Towards macroscopic optical invisibility devices: geometrical optics of complex materials

Y. A. Urzhumov[1], D. R. Smith[2]

[1]Department of Electrical and Computer Engineering, Duke University,
P.O. Box 90271, Durham, North Carolina, 27708
email: yaroslav.urzhumov@duke.edu
[2]Center for Metamaterials and Integrated Plasmonics, Pratt School of Engineering, Duke University, 101 Science Drive, Durham, North Carolina, 27708
Fax: + 1–919-681-2055; email: drsmith@ee.duke.edu

Abstract

Recently, a path towards macroscopic, transparent optical cloaking devices that may conceal objects spanning millions of wavelengths has been proposed [1]. Such devices are designed using transformation optics (TO) [2,3]. In this paper, we offer further analysis and improvements to the concept using the method of geometrical optics extended to complex photonic media with an arbitrary dispersion relation. A technique for solving the highly nonlinear partial differential equation of the eikonal using the finite element method is presented. Aberrations caused by the non-quadratic part of the dispersion relation are demonstrated quantitatively in a numerical experiment. An analytical argument based on the scalability of the eikonal phase is presented, which points towards a solution that removes this type of aberration in each order of the *k*-perturbation theory, thus restoring the perfect cloaking solution.

1. Introduction and motivation

The use of complex material properties and distributions for mimicking certain coordinate transformations is a novel concept usually referred to as the transformation optics (TO) [1,2,3,7]. This concept has been successfully applied to the development of cloaking devices [1,2,7], superlenses, flattened Luneburg [4] and flattened Maxwell [5] lenses, and a plethora of other electromagnetic devices for frequencies ranging from RF to the optical [3].

In this paper, we concentrate on, perhaps, the most mysterious and controversial of all TO-based devices – the so-called *cloak of invisibility* [1,2,7]. Its implementation requires materials with extreme electromagnetic properties – superluminal phase velocities and strong anisotropy of refractive index [10]. Although limited forms of invisibility, such as invisibility near a ground plane [6], or invisibility for one angle of incidence [7,8], may be achievable with isotropic media, large medium anisotropy is the key to the full, *omnidirectional* invisibility [9,10].

In the optical range, neither superluminal velocity nor large anisotropy is readily available in naturally occurring transparent media, such as crystals and glasses. Metamaterials proposed for the microwave invisibility cloaks [11] are not easily scalable to the visible wavelengths. They rely on resonant phenomena in metal-dielectric composite structures [11], which become prohibitively lossy in the optical range [12]. Despite a significant effort dedicated to the development of gain-assisted optical metamaterials [13], nearly lossless optical media with very large superluminal velocity and strong anisotropy have not been demonstrated so far.

Our search for lossless optical media with the properties required for omnidirectional cloaking led us to dielectric photonic crystals (PCs). Waves propagating in photonic crystals have a well-defined phase velocity, and thus a well-defined refractive index. The attractive property of PCs for their use in TO devices is the possibility to have arbitrarily large phase velocity and a negligibly small attenuation constant *simultaneously* – a combination of properties prohibited in all homogeneous and homogenizable media by the Kramers-Kronig relationships.

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Abandoning homogenizable media relieves us from the bounds imposed on constitutive parameters by causality. However, this relief comes at a price: non-homogenizable media do not possess well-defined wave impedance, which makes their reflectance unpredictable without the knowledge of the microstructure of the surface. Maintaining hope that efficient surface layers can be designed for the coupling of free-space radiation into the photonic crystal medium, in this study we concentrate on the design of the proper dispersion properties which would enable perfect cloaking at optical frequencies.

2. Eikonal equation modelling of complex electromagnetic media

The optical eikonal equation describes the propagation of monochromatic waves in transparent media. It solves only for the phase of the time-harmonic electromagnetic field, but not the intensity or polarization. The eikonal equation can be generally stated as a Hamiltonian conservation law,

$$H(\vec{k}) \equiv H(\vec{\nabla}\Phi) = const \tag{1}$$

where $H(\boldsymbol{k},\boldsymbol{r})=F(\omega(\boldsymbol{k},\boldsymbol{r}))$ is the optical Hamiltonian, $F(\omega)$ is an arbitrary monotonic function of frequency, and $\omega(\boldsymbol{k},\boldsymbol{r})$ is the dispersion relation of light in the medium at the position $\boldsymbol{r}$. The wave vector $\boldsymbol{k}$ plays the role of canonical momentum in the optical Hamiltonian formalism. The dispersion relation is allowed to vary with position adiabatically, i.e. on a spatial scale greater than one wavelength. With a choice of scaling function

$$F(\omega) = \omega^2, \tag{2}$$

and the dispersion relation $\omega(\boldsymbol{k},\boldsymbol{r})=|\boldsymbol{k}|/n(\boldsymbol{r})$ of a homogeneous, isotropic medium with refractive index $n(\boldsymbol{r})$, the eikonal equation (1) becomes

$$\left(\frac{\nabla\Phi}{n}\right)^2 = const \tag{3}$$

Written in the most general form (1), the eikonal equation describes light propagation not only in locally homogenizable media, with or without anisotropy, but also in exotic media with spatial dispersion, including metamaterials and photonic crystals. The only two requirements for the applicability of this description are: (a) the medium is transparent (negligible loss), and (b) a single-valued dispersion relation exists locally at all points. Single-valued dispersion relation is typical for the “impedance-matched” ($\varepsilon=\mu$) media arising from the TO concept.

Equation (1) is highly nonlinear even for the simplest case of wave propagation in vacuum, as seen from Equation (3) with $n=1$. Special techniques including artificial diffusion, damped Newton iteration solver and parametric continuation are required to solve it numerically with the Finite Element Method. We have tested this technique on several optical structures, including the most extreme case of the optical cloak. The cloak is the most challenging case, not only due to its very large anisotropy, but also due to the divergence of refractive index gradient at the cloaked boundary. Nevertheless, we are able to solve Equation (1) numerically, both in regular media with a standard quadratic dispersion relation, as well as in exotic media whose dispersion relation can be approximated as a sum of standard (quadratic) and non-standard (quartic) terms:

$$\omega^2(\vec{k}) = \omega_0^2 + \frac{k_r^2}{n_r^2} + \frac{k_\varphi^2}{n_\varphi^2} + \alpha_{abcd}k_a k_b k_c k_d \tag{4}$$

The expansion of the dispersion relation in the powers of k is known as the *k*-perturbation theory [14]. Deviations from the standard, quadratic Hamiltonian affects the performance of TO devices; therefore it is important to find a way to compensate the aberrations resulting from spatial dispersion. We have shown theoretically, and demonstrated in numerical simulations (Fig. 1), that the non-quadratic terms in the dispersion relation can be manipulated in each order of the *k*-perturbation theory, to regain perfect performance of any TO-based device that was designed initially for the standard, non-spatially-dispersive media. The analytical approach uses coordinate-dependent rescaling of the effective time (optical path) along the rays, and is based on the invariance of Hamiltonian optics with respect to functional transformations of the Hamiltonian.

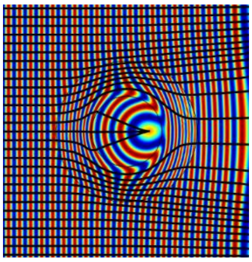
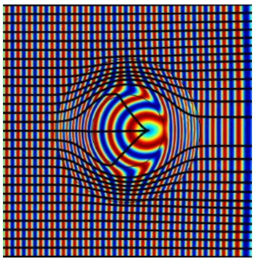
 (a)  (b)

Fig.1. Plane wave propagation through a two-dimensional cloak with a non-standard dispersion relation (4), modelled using the eikonal equation (1) with artificial diffusion. Left: the refractive index distribution of the standard cloak is used; rays experience deviations due to the presence of a quartic term in dispersion relation. Right: the coefficients in the quartic term are chosen judiciously such that the cloak operation remains perfect.

4. Conclusion

We have introduced a novel numerical method for the study of wave propagation in optically transparent media with a very general, possibly anisotropic dispersion relation. We have applied the technique to the problem of a photonic crystal based optical cloak, whose dispersion relation is affected by strong spatial dispersion. We show that an exact solution to the cloaking problem exists even when using media with strong spatial dispersion. We suggest that the detrimental effects of spatial dispersion could be removed or minimized by the optimization of the dispersion relation in each order of the *k*-perturbation theory.

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