorporate students' answers into subsequent questions)*, probe* (teachers stimulate students to further elaborate), and *refocus* (get students back on task).

The previous framework was used as an organizer to classify and describe the interactions between teacher and students, also considering who was the actor of the IWB actions. For this reason, discourse moves and IWB actions have been crossed, so that it would be possible to clarify the impact of the IWB on teacher-students interaction. Although the framework could be described as teacher-centered, through an in-depth qualitative investigation it was possible to analyse extended possibilities for learning, for example whether the students were encouraged to suggest their solutions and to discuss them with the teacher and other students. As stated by Klette (2009), the IRF pattern conveys more learning opportunities and patterns of dialogic interaction (both teacher-student and student-student) than identified in earlier studies.

The second precondition was that discourse moves would show a prevalence of open questioning, general discussion, spontaneous contributions, with an open instructional approach by the teacher and students engaging directly with the IWB.

In light of these two preconditions, the objective of this study was to investigate and examine what type of support IWB can provide in developing high-level mathematical tasks and promoting discourse interactivity.

This was examined by applying a framework based on the work of Kennewell and Beauchamp (2007), who investigated how teachers use IWB features to enhance learning activities in a set of observed mathematical lessons. Kennewell and Beauchamp (2007) identified teacher and learner pedagogical actions supported by IWB features and built a classification framework that mapped the impact of these actions on learning. These activities focused on learner–IWB interaction, teacher–IWB interaction, learner–teacher and learner–learner interaction through the IWB. Examining how these actions were used in classroom activities and linking them to the actors (students, teachers) provided an exhaustive overview of ways in which IWB contributed to the development of high level mathematical tasks and discourse interactivity.

Kennewell and Beauchamp's actions included *Composing* (use IWB as a tool to develop or record ideas), *Editing* (modify the data stored and displayed), *Selecting* (choose resources or procedures), *Comparing* (compare features of the same object from different angles or contrast different items), *Retrieving* (access stored resources), *Apprehending* (students interpret the display, i.e. text, images, sound, diagrams), *Focusing* (draw attention to particular aspects of a process or representation), *Transforming* (alter the way data are displayed), *Collating* (bring together a variety of items from different sources), *Annotating* (add notes to a process or representation), *Repeating* (iterate an automated or stored process), *Modelling* (simulate a process by representing relationships between variables), *Revisiting* (return to an activity with a different focus), *Undoing* (reverse an action). Because external software is often used in mathematics lessons, the framework was adjusted by dividing the action *Composing* into two categories for the present study: C*omposing using software* (elaboration through external software) and *Composing without software* (elaboration without external software).

Method

The present study might be said to use a developmental research method, a qualitative approach similar to design-based research with a number of lessons designed by teachers and a researcher in a first phase, and implemented and discussed in a second phase. Development involves interaction and collaboration with research participants to approximate interventions (Wang & Hannafin, 2005). The research is context specific with conclusions typically taking the form of lessons drawn from the development of a specific environment, as well as conditions that improve the effectiveness of that environment. As Verschaffel and Greer (2013) note, "design experiments typically involve a kind of interdisciplinary teamwork that evolves among practitioners, researchers, teacher educators and community partners around the design, implementation and analysis of changes in practice" (p. 555). For this study, the researcher promoted intervention design and progressively refined this together with the participants. He acted as stimulus in the preliminary discussion and assisted teachers in designing the lessons. During discussions about the taught lessons, the researcher highlighted positive and negative aspects, and this way pushed the participants to carefully reflect on the three main elements of the above-mentioned theoretical framework, namely task enactment, discourse interactivity and IWB support.

Three mathematics teachers (Giovanna, Ida and Stefano) in secondary schools in Italy participated in the study in the period between January and May of 2014. The study comprised three phases: the first was to design and implement a cycle of lessons for each teacher, while the second was to teach the designed lessons and test them in the classroom. In a third phase, the implemented lessons were analysed and discussed by the teachers and researcher.

The first phase (January - March 2014) included a series of discussions with the teachers about the general lesson outlines. The researcher highlighted the potential of IWB for facilitating high-level mathematical tasks and promoting classroom interactivity in these talks. All the teachers participated in four group discussions. In addition, the researcher met with every teacher individually twice to discuss the preparation of the lessons. The teachers prepared detailed lesson plans that included learning goals, mathematical tasks and different discourse moves. Particular attention was paid to the role of IWB in enhancing the lessons. The discussions with the teachers were tape-recorded.

In a second phase (March - May 2014), the prepared lessons were taught in the presence of the researcher and videotaped by him. Each teacher taught four lesson hours. The mathematical tasks were not analysed as they had been decided on and agreed to by all the teachers in the first phase. Discourse moves, however, were analysed to verify the quality of discourse interaction between teacher and students, and amongst the students themselves. The lessons were videotaped and the recordings coded according to the two frameworks mentioned above, namely IWB support and interactivity in the classroom through the forms of the discourse. The coding registered what was happening in the classroom every 30 seconds, from the beginning to the end of a lesson. A note was made of the actor – either teacher or student – for every type of discourse move and IWB action.

The analysis followed the discussed classification for each element and was focused on obtaining a detailed picture of the teaching and learning setting, highlighting how the IWB was used to facilitate high-level mathematical tasks and revealing the nature of the dialogic interactive discourse.

The frameworks used for the analysis are summarised in Table 1.

Table 1. Frameworks used in the analysis.

Discourse moves *Initiation:* • *Teacher's open question* • *Teacher's closed question* • *Student's question* • *Explain* *Response* • *Single student answer* • *Choral response* • *Spontaneous contribution* • *General* *discussion Feedback* • *Repeat question* • *Uptake question* • *Probe* • *Refocus*	
IWB actions • *Composing without software* • *Composing using software* • *Editing* • *Selecting* • *Comparing* • *Retrieving* • *Apprehending*	• *Focusing* • *Transforming* • *Collating* • *Annotating* • *Repeating* • *Modelling* • *Revisiting* • *Undoing*

Results

The following section discusses how the teachers and researcher developed optimal IWB uses. First, how the potential role of the IWB was discussed in the preparatory phase between the teachers and researcher and planning of the lessons. Second, how the three participating teachers actually used IWB in their classrooms and how this use reflected the teachers' intentions.

1. Preparatory phase: general discussions, meetings with individual teachers, lesson planning.

The general discussions centred on use of IWB to support mathematical learning and classroom discourse interactivity. Detailed lesson plans were prepared that specified particular classroom activities and included IWB actions particularly effective in sustaining teaching and learning activities. As high-level mathematical tasks and high-level classroom interaction were considered prerequisites, the discussion first focused on these two prerequisites and then explored issues related to IWB use. Discussions covered the following topics in order of mention: (i) the topics to be taught and related mathematical tasks; (ii) promoting high-level classroom interaction around these topics and tasks; and (iii) the supporting role of the IWB in developing the chosen topics.

The topics were selected taking into consideration the curricula followed by the teachers and paying attention to the inclusion of *high-level mathematical tasks,* i.e. *procedures with connections to concepts and meaning* or *doing mathematics,* to positively influence the kinds of thinking processes that would succeed in engaging students (Stein et al., 1996). The topics were also chosen in view of their suitability to be implemented through IWB. Not all the topics were fit to be implemented through IWB. It is for instance difficult to use IWB affordances when pure algebraic procedures are used. The topics should allow multiple visualisations and manipulations using mathematical software. For instance, concepts from analytical and Euclidean geometry as well as statistical topics and analytical functions perfectly lent themselves to IWB use. Different topics were discussed and analysed in relation to their suitability for implementing IWB use.

In view of the lesson plans followed by the teachers involved in the study, the following topics were chosen:

- constructing a circle, exploring relationships between circles and lines (tangents), and between circles and triangles. This task was classified by the team as *doing mathematics,* (Giovanna);
- investigating 3D geometry concepts and properties, also in relation to 2D geometry (planes, lines and angles in the plane and in space), studying regular polyhedra (platonic solids), proofing Cavalieri's principle and Galileo's bowl (volume of the sphere). These tasks were classified as *procedures with connections to concepts and meaning* (Ida);
- constructing an ellipse and exploring its features, geometrically and analytically, and exploring real-life ellipse applications. These types of tasks were categorised under *procedures with connections to concepts and meaning* (Stefano).

All the above tasks were considered high-level tasks, in accordance with Stein and Smith (1998) framework. The three teachers planned a cycle of four one-hour lessons.

Different methods of involving students in the development of the lessons were discussed at length during the meetings with the teachers. There was wide agreement on the need to enhance the dialogue amongst students as well as between teachers and students through open questioning and probes. Problem-solving activities, when possible done in small groups of students, were considered a powerful means to promote discussion and argumentation, to choose alternative solution methods, to improve students' collaboration and self-confidence, and to encourage more critical and elaborate contributions from students. The direct involvement of the students in managing IWB was also considered a crucial point that stimulates students' participation during lessons. IWB use can promote students' involvement by serving as a tool that allows students to perform mathematical software manipulations, and to actively engage in researching materials, ideas and multimedia resources.

The teachers considered different ways to stimulate students' involvement. Giovanna suggested students first work in small groups during problem-solving activities, then present their solutions at the IWB and discuss these with the entire class.

> *Excerpt 1* (Discussion on January 27, 2014)
> *Giovanna*: I think to make students working on different problems of the same difficulties. I will divide students in small groups, and at the beginning of the lessons I would give them a short time to reflect about the problems. They have to find quickly a solution draft, and then develop it at the IWB.
> *Ida*: Do you allow them to use computers while reflecting?
> *Giovanna*: No, they should elaborate just an idea of the solution, using paper and pencil, and then check and elaborate this idea at the IWB, with the contribution of the whole classroom.
> *Stefano*: Thus will all the classroom be involved in solution process of all the problems?
> *Giovanna*: Yes, this is exactly my intention, so students will not lose concentration.

The other two teachers preferred a model in which the topic was developed through general classroom discussion, starting from open questioning. Students would be expected to retrieve materials from Internet, build tables of comparison and engage in problem-solving activities during this discussion.

The discussion subsequently focused on the role of IWB in the development of the lessons.

A number of IWB actions were identified as particularly effective in mathematics teaching in a full classroom context in a previous study (De Vita, Verschaffel & Elen, 2014):

- use of dynamic geometry software such as Geogebra or Excel spreadsheet (*composing using software*). Geogebra software can be used for exploration and elaboration purposes, for theorem demonstrations and to search for connections between concepts. As one teacher said during one of the discussions: 'It gets students used to see mathematical objects move';
- analysis of the same object from different points of view (*comparing*), for instance, the algebraic representation of a function and its graph;
- demonstrations of how graphs change in accordance to modified data (*transforming*), for instance, showing different kinds of parabola graphs depending on numerical coefficients;
- simulations of a process by representing the relationships between variables *(modelling*), for instance, exponential growth.

The participating teachers and researcher viewed these actions as especially critical while planning the cycle of lessons to be taught.

> *Excerpt 2* (Discussion on January 27, 2014)
> *Stefano:* We should maximally exploit the potentialities of mathematical software as Geogebra. This software allows dynamical representations, and students find very useful and

explicative to look at a figure and to its transformations when you apply for instance a symmetry, or when you change some parameters and the figure changes, showing relationships between its algebraic and its graphic form.

Giovanna: I agree with you, but I think we should get students used to working themselves with the software. It is more useful for them and they learn much more rather than only looking at the teacher.

Ida: Students should also learn to compare different instances in a process, looking for different examples that could show similarities and contrasts.

In addition to these actions particularly useful in explaining mathematical properties, a number of other actions were considered useful in stimulating students' participation and exerting a positive influence on students' reasoning. The Internet is a good source of materials, exercises, graphs and animations. The teachers supported the idea that they themselves as well as students use the Internet – the teachers to find resources, and students, whether individually or in the classroom through the IWB, for research purposes and to store interesting resources (*retrieving* and *revisiting*). The teachers deemed Internet searches to be particularly suited to Ida's planned lessons on 3D geometry, as highlighted below in this excerpt of a recorded discussion between the teachers and researcher about the best method to help students visualise a 3D space:

Excerpt 3 (Discussion on February 5, 2014)

Ida: It could be interesting to involve students in researching and selecting materials from the Internet, it could be a useful exercise for them, with the help of the teacher. I think I will do an accurate selection of resources before the lesson, but I will also leave space for students' researches.

Giovanna: In the 3D geometry there is always the problem to draw figures from scratch; using a 3D geometry software, like Cabri 3D, it is difficult because the software requires particular skills and it is difficult to manage, especially in the classroom.

Stefano: Likely the best solution is to look for images, animations; there are many websites where it is possible to find good materials, also interactive animations.

The idea of having students make their own IWB notebooks to collect, organise, enhance and highlight external software manipulations (e.g. Geogebra) as well as materials imported from other sources was also offered as an interesting action during the lesson planning. The following excerpt of a discussion between the teachers and researcher illustrates the importance ascribed to these student IWB notebooks:

Excerpt 4 (Discussion on February 5, 2014)

Giovanna: I thought that it could be interesting to record all the manipulations by Geogebra in an IWB file, showing the development of reasoning, adding comments to the figures, perhaps also with analytical examples.

Stefano: I think so, there is only the problem that figures by Geogebra reported in an IWB file become static, they lose the dynamicity of the original software.

> *Giovanna*: yes, but we could store both files, the original one by Geogebra and the IWB one. So students could have all the path done previously, and use it for revisiting the topics we treated. I think this is an important affordance that IWB offers, the possibility to review the work done, and in case to add or correct something. It is also a great opportunity for the classroom, to have a kind of library …
>
> *Ida*: I think to do the same thing in my work about 3D geometry, to take note of all the elaborations and save them. Elaboration by the students focuses them on the topic and helps them to clarify their ideas. More, taking notes allows the classroom to review the previous steps in case of need…

In addition, having students construct tables of comparison was seen as a useful means of promoting classroom discussion and involving students in the taught topics, for instance, the respective properties of objects in 2D and 3D geometry. These actions were categorised as *composing without software* or *annotating*.

The teachers' respective plans to use IWB affordances in their lessons at the end of the discussions was as follows:

- Giovanna: intensive use of Geogebra (*composing using software*), manipulations by students, *retrieval* and *annotation* of pictures from geometrical dynamic software. Also, students will build notebooks (IWB files) in which the instructional path is summarised for the purposes of further elaborations, or to be *retrieved* at a later stage to revisit information.
- Ida: *retrieval* of multimedia (movies, pictures); students will build tables that *compare* the characteristics of 2D and 3D spaces. Also, use of geometrical software 3D (Cabri3D) and *modelling*.
- Stefano: students will intensively use Geogebra with the IWB (*composing using software*), *annotate* pictures from Geogebra, *retrieve* multimedia (movies and pictures), *compare* different ellipse characteristics, *model* and *transform*.

Both the teachers and researcher deemed this planning generally satisfactory.

2. *Overview of the taught lessons*

In this section, we will report how the IWB was actually used in the three classrooms, to promote high-level mathematical tasks as well as teacher-student interaction and interaction between students themselves.

Giovanna

The first teacher (Giovanna) developed the lessons through problem-solving activities that students worked on in small groups. The students were in their third year at a Scientific Lyceum (ages 15 to 16). Four one-hour lessons were recorded. As planned, the lessons covered circle properties, relationships between circles and lines (tangents) and relationships between circles and triangles (both circumscribed and inscribed).

The lessons aimed to develop students' reasoning abilities through use of Geogebra, first graphically and subsequently by linking graphics to analytical geometry (calculus). Students were stimulated to do mathematics by high-level tasks (Stein and Smith, 1998). These are demanding tasks that involved making and testing conjectures and required framing problems.

The teacher prepared the problems in advance and organised them according to increasing difficulty. In a first preparatory phase, the teacher assigned problems to small groups of students (five groups, with each group assigned a different problem). This phase was intended as an 'exploratory' phase, in which students worked independently using paper and pencil, with enough time (around ten minutes) to reflect on and propose solutions that they could later check and elaborate on at the IWB. Each group subsequently used the software Geogebra to check and improve their solutions if applicable, and to explore alternative solutions together with the other students and the teacher. The software was intensively used as a laboratory and the IWB, which focused the attention of the whole classroom on the particular problem at hand, promoted classroom discussion. Students' explanations stimulated and challenged other students, whose interventions became resources for their own learning.

Students were presented with different kinds of problems of increasing difficulty levels, for instance, from construction of a circle when given three points (an easy problem given during the first lesson, see Fig. 1a), to the construction of a tangent from an external point (a more difficult problem given during the third lesson, see Fig. 1b).

Figure 1a. Elaboration by Geogebra

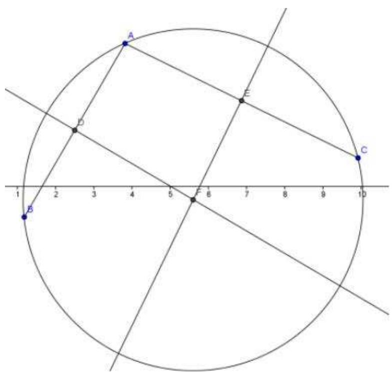
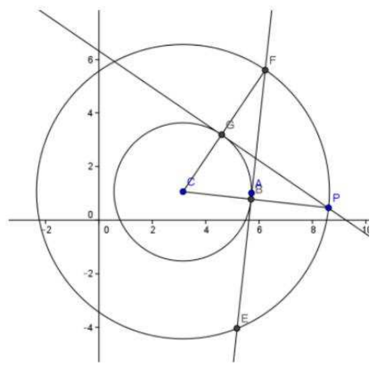


Figure 1b. Elaboration by Geogebra



When a solution was deemed to be correct, it was recorded in the form of an IWB notebook, with software pictures and annotations, as shown in Figures 2 and 3. This way, a problem-solving track was recorded and saved for every problem, one that could be re-used whenever necessary.

Figure 2. Student presenting an elaboration by Geogebra

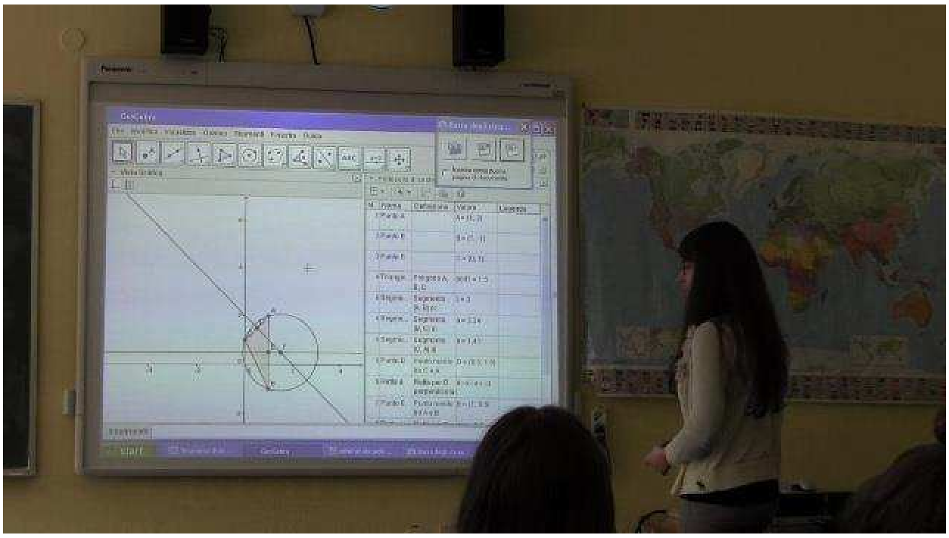


Figure 3. Student creating an IWB notebook

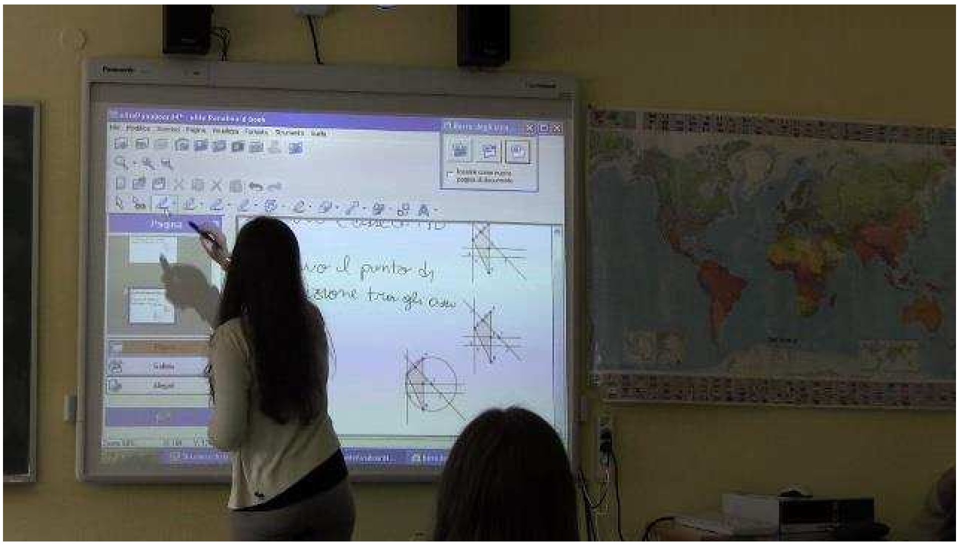


Table 2 summarises the data and shows the percentage rate at which each IWB action occurs in correspondence with the corresponding discourse moves out of the total amount of coded actions during the lessons (such a table was made for each teacher separately). For instance, *composing without software* accounts for 12% of the total amount of coded IWB actions, and this percentage divides into 2.8% *open questions*, 6.2% *repeat questions* and 3% *probe*.

The teacher stimulated students by using *open questions* (6.9%), offering suggestions and guiding the lesson progress through *repeat questions* (questions that focus students' attention on the key mathematical ideas, 11.6%) and *probe* (teacher stimulates students for further elaboration, 15.6%). As the table demonstrates, the teacher made ample use of the IWB to stimulate students and offer them suggestions.

Students made a continuous use of the IWB. The table divides students' contributions, both through Geogebra constructions and the creation of IWB notebooks, into spontaneous contributions (further divided into composing using software, which accounted for 39.5% of the coding; retrieving pictures from the geometrical dynamic software, 10.5%; and annotating them, 14.1%). The quality of these spontaneous contributions was high as students were directly engaged in difficult tasks and they experimented with different solutions using geometrical dynamic software.

Table 2. IWB actions and discourse moves (%)–Giovanna

IWB actions	*Composingwithoutsoftwa re*	*Composing usingsoftware*	*Comparing*	*Retrieving*	*Transforming*	*Annotating*	*Modelling*	*Revisiting*	*Total*
Discourse moves									
Initiation									
• open questions	2.8	4.1							6.9
• closed questions	0								0
• students' questions	0								0
• explain	0								0
• direct	0								0
Subtotal									6.9
Response									
• single student answer	0								0
• choral response	0								0
• spontaneous contribution	0	39.5		10.5		14.1			64.1
• general discussion	0	1.2				0.6			1.8
Subtotal									65.9
Feedback									
• repeat questions	6.2	2			1.7		1.7		11.6
• uptake questions	0								0
• probe	3.0	3.2		4.5		4.9			15.6
• refocus	0								0
Subtotal									27.2
Total	12	50	-	15	1.7	19.6	1.7	-	100

The lessons appeared interactive with significant involvement and participation from students, and collaboration between teacher and students. The effective use of classroom discourse made students' mathematical reasoning visible and opened it up to discussion. Students shared their own ideas, which stimulated, challenged and expanded the thought processes of their fellow students. They played a major role both in the theoretical development and in managing the IWB. The joint elaboration by students and teacher resulted in an instructional output that might be useful for further progress, or revisited at a later stage for reviewing purposes. During the IWB actions, students acted directly at the IWB in most cases (66.7%). During the work at the IWB, the teacher gave students time and opportunities to reflect on and develop their own solutions. Students were encouraged to solve the problems independently, and the teacher's hints and suggestions engaged students but did not directly provide a solution to the problems. The IWB acted as a powerful catalyst; it focused the attention of the full class on the screen where all the activity was happening.

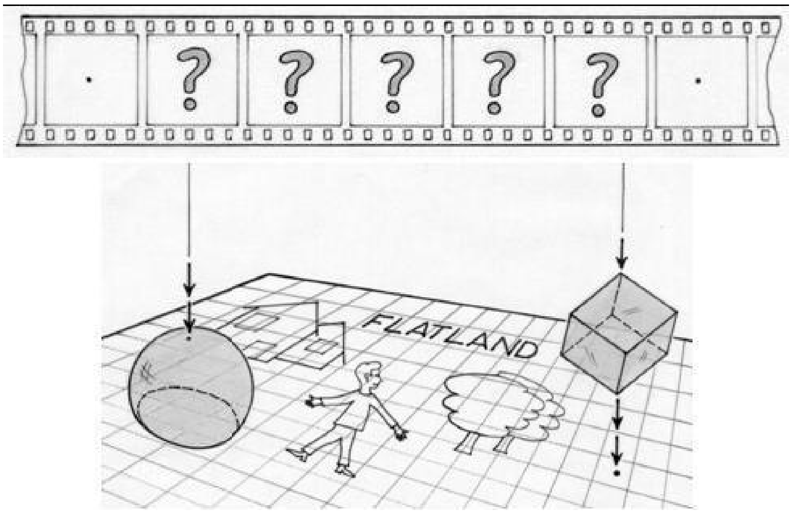
Ida

The second teacher (Ida) structured her lessons around a discussion with the entire classroom on geometrical concepts in the plane and in space. Her classroom was composed of students in their fourth year at a Scientific Lyceum (ages 16 to 17). Four one-hour lessons were recorded. The mathematical tasks focused on 3D geometry concepts and properties, also in relation to 2D geometry (planes, lines and angles in the plane and in space), construction of regular polyhedra (platonic solids), proof of Cavalieri's principle and Galileo's bowl (volume of the sphere). As previously planned by the teachers and researcher, the tasks were classified as *procedures with connections to concepts and meaning*, as defined by Stein and Smith (1998).

The teacher developed an instructional path that explored the main features of 3D spaces. The path started with a movie retrieved from the Internet that served as an introduction to 2D and 3D spaces. Inspired by a satirical novella by E. A. Abbott (1884), '***Flatland***' introduces viewers to a two-dimensional world inhabited by geometric figures, where women are simple line segments, while men are polygons with a varying number of sides. The narrator is a square, a member of the caste of gentry and professionals, who guides viewers through some of the implications of two-dimensional life. He is visited by a three-dimensional sphere that passes through Flatland. The sphere sections that pass through Flatland of course are circles. The square cannot comprehend this until he visits the three-dimensional world of Spaceland for himself.

After this movie was showed, the teacher assigned the students a problem-solving activity: What if a cube passed through Flatland moving in the direction of one of its diagonals?

Figure 4. Problem solving activity: a cube crossing a plan

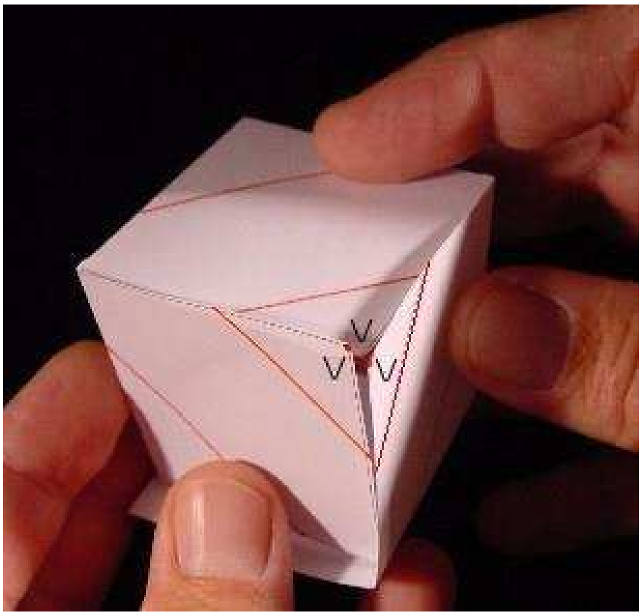


Students attempted to solve the problem at the IWB, conjecturing and drawing, and constructing paper models, as shown in Excerpt 5.

> *Excerpt 5* (April 24, 2014)
> *Student 1*
> First we draw a cube (*draws the cube at the IWB),* then we draw the plan crossing the cube *(he draws the plan)*. When it crosses, it generates triangles...

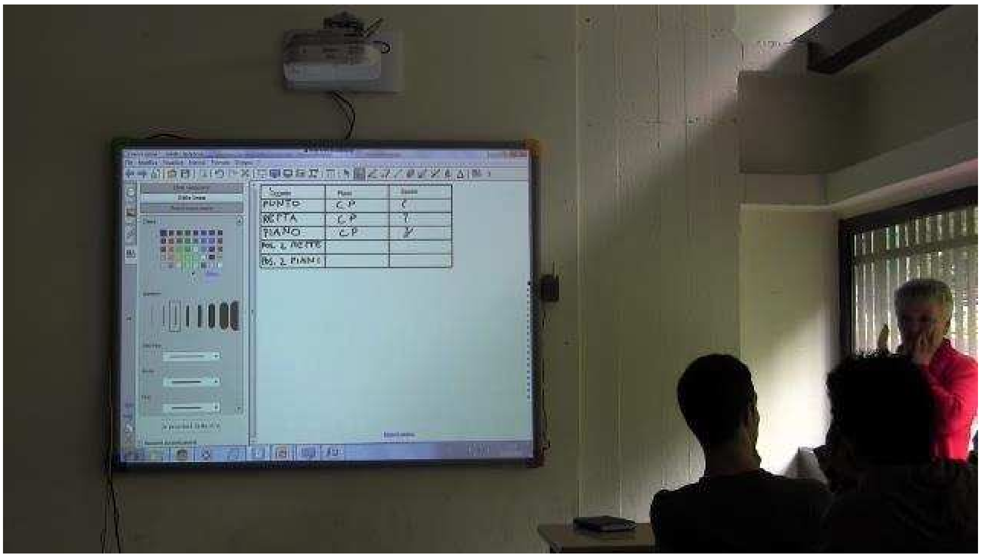
Teacher
How are these triangles in your opinion?
Student 1
Isosceles...no... equilateral, maybe?
Student 2
The cube has all the surfaces equal, the plan progresses at a constant speed, so at a certain point the crossings become squares.
Student 1
Then the figures progress in inverted order.
Student 3
We could try to build a paper model...
Teacher
Ok, let’s try.... (*Two students build a paper cube*)
Teacher
Let’s see on this cube how the grossing lines appear (*students draw on the cube the lines, as shown in Fig. 5*)
Teacher
Were the previous conclusions right?
Student 4
It does not seem right; at a certain point it seems to be a hexagon... (*draws a cube at the IWB and shows the different states in succession*). First, it is a point, in the vertex, then it becomes a triangle, and then a hexagon... but I am not sure...
Student 5
In my opinion, the square is wrong, the crossing cannot be a square.
Teacher
Well, at the moment we have two versions, one with the square and one with the hexagon... (*the teacher summarises the two versions)*
Who agrees with the first version? (*nobody answers*) Who agrees with the second one? (*the majority of the classroom agrees)*
(*the discussion continues for a while; the teacher suggests searching for solutions on the Internet*)
Student 6
(searches for solutions on the Internet at the IWB; several pictures and animations are retrieved and discussed by the entire classroom)

Figure 5. A possible solution: drawing crossing lines on a paper cube



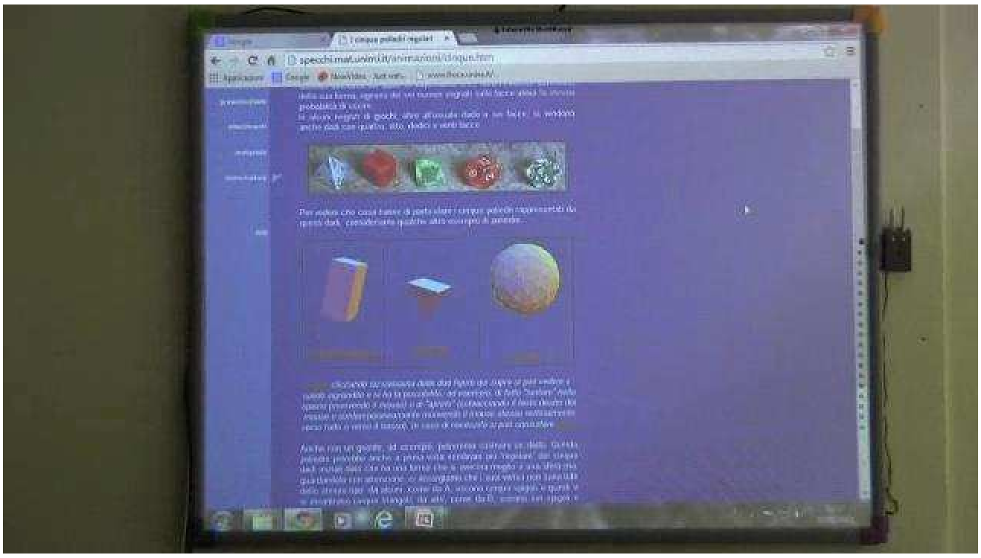
During the subsequent lesson, students built a table at the IWB that compared features of 2D and 3D spaces through a classroom discussion (Figure 6).

Figure 6. Ida's classroom: table comparing the features of 2D and 3D spaces



Students searched for and retrieved pictures, figures and animations online (see Figure 7). Students engaged with the topic of regular polyhedra and the mutual relationships between a number of vertexes (corners), edges and faces (Euler's formula).

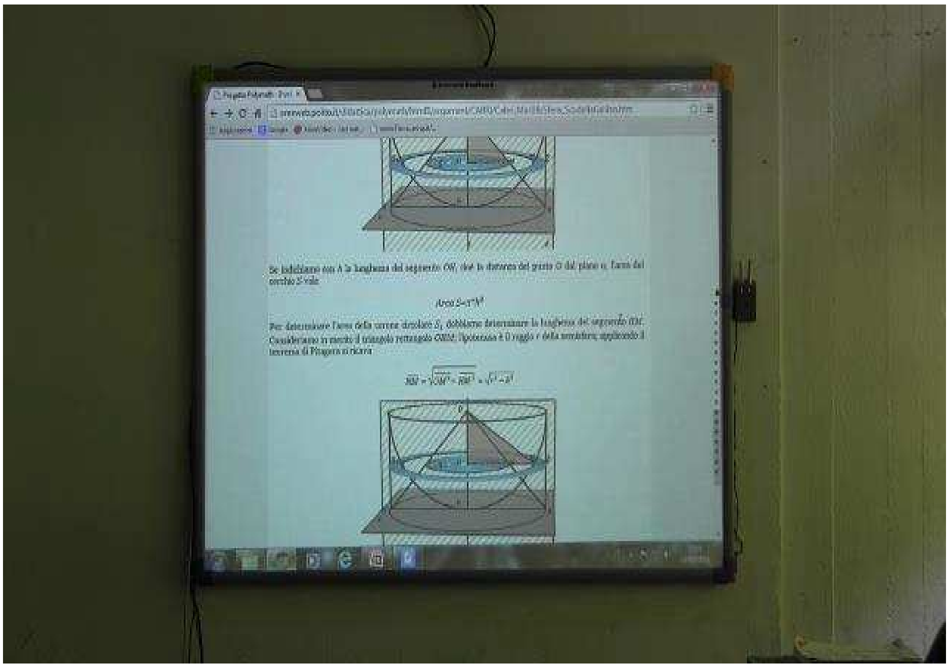
Fig 7. Ida's classroom: Polyhedra pictures (retrieved from the Internet: http://specchi.mat.unimi.it/animazioni/cinque.htm)



Finally, in collaboration with the teacher and using geometrical software animations, the students proved two main topics in 3D geometry (as shown in Fig.8):

- **Cavalieri's principle**, or the **method of indivisibles:** if two solids are intersected by parallel planes in cross sections of equal areas, then the two solids have equal volumes.
- **Galileo' bowl**, applying Cavalieri's principle: Galileo devised a method to calculate sphere volume (shown in Figure 8).

Fig 8. Galileo's bowl (retrieved from the Internet:http://www.slideshare.net/annamariaorlandi/il-volume-della-sfera-e-la-scodella-di-galileiprova-zacademy)



The lessons were structured as an extended class discussion, stimulated by the teacher's *open questions* (6.6%). All the students actively participated in the course of the four lessons, completing comparison tables, drawing models, and proposing and discussing hypotheses (*general discussion* 54.1%). The students used the IWB as an instrument to collect input from different sources and to build a synopsis of 3D geometry. They used it to show multimedia (movies, pictures), to create tables that *compared* 2D and 3D spaces characteristics and to show their drawing models, which used animations retrieved from the Internet (as well as animations developed with the 3D geometrical software programme Cabri3D). IWB actions such as c*omposing without software, retrieving* and *annotating* are the most heavily present in the coding, mainly in the form of general classroom discussions that involved all the students. Students were directly engaged at the IWB in most cases (61.8%).

Practical activities were sometimes part of the lesson (e.g. construction of a paper cube to better visualise a problem), but in general the IWB represented the actual centre of the classroom activities, engaging all students and stimulating them to participate.

The teacher prepared the structure of the lessons in advance, but students were given ample room to intervene and hypothesise. They were continuously involved in the lessons in a participative and original way – both through direct interventions at the IWB and classroom discussions. The teacher encouraged and pushed forward the discussion by *probing* (24.7%), mainly by retrieving different resources from the Internet (18.4 %).

Table 3 summarises the data and shows the rate (as a percentage of the total amount of coded actions) at which each IWB action occurs in correspondence with the corresponding discourse moves during the lessons. For instance, *composing without software* accounts for 28.6% of the total amount of coded IWB actions, and this percentage divides into 6.6% *open questions*, 20.6.% *general discussion and* 1.2% *probe*.

Table 3. IWB actions and discourse moves (%) – Ida

IWB actions	*Composingwithoutsoftwa re*	*Composing usingsoftware*	*Comparing*	*Retrieving*	*Transforming*	*Annotating*	*Modelling*	*Revisiting*	*Total*
Discourse moves									
Initiation									
• open questions	6.6								6.6
• closed questions									0
• students questions									0
• explain			2.6					0.2	2.8
• direct									0
Subtotal									9.4
Response									
• single student answer									0
• choral response				1.6					1.6
• spontaneous contribution						1.9	4.2		6.1
• general discussion	20.6		3.5	19.4		10.6			54.1
Subtotal									61.8
Feedback									

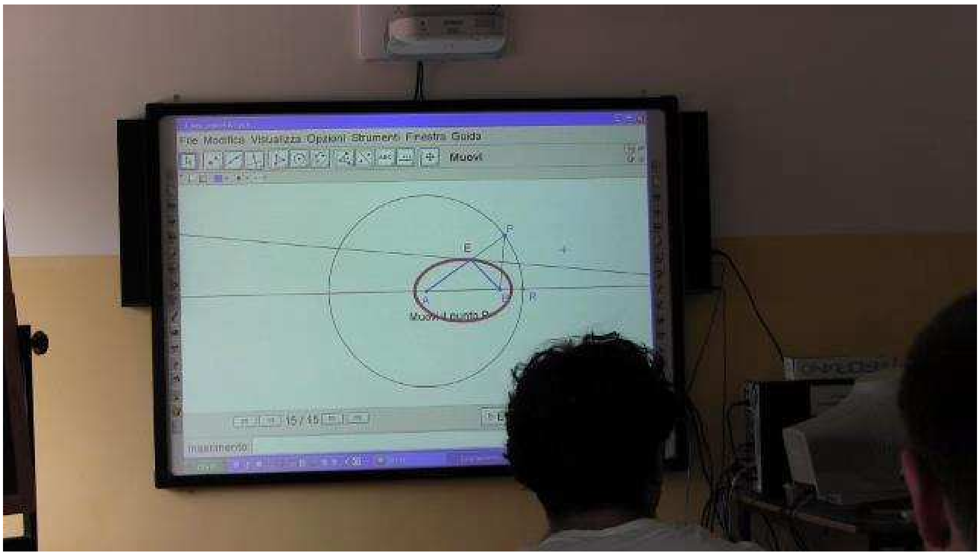
• repeat questions			3.9						3.9
• uptake questions									0
• probe	1.2		0,2	18.4		4.9			24.7
• refocus									0
Subtotal									28.8
Total	28.6		10.2	39.4		17.4	4.2	0.2	100

Stefano

The third teacher (Stefano) focused his lessons on the ellipse. In his lessons, the teacher drew heavily on previously prepared Geogebra files that explored the ellipse construction and its main properties in depth. His classroom was composed of third-year students at a Scientific Lyceum (ages 15 to 16). Four one-hour lessons were recorded.

The first two lessons were devoted to construction of the ellipse through different geometrical methods as well as the calculation of its analytical expression. Using the Geogebra programme, the teacher showed how to use geometry properties to draw the curve.

Figure 8. Ellipse construction using Geogebra.



With guidance from the teacher, students transformed the curve by calculating the analytical expression. By changing the distance between the focuses, students discovered the limit position of circle. The teacher also showed other geometrical transformations, for instance, how to modify the position of the ellipse in the plan and relating it to the analytical equation of the curve.

In the subsequent lessons, the teacher illustrated phenomena related to the ellipse as well all its real-world properties, for instance, Kepler's laws on planet orbits and ellipse applications in reflecting light or sound from one focus to another one.

The teacher primarily steered the direction of the lessons. Results of the analysis showed that one of the preconditions, namely the quality of interaction between students and teacher, was not fulfilled. As illustrated in Table 4, the teacher's role was dominant and it prevailed over the role of the students. *Initiation* was mainly coded as *closed question* (7.6%), and *Response* showed a prevalence of *Single student answers* (42%). This data is likely an indicator of a model in which the teacher guides students through a reasoning process in a constrained way that leaves few spaces for independent thinking.

The IWB was mainly used by the teacher (75.8%). In addition, when a student operated the IWB, he or she always followed the instruction of the teacher. The IWB was often used as a normal blackboard to perform calculations.

Table 4. IWB actions and discourse moves (%) – Stefano

IWB actions	*Composingwithoutsoftware*	*Composing usingsoftware*	*Comparing*	*Retrieving*	*Transforming*	*Annotating*	*Modelling*	*Revisiting*	*Total*
Discourse moves									
Initiation									
• open questions	5								5
• closed questions	4.8				2.8				7.6
• students questions									0
• explain									0
• direct									-
Subtotal									12.6
Response									
• single student answer	21.4	12.5		6.1	2.0				42
• choral response	5.3	15.4	3	2.2	1				26.9
• spontaneous contribution		1.2							1.2
• general discussion	8.4	3.6							12
Subtotal									82.1
Feedback									
• repeat questions				0.3					0.3
• uptake questions									0
• probe		3.8		0.9		0.3			5
• refocus									0
Subtotal									5.3
Total	44.9	36.5	3	9.5	5.8	0.3	-	-	100

Though the teacher subscribed to the outlined conditions in the preparatory phase, the lessons actually taught significantly departed from the agreed plans for the lessons. At the level of interaction between teacher and students, the latter were not stimulated to assume an active role during the lessons; rather, they were steered by the teacher both in the theoretical development and in the use of geometrical software. Geogebra files were prepared by the teacher in advance and shown to the students, who were offered no opportunities to work on them independently. The pictures, too, were selected by the teacher in advance and then presented to the students, who were not once allowed to present something on the IWB themselves. Students were led through the reasoning process step by step and the teacher left little room for cognitive autonomous processing, therefore lowering the level of the tasks to less demanding forms of cognitive activity, as the use of procedures without connection to the concepts and without an actual reasoning. It seems likely that the teacher struggled to adjust his directive teaching style and to allow more room for students' initiative.

As the main aim of this study was to investigate how IWB can provide support with high-level mathematical tasks and how it might promote discourse interactivity, Stefano's lessons were not considered relevant to the aim of this research because his lessons did not promote interactivity, while the IWB proved to be used in a rather directive manner.

Discussion and conclusions

After the teachers had taught their lessons, the recorded lessons were watched (fragments were selected together by the teacher who taught the lesson and the researcher). The teachers and the researcher reviewed and commented on these lesson fragments in an extended discussion that aimed to offer both a number of final considerations and to refine classroom teaching and learning as part of the study.

The three case studies showed significant differences both in the implementation of the lessons and in IWB use.

The first teacher (Giovanna) made extensive use of geometrical software to stimulate students' mathematical reasoning. Students who used this software engaged in independent solving of demanding tasks, and developed them sharing with the classroom their elaborations and their reflections. Having students work in small groups on problem-solving tasks promoted discussion and argumentation when alternative solution methods were explored, improved students' collaboration and self-confidence, and stimulated students to make more critical and extensive contributions. Works by the small groups were developed and discussed by the other students, this way becoming a subject that the entire classroom engaged with. The IWB was continuously at the centre of the activities, at the heart of the discussion; it contributed to the development of reasoning, both as an instrument of geometrical visualisation and as a tool that enhanced dynamic exploration of different conjectures. The aim of integrating dynamic geometry software into mathematics lessons in this case appeared to be fully met, with new possibilities for the development of mathematical curricula arising (Ruthven, 2008). These lessons appear to be a good example of a process that Ruthven (2008) calls 'instrumental evolution', through which new tool-mediated forms of mathematical thinking emerge within a mathematical community. Students were expected to use dynamic geometry with the aim of promoting mathematically productive activities. Discourse interaction between teacher and students sustained students' learning by involving them in the formulation and verification of concepts, by further building on students' responses, and by elevating their reasoning and thinking to an appropriate level.

The second teacher, Ida, developed an instructional path in the form of an extended class discussion that involved all the students; they actively participated in this discussion, completed comparison tables, drew models, and proposed and discussed hypotheses. The teacher planned her lessons with the aim of presenting the topic from different points of view (visualisation, exploration, theory), and exploiting different activities. The structure of the lessons, prepared in advance, gave students ample room for interventions and conjectures, both through direct interventions at the IWB and classroom discussions. As Walshaw and Anthony (2008) highlighted, discussion of students' contributions allowed students to see mathematics as a collective construction, sustained students' learning by involving them in the formulation and verification of concepts, and helped students conceptualise mathematical activities. IWB was used as an instrument to collect input from different sources and to build a synopsis of the 3D geometry together with the students. It served as a repository for the different suggestions and contributions, which were organised in a structured manner. The teacher played the role of orchestrator by organising and coordinating students' work, by systematically using the IWB as an instrument for teacher and student work, by guiding the students in the discovery of the geometrical 3D properties and by directing the classroom interactions. As Drijvers et al. (2010) state, the teacher played a didactical performance, involving the ad hoc decisions taken while teaching on how to actually perform, how dealing with different aspects of the mathematical tasks and of the IWB technologies.

The teacher in the third case study (Stefano) subscribed to the required preconditions (high-level mathematical tasks and quality of discourse interaction) in the preparatory phase, but his actual lessons significantly departed from the plans at the level of the quality of the discourse interaction. Though the lessons were well prepared and structured, students were not stimulated to assume an active role during the lessons; instead, they were directed by the teacher both in theoretical elaboration and in their use of the geometrical software. An in-depth analysis of the lessons from the interactivity point of view showed how the teacher did not leave enough room for students' autonomous elaborations, and for autonomous reasoning, precluding an effective conceptual evolution. Anthony (1996) describes this scenario as "path smoothing", with the teacher taking students through the chain of reasoning and students merely filling in the gaps with low-level interventions. This strategy does not lead to sustained learning because it reduces a problem to what the learner can already do, with few opportunities for cognitive processing (Anthony, 1996). As discussed above, Stefano's lessons were not deemed to be relevant to the aim of the research for this reason.

The first and second teacher in different ways demonstrated a skilful and competent ability to orchestrate classroom activities. Through detailed advance planning and careful manipulation of the classroom setting, they supported the actions carried out by students and sustained the development of collective learning. The orchestration competence concerned both the ability to orchestrate the didactical dialogue through an intensive interplay between teacher and students, as well as the ability to manage the different IWB affordances such as multiple visualisations, use of mathematical software and Internet resources.

The two teachers pursued two different patterns. The first pattern consists in using IWB to support problem-solving activities through extensive use of geometrical or other mathematical software. The potential uses of dynamic geometry and computer algebra software are emphasised by employing them in an IWB setting that makes the classroom environment more favourable to socialization, by establishing modalities for classroom activities that go beyond laboratory work on individual computers, and by using the IWB affordances for class discussion and reflection. IWB allows dynamic visualisation of the tasks and engages students to participate at the classroom level. Students are stimulated by the teacher to perform tasks, without a solution to the problem being provided. This model requires relatively low preparation from the teacher, but it does call for the ability to guide 'in the moment' activities, and to scaffold and respond to students' conjectures and explorations.

The second pattern consists in using IWB as a notepad, as a kind of advanced organiser, that the teacher, in collaboration with the students, 'tailors' by following a thread and offering links to external sources, mathematical and geometrical constructions, and problems or activity proposals.

By following this thread, students move forward on their educational path through discussions with their fellow classmates and with the teacher. This calls for significant preparation from the teacher, both at the level of required preparation time and the work of retrieving and organising the materials.

What seems to be important in both patterns is the development of a strong synergy between IWB affordances and students' interaction with the IWB. IWB affordances in themselves might not suffice to push classrooms towards this constructive activity. An IWB might easily be used merely as an overhead projector or as a normal blackboard with some technological affordances when it is not supported by a lesson design that calls on students to participate in the knowledge building. The IWB appears to play a more effective role when students work directly at the IWB and explore improved ways to accomplish tasks and discuss their findings with the classroom. When students are involved in a constructive activity, the IWB appears as a powerful tool that makes possible rapid shifts between different registers – from mathematical software manipulations to annotations and organisation of materials, and from retrieval of pictures, movies and animations from the Internet to building notebooks. Students and teacher are thus allowed to alternate between different points of view and different visualisations of the same topic.

The results show that the IWB might assume the role of an instrument that can promote new forms of mathematical thinking in the classroom. These new forms of mathematical thinking are centred on i) engaging students in processes of mathematical thinking, reasoning about core mathematical concepts, building new mathematical knowledge through problem-solving, making and investigating mathematical conjectures, and on ii) classroom discourse in which students coproduce mathematical reasoning and reflect on important mathematical concepts. A systematic combination of the two previous elements with the IWB affordances offers new potential schemes that combine great versatility with deep conceptualisation. The distinctive affordances of the IWB instrument may promote and support the development of qualitatively different mathematical actions and activities. This cultural process is what Ruthven (2008) described as "instrumental evolution".

As a limitation of this study, we point out that the analytic framework used for the interactions is not completely adequate to understand the complexity of the relationships between the teacher and the students, and between the students themselves. Despite the more teacher-centered oriented framework, the quality of interactions is not limited to teacher initiated interactions. Likely, the interactivity mediated by a technological tool as the IWB requires a more flexible framework, in which actions performed at the IWB and discourse moves could be analysed in an integrated way. Such a framework would allow a better understanding of the quality of teacher facilitation and of the dynamics that occur in the classroom.

The "instrumental evolution" also requires a considerable degree of technical skills as well as the ability to adequately use the relevant tools (software, Internet searches, etc.) that may perhaps only be reached after daily and extended IWB use by the students and the teacher. These are demanding requirements and this type of lesson preparation and enactment requires more time than frontal lessons do. This may slow the lesson pace and extend the teaching and learning time. There is consequently a need for accurate calibration of these activities, also considering curriculum constraints. Often, a curriculum requires teachers to cover too large a number of topics, and it would be difficult to apply a laboratorial teaching method to each topic. As one of the teachers involved in the project said: "Fast and well do not go together".

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