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Towards Golden Rule of Capital Accumulation: A Genetic Algorithm Approach

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Abstract. The current study deals with maximizing consumption per worker in connection with the economic growth of society. The traditional Solow model based approach is well-studied and computationally complex. The present work proposes a Genetic Algorithm (GA) based consumption maximization in attaining the Golden rule. An objective function derived from traditional Solow model based on depreciation rate and amount of accumulated capital is utilized. The current study considered a constant output per worker to incorporate a constant efficiency level of labor. Different ranges of Depreciation rate and accumulated capital are tested to check the stability of the proposed GA based optimization process. The mean error and standard deviation in optimization process is utilized as a performance metric. The experimental results suggested that GA is very fast and is able to produce economically significant result with an average mean error 0.142% and standard deviation 0.021%.

Keywords: Accumulated capital, Depreciation rate, Golden rule, Genetic Algorithm, Metaheuristic, Solow model

1 Introduction

Economic growth of society directly impacts economy of a country. Economic growth depends on several factors; one of them is consumption. Thus, consumption maximization is an imperative task to ensure ever growing economic stability. Solow model [11] deals with the growth of the economy in terms of basic production, investment and depreciation. The steady level of capital plays a vital role in the same. Maximization of consumption is traditionally done by employing a long time consuming and computationally complex Solow model based mathematical approach. Motivated by this, in the current work we consider Genetic Algorithm (GA) to maximize consumption. Typically, the economic problems can be framed as an optimization problem, such as cost minimization, and revenue maximization. Thus, efficient optimization methods are required to deliver accurate results. Consequently, several optimization algorithms can be involved to solve the required maximum profit level, such methods include the genetic algorithm (GA), particle swarm optimization (PSO), firefly algorithm, and cuckoo search algorithm (CS). Several studies have established the ingenuity and accuracy of GA [1-4]. Generally, the GA has numerous advantages including its flexibility to model the problem's constraints, and its easy convergence to the optimal solution inspired by Darwinian principle [6]. Nicoară [12] revealed about the GA relevance compared to the traditional methods for manufacturing structure optimization. Geisendorf [13] employed the GA to solve Resource Economic problem using two different assumptions to calculate the optimal extraction rate in order to achieve optimal benefits. Arifovic [14] solved decision rules of future production and sales by employing the GA in a competitive cobweb model in a market of single product. The simulation results indicated that the GA is capable of capturing different features from the experimental nature of the subjects under consideration. Riechmann [15] established that the GA can be connected with the evolutionary game theory. Hommeset al. [16] reveled that GA can converge to a series of near Nash equilibrium solutions, where the heterogeneous agent behavior has been modeled using GA.

The rest of the work is arranged as follows; first in section 2 the basic economic background is introduced and mathematically explained. Next in section 3 the economical steady state is introduced and explained. In section 4 Golden rule of capital based on the Solow model is formulated. The appropriateness of choosing the objective function is mathematically established. Section 5 introduces the GA based proposed method. Section 6 reported the experimental set up of GA and finally, Section 7 reveals experimental results. It discusses the economic significance of the obtained results as well.

2 Basic Economic Background

The Solow model deals with growth of economy. It includes the living standard of every citizen currently living in the economy. Their living standard depends on various kinds of determinant; one of them is their income. The consumption increases with increasing earning of citizens thereby raising the overall consumption of economy. These results in potential growth in economic systems as the consumption rate of citizens have increased inside that economy. Consequently, consumption turns out to be a major indicator of growth. At maximum consumption rate, economic growth will take place. The golden rule of capital accumulation is a tool which has used to maximize the consumption. Golden rule actually indicates steady state with maximum consumption. The aforesaid formalism is mathematically established using a basic Coubb-Douglaus production function [11].

$$Q = f(L, K)$$

$$\frac{Q}{L} = f(\frac{L}{L}, \frac{K}{L})$$

$$y = f(1, k)$$

$$y = f(k) \quad (1)$$

Where,'k' denotes capital per worker,'y' denotes output per worker, 'c' is consumption per worker, 'i' is investment per worker, 'Q' is total output, 'L' denotes total labor and'K' is total capital.

Suppose, a firm earns 'y' and saves 's' fraction of 'y'. $(1 - s)$ fraction of 'y' goes to its consumption. Hence;

$$c = (1 - s) * y \quad (2)$$

Where, 'y' is income per worker. And the relation between income per worker and consumption is given by;

$$y = c + i \quad (3)$$

From (2) and (3) we get;

$$y = (1 - s) * y + i$$

$$i = s * y$$

$$i = s * f(k) \quad (4)$$

Investment per worker depends on the savings rate of the firm (s). There is a positive relationship between investment per worker and savings rate. As saving rate rises, investment per worker also rises. An increasing savings rate shifts the investment per worker vs. capital per worker curve upward as depicted in Figure 2.

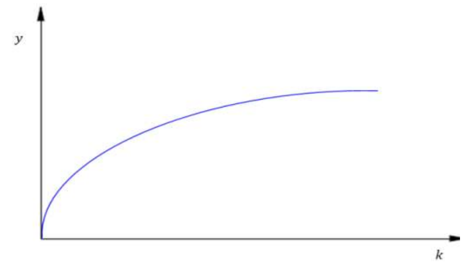

Fig. 1. Capital per worker vs. output per worker curve

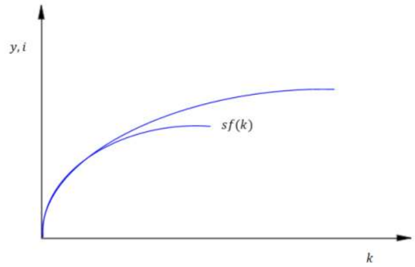

Fig. 2. Combining output per worker and investment per worker curve. Where, $sf(k)$ denotes investment per worker curve

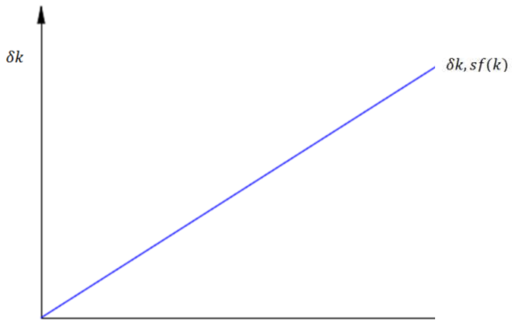

Fig. 3. Relationship between capital and total depreciation

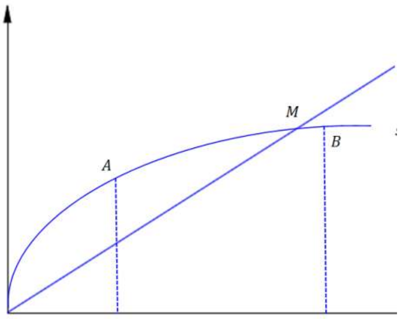

Fig. 4. Combining Figure 1 and Figure 3. Depicting economic steady state

The relationship between change in capital per worker and depreciation is given as;

$$\Delta k = i - \delta * k \tag{5}$$

From equation (4) we get;

$$\Delta k = s * f(k) - \delta * k$$

Figure 3 depicts that depreciation is increased with increased capital per worker.

3 Economical Steady State

From the Figure 4 we depict how the economy approaches the steady level of capital. In Figure 4 we have shown the steady level is at point'M'. Now, to examine whether at point 'M' the economy reached steady state level of capital or not we take two points. The first point is point 'A'which is below the steady state level. And the second point is point 'B' which is above the steady state level. At point 'A'it can be observed that the investment curve is steeper than the depreciation, so here investment is greater than the depreciation thus, if investment takes place it would enrich the capital stock which would lead to higher output. The capital increases from the point 'A'and moves toward point 'M' while in case of the point 'B' the depreciation is steeper than the depreciation.If investment takes place it would shrink the capital stock as depreciation is far greater than the investment. Thus, the level of capital will move downward to the point'M'.

4 Golden Rule level of Capital

After achieving the steady state level of capital, there is a state where we maximize the consumption per worker that is generally known as the Golden rule level of capital [11, 18]. The Golden rule addresses the question of presence of any growth in an economy. There are many factors which decide the growth of an economy. The current study focuses on the consumption. Maximization of consumption is an imperative task in order to achieve economic growth. The Golden rule of capital indicates a state (a unique value of depreciation rate and amount of capital accumulated) that ensures maximum consumption level, thereby assures a strong economic growth [18].

From equation 3 and 5 we get;

$$c = y - i \tag{6}$$

$$\Delta k = i - \delta * k$$

$$i = \delta * k$$

As, the difference between output and depreciation curve is zero (At point 'M' of Figure 4) Hence, $\Delta k = 0$

$$c = f(k^*) - \delta * k^* \tag{7}$$

Here, $i = \delta * k^*$ as stated in Golden rule of Capital [11]. Now differentiating with respect to k^*we get;

$$\frac{dc}{dk^*} = f'(k^*) - \delta$$

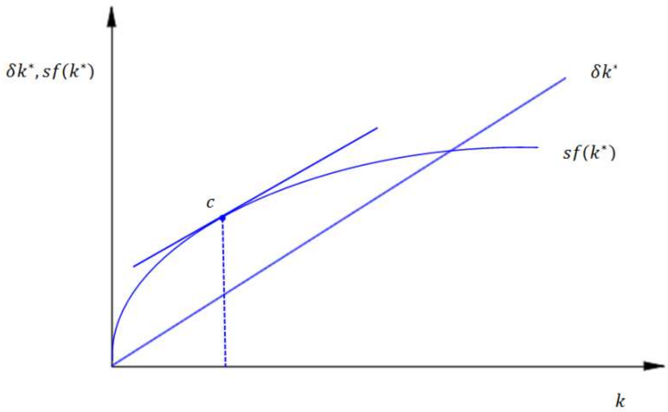

Fig. 5. Golden rule level of capital accumulation. At point 'c'$MP_k = \delta$ which indicates that the rate of change of output with respect to capital (MP_k) is equal to the slope of the depreciation line. Thus it is achieving the Golden rule of capital accumulation.

At maxima we have;

$$\frac{dc}{dk^*} = 0$$

$$f'(k^*) - \delta = 0$$

$$f'(k^*) = \delta$$

$$MP_k = \delta \tag{8}$$

Change in output due to one unit increase of capital is called the marginal product of capital and is denoted as MP_k. In Figure 1 output per worker vs. capital curve has already been described. The slope of this curve is $\frac{dy}{dk}$ which is denoted by MP_k. Initial condition to achieve the Golden rule level of capital is; marginal product of capital should be equals to depreciation rate. To decide if the Golden rule of capital is steady or not; we consider two different cases.

i. **Case 1**

$$MP_k - \delta < 0$$

Where, if one unit of capital is added, output will increase less than the depreciation. If level of capital is increased consumption will fall.

ii. Case 2

$$MP_k - \delta > 0$$

Where, if one unit of capital is added, output will increase more than the depreciation. If level of capital is increased consumption will rise.As none of Case 1 and 2 produces a steady state hence, equation 8 is only condition which satisfy Golden rule level of capital.

5 Genetic Algorithm based methodology

The GA is a parallel optimization procedure that relies on evolution for optimizing a group of solutions at once [5]. The model is highly inspired from Darwinian principle of Natural Evolution, and involves a population which participates in finding the solution of a particular problem under consideration. In the proposed work, the GA is applied to determine the optimal consumption value, where the GA proves its effectiveness for superior convergence toward global optimization compared to other global search algorithm. The overall GA algorithm is as follows [6] mentioning types of different operators. Each chromosome represents a potential solution in mating pool. The chromosome representation considered in the current study is as follows;

```
Algorithm: Genetic Algorithm
Start
Generate random chromosomes' population that represents
solutions
Evaluate the population-fitness
Create new population using the following steps:
Select two parent chromosomes form the population based
on their fitness (Roulette)
Crossover the parents to generate new offspring (Single
Point)
Mutate new offspring at each locus (Gaussian)
Place new offspring in the new population
Use new generated population
Test: If the end condition is satisfied, stop, and re-
turn the best solution in present population
Stop
```

A binary coded chromosome representation technique is adapted. The chromosome has two major parts. The first part depicts δ (depreciation rate) and the next part depicts k^* (amount of capital accumulated). The chromosome representation is depicted in Figure 6. Each part is numerically a sequence of binary digits. The length of each such part is n. The length depends on the search range of the corresponding variable.

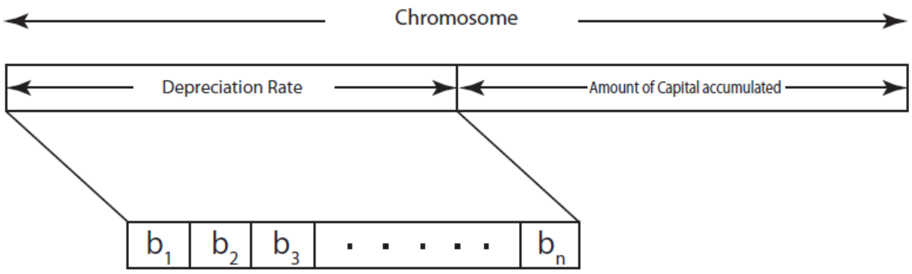

Fig. 6. Chromosome representation of Genetic algorithm

As reported in the preceding algorithm, it is clear that each member of the population represents a potential solution of the problem under concern. Further, each of them is associated with a fitness value indicating the superiority of that particular solution. The fitness value is calculated by following equation 7 described in section 4.

Solutions at any stage participate in Darwinian reproduction, survival of the fittest and other genetic operations to produce successors. The GA tries to find out the best solution at every generation by breeding the best solutions from the previous generation. The candidate solutions under consideration are actually artificial chromosomes which are inspired from the DNA structure. In practice, these chromosomes have fixed length strings (binary/real coded), where each location (gene) holds information about one of the variables associated in the problem. In the current study real coded fixed length formalism is utilized. Each chromosome has two different components one depicting δ (Depreciation rate) and another is k^* (amount of capital accumulated). The range of each of the components varies from context to context [18].

GA the population (initially random) by applying genetic operators already defined to generate new solutions from them. The genetic operators such as crossover, mutation and several other versions of these two are popular [7-8].Crossover can be of different types; broadly Single point or multi point. The current study used a single point crossover [9] method. A Gaussian type mutation strategy is adopted [10] and employed. The probability value of mutation is determined by a simple trial and error method and is described in next section. Based on the preceding genetic algorithm procedure, Figure 7 demonstrated the proposed approach for consumption maximization in achieving Golden rule of capital accumulation using the GA.

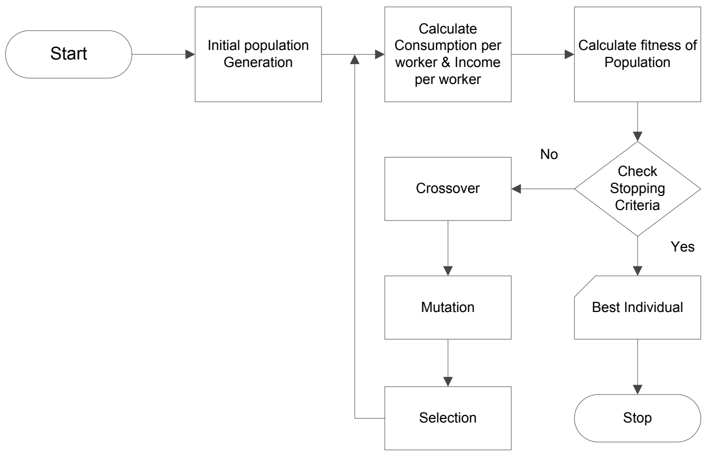

Fig. 7. The profit maximization process using Genetic Algorithm

6 Results & Discussion

The experiments are carried out using Intel Core i3 4GB Machine. A real coded Genetic algorithm is implemented using the algorithmic set up which are obtained by running the GA 100 times with different parameters and is described in Table 1. Depending on the difficulty of the problem being solved the initial size of population is set to 200 and maximum number of generation is set to 500. The selection procedure during iteration is Roulette. Single point cross over strategy is adopted with a crossover probability of 0.15 which indicates that a significantly low probability would be sufficient to ensure the better quality population, while it is kept low in order to prevent the process from being trapped in local optima. The solution generated by the proposed system is a pair of values consisting δ (depreciation rate) and k^* (amount of capital accumulated) with in the given range of search. The output is approximate a point in search space for which the objective function is highly close to global optima.

Table 1. Genetic algorithm setup for consumption maximization

Size of Population	200
Maximum number of generation	500
Type of Selection	Roulette

Type of Crossover	Single point crossover
Crossover probability	0.15
Mutation type	Gaussian
Stall Time Limit	25 seconds

The experiments are run for 50 times for each set up of input ranges and the best results are reported. The objective function is already described and explained in section 4 (equation 7). $f(k^*)$ denotes output per worker. This is kept constant to ensure constant efficiency level of labor [18] and in the current study this is set to 2000. δ denotes depreciation rate and k^* denotes amount of capital accumulated.

7 Experimental results

Table 2 reports the performance of Genetic Algorithm. Performance analysis is done using the Mean Error and Standard Deviation. The results are tabulated in form mean error (%) ± standard deviation (%). Different ranges of δ and k is used. The ranges are written in form $[lower\ bound, upper\ bound]$. As suggested in [11] the range of δ is kept low and k is kept high. The mean error for ranges [0.01,0.1], [1,1000] of δ and $k$respectively the mean error is 0.257% and standard deviation is 0.005%.Next for ranges [0.1,0.9] and [1,1000] the achieved mean error and standard deviation are 0.192% and 0.043% respectively. The average of these different set up is reported as well. Average mean error achieved is 0.142% and standard deviation is 0.021%.

Table 2. Performance analysis of GA

Range of $\boldsymbol{\delta}$	Range of $\boldsymbol{k^*}$	Mean Error (%) $\pm$Standard Deviation (%)
$[0.01, 0.1]$	$[1, 1000]$	$0.257\% \pm 0.005\%$
$[0.1, 0.9]$	$[1, 1000]$	$0.192\% \pm 0.043\%$
$[0.25, 1]$	$[250, 750]$	$0.048\% \pm 0.003\%$
$[0.01, 0.9]$	$[250, 850]$	$0.072\% \pm 0.036\%$
	Average	$\mathbf{0.142\%} \pm \mathbf{0.021\%}$

Figure 8 depicts the convergence graph of Genetic Algorithm for ranges of δ and k [0.01, 0.1] and [1, 1000] respectively. GA took 52 iterations to converge. Figure 8 depicts a similar plot with ranges [0.1, 0.9] and [1, 1000] of δ and k respectively. It took 54 iterations to converge.The plots reveal that GA is extremely fast in maximizing the objective function and is better than traditional methods [18] because these involve calculation of mathematical differentiation of complex functions and is proved to be NP - Hard (exponential complexity)

[17]. GA is faster as it takes polynomial time to converge [3, 4]. This is supported by the experimental results and it establishes that GA based method to be faster than traditional method as well.

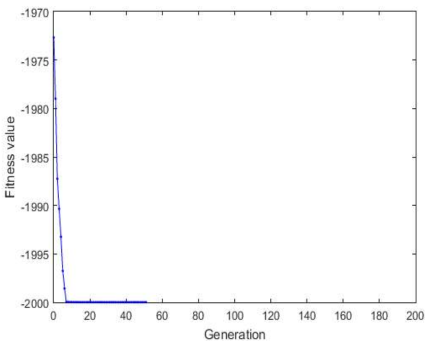

Fig. 8. GA convergence plot with ranges of δ and k being [0.01-0.1] and [1-1000]

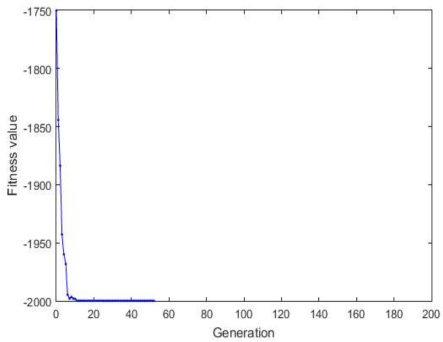

Fig. 9. GA convergence plot with ranges of δ and k being [0.1-0.9] and [1-1000]

7.1 Economic Significance

The results obtained by Genetic algorithm are economically significant. The optimum results indicated that in maximizing the consumption per worker the value of depreciation rate (δ) tends to the lower bound of the given range which indicates a lower depreciation rate. From economical point of view lower depreciation rate is expected [11] and in addition the capital accumulation tends to the higher bound of given range that indicates an increased accumulated capital that is equally significant [18].

Conclusion

Economic growth takes a vital role in sustainable development of a society and impacts the economic stability of a country to a greater extent. The current study considered the consumption maximization problem which is a major factor in economic growth. The traditional Solow model based approach is time consuming and computationally complex due to high involvement of mathematical operations and is proved to be NP-Hard. In the current study a well-known Darwinian principle inspired metaheuristic Genetic Algorithm is employed in maximizing the consumption (The Golden rule of capital accumulation) in terms of depreciation and capital accumulation. Experimental results have indicated that the proposed GA based method is highly fast and produces economically significant results in achieving the Golden rule of capital accumulation.

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