



# Towards better coordination of the supply chain

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## Abstract

This paper analytically examines the improvement of supply chain coordination (SCC) through more effective information exchange and consistent forecasting. It shows the negative impact that independent actions taken by members of a conventional supply chain typically have on order release volatility and forecast error volatility. Such increases in variation are argued to pose tremendous planning and utilization problems. This paper demonstrates when and to what extent such fluctuations can be controlled through collaboration within the supply chain. © 2000 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

Effective supply chain management requires coordination among the various channel members including retailers, manufacturers, and intermediaries. Programs such as vendor managed inventory (VMI) and continuous replenishment planning (CRP) have been advocated by some as promising approaches to supply chain coordination (SCC) (cf. Lee et al., 1997a; Vergin and Barr, 1999). These programs enable the seller to monitor inventory levels at the buyer's stock-keeping locations and assume responsibility for requisite inventory replenishments needed to achieve specified inventory-turn targets and customer-service levels (Waller et al., 1999).

The notion underlying such programs as VMI and CRP has been around for a long time, but recent advances in technology, including electronic data interchange and the Internet, are now making these programs feasible. For instance, two major barriers associated with managing

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another organization's inventory at remote locations – knowing the exact inventory status and predicting the demand activity – are being overcome as a result of the real-time capture and transmission of transaction information (Stank et al., 1999).

Supporters view these programs as the wave of the future and believe they will revolutionize the distribution channel (cf. Burke, 1996). However, while there is some empirical evidence to support such a position (cf. Stank et al., 1999; Vergin and Barr, 1999; Waller et al., 1999), a systematic evaluation of these programs in light of their impact on the channel members involved does not exist in the literature. For example, while VMI in the health care industry is becoming increasingly more popular, confusion exists as to when and why it is effective (Gerber, 1987).

Previous studies demonstrated, either empirically in a variety of industries or theoretically under reasonable assumptions of production behavior, that the variance of order releases tends to be larger than that of sales and that the distortions tend to increase upstream (away from final demand), a phenomenon termed “the bullwhip effect” (Lee et al., 1997b). For instance, Bishop et al. (1984) provided empirical evidence regarding how fluctuations in demand for fossil fuels caused precipitous changes in demand for fuel producing, turbo machinery. Blinder (1986) documented similar behavior in 20 different sectors of the economy, explaining it from a macro-economic perspective. Lee et al. (1997b) identified four sources of the bullwhip effect. In addition to providing a comprehensive review of the literature to date, Metters (1997) analyzed the impact of the bullwhip effect on profitability. Other significant efforts include Abel (1985), Caplin (1985), Kahn (1987), and Sterman (1989). Some of these studies (cf. Lee et al., 1997b; Waller et al., 1999) have even suggested possible approaches for reducing the bullwhip effect, though few have actually shown that they work. The research question posed here is: can the bullwhip effect be mitigated through supply chain coordination efforts such as VMI or CRP?

Chen et al. (2000) provided an initial investigation of possible solutions to the bullwhip effect. Using a  $p$ -period moving average forecast of demand, they considered one specific demand scenario and the impact of lead times but did not examine forecast errors. This paper complements their work by incorporating forecast uncertainty and alternative demand scenarios into the analysis. Thus, the purpose of the current research is twofold: (1) to evaluate the effectiveness of supply chain coordination programs in terms of linking information flows and reducing both the bullwhip effect and safety stocks; and (2) to investigate how these programs function with stationary and non-stationary demand patterns. This study also identifies some basic principles and guidelines for effective SCC.

## 2. The buyer–seller dyad

Consider a generic dyad consisting of only one buyer and one seller that resembles a wide variety of transactional relationships including retailer–manufacturer. Now, suppose that the buyer (in this case, retailer) is the only customer of the seller (manufacturer), which means that the retailer's purchase order becomes the manufacturer's observed demand. Both parties operate in an infinite discrete time horizon ( $t$ ), where  $t = -\infty, \dots, 0, 1, 2, \dots, \infty$ . The retailer faces actual, customer demand  $D_t$  but is unaware of its exact distribution and thus has to resort to a forecast. Both stationary and non-stationary (but with no observable trend or seasonality) demand

processes are examined. Use of demand forecasting and consideration of two different demand patterns are features of this study distinguishing it from previous ones.

In the conventional environment, retailers and manufacturers independently forecast their respective demands for the next period at the end of each period and then place purchase orders with their respective upstream channel members. At the beginning of the next period, the shipments arrive at their respective destinations just in time for the retailer's receipt to satisfy its demand realized during the period and for the manufacturer's receipt to satisfy the retailer's order received during the period. Any unsatisfied demand in the current period is back-ordered and satisfied with the arriving replenishment in the next period. It is assumed that both the retailer and manufacturer rely on a simple exponentially weighted moving average to forecast demand. Accordingly, the forecast for the retailer is

$$X_t = \alpha D_{t-1} + (1 - \alpha)X_{t-1} = \sum_{k=1}^{\infty} \alpha(1 - \alpha)^{k-1} D_{t-k} \quad (1)$$

and for the manufacturer is

$$Y_t = \beta O_{t-1} + (1 - \beta)Y_{t-1} = \sum_{k=1}^{\infty} \beta(1 - \beta)^{k-1} O_{t-k}, \quad (2)$$

where  $\alpha$  is the retailer's smoothing constant ( $0 < \alpha < 1$ ),  $\beta$  the manufacturer's smoothing constant ( $0 < \beta < 1$ ), and  $O_{t-1}$  is the retailer's purchase order release in the preceding period.

This forecasting method is popular in practice due to its simplicity, computational efficiency, reasonable accuracy, and ease of adjusting the forecast responsiveness (Montgomery and Johnson, 1997). Indeed, exponentially weighted moving averages are the most complicated forecasting techniques that some notable supply chain software developers currently include in their offerings. It is also appropriate here since no apparent trends or seasonalities in the demand patterns arise.

It is assumed that each party determines the value of its own smoothing constant according to its perception of demand and other trade-offs. For instance, a company could use historical data to find the smoothing constant that minimizes the forecast's mean squared error. In principle, the higher the smoothing constant, the more responsive the forecast is to fundamental changes in demand, but also the higher its forecast variance will be. It is also assumed that  $\alpha \geq \beta$ , since the manufacturer may suspect that some fluctuation in the retailer's order does not represent a fundamental change in customer demand and thus may not want its forecast to be too responsive to recent changes in the retailer's order pattern. As will be seen, this last assumption ( $\alpha \geq \beta$ ) is not needed for some results, and even when the assumption is necessary, the case of  $\alpha < \beta$  is discussed. (Note that the purpose of this paper is to investigate the effect of supply chain collaboration given the smoothing constants; the determination of their optimal values is not within its scope.)

Regarding their reordering policy, both retailer and manufacturer are assumed to implement some form of the popular order-up-to-S inventory system. That is, the retailer sets its order-up-to inventory position for period  $t$  as

$$SB_t = X_t + Z_B * \sigma_B, \quad (3)$$

where  $Z_B * \sigma_B$  is the retailer's safety stock,  $\sigma_B = [\text{Var}(X_t - D_t)]^{1/2}$  the standard deviation of the retailer's demand forecast error, which can be obtained from historical data, and  $Z_B$  is the retailer's safety factor, which depends on its targeted service level.

Thus, the retailer's purchase order release quantity is

$$O_t = SB_t - SB_{t-1} + D_{t-1} = X_t - X_{t-1} + D_{t-1}. \quad (4)$$

Similarly, the manufacturer updates its required inventory position at period  $t$  as

$$SS_t = Y_t + Z_S * \sigma_S, \quad (5)$$

where  $Z_S * \sigma_S$  is the manufacturer's safety stock,  $\sigma_S = [\text{Var}(Y_t - O_t)]^{1/2}$  the standard deviation of the manufacturer's demand forecast error, which can be estimated from historical data, and  $Z_S$  is the manufacturer's safety factor, which depends on its targeted service level. Thus, the manufacturer's order release quantity is

$$Q_t = SS_t - SS_{t-1} + O_{t-1} = Y_t - Y_{t-1} + O_{t-1}. \quad (6)$$

### 2.1. The case of non-stationary, serially correlated demand

Suppose that the retailer's actual demand is an AR(1) process

$$D_t = d + \rho D_{t-1} + u_t, \quad (7)$$

where  $d$  is a constant underlying the actual demand pattern,  $\rho$  the serial correlation coefficient ( $-1 < \rho < 1$ ), and  $u_t$  is the random error, which is an independently and identically distributed random variable with mean 0 and variance  $\sigma^2$  and is uncorrelated with anything known at time  $t - 1$  (i.e.,  $\text{Cov}(u_t, D_{t-k}) = 0$  for  $k = 1, 2, \dots, \infty$ ).

Basic properties of this demand process are given in (A.1)–(A.5) of Appendix A. Note that this demand pattern is potentially unstable over time since there is no guarantee that  $E(D_t) = E(D_{t-1})$ , though  $E(D_t)$  will fluctuate around the long run average  $d/(1 - \rho)$  as  $t \rightarrow \infty$ .

It was established in (3) and (5) that each party's safety stock is proportional to its respective variance of forecast errors. On the other hand, certain production activities for each party are usually directly related to order size, which in turn means that resource utilization will be adversely affected by the variance of the party's order releases. For instance, if the seller is a manufacturer producing whatever order size specified, there will be significant downtime in labor and machinery when the order is small and substantial overtime and overutilization of capacity when the order is large. Workforce scheduling and capacity investment decisions are extremely difficult in these situations. Alternatively, if the seller is an intermediary purchasing from another upstream channel member, it will pass the resource utilization problem on to that member. For these reasons, the variances of order releases and forecast errors for both parties are of interest. While only the relevant results are discussed here, most of the derivations are presented in Appendix A.

#### 2.1.1. Before collaboration

Before any supply chain coordinating arrangement is entered into, the retailer and manufacturer manage their inventories independently as explained above, with the manufacturer in a less favorable position regarding information. The manufacturer relies on historical order data from the retailer to predict both future ordering patterns of the retailer and true demand patterns of the retailer's customers. Obviously, the manufacturer's understanding of the retailer's actual demand pattern is distorted by the latter's demand forecasting and ordering behavior. Therefore, it is

difficult to closely synchronize the manufacturer's delivery at the retailer's location with demand. A mismatch between delivery and demand also causes fluctuations in order releases. The following two lemmas provide some understanding of the nature of this mismatch and the fluctuation of order releases.

**Lemma 1.1.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face non-stationary, serially correlated demand, then the variance of the manufacturer's order releases is higher than that of the retailer's, which in turn is higher than the variance of actual demand.*

**Proof.** In Appendix A, (A.10) and (A.22) together indicate that  $\text{Var}(Q_t) > \text{Var}(O_t) > \text{Var}(D_t)$ . This result holds true regardless of the relative values of the smoothing constants.  $\square$

Confirming the existence of a bullwhip effect along the supply chain when both the retailer and manufacturer conduct forecasting and purchasing/production decisions independently, this lemma can be explained as follows. The retailer adjusts its own order release using the procedure identified in (4), which means that its order release contains some forecast variability in addition to demand variability. Therefore, the higher the smoothing constant used by the retailer, the faster its forecast adjustment process will be, and consequently the higher the variance of the retailer's forecasts and order releases.

By the same token, the manufacturer forecasts its own demand pattern from the retailer's order release process based on (2) and in turn adjusts its order release quantity based on (6). Just as the retailer cannot perfectly forecast the actual demand pattern, the manufacturer is unable to exactly predict the retailer's order quantity. This leads to extra variability in the manufacturer's order release beyond that of the retailer's. And, the higher the manufacturer's smoothing constant, the higher the variability in the manufacturer's order release relative to the variability in the retailer's order release.

**Lemma 1.2.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face non-stationary, serially correlated demand, then the forecast errors of the manufacturer have greater variance than those of the retailer.*

**Proof.** From (A.23) in Appendix A,  $\text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) < 0$  if and only if

$$\rho > (3\alpha + \beta - \alpha\beta)/[(\alpha + 2\beta - 3\alpha\beta) + (\alpha^2 - \alpha^2\beta) + (\beta - \beta^2) + \alpha\beta^2] > 0. \quad (8)$$

Since  $\alpha \geq \beta$  by assumption,  $\rho > 1$  which is not possible; therefore,  $\text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) > 0$ .  $\square$

Condition (8) indicates that if the retailer and manufacturer choose their exponential smoothing constants to fit their own needs, there will be few situations in which the variance of the manufacturer's forecast errors is less than that of the retailer's. Consider the following: in the common scenario assumed earlier, where the manufacturer chooses a smaller smoothing constant than the retailer does (i.e.,  $\alpha \geq \beta$ ), it can be established that condition (8) calls for  $\rho > 1$ , which

can never be attained. Moreover, when both parties use the same exponential smoothing constant ( $\alpha = \beta$ ), the above condition becomes  $\rho > (4 - \alpha)/(4 - 3\alpha) > 1$ , again making the manufacturer's variance of forecast errors always higher than the retailer's. Only in relatively extreme cases under  $\alpha < \beta$ , such as  $\alpha \rightarrow 0$ , does the condition become  $\rho > 1/(3 - \beta)$ . In other words, unless the serial correlation is higher than a condition established by the actual values of the smoothing constants when  $\alpha < \beta$  (which is not satisfied for most levels of  $\rho$  given practical values of  $\alpha$  and  $\beta$ ), the variance of the manufacturer's forecast errors will be higher than that of the retailer's. This implies that the safety stock of the manufacturer will be proportionally higher than that of the retailer, according to (3) and (5) above.

Similar to the two parties' variances of order releases, the higher the smoothing constants, the greater the difference between the variances of the manufacturer's and retailer's forecast errors in most scenarios (e.g.,  $\rho < 0.5$  and  $\alpha > 0.1$ ). Only in the limiting case of  $\alpha \rightarrow 0$  and  $\rho > 1/(3 - \beta)$  with  $\alpha < \beta$  may the order of difference be slightly reversed.

From the analysis, it is clear that an increase in safety stock is inevitable if the parties involved act in their own interests with little regard to or control over the operations of their trading partners. Information exchange is largely inadequate in this kind of environment as channel members have little knowledge of the bases for each other's decisions. As Nolan (1997) noted, forecasting accuracy for distributors in the metal center industry was poor because their customers had widely fluctuating production schedules prior to the adoption of a VMI program. Accordingly, it is quite fitting to attempt to minimize the resource utilization and safety stock accumulation problems in the supply chain through better coordination and communication. Collaborative supply chain programs considered here stipulate that the retailer yields some of its inventory to the control and coordination of the manufacturer. While satisfying the retailer's original goals of customer service and inventory turnover, the manufacturer is then left to identify proper approaches in order to streamline its own operations. Therefore, analysis of the effect of a switchover to a coordination program is in order.

### 2.1.2. *After collaboration*

The adoption of a coordination program such as VMI or CRP normally brings about many organizational and technological changes. For instance, the retailer's and manufacturer's forecasting systems typically generate widely different estimates of expected demand prior to implementation of such programs (Davis, 1995). A difference in information bases is usually one main cause of this divergence. However, these coordination programs, coupled with electronic data interchange, enable both sides to share real-time information (Johnson, 1997). This electronic transfer of data represents a critical part of the forecasting process (Burke, 1996).

In practice, the forecasting responsibility may reside with the retailer in some collaborative supply chain cases (Hughes, 1996; Lamb, 1997) but with the manufacturer in others (Nolan, 1997). Regardless of which party has final responsibility, a single, consistent forecast used by both is often a key to success of SCC. This notion is examined below assuming, without loss of generality, that the manufacturer performs the forecast.

With the implementation of a coordination program, the manufacturer gains equal access to the retailer's actual demand information, adopts a one-forecast policy for both parties (i.e.,  $X_t = Y_t$ ), and determines the order releases for both parties. With regard to forecasting, either the

retailer's or the manufacturer's previous smoothing constant could be used or an optimal smoothing constant could be found using some established criterion. If the manufacturer wants to achieve the inventory turn targets and customer service levels previously achieved by the retailer, the manufacturer should use the retailer's original smoothing constant ( $\alpha$ ). This simple collaborative program ( $\gamma = \alpha$ , where  $\gamma$  is the smoothing constant used upon collaboration) is evaluated below against the conventional system in terms of reducing amplifications of safety stock and order release volatility within the supply chain.

The direct effect of adopting the retailer's smoothing constant in a SCC program is that the variances of order releases and forecast errors of the retailer before and after implementation are not affected – compare (A.8) and (A.10) with (A.25) and (A.26). Thus, the focus is on the manufacturer.

The first performance measure of concern is the variance of order releases. Though the conditions leading to a decrease in the variance of the manufacturer's order releases due to coordination can be established, their expressions are too complex to be presented here; therefore, the discussion revolves mainly around numerical analyses (available upon request). Specifically, according to numerical results, coordination results in a decrease in the manufacturer's order release variability for all  $\rho \leq 0$  (note that  $0 < \gamma = \alpha < 1$  and  $0 < \beta < 1$ ). When  $\rho > 0$ , there are some threshold levels for  $\alpha$  and  $\beta$  above which coordination also results in a decrease in the variance of the manufacturer's order releases. These threshold values can be exceeded for simple reasons. For instance, the retailer may choose a higher value for its smoothing constant to be more responsive (e.g.,  $\alpha = 0.4$ ), realizing that actual demand might be highly, positively correlated ( $\rho = 0.5$ ). The manufacturer may also choose a relatively high smoothing constant before collaboration for similar reasons. In other words, when end customer demand is negatively or not serially correlated, variance reductions associated with the manufacturer's order releases can be easily accomplished through coordination. If the demand is positively serially correlated, such reductions are present only when the original values of the smoothing constant used by the retailer and the manufacturer are above certain values related to the serial correlation of demand. This indicates that, in terms of reducing the manufacturer's safety stocks, coordination is always effective when demand is not positively correlated, but is effective otherwise only if the smoothing constants used prior to collaboration exceed certain levels.

**Proposition 1.1.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face non-stationary, serially correlated demand, then coordination (where the manufacturer performs the forecasting and ordering activities for both parties using the retailer's smoothing constant) will result in lower forecast errors for the manufacturer.*

**Proof.** From (A.29) in Appendix A,  $\text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma = \alpha} < 0$  if and only if

$$\rho > (3\alpha + \beta - \alpha\beta) / [(\alpha + 2\beta - 3\alpha\beta) + (\alpha^2 - \alpha^2\beta) + (\beta - \beta^2) + \alpha\beta^2] > 0. \quad (9)$$

Since  $\alpha \geq \beta$  by assumption,  $\rho > 1$  which is not possible; therefore,  $\text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma = \alpha} > 0$ . Note that condition (9) is exactly the same as (8) in Lemma 1.2 since  $\text{Var}(X_t - D_t) = \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma = \alpha}$  according to (A.28).  $\square$

This proposition sufficiently demonstrates that under most reasonable levels of  $\rho$  (see the discussion pertaining to Lemma 1.2), coordination is very effective in reducing the safety stock requirements of the manufacturer, regardless of the range of smoothing constants used by either party before collaboration. In common situations where the retailer chooses a smoothing constant greater than or equal to that of the manufacturer's ( $\alpha \geq \beta$ ), coordination will always lead to a reduction in safety stock requirements for the manufacturer regardless of the demand correlation. Even in the less likely case where  $\alpha < \beta$  and  $\rho$  is highly positive, a reasonable size of  $\alpha$  (e.g.,  $\alpha > 0.1$ ) means that safety stock reduction through coordination is still easily achievable. Usually the higher either the retailer's or the manufacturer's original smoothing constant, the more effective coordination will be in reducing supplier safety stocks. Note that by adopting the retailer's smoothing constant upon collaboration, the safety stock carried at the retailer's location remains the same.

### 2.1.3. Numerical illustration of collaboration effects

A numerical example illustrates the effects of supply chain coordination on variance reduction from the manufacturer's perspective. Reductions due to collaboration in both the manufacturer's variance of order releases,  $\text{Var}(Q_t)$ , and the manufacturer's variance of forecast errors,  $\text{Var}(Y_t - O_t)$ , are shown in Table 1 for a wide range of  $\alpha, \beta$  and  $\rho$ . The variance of order releases column compares the results before coordination (A.16), with those after coordination (A.31), on a percentage basis. The variance of forecast errors column compares the before (A.21), and after (A.28), results on a similar basis.

From the table, significant variance reductions for both order releases and forecast errors in the range of 20–90% are found for reasonable ranges of  $\alpha, \beta$  and  $\rho$ . Consistent with the preceding discussion, substantive reductions in the variances arise when  $\rho \leq 0$  or when  $\rho > 0$  and  $\alpha$  and  $\beta$  are above threshold levels. Note that in select cases where the serial correlation is extremely high and both  $\alpha$  and  $\beta$  are relatively low (and therefore unresponsive), coordination can actually make the variance of order releases worse. Likewise, in few cases where  $\alpha < \beta$  and  $\alpha$  is small, coordination results in a greater variance of forecast errors when the serial correlation is extremely high.

### 2.2. The case of stationary, one-lag correlated demand

At this point, a logical question is whether the demand pattern substantially affects the role of coordination in improving the two performance measures. The concern is whether the non-stationarity of demand adds excessive variation to the retailer's and manufacturer's forecast errors and order releases in such a way that the validity of the above observations is limited to a specific demand pattern. For this reason, a less complex, stationary demand pattern is now evaluated, with the derivations presented in Appendix B. Accordingly, consider a simple demand process that is stable over time and characterized by only one-lag serial correlation

$$E(D_t) = E(D_{t-1}) = \text{constant}, \quad (10)$$

the basic properties of which are shown in (B.1)–(B.5) of Appendix B.

It turns out that, for the common range of parameters discussed (e.g.,  $|\rho| < 0.5$ ), the pattern of differences between the variances of concern in stationary demand is extremely similar to that of concern in non-stationary demand. However, the pattern near some critical points (usually the



Table 1  
Numerical results for non-stationary, serially correlated demand

$\alpha$	$\beta$	$\rho$	Percent reduction in manufacturer's variance of order releases	Percent reduction in manufacturer's variance of forecast errors
0.1	0.1	-0.8	92.03	18.10
0.1	0.5	-0.8	96.42	47.39
0.1	0.9	-0.8	98.49	70.24
0.5	0.1	-0.8	92.82	42.85
0.5	0.5	-0.8	96.79	63.58
0.5	0.9	-0.8	98.66	79.50
0.9	0.1	-0.8	90.73	57.58
0.9	0.5	-0.8	95.89	73.21
0.9	0.9	-0.8	98.30	85.08
0.1	0.1	-0.4	70.11	17.98
0.1	0.5	-0.4	85.79	42.95
0.1	0.9	-0.4	93.29	63.75
0.5	0.1	-0.4	82.17	44.48
0.5	0.5	-0.4	91.71	62.50
0.5	0.9	-0.4	96.17	76.58
0.9	0.1	-0.4	86.96	58.92
0.9	0.5	-0.4	94.07	73.12
0.9	0.9	-0.4	97.36	83.81
0.1	0.1	0	31.81	17.77
0.1	0.5	0	64.76	35.29
0.1	0.9	0	80.79	52.58
0.5	0.1	0	64.76	47.09
0.5	0.5	0	82.58	60.87
0.5	0.9	0	90.91	72.32
0.9	0.1	0	80.79	61.05
0.9	0.5	0	90.91	72.99
0.9	0.9	0	95.58	82.18
0.1	0.1	0.4	-52.09	17.32
0.1	0.5	0.4	9.58	19.00
0.1	0.9	0.4	40.86	28.83
0.5	0.1	0.4	31.01	51.88
0.5	0.5	0.4	62.03	58.14
0.5	0.9	0.4	76.92	65.53
0.9	0.1	0.4	68.82	64.91
0.9	0.5	0.4	84.05	72.79
0.9	0.9	0.4	91.32	79.99
0.1	0.1	0.8	-382.68	15.68
0.1	0.5	0.8	-279.72	-39.59
0.1	0.9	0.8	-218.06	-56.27
0.5	0.1	0.8	-64.46	63.65
0.5	0.5	0.8	-16.58	52.63
0.5	0.9	0.8	11.71	52.99
0.9	0.1	0.8	34.73	74.08
0.9	0.5	0.8	58.16	72.45
0.9	0.9	0.8	72.54	76.91

limiting case such as  $\alpha \rightarrow 0$  or  $\beta \rightarrow 1$ ) differs between the two demand scenarios as the following lemmas indicate.

**Lemma 2.1.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face stationary, one-lag correlated demand, then the variance of the retailer's order releases is always higher than the variance of actual demand. Moreover, the variance of the manufacturer's order releases is higher than that of the retailer's, except when*

$$\rho > \rho_0 = (2\alpha + 3\alpha^2 + 2\beta + \alpha\beta - \alpha^2\beta)/(4\alpha^3 + 2\alpha\beta + 3\alpha^2\beta - 2\alpha^3\beta + 2\beta^2 + \alpha\beta^2 - \alpha^2\beta^2) > 0. \quad (11)$$

**Proof.** To show that  $\text{Var}(O_t) > \text{Var}(D_t)$  is unconditional, but that  $\text{Var}(Q_t) > \text{Var}(O_t)$  depends on (11), see (B.10) and (B.22) in Appendix B.  $\square$

Note that (11) is usually not satisfied with most practical levels of  $\alpha$ ,  $\beta$ , and  $\rho$  as the critical value,  $\rho_0$ , is higher than 1 in most cases of  $\alpha$  and  $\beta$  values and slightly below 1 only when  $\alpha \rightarrow 1$  or  $\beta \rightarrow 1$ . That is,  $\rho > \rho_0$  is feasible only when  $\alpha \rightarrow 1$  or  $\beta \rightarrow 1$ . In other words, when demand is stationary, there is a slight chance that the bullwhip effect may not arise in extreme situations where  $\alpha \rightarrow 1$  and  $\beta \rightarrow 1$ . Recall that when demand is non-stationary, the bullwhip effect always exists according to Lemma 1.1.

**Lemma 2.2.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face stationary, one-lag correlated demand, then forecast errors of the manufacturer have greater variance than those of the retailer if and only if*

$$(3\alpha + \beta - \alpha\beta) + \rho(2\alpha - 4\alpha^2 - 2\beta - 2\alpha\beta + 2\alpha^2\beta) > 0. \quad (12)$$

**Proof.** According to (B.23) in Appendix B,  $\text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) > 0$  when (12) holds.  $\square$

Numerical experiments reveal that (12) is typically satisfied, except when highly positive serial correlation ( $\rho > 0.5$ ) is coupled with  $\beta \rightarrow 1$  or  $\alpha \rightarrow 0$ . Comparing the results in Lemmas 1.2 and 2.2 suggests that safety stock amplifications along the supply chain under non-stationary demand are more prevalent than under stationary demand.

As with non-stationary demand, in the case of stationary demand, benefits can again be obtained in many situations upon implementation of a coordination program.

**Proposition 2.1.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face stationary, one-lag correlated demand, then coordination (where the manufacturer performs the forecasting and ordering*

activities for both parties using the retailer's smoothing constant) will result in lower forecast errors for the manufacturer if and only if

$$(3\alpha + \beta - \alpha\beta) + \rho(2\alpha - 4\alpha^2 - 2\beta - 2\alpha\beta + 2\alpha^2\beta) > 0. \quad (13)$$

**Proof.** To show that  $\text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma=\alpha} > 0$  when (13) holds, see (B.29) in Appendix B. Note that (13) is exactly the same as (12) in Lemma 2.2 since  $\text{Var}(X_t - D_t) = \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma=\alpha}$  according to (B.28).  $\square$

Similar to the condition in Lemma 2.2, condition (13) simply implies that there are a lot of opportunities for coordination to reduce supplier safety stocks from conventional-system levels, unless the smoothing constants used prior to collaboration are in their limiting cases, such as  $\beta \rightarrow 1$  or  $\alpha \rightarrow 0$ .

**Lemma 2.3.** *If the retailer and manufacturer both use order-up-to-S inventory systems coupled with simple exponentially weighted moving average forecasting systems and face stationary, one-lag correlated demand, then coordination (where the manufacturer performs the forecasting and ordering activities for both parties using the retailer's smoothing constant) will result in less variance in order releases for the manufacturer if and only if*

$$\begin{aligned} & \rho(4\alpha^2 - 2\alpha^3 + 2\alpha^4 + 4\alpha\beta - 8\alpha^2\beta + 13\alpha^3\beta - 3\alpha^4\beta + 2\alpha\beta^2 + 9\alpha^2\beta^2 - 6\alpha^3\beta^2 + \alpha^4\beta^2 \\ & + 4\beta^3 + 2\alpha\beta^3 - 2\alpha^2\beta^3) < (2\alpha^3 + 4\alpha\beta + 8\alpha^2\beta - 3\alpha^3\beta + 4\beta^2 + 2\alpha\beta^2 - 3\alpha^2\beta^2 + \alpha^3\beta^2). \end{aligned} \quad (14)$$

**Proof.** According to (B.32) in Appendix B,  $\text{Var}(Q_t) - \text{Var}(Q_t) |_{\text{SCC}, \gamma=\alpha} > 0$  when (14) holds.  $\square$

Comparing the two demand scenarios, coordination will be effective in reducing the safety stock requirements of the manufacturer while keeping the retailer's unchanged when the manufacturer's variance of forecast errors is greater than that of the retailer's before collaboration, whether demand is stationary or not. Note that  $\alpha \geq \beta$  is not required in either case. This observation provides some insight into identifying potentially fruitful areas for coordination. If safety stock reduction is the primary objective of the manufacturer when adopting a coordination program, then the manufacturer will be successful if its forecast errors in the conventional channel are greater than those of the retailer.

Moreover, in a conventional environment, having stationary and serially uncorrelated demand does not eliminate the safety stock amplifications and bullwhip effect within the supply chain. The implication of this is that serially correlated and non-stationary demands do NOT cause the bullwhip effect and safety stock amplifications as one might suspect – they just aggravate the amplification effects further! Nor does the role of coordination in reducing safety stock amplifications and the bullwhip effect disappear when demand is stationary and not serially correlated. This suggests that the role of collaboration is not limited to non-stationary, serially correlated demands.

### 3. Conclusions and implications

It has been analytically shown that the bullwhip effect of order releases and amplifications of safety stock arise within the supply chain even when level demand patterns with no trend and

seasonality are considered and a simple exponentially weighted moving average is used to forecast. A simple program of coordination was examined to determine how it can help alleviate such negative effects (and in the process reduce related operating costs). Several interesting observations were noted. First, when the manufacturer's forecasting errors are greater than those of the retailer's before collaboration, coordination will be effective in reducing the manufacturer's safety stocks. Coordination usually reduces fluctuations in order releases as well. Secondly, the smoothing constants adopted by the retailer and manufacturer determine the extent of the effect of coordination in terms of reducing both safety stock levels and the variances of order releases. Thirdly, coordination is effective in the case of either non-stationary or stationary demand, though in some limiting situations the advantage is greatly reduced (in a few extreme cases, this simple collaboration scheme can even make things worse).

These findings support the tenet that effective communication between the retailer and manufacturer is one of the keys to the success of SCC. A simple switch to a one forecast policy results in the removal of waste (safety stock) and the improvement of efficiency (resource utilization). Further benefits could be achieved if the forecasting and other scheduling methods can be fine-tuned to suit the needs of both companies.

In this paper, a supply chain perspective was taken to examine the role and effectiveness of a coordination program in reducing the resource wastes of both retailer and manufacturer. The simple collaborative program evaluated here indicates that the manufacturer is the main beneficiary of coordination in terms of safety stock and resource waste reduction, while there is little effect on the retailer in these two areas. On the other hand, under a collaborative program such as VMI or CRP, the retailer does not incur costs related to demand forecasting, order placement, and inventory holding, thereby providing strong incentives for it to adopt a SCC program with the manufacturer. Nevertheless, future research is needed to determine additional incentives that the manufacturer could offer to ensure retailer participation. Further research could also examine other, more sophisticated approaches to coordination where both parties are actively involved.

Of course, coordination may lead to other, longer-term benefits for both sides beyond those discussed in this paper. For instance, through a successful coordination program, the manufacturer could, over time, retain customers by building customer loyalty and reduce costs by re-configuring operations using better information (cf. Vergin and Barr, 1999). The retailer could obtain an uninterrupted supply of product to help keep its operations going, reduce inventory and/or increase inventory turns, and reduce labor costs (Lamb, 1997). Whatever the gains received and however, they are shared, as illustrated in this paper, better coordination of the supply chain can be quite advantageous.

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## Appendix A. Analytical results for non-stationary, serially correlated demand

The basic properties of non-stationary, serially correlated demand are

$$D_t = d + \rho D_{t-1} + u_t, \quad (\text{A.1})$$

$$\text{Var}(D_t) = \text{Var}(D_{t-1}) = \sigma^2 / (1 - \rho^2), \quad (\text{A.2})$$

$$\text{Var}(D_t - D_{t-1}) = 2\sigma^2 / (1 + \rho), \quad (\text{A.3})$$

$$\text{Cov}(D_t - D_{t-1}, D_t) = \sigma^2 / (1 + \rho) \quad (\text{A.4})$$

and

$$\text{Cov}(D_t, D_{t-k}) = \rho^k \sigma^2 / (1 - \rho^2), \quad k = 0, 1, 2, \dots \quad (\text{A.5})$$

### A.1. Before collaboration

The retailer and manufacturer manage their own inventories and conduct their own forecast independently. From the retailer's forecast in (1), it can be shown that

$$\text{Cov}(X_t, D_t) = \text{Cov} \left[ \sum_{k=1}^{\infty} \alpha(1 - \alpha)^{k-1} D_{t-k}, D_t \right] = \frac{\sigma^2}{1 - \rho^2} \frac{\alpha\rho}{1 - \rho(1 - \alpha)}. \quad (\text{A.6})$$

Similarly, the variance of the retailer's forecasts is

$$\begin{aligned} \text{Var}(X_t) &= \text{Var}[\alpha D_{t-1} + (1 - \alpha)X_t] \\ &= \alpha^2 \text{Var}[D_{t-1}] + (1 - \alpha)^2 \text{Var}[X_{t-1}] + 2\alpha(1 - \alpha) \text{Cov}[D_{t-1}, X_{t-1}] \\ &= \frac{\sigma^2}{1 - \rho^2} \frac{1 + \rho(1 - \alpha)}{1 - \rho(1 - \alpha)} \frac{\alpha}{2 - \alpha}. \end{aligned} \quad (\text{A.7})$$

The variance of the retailer's forecast errors can then be obtained as

$$\begin{aligned} \text{Var}(X_t - D_t) &= \text{Var}[D_t] + \text{Var}[X_t] - 2\text{Cov}[D_t, X_t] \\ &= \frac{\sigma^2}{1 - \rho^2} \frac{1 - \rho}{1 - \rho(1 - \alpha)} \frac{2}{2 - \alpha} \end{aligned} \quad (\text{A.8})$$

with

$$\text{Cov}(X_t, X_{t-1}) = \frac{\sigma^2}{1 - \rho^2} \frac{1 + \rho - \alpha}{1 - \rho(1 - \alpha)} \frac{\alpha}{2 - \alpha} \quad (\text{A.9})$$

and the variance of the retailer's order releases can be expressed as

$$\begin{aligned} \text{Var}(O_t) &= \text{Var}[(1 + \alpha)D_{t-1} - \alpha X_{t-1}] \\ &= (1 + \alpha)^2 \text{Var}[D_{t-1}] + \alpha^2 \text{Var}[X_{t-1}] - 2\alpha(1 + \alpha) \text{Cov}[D_{t-1}, X_{t-1}] \\ &= \frac{\sigma^2}{1 - \rho^2} \left\{ 1 + \frac{1 - \rho}{1 - \rho(1 - \alpha)} \frac{4\alpha}{2 - \alpha} \right\} > \frac{\sigma^2}{1 - \rho^2} = \text{Var}(D_t). \end{aligned} \quad (\text{A.10})$$

Since  $\text{Cov}(u_t, D_{t-k}) = 0$  for  $k = 1, 2, \dots, \infty$

$$\text{Cov}(O_t, O_{t-1}) = \frac{\sigma^2}{1-\rho^2} \frac{\rho^2(-2-\alpha+3\alpha^2+\alpha^5) + \rho(2+3\alpha-2\alpha^2) - 2\alpha^2}{1-\rho(1-\alpha)} \frac{1}{2-\alpha}. \quad (\text{A.11})$$

The manufacturer's demand is the retailer's order release. Since the manufacturer forecasts its demand based on (2), (6) can be re-written as

$$\begin{aligned} Q_t &= (1+\beta)O_{t-1} - \beta Y_{t-1} = (1+\beta)O_{t-1} - \sum_{l=1}^{\infty} \beta^2(1-\beta)^{l-1} O_{t-l-1} \\ &= (1+\alpha)(1+\beta)D_{t-2} + \sum_{k=1}^{\infty} [C1(1-\alpha)^{k-1} + C2(1-\beta)^{k-1}] D_{t-k-2}, \end{aligned} \quad (\text{A.12})$$

where  $C1$  and  $C2$  are parameters dependent on  $\alpha$  and  $\beta$ , assuming  $\alpha \neq \beta$  for the moment (for the special case of  $\alpha = \beta$ , the value of  $Q_t$  can be easily obtained by taking the limit of  $\alpha \rightarrow \beta$  in the above expression), and

$$C1 = \frac{\alpha^3\beta}{\beta-\alpha} - \alpha^2, \quad (\text{A.13})$$

$$C2 = -\frac{\alpha\beta^3}{\beta-\alpha} - \beta^2, \quad (\text{A.14})$$

then the variance of the manufacturer's order releases can be expressed as

$$\begin{aligned} \text{Var}(Q_t) &= (1+\alpha)^2(1+\beta)^2\text{Var}(D_{t-2}) + \sum_{k=1}^{\infty} [C1(1-\alpha)^{k-1} + C2(1-\beta)^{k-1}]^2\text{Var}(D_{t-k-2}) \\ &\quad + 2 \sum_{k=1}^{\infty} [C1(1-\alpha)^{k-1} + C2(1-\beta)^{k-1}](1+\alpha)(1+\beta)\text{Cov}(D_{t-2}, D_{t-k-2}) \\ &\quad + 2 \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \{ [C1(1-\alpha)^{k-1} + C2(1-\beta)^{k-1}] * [C1(1-\alpha)^{l-1} \\ &\quad + C2(1-\beta)^{l-1}] \text{Cov}(D_{t-k-2}, D_{t-l-2}) \}, \end{aligned} \quad (\text{A.15})$$

which can be transformed into the following simplified form:

$$\begin{aligned} \text{Var}(Q_t) &= \sigma^2/(1-\rho^2) * (-4\alpha - 6\alpha^2 - 4\beta - 8\alpha\beta - 3\alpha^2\beta - 6\beta^2 - 3\alpha\beta^2 + \alpha^2\beta^2 + 8\alpha\rho \\ &\quad + 8\alpha^2\rho + 2\alpha^3\rho + 8\beta\rho + 8\alpha\beta\rho + \alpha^3\beta\rho + 8\beta^2\rho - 3\alpha^3\beta^2\rho + 2\beta^3\rho + \alpha\beta^3\rho \\ &\quad - 3\alpha^2\beta^3\rho - 4\alpha\rho^2 - 2\alpha^2\rho^2 - 2\alpha^3\rho^2 - 4\beta\rho^2 - \alpha^2\beta\rho^2 + \alpha^3\beta\rho^2 - 2\beta^2\rho^2 \\ &\quad - \alpha\beta^2\rho^2 + 7\alpha^2\beta^2\rho^2 - 2\beta^3\rho^2 + \alpha\beta^3\rho^2 + \alpha^3\beta^3\rho^2)/[(2-\alpha)(2-\beta)(-\alpha-\beta+\alpha\beta) \\ &\quad * (1-\rho+\alpha\rho)(1-\rho+\beta\rho)]. \end{aligned} \quad (\text{A.16})$$

The special case of  $\beta = \alpha$  is given by

$$\begin{aligned} \text{Var}(Q_t) |_{\beta=\alpha} &= (-8 - 20\alpha - 6\alpha^2 + \alpha^3 + 16\rho + 24\alpha\rho + 4\alpha^2\rho + 2\alpha^3\rho - 6\alpha^4\rho - 8\rho^2 - 4\alpha\rho^2 \\ &\quad - 6\alpha^2\rho^2 + 9\alpha^3\rho^2 + \alpha^5\rho^2)/[(-2+\alpha)^3(1-\rho+\alpha\rho)^2]\text{Var}(D_t). \end{aligned} \quad (\text{A.17})$$

On the other hand, the manufacturer’s forecast in (2) can be written as

$$Y_t = \sum_{k=1}^{\infty} [C3(1 - \alpha)^{k-1} + C4(1 - \beta)^{k-1}] D_{t-k-1}, \tag{A.18}$$

where C3 and C4 are parameters dependent on  $\alpha$  and  $\beta$ , and

$$C3 = \frac{-\alpha^3}{\beta - \alpha}, \tag{A.19}$$

$$C4 = \frac{\alpha\beta^2}{\beta - \alpha} + \beta. \tag{A.20}$$

The variance of the manufacturer’s forecast errors can then be obtained as

$$\begin{aligned} \text{Var}(Y_t - O_t) = \text{Var}(D_t) [ & 2(1 - \rho)(2\alpha + 3\alpha^2 + 2\beta + \alpha\beta - \alpha^2\beta - 2\alpha\rho - \alpha^2\rho - \alpha^3\rho \\ & - 2\beta\rho + \alpha\beta\rho + 2\alpha^2\beta\rho + \alpha^3\beta\rho)] / [(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta) \\ & * (1 - \rho + \alpha\rho)(1 - \rho + \beta\rho)], \end{aligned} \tag{A.21}$$

while the difference between the manufacturer’s and retailer’s variance of order releases is

$$\begin{aligned} \text{Var}(Q_t) - \text{Var}(O_t) = \text{Var}(D_t) [ & 4\beta(1 - \rho)(2\alpha + 3\alpha^2 + 2\beta + \alpha\beta - \alpha^2\beta - 2\alpha\rho - \alpha^2\rho \\ & - \alpha^3\rho - 2\beta\rho + \alpha\beta\rho + 2\alpha^2\beta\rho + \alpha^3\beta\rho)] / [(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta) \\ & * (1 - \rho + \alpha\rho)(1 - \rho + \beta\rho)] > 0 \end{aligned} \tag{A.22}$$

and the difference between the manufacturer’s and retailer’s variance of forecast errors is

$$\begin{aligned} \text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) = \text{Var}(D_t) [ & 2(\alpha + \beta)(1 - \rho)(3\alpha + \beta - \alpha\beta - \alpha\rho - \alpha^2\rho \\ & - 3\beta\rho + 3\alpha\beta\rho + \alpha^2\beta\rho + \beta^2\rho - \alpha\beta^2\rho)] / [(2 - \alpha)(2 - \beta) \\ & * (\alpha + \beta - \alpha\beta)(1 - \rho + \alpha\rho)(1 - \rho + \beta\rho)]. \end{aligned} \tag{A.23}$$

Note that  $\text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) < 0$  if and only if

$$\rho > (3\alpha + \beta - \alpha\beta) / [(\alpha + 2\beta - 3\alpha\beta) + (\alpha^2 - \alpha^2\beta) + (\beta - \beta^2) + \alpha\beta^2] > 0.$$

### A.2. After collaboration

Assume that the manufacturer will manage the inventory at the retailer’s location. Accordingly, the manufacturer will forecast and order for both parties

$$Y_t = X_t \quad \text{and} \quad X_t = \gamma D_{t-1} + (1 - \gamma) X_{t-1}, \tag{A.24}$$

where  $\gamma$  is the smoothing constant used by the manufacturer upon implementation of the SCC program. Naturally, the variance characteristics of the retailer are similar to those in (A.6)–(A.11) with  $\alpha$  replaced by  $\gamma$ . For instance

$$\text{Var}(X_t - D_t)|_{\text{SCC}} = \text{Var}(D_t) \frac{1 - \rho}{1 - \rho(1 - \gamma)} \frac{2}{2 - \gamma} \tag{A.25}$$

and

$$\text{Var}(O_t)|_{\text{SCC}} = \text{Var}(D_t) \left( 1 + \frac{1-\rho}{1-\rho(1-\gamma)} \frac{4\gamma}{2-\gamma} \right). \quad (\text{A.26})$$

Combining (A.24) with (4) and (6), the manufacturer's forecast error and order release will be revised. In particular, since under coordination

$$Y_t - O_t = X_t - O_t = X_{t-1} - D_{t-1}, \quad (\text{A.27})$$

the variance of the manufacturer's forecast errors becomes

$$\text{Var}(Y_t - O_t)|_{\text{SCC}} = \text{Var}(X_{t-1} - D_{t-1})|_{\text{SCC}} = \text{Var}(D_t) \frac{1-\rho}{1-\rho(1-\gamma)} \frac{2}{2-\gamma} \quad (\text{A.28})$$

and the difference between the manufacturer's variance of forecast errors before and after collaboration is

$$\begin{aligned} \text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma=\alpha} & \\ &= \text{Var}(D_t) [2(\alpha + \beta)(1 - \rho)(3\alpha + \beta - \alpha\beta - \alpha\rho - \alpha^2\rho \\ &\quad - 3\beta\rho + 3\alpha\beta\rho + \alpha^2\beta\rho + \beta^2\rho - \alpha\beta^2\rho)] / [(2 - \alpha)(2 - \beta) \\ &\quad * (\alpha + \beta - \alpha\beta)(1 - \rho + \alpha\rho)(1 - \rho + \beta\rho)]. \end{aligned} \quad (\text{A.29})$$

Note that  $\text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma=\alpha} < 0$  if and only if

$$\rho > (3\alpha + \beta - \alpha\beta) / [(\alpha + 2\beta - 3\alpha\beta) + (\alpha^2 - \alpha^2\beta) + (\beta - \beta^2) + \alpha\beta^2] > 0.$$

On the other hand, the manufacturer's order release under coordination is

$$\begin{aligned} Q_t &= X_t - X_{t-1} + O_{t-1} \\ &= \gamma D_{t-1} + (1 + \gamma - \gamma^2) D_{t-2} + (\gamma - 2) \gamma^2 \sum_{k=1}^{\infty} (1 - \gamma)^{k-1} D_{t-k-2} \end{aligned} \quad (\text{A.30})$$

with a variance of

$$\text{Var}(Q_t) |_{\text{SCC}} = \text{Var}(D_t) [2 + 2\rho - \gamma(1 + \rho + \gamma\rho)] / [(2 - \gamma)(1 - \rho + \gamma\rho)]. \quad (\text{A.31})$$

## Appendix B. Analytical results under stationary, one-lag correlated demand

The basic properties of stationary, one-lag correlated demand are

$$E(D_t) = E(D_{t-1}) = \text{constant}, \quad (\text{B.1})$$

$$\text{Var}(D_t) = \text{Var}(D_{t-1}) = \sigma^2, \quad (\text{B.2})$$

$$\text{Cov}(D_t, D_{t-1}) = \rho\sigma^2, \quad (\text{B.3})$$

$$\text{Cov}(D_t, D_{t-k}) = 0 \quad \text{for } k > 1 \quad (\text{B.4})$$

and

$$\text{Var}(D_t - D_{t-1}) = 2(1 - \rho)\sigma^2. \quad (\text{B.5})$$



### B.1. Before collaboration

The retailer and manufacturer again manage their own inventories and conduct their own forecasts independently. From the retailer's forecast in (1), it can be shown that

$$\text{Cov}(X_t, D_t) = \text{Cov}\left[\sum_{k=1}^{\infty} \alpha(1-\alpha)^{k-1} D_{t-k}, D_t\right] = \alpha\rho\sigma^2 = \text{Var}(D_t)\alpha\rho. \quad (\text{B.6})$$

Similarly, the variance of the retailer's forecasts is

$$\text{Var}(X_t) = \text{Var}[\alpha D_{t-1} + (1-\alpha)X_t] = \text{Var}[D_t][1 + 2\rho(1-\alpha)]\frac{\alpha}{2-\alpha}. \quad (\text{B.7})$$

The variance of the retailer's forecast errors can then be obtained as

$$\begin{aligned} \text{Var}(X_t - D_t) &= \text{Var}[D_t] + \text{Var}[X_t] - 2\text{Cov}[D_t, X_t] \\ &= \text{Var}[D_t](1 - \alpha\rho)\frac{2}{2-\alpha} \end{aligned} \quad (\text{B.8})$$

with

$$\text{Cov}(X_t, X_{t-1}) = \text{Var}(D_t)[(1-\alpha)(1+2\rho) + \alpha^2\rho]\frac{\alpha}{2-\alpha} \quad (\text{B.9})$$

while the variance of the retailer's order releases can be expressed as

$$\begin{aligned} \text{Var}(O_t) &= \text{Var}[(1+\alpha)D_{t-1} - \alpha X_{t-1}] \\ &= \text{Var}(D_t)\left[1 + (1-\alpha\rho)\frac{4\alpha}{2-\alpha}\right] > \text{Var}(D_t) \end{aligned} \quad (\text{B.10})$$

with

$$\text{Cov}(O_t, O_{t-1}) = \text{Var}(D_t)[\rho(2 + 3\alpha - 2\alpha^2 + 2\alpha^3) - 2\alpha^2]\frac{1}{2-\alpha}. \quad (\text{B.11})$$

Once again, the manufacturer's demand is the retailer's order release. Since the manufacturer forecasts its demand based on (2), (6) can be re-written as

$$\begin{aligned} Q_t &= (1+\beta)O_{t-1} - \beta Y_{t-1} = (1+\beta)O_{t-1} - \sum_{l=1}^{\infty} \beta^2(1-\beta)^{l-1} O_{t-l-1} \\ &= (1+\alpha)(1+\beta)D_{t-2} + \sum_{k=1}^{\infty} [C1(1-\alpha)^{k-1} + C2(1-\beta)^{k-1}]D_{t-k-2}, \end{aligned} \quad (\text{B.12})$$

where  $C1$  and  $C2$  are parameters dependent on  $\alpha$  and  $\beta$ , assuming  $\alpha \neq \beta$  for the moment (for the special case of  $\alpha = \beta$ , the value of  $Q_t$  can be easily obtained by taking the limit of  $\alpha \rightarrow \beta$  in the above expression), and

$$C1 = \frac{\alpha^3\beta}{\beta-\alpha} - \alpha^2, \quad (\text{B.13})$$

$$C2 = -\frac{\alpha\beta^3}{\beta-\alpha} - \beta^2, \quad (\text{B.14})$$

then the variance of manufacturer's order releases can be expressed as

$$\begin{aligned} \text{Var}(Q_t) &= (1 + \alpha)^2(1 + \beta)^2\text{Var}(D_{t-2}) + \sum_{k=1}^{\infty} [C1(1 - \alpha)^{k-1} + C2(1 - \beta)^{k-1}]^2\text{Var}(D_{t-k-2}) \\ &\quad + 2 \sum_{k=1}^{\infty} [C1(1 - \alpha)^{k-1} + C2(1 - \beta)^{k-1}](1 + \alpha)(1 + \beta)\text{Cov}(D_{t-2}, D_{t-k-2}) \\ &\quad + 2 \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \{ [C1(1 - \alpha)^{k-1} + C2(1 - \beta)^{k-1}] * [C1(1 - \alpha)^{l-1} \\ &\quad + C2(1 - \beta)^{l-1}] \text{Cov}(D_{t-k-2}, D_{t-l-2}) \}, \end{aligned} \quad (\text{B.15})$$

which can be transformed into the following simplified form

$$\begin{aligned} \text{Var}(Q_t) &= \text{Var}(D_t) * (4\alpha + 6\alpha^2 + 4\beta + 8\alpha\beta + 3\alpha^2\beta + 6\beta^2 + 3\alpha\beta^2 - \alpha^2\beta^2 - 8\alpha^3\rho \\ &\quad - 8\alpha^2\beta\rho - 4\alpha^3\beta\rho - 8\alpha\beta^2\rho - 8\alpha^2\beta^2\rho + 4\alpha^3\beta^2\rho - 8\beta^3\rho - 4\alpha\beta^3\rho \\ &\quad + 4\alpha^2\beta^3\rho) / [(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta)]. \end{aligned} \quad (\text{B.16})$$

The special case of  $\beta = \alpha$  is given by

$$\text{Var}(Q_t) |_{\beta=\alpha} = \text{Var}(D_t) * (-8 - 20\alpha - 6\alpha^2 + \alpha^3 + 32\alpha^2\rho + 16\alpha^3\rho - 8\alpha^4\rho) / (-2 + \alpha)^3. \quad (\text{B.17})$$

On the other hand, the manufacturer's forecast in (2) can be written as

$$Y_t = \sum_{k=1}^{\infty} [C3(1 - \alpha)^{k-1} + C4(1 - \beta)^{k-1}] D_{t-k-1}, \quad (\text{B.18})$$

where  $C3$  and  $C4$  are parameters dependent on  $\alpha$  and  $\beta$ , and

$$C3 = \frac{-\alpha^3}{\beta - \alpha}, \quad (\text{B.19})$$

$$C4 = \frac{\alpha\beta^2}{\beta - \alpha} + \beta. \quad (\text{B.20})$$

The variance of the manufacturer's forecast errors can then be obtained as

$$\begin{aligned} \text{Var}(Y_t - O_t) &= \text{Var}(D_t) [2(2\alpha + 3\alpha^2 + 2\beta + \alpha\beta - \alpha^2\beta - 4\alpha^3\rho - 2\alpha\beta\rho - 3\alpha^2\beta\rho + 2\alpha^3\beta\rho \\ &\quad - 2\beta^2\rho - \alpha\beta^2\rho + \alpha^2\beta^2\rho)] / [(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta)], \end{aligned} \quad (\text{B.21})$$

while the difference between the manufacturer's and retailer's variance of order releases is

$$\begin{aligned} \text{Var}(Q_t) - \text{Var}(O_t) &= \text{Var}(D_t) [4\beta(2\alpha + 3\alpha^2 + 2\beta + \alpha\beta - \alpha^2\beta - 4\alpha^3\rho - 2\alpha\beta\rho - 3\alpha^2\beta\rho \\ &\quad + 2\alpha^3\beta\rho - 2\beta^2\rho - \alpha\beta^2\rho + \alpha^2\beta^2\rho)] / [(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta)]. \end{aligned} \quad (\text{B.22})$$

Note that  $\text{Var}(Q_t) - \text{Var}(O_t) < 0$  if and only if

$$\rho > (2\alpha + 3\alpha^2 + 2\beta + \alpha\beta - \alpha^2\beta) / (4\alpha^3 + 2\alpha\beta + 3\alpha^2\beta - 2\alpha^3\beta + 2\beta^2 + \alpha\beta^2 - \alpha^2\beta^2) > 0.$$

The difference between the manufacturer’s and retailer’s variance of forecast errors is

$$\text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) = \text{Var}(D_t)[2(\alpha + \beta)(3\alpha + \beta - \alpha\beta + 2\alpha\rho - 4\alpha^2\rho - 2\beta\rho - 2\alpha\beta\rho + 2\alpha^2\beta\rho)]/[(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta)]. \quad (\text{B.23})$$

Note that  $\text{Var}(Y_t - O_t) - \text{Var}(X_t - D_t) > 0$  if and only if

$$(3\alpha + \beta - \alpha\beta) + \rho(2\alpha - 4\alpha^2 - 2\beta - 2\alpha\beta + 2\alpha^2\beta) > 0.$$

### B.2. After collaboration

Assume that the manufacturer will manage the inventory at the retailer’s location. Accordingly, the manufacturer will forecast and order for both parties

$$Y_t = X_t \quad \text{and} \quad X_t = \gamma D_{t-1} + (1 - \gamma)X_{t-1}, \quad (\text{B.24})$$

where  $\gamma$  is the smoothing constant used by the manufacturer upon implementation of the SCC program. The variance characteristics of the retailer are again similar to those in (B.6)–(B.11) with  $\alpha$  replaced by  $\gamma$ . For instance

$$\text{Var}(X_t - D_t)|_{\text{SCC}} = \text{Var}(D_t)(1 - \gamma\rho) \frac{2}{2 - \gamma} \quad (\text{B.25})$$

and

$$\text{Var}(O_t)|_{\text{SCC}} = \text{Var}(D_t) \left( 1 + (1 - \gamma\rho) \frac{4\gamma}{2 - \gamma} \right). \quad (\text{B.26})$$

Combining (B.24) with (4) and (6), the manufacturer’s forecast error and order release will be revised. In particular, since under coordination

$$Y_t - O_t = X_t - O_t = X_{t-1} - D_{t-1}, \quad (\text{B.27})$$

the variance of the manufacturer’s forecast errors becomes

$$\text{Var}(Y_t - O_t)|_{\text{SCC}} = \text{Var}(X_{t-1} - D_{t-1})|_{\text{SCC}} = \text{Var}(D_t)(1 - \gamma\rho) \frac{2}{2 - \gamma} \quad (\text{B.28})$$

and the difference between the manufacturer’s variance of forecast errors before and after collaboration is

$$\text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma=\alpha} = \text{Var}(D_t)[2(\alpha + \beta)(3\alpha + \beta - \alpha\beta + 2\alpha\rho - 4\alpha^2\rho - 2\beta\rho - 2\alpha\beta\rho + 2\alpha^2\beta\rho)]/[(2 - \alpha)(2 - \beta)(\alpha + \beta - \alpha\beta)]. \quad (\text{B.29})$$

Note that  $\text{Var}(Y_t - O_t) - \text{Var}(Y_t - O_t) |_{\text{SCC}, \gamma=\alpha} > 0$  if and only if

$$(3\alpha + \beta - \alpha\beta) + \rho(2\alpha - 4\alpha^2 - 2\beta - 2\alpha\beta + 2\alpha^2\beta) > 0.$$

On the other hand, the manufacturer’s order release under coordination is

$$\begin{aligned} Q_t &= X_t - X_{t-1} + O_{t-1} \\ &= \gamma D_{t-1} + (1 + \gamma - \gamma^2)D_{t-2} + (\gamma - 2)\gamma^2 \sum_{k=1}^{\infty} (1 - \gamma)^{k-1} D_{t-k-2} \end{aligned} \quad (\text{B.30})$$

with a variance of

$$\text{Var}(Q_t) |_{\text{SCC}} = \text{Var}(D_t)[1 + 2\gamma + 2\gamma\rho(1 - \gamma)] \quad (\text{B.31})$$

and the difference between the manufacturer's variance of order releases before and after collaboration is

$$\begin{aligned} \text{Var}(Q_t) - \text{Var}(Q_t) |_{\text{SCC}, \gamma=\alpha} = & \text{Var}(D_t)[2(-2\alpha^3 - 4\alpha\beta - 8\alpha^2\beta + 3\alpha^3\beta - 4\beta^2 \\ & - 2\alpha\beta^2 + 3\alpha^2\beta^2 - \alpha^3\beta^2 + 4\alpha^2\rho - 2\alpha^3\rho + 2\alpha^4\rho + 4\alpha\beta\rho \\ & - 8\alpha^2\beta\rho + 13\alpha^3\beta\rho - 3\alpha^4\beta\rho + 2\alpha\beta^2\rho + 9\alpha^2\beta^2\rho - 6\alpha^3\beta^2\rho + \alpha^4\beta^2\rho \\ & + 4\beta^3\rho + 2\alpha\beta^3\rho - 2\alpha^2\beta^3\rho)/[(2 - \alpha)(2 - \beta)(-\alpha - \beta + \alpha\beta)]. \end{aligned} \quad (\text{B.32})$$

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