Tracing Active and Reactive Power between Generators and Loads Using Real and Imaginary Currents

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Abstract - In a competitive environment, usage allocation questions must be answered clearly and unequivocally. To help answer such questions, this paper proposes a method for determining how much of the active and reactive power output of each generator is contributed by each load. This method takes as its starting point a solved power flow solution. All power injections are translated into real and imaginary currents to avoid the problems arising from the non-linear coupling between active and reactive power flows caused by losses. The method then traces these currents to determine how much current each source supplies to each sink. These current contributions can then be translated into contributions to the active and reactive power output of the generators. It is also shown that the global contribution of a load can be decomposed into contributions from its active and reactive parts. This decomposition is reasonably accurate for the reactive power generation. To determine the contributions to active power generation, the previously-described method based on the active power flows is recommended.

Keywords: Power systems economics, usage allocation, contributions, reactive power.

I. INTRODUCTION

What fraction of the reactive capability of a generator is used to supply a particular load? In a vertically-integrated power system, questions like this one are of little practical importance. On the other hand, in a competitive environment, such "usage allocation" questions must be answered clearly and unequivocally to ensure that the market is fair and efficient.

A previous paper [1] has shown how active power can be traced from generators to loads. Having determined where the power goes, one can compute how much power flows from a given generator to each load or from all generators to a particular load. It is also possible to determine how many MWs each load or generator contributes to the active flow in a branch. These physical "contributions" form a basis upon which the cost of building and maintaining each component of the network could be allocated among its users [2].

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Computation of the contributions defined and used in [1,2] is possible only if the quantities being allocated are linearly additive. This implies that active and reactive powers should be considered separately. Addressing issues such as the allocation of line capacity solely on the basis of active power flows is a reasonable and defensible approximation. On the other hand, neglecting the effects of active power flows when dealing with reactive power issues is not sensible: in a heavily loaded power system, even if all loads have a unity power factor, the generators must produce a significant amount of reactive power to supply the reactive losses caused by the active flows.

Applying the principles of the contributions method to reactive problems therefore requires the simultaneous consideration of both active and reactive effects. However, combining independently computed active and reactive power contributions is not possible because of the interaction introduced by the losses and the fact that the line flows usually have different power factors. To get around this difficulty, complex power injections can first be translated into real and imaginary current injections. Then, since there are no current losses (if shunt elements are handled properly) real and imaginary current contributions can be computed independently. Finally, active and reactive power contributions can be reconstructed from the current contributions. The next section of this paper explains the concepts and algorithms underlying the computation of current contributions. The following section discusses how these current contributions can be translated into power contributions. Finally, as an illustration of the possible applications of this method, it is shown how much of the reactive power output of each generator in a 30-bus system can be traced to each load.

II. CONCEPTS AND METHODS

Computing contributions is possible only in networks without loops. Several abstract concepts must therefore be introduced to make possible the transformation of an arbitrary transmission network into an acyclic graph. These concepts and the associated method are a generalization of the approach described in [1]. The interested reader is refered to that paper for a more detailed description of the basic concepts and algorithms.

The following paragraphs describe the concepts and methods which should be used to compute the contributions of each load to the active and reactive power output of a particular generator. Using a symmetrical set of concepts and methods, it is possible to determine the contribution that each source makes to the active and reactive powers consumed by a particular load. Since the latter set of contributions appears to be less useful, their derivation has been omitted.

Current sources and sinks

The proposed method takes as its starting point a snapshot of the state of the power system, i.e. the solution of a power flow or state estimation computation. All active and reactive injections and flows in this solution are translated into complex currents expressed in rectangular form. Injections are represented as sources or sinks of real and imaginary current:

- Generators are sources of real current but may be sources or sinks of imaginary current depending on their power factor and the sign of their voltage angle.
- Loads are sinks of real current but may be either sources or sinks of imaginary current.
- Shunt reactors and capacitors are normally and respectively sinks and sources of imaginary current. They are also either sources or sinks of real current unless they are connected to the reference bus.
- The shunt capacitances of the π model of a transmission line must be included in the real and imaginary sources or sinks located at the busses where the line terminates. Failure to include these capacitances would make the results erroneous, particularly at light loads.

While one could offset current sinks against current sources at each bus, it seems preferable to maintain their individuality. A bus to which are connected both generation and load could therefore be home to both real and imaginary current sources as well as both real and imaginary current sinks. To obtain an exact balance of real and imaginary currents, it is essential that the injections corresponding to the equivalent shunt admittances of all branches be included in these sources and sinks.

Real and imaginary current networks

Since the real and imaginary components of the current are orthogonal, Kirchoff's current law applies to each of them separately. No physical device can transform a real current into an imaginary current or vice-versa. For a given power flow solution, real and imaginary currents are therefore totally decoupled. For the purpose of analysing flows between current sources and current sinks, the actual network can be treated as the conjunction of two separate networks. The real current network connects the real current sources to the real current sinks and its branches carry only the real component of the branch currents. Similarly, the imaginary current network carries the imaginary component of the branch currents from the imaginary current sources to the imaginary current sinks.

Domain of a source

The domain of a source is defined as the set of buses which are reached by current from this source. In most cases, this domain covers only a fraction of the network and can be determined using a simple search algorithm. This algorithm is summarized as follows:

Starting from the bus to which the source is connected, recursively add to the domain of this source all the buses which can be reached from a bus already in the domain by following a branch in the same direction as the current in that branch.

A bus usually belongs to the domain of several real current sources and to the domain of several imaginary current sources.

Source commons

Having determined the domains of all the real (or imaginary) current sources and using a node coloring algorithm, it is a fairly simple matter to determine the sets of contiguous buses which are supplied by the same sources. Such buses form what will be called a source common. It should be noted that each bus belongs to one and only one real source common and to one and only one imaginary source common.

State graph

Currents always flow from source commons supplied by a smaller number of sources to source commons supplied by a larger number of sources. These commons can therefore be arranged in an acyclic state graph. A link between two commons in this graph represent all the lines and cables connecting these two commons. Note that the state graph for the real currents is usually quite different from the state graph for the imaginary currents. In this transformed representation of the power system, the currents trickle down from the commons at the roots of the state graphs as they combine with the currents from other sources.

Proportionality assumption

The inflow of a common is defined as the amount of current flowing into this common from sources inside the common or through links from other commons. Similarly, the outflow is defined as the amount of current absorbed by sinks inside the common or flowing on into other commons. In order to be able to trace the amount of current flowing from a source to the various sinks in its domain, the following proportionality assumption must be made:

For a given common, if the proportion of the inflow which can be traced to source i is x_i, then the proportion of the current sunk in that common or flowing out of this common which can be traced to source i is also x_i.

Like all postulates, this assumption can neither be proven or disproven and its only justification is that it appears more reasonable than any other possible assumption. Other assumptions would imply that the current traceable to some sources is disproportionately absorbed in the sinks located in a common while the current traceable to other sources is disproportionately transmitted to further commons. Since all busses within a common are reached by current traceable to the same set of sources, these competing assumptions do not seem to have any reasonable physical basis. It must also be stressed that the proposed method is based on an analysis of a snapshot of the state of the power system. It says nothing about the effect that a *change* in load might have on the state of the system.

Sink currents contributions to source currents

The computation of the contributions starts from the root nodes of the state graph where the contribution of the local sources is 100%. As it then proceeds layer by layer towards the leaf nodes, it is governed by the following equations:

$$I_k^n = \sum_{j \in \Phi_k^n} F_{jk}^n + \sum_{i \in \Psi_k^n} S_i^n \tag{1}$$

$$F_{ijk}^n = C_{ij}^n \cdot F_{jk}^n \tag{2}$$

$$C_{ik}^n = \frac{\sum_{j \in \Psi_k^n} F_{ijk}^n}{I_k^n} \quad \text{if source } i \text{ is not in common } k \tag{3-a}$$

$$C_{ik}^n = \frac{S_i^n}{I_k^n} \quad \text{if source } i \text{ is in common } k \tag{3-b}$$

where the following notations have been used:

I_k^n: Inflow of common k

S_i^n: Magnitude of source i

Φ_k^n: Set of commons located upstream from common k in the state graph

Ψ_k^n: Set of sources located in common k

F_{jk}^n: Current on the link between commons j and k

F_{ijk}^n: Current between commons j and k due to source i

C_{ij}^n: Contribution of source i to common j

The superscript n takes the value x or y depending on whether real or imaginary contributions are being calculated.

Sink contributions to active and reactive generations

Since there are no current losses, the currents flowing out of real and imaginary sources are absorbed entirely by the sinks contained in their domains. Hence, they can be expressed as the sum of the currents absorbed by the sinks in their domains weighted by the appropriate contributions:

$$I_u^x = \sum_{k \in D_u^x} C_{uk}^x I_k^x \tag{4}$$

$$I_v^y = \sum_{k \in D_v^x} C_{vk}^y I_k^y \tag{5}$$

where the subscripts u and v represent real and imaginary current sources respectively and D represent their domains.

The active and reactive power outputs of a generator can then be expressed as follows:

$$\begin{aligned} P_g &= \mathrm{Re}(\overline{V_g}\,\overline{I_g}^*) \\ &= V_g^x I_g^x + V_g^y I_g^y \\ &= V_g^x \alpha_{gu}^x I_u^x + V_g^y \alpha_{gv}^y I_v^y \\ &= V_g^x \alpha_{gu}^x \sum_{k \in D_u^x} C_{uk}^x I_k^x + V_g^y \alpha_{gv}^y \sum_{k \in D_v^y} C_{vk}^y I_k^y \end{aligned} \tag{6}$$

$$\begin{aligned} Q_g &= \mathrm{Im}(\overline{V_g}\,\overline{I_g}^*) \\ &= V_g^y I_g^x - V_g^x I_g^y \\ &= V_g^y \alpha_{gu}^x I_u^x - V_g^x \alpha_{gv}^y I_v^y \\ &= V_g^y \alpha_{gu}^x \sum_{k \in D_u^x} C_{uk}^x I_k^x - V_g^x \alpha_{gv}^y \sum_{k \in D_v^y} C_{vk}^y I_k^y \end{aligned} \tag{7}$$

The factors α_{gu} and α_{gv} reflect the fact that several devices (e.g. generators and capacitors) are occasionally lumped into real and imaginary current sources. The results obtained for each source must then be scaled by these factors when computing the results for physical devices.

If a load contributes 100% of the currents absorbed by a pair of real and imaginary current sinks, its relative contributions to the real and reactive outputs of generator g is obtained by extracting the corresponding terms in the summations (6) and (7):

$$p_{gk} = \frac{V_g^x \alpha_{gu}^x C_{uk}^x I_k^x + V_g^y \alpha_{gv}^y C_{vk}^y I_k^y}{P_g} \tag{8}$$

$$q_{gk} = \frac{V_g^y \alpha_{gu}^x C_{uk}^x I_k^x - V_g^x \alpha_{gv}^y C_{vk}^y I_k^y}{Q_g} \tag{9}$$

where the current contributions C_{uk}^x or C_{vk}^y are taken to be zero if this load is not in the domain of the real or imaginary current sources corresponding to the generator. It is important to note that these expressions implicitly take into account the active and reactive power losses caused by the flow of current from source to sink.

Choice of reference for the angles

As always in power systems analysis, the reference for the angles can be chosen arbitrarily. This choice obviously has a direct effect on how the branch and injection currents are divided into real and imaginary components. It will therefore also affect the size and shape of the domains and the definition of the commons. On the other hand, experience has shown that it has only a second order effect on the contribution coefficients defined in (8) and (9). So far, no particular criteria for choosing this reference has been identified.

Load contributions to active and reactive generations

Equations (8) and (9) quantify the contribution of each load taken as a whole. In some cases, it may be interesting to divide this contribution into a component linked to the active part of each load and another component linked to its reactive part. This is achieved by noting that the sink current can be expressed in terms of the active and reactive load as follows:

$$\begin{pmatrix} I_k^x \\ I_k^y \end{pmatrix} = \frac{1}{\left(V_k^x\right)^2 + \left(V_k^y\right)^2} \begin{pmatrix} V_k^x & V_k^y \\ V_k^y & -V_k^x \end{pmatrix} \begin{pmatrix} P_k \\ Q_k \end{pmatrix} \tag{10}$$

Replacing the sink currents in (8) and (9) by the values from (10), gives:

$$p_{gk} = p_{gk}^P + p_{gk}^Q \tag{11}$$

$$q_{gk} = q_{gk}^P + q_{gk}^Q \tag{12}$$

with:

$$p_{gk}^P = \left(\frac{V_g^x \alpha_{gu}^x C_{uk}^x V_k^x + V_g^y \alpha_{gv}^y C_{vk}^y V_k^y}{V_k^2} \right) \frac{P_k}{P_g} \tag{13}$$

$$p_{gk}^Q = \left(\frac{V_g^x \alpha_{gu}^x C_{uk}^x V_k^y - V_g^y \alpha_{gv}^y C_{vk}^y V_k^x}{V_k^2} \right) \frac{Q_k}{P_g} \tag{14}$$

$$q_{gk}^P = \left(\frac{V_g^y \alpha_{gu}^x C_{uk}^x V_k^x - V_g^x \alpha_{gv}^y C_{vk}^y V_k^y}{V_k^2} \right) \frac{P_k}{Q_g} \tag{15}$$

$$q_{gk}^Q = \left(\frac{V_g^y \alpha_{gu}^x C_{uk}^x V_k^y + V_g^x \alpha_{gv}^y C_{vk}^y V_k^x}{V_k^2} \right) \frac{Q_k}{Q_g} \tag{16}$$

Gaining some insight into the meaning of these equations is easier if they are applied to the two-bus system shown on Fig. 1. In this system, if the injections due to the equivalent shunt elements of the lines are ignored or lumped into the generation and load, the source factors are equal to unity and the computation of the contributions is trivial since:

$$\begin{pmatrix} I_g^x \\ I_g^y \end{pmatrix} = \begin{pmatrix} I_k^x \\ I_k^y \end{pmatrix} \tag{17}$$

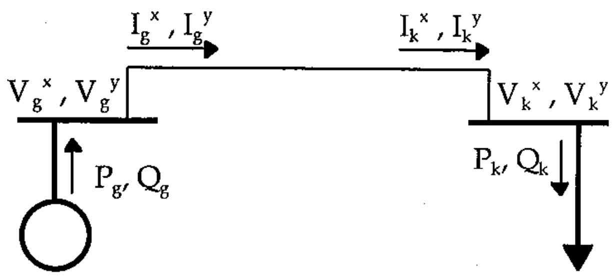

Fig. 1: Two-bus example

Using the polar representation of the voltage phasors, the active and reactive power generations can be expressed in terms of the active and reactive loads as follows:

$$\begin{pmatrix} P_g \\ Q_g \end{pmatrix} = \frac{V_g}{V_k} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} P_k \\ Q_k \end{pmatrix} \tag{18}$$

On this two-bus system, the active and reactive power balances can also be expressed in the usual way:

$$\begin{cases} P_g = \left(P_k + \dfrac{R}{V_k^2} P_k^2 \right) + \dfrac{R}{V_k^2} Q_k^2 \\ Q_g = \dfrac{X}{V_k^2} P_k^2 + \left(Q_k + \dfrac{X}{V_k^2} Q_k^2 \right) \end{cases} \tag{19}$$

Equations (18) and (19) both decompose the active and reactive generations in terms of the active and reactive load. The non-linearity of the decomposition of (19) prevents its application to larger networks. On the other hand, (18) is linear and does not explicitly involve any network parameters. Equations (11-16) can therefore be viewed as the generalization of (18) to networks of arbitrary complexity.

Equation (19) gives an exact causal relationship between the active and reactive loads and the active and reactive generations. Unfortunately, the same cannot be said of (18). This linear decomposition is correct but its causal interpretation (i.e. using its components to determine how much each component of the load affects the output of the generator) is only approximate. To give the reader an idea of the nature and magnitude of this approximation, the components of both sets of equations have been computed for a system with typical parameters and are shown in Tables 1 and 2. One can observe that:

- The coefficient linking the reactive power load to the active power generation in (18) is negative. This suggest that the linear decomposition of (18) is probably not useful for the active power.
- On the other hand, the linear decomposition of (18) is fairly accurate for the reactive power generation.

P_k	Q_k	P_g	$P_k + \frac{R}{V_k^2} P_k^2$	$\frac{R}{V_k^2} Q_k^2$	$\frac{V_g}{V_k} \cos\theta\, P_k$	$\frac{V_g}{V_k} \sin\theta\, Q_k$
10.0	0.0	10.0100	10.0100	0.0	10.010	0.0
10.0	4.8432	10.0125	10.0101	0.0024	10.0591	-0.0466
100.0	48.4322	101.4210	101.1510	0.2700	106.7258	-5.3047
100.0	75.0000	101.9278	101.2338	0.6940	110.4870	-8.5592

Table 1: Comparison of the linear and non-linear decompositions of the active power generation in a 2-bus system with the following parameters: R = 0.01 pu, X = 0.1 pu, V_g = 1.0 pu.

P_k	Q_k	Q_g	$\frac{X}{V_k^2} P_k^2$	$Q_k + \frac{X}{V_k^2} Q_k^2$	$-\frac{V_g}{V_k} \sin\theta\, P_k$	$\frac{V_g}{V_k} \cos\theta\, Q_k$
10.0	0.0	0.1002	0.1002	0.0	0.1002	0.0
10.0	4.8432	4.9682	0.1012	4.8670	0.0963	4.8719
100.0	48.4322	62.6425	11.5104	51.1322	10.9529	51.6896
100.0	75.0000	94.2775	12.3376	81.9399	11.4123	82.8652

Table 2: Comparison of the linear and non-linear decompositions of the reactive power generation in a 2-bus system with the following parameters: R = 0.01 pu, X = 0.1 pu, V_g = 1.0 pu.

- This accuracy improves as the power factor of the load tends towards unity. The error is obviously zero for unity power factor loads.

III. TEST RESULTS

Tracing the real and imaginary components of the currents from sources to sinks makes it possible to compute the exact contribution of each load to the active and reactive output of each generator. The approximate linear decomposition can then be used to separate the contributions of the active and reactive components of the load. Tables 3 and 4 illustrate the results of the method for a 30-bus system and the power flow conditions summarized by the data of Tables 5 and 6.

Table 3 shows the relative contributions of the loads at each bus in the system to the active power output of the generator located at bus 11. It also shows the decomposition of these contributions into their active and reactive components. As could be anticipated from the discussion of the two-bus example, the reactive component of this decomposition are negative. Table 4 provides the same information for the reactive power output of the same generator. In this example, while some components are negative, all overall load contributions are positive. Overall contributions can occasionally be negative. For example, the active load at bus 5 contributes -0.4% of the reactive power output of the generator located at bus 8. This is larger than the contribution of the reactive load at the same bus (0.26%). The overall contribution of the load at bus 5 to the reactive generation at bus 8 is therefore -0.14%.

Load bus	p_{gk}	p_{gk}^P	p_{gk}^Q
10	6.94	7.12	-0.18
24	7.23	7.75	-0.52
30	9.03	9.26	-0.23
29	1.99	2.08	-0.09
26	2.84	3.05	-0.21
21	16.89	17.86	-0.97
17	2.75	2.95	-0.20
20	2.19	2.25	-0.06
19	5.57	5.78	-0.21
7	22.54	23.38	-0.84
5	21.69	22.13	-0.44

Table 3: Contributions of the loads to the active power output of the generator located at bus 11. All values in %.

Load bus	q_{gk}	q_{gk}^P	q_{gk}^Q
10	7.47	4.71	2.76
24	7.39	4.87	2.52
30	5.33	5.46	-0.13
29	1.16	1.22	-0.06
26	1.68	1.80	-0.12
21	28.01	12.32	15.69
17	10.80	2.55	8.25
20	2.54	1.56	0.98
19	9.31	4.47	4.84
7	13.29	13.78	-0.49
5	12.79	13.05	-0.26

Table 4: Contributions of the loads to the reactive power output of the generator located at bus 11. All values in %.

Branch		P_{FROM}	Q_{FROM}	P_{TO}	Q_{TO}	Branch		P_{FROM}	Q_{FROM}	P_{TO}	Q_{TO}
1	2	46.796	0.382	-46.420	-5.132	15	18	7.052	1.376	-6.999	-1.268
1	3	19.502	1.230	-19.344	-5.107	18	19	3.799	0.368	-3.790	-0.349
2	4	8.728	-0.206	-8.687	-3.697	19	20	-5.710	-3.051	5.724	3.079
3	4	16.944	3.907	-16.907	-4.716	10	20	7.994	3.936	-7.924	-3.779
2	5	43.169	10.115	-42.305	-10.922	10	17	3.127	5.148	-3.116	-5.119
2	6	12.823	1.374	-12.730	-5.166	10	21	17.084	9.164	-16.961	-8.898
4	6	17.886	7.034	-17.845	-7.864	10	22	8.463	4.032	-8.402	-3.908
5	7	-21.045	0.118	21.245	-1.711	21	22	-0.539	-2.302	0.540	2.304
6	7	44.562	8.980	-44.045	-9.189	15	23	7.572	1.956	-7.514	-1.838
6	8	-14.515	-2.915	14.539	2.028	22	24	7.862	1.604	-7.791	-1.493
6	9	-11.120	2.814	11.120	-2.560	23	24	4.314	0.238	-4.290	-0.188
6	10	3.588	1.709	-3.588	-1.627	24	25	3.381	-0.677	-3.358	0.716
9	11	-50.000	-1.580	50.000	6.468	25	26	3.546	2.369	-3.500	-2.300
9	10	38.880	4.140	-38.880	-2.561	25	27	-0.188	-3.085	0.198	3.105
4	12	0.107	-0.221	-0.107	0.221	28	27	13.490	7.327	-13.490	-6.454
12	13	-50.000	-18.635	50.000	22.300	27	29	6.194	1.677	-6.105	-1.508
12	14	8.568	2.099	-8.480	-1.916	27	30	7.097	1.673	-6.930	-1.357
12	15	20.843	6.091	-20.556	-5.526	29	30	3.705	0.608	-3.670	-0.543
12	16	9.497	2.724	-9.412	-2.545	8	28	5.461	-1.013	-5.443	-3.532
14	15	2.280	0.316	-2.269	-0.306	6	28	8.059	2.442	-8.047	-3.795
16	17	5.912	0.745	-5.884	-0.681						

Table 5: Branch flows in the 30-bus test system. All quantities are in MW or MVAR.

Bus	P_G	Q_G	P_L	Q_L	Q_C	V	θ
1	66.297	1.611	0.000	0.000	5.258	1.060	0.000
2	40.000	18.850	21.700	12.700	9.305	1.050	-1.352
3	0.000	0.000	2.400	1.200	2.692	1.046	-1.784
4	0.000	0.000	7.600	1.600	2.944	1.042	-2.091
5	30.850	8.196	94.200	19.000	3.173	1.010	-5.664
6	0.000	0.000	0.000	0.000	4.595	1.037	-2.436
7	0.000	0.000	22.800	10.900	1.940	1.018	-4.275
8	50.000	31.015	30.000	30.000	2.801	1.040	-2.128
9	0.000	0.000	0.000	0.000	0.000	1.031	-1.198
10	0.000	0.000	5.800	2.000	20.092	1.028	-3.508
11	50.000	6.468	0.000	0.000	0.000	1.040	4.363
12	0.000	0.000	11.200	7.500	0.000	1.043	-2.106
13	50.000	22.300	0.000	0.000	0.000	1.070	1.491
14	0.000	0.000	6.200	1.600	0.000	1.028	-3.140
15	0.000	0.000	8.200	2.500	0.000	1.022	-3.350
16	0.000	0.000	3.500	1.800	0.000	1.029	-2.976
17	0.000	0.000	9.000	5.800	0.000	1.023	-3.561
18	0.000	0.000	3.200	0.900	0.000	1.012	-4.122
19	0.000	0.000	9.500	3.400	0.000	1.009	-4.384
20	0.000	0.000	2.200	0.700	0.000	1.013	-4.224
21	0.000	0.000	17.500	11.200	0.000	1.016	-4.035
22	0.000	0.000	0.000	0.000	0.000	1.017	-4.043
23	0.000	0.000	3.200	1.600	0.000	1.011	-4.090
24	0.000	0.000	8.700	6.700	4.342	1.005	-4.729
25	0.000	0.000	0.000	0.000	0.000	1.001	-5.435
26	0.000	0.000	3.500	2.300	0.000	0.983	-5.869
27	0.000	0.000	0.000	0.000	0.000	1.007	-5.605
28	0.000	0.000	0.000	0.000	2.984	1.034	-2.666
29	0.000	0.000	2.400	0.900	0.000	0.987	-6.874
30	0.000	0.000	10.600	1.900	0.000	0.976	-7.786

Table 6: Bus injections and voltages for 30-bus test system. Q_C represents the reactive injection due both to the capacitor banks and to the equivalent shunt susceptances of the lines. Injections are in MW or MVAR. Voltage magnitudes are in pu, voltage angles are in degrees.

IV. CONCLUSION

This paper proposes a method for computing the contribution of each load to the active and reactive power output of each generator. It is argued that this can be done accurately only if all power injections are translated into real and imaginary currents. The method then traces these currents to determine how much current each source supplies to each sink. These current contributions can then be translated into contributions to the active and reactive power output of the generators. It is also shown that the global contribution of a load can be decomposed into contributions from its active and reactive parts. This decomposition is reasonably accurate for the reactive power generation. For the active power generation, it is reasonable to neglect the effects of reactive flows on active losses and the previously-described method based on the tracing of the active flows [1] might be preferred because it has the advantage of being simpler and more intuitive.

V. REFERENCES

[1] D S Kirschen, R N Allan, G Strbac, "Contributions of Individual Generators to Loads and Flows," IEEE Transactions on Power Systems, Vol. 12, No. 1, February1997.

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VI. BIOGRAPHIES

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Discussion

J. W. Bialek (University of Durham, England): The authors present an interesting approach to the problem of network cost allocation by tracing the flow of real and imaginary currents. First of all one should note that the real and reactive current, unlike the real and reactive power, is a mathematical fiction which has no physical meaning. To be more precise, it has a physical meaning in a simple two-bus case analysed by the authors when, if one of the terminal voltage phasors is chosen as the reference, the real component of the current carries real power while the imaginary component of the current carries reactive power. However when a large meshed system is considered, the reference angle may generally be at any position with respect to the busbar voltage at question and the real and imaginary currents cannot be interpreted in any physical way. Moreover, as the choice of reference angle is arbitrary, the methodology gives non-unique results. This is very dangerous in the context of cost allocation as any network user disadvantaged by the choice of a particular reference angle may challenge the choice.

Secondly let us compare the decomposition expressed by equations (18) and (19). The decomposition expressed by equation (19) has a clear physical meaning as components RP_k^2/V_k^2 and RQ_k^2/V_k^2 express the real power loss on the line resistance due to the flow of real and reactive power while the components XP_k^2/V_k^2 and XQ_k^2/V_k^2 express the reactive power loss on the line reactance due to the flow of real and reactive power. Unfortunately no such a physical meaning can be associated with the components expressed by equation (18). Clearly the decomposition (18) for real power generation is nonsensical as the reactive demand component is negative. The authors claim that the decomposition (18) for the reactive power generation gives results close to that due to decomposition (19) but the error for the active component even in this simple case is about 8%. One can question what the accuracy for large systems is. The suspicion is that it may not be high as the authors reported that the overall contributions to reactive power generation can be occasionally negative. This again seems to be nonsensical. To summarise, the methodology presented seems to be interesting and mathematically correct but it has no physical interpretation and therefore may be very difficult to implement in practice.

One should also point out to an earlier attempt to solve a similar problem in which the network usage costs were allocated based on tracing the flow of real and reactive power [A, B]. In this approach the problem of line reactive loss has been accommodated by creating additional fictitious line nodes responsible for the line's reactive power generation or consumption. The tracing of real and reactive was done separately, but using the same tracing methodology, and then the results combined to give the overall charges. This approach has a clear physical meaning as for each real or reactive power injection, one can determine the flow of power in the network and the corresponding sinks. Also the result is unique as the methodology does not depend on the choice of the reference bus or angle. The disadvantage of the approach is that the size of the problem increases due to creation of additional nodes. However the tracing algorithm is computationally very effective and the overall computational complexity is not excessive. I would be grateful for the authors' comparison between the two approaches.

References:

A. J. Bialek 'Tracing the flow of electricity' *IEE Proc. Gen. Transm. Distrib.*, vol. 143, pp. 313-320, July 1996.

B. J. Bialek "Allocation of Transmission Supplementary Charge to Real and Reactive Loads" paper PE-988-PWRS-0-07-1997 published in *IEEE Trans. on Power Systems*, Vol. 13, No.3, August 1998, pp. 749-754.

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D. S. Kirschen, G. Strbac (UMIST, Manchester, United Kingdom): Mr Bialek's discussion emphasizes some of the difficulties associated with the proposed method. We had ourselves noted these difficulties and discussed them in the paper and we do not believe that they invalidate or fundamentally undermine the method.

We are glad for the opportunity to compare our method with the one that Mr Bialek proposed in [A, B]. If we first consider the case where only the active power is traced through the network and "allocated" to the various sources and sinks, Mr Bialek's method is roughly similar to the method that we originally described in [1]. We believe that our method is more rigorous as it does not require the assumption that the network is lossless and that the gross and net flows (i.e. the flows adjusted to take into account the fact that there are losses) are distributed in the same way as the actual flows.

On the other hand, the treatment of reactive power flows and losses in [A, B] is fundamentally flawed because it is based on the premise that active and reactive power flows can be analyzed independently. This assumption is untenable as it amounts to saying that active power flows do not cause reactive losses and vice versa. The fictitious nodes that are introduced to account for the reactive losses provide a convenient mean of making the books balance. However, lumping the losses into those nodes does not answer the original question: How much of the active and reactive flows, injections and losses is attributable to each source and each sink?

It is only through a careful analysis of the interactions between active and reactive flows, such as the one our paper provides, that this question can be answered rigorously.

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