Tracing the flow of electricity

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Abstract: Continuing trend towards deregulation and unbundling of transmission services has resulted in the need to assess what the impact of a particular generator or load is on the power system. A new method of tracing the flow of electricity in meshed electrical networks is proposed which may be applied to both real and reactive power flows. The method allows assessment of how much of the real and reactive power output from a particular station goes to a particular load. It also allows the assessment of contributions of individual generators (or loads) to individual line flows. A loss-apportioning algorithm has also been introduced which allows the break down of the total transmission loss into components to be allocated to individual loads or generators. The method can be useful in providing additional insight into power system operation and can be used to modify existing tariffs of charging for transmission loss, reactive power and transmission services.

1 Introduction

The mesh structure of high-voltage transmission networks provides a large number of possible routes by which electrical power can flow from the sources (generators) to the sinks (grid supply points, here referred to as loads). Tracing the connections using the load flow program is not possible as changing a demand or generation at any node would result in a corresponding change of generation coming from the marginal (swing) plant. Hence, the conventional wisdom is that with an integrated system it is not possible to trace electricity from a particular generator to a particular supplier [1]. It is only possible to determine relation between the generators (or loads) and the flows in transmission lines by means of sensitivity analysis, that is by determining how a change in a nodal generation/demand influences the flow in a particular line [2, 3]. The sensitivity analysis, however, does not answer the question where the power goes; it answers the question how would line flows change following a change in the nodal generation/demand.

Until very recently, the question of tracing electricity was of a limited interest. The electrical supply industry tended to be integrated vertically almost everywhere and power exchanges between utilities were determined by contracts. Since the 1980s, however, the increased deregulation of the industry in almost every corner of the world has posed many new questions to electrical engineers. In the USA, the introduction of wholesale 'wheeling' has created a problem of pricing the transmission services based conceptually on the distance the wheeled power travels via the third-party utility [4]. In many countries the transmission is seen as a separate business of transporting electricity from any generator to any area supplier. This concept led to the creation of the National Grid Company in the UK. Also in the USA, the federal regulation seems to be heading in the same direction [5]. It is widely recognised that the proper regulatory framework of the transmission is of a vital importance as 'the market power through control of transmission is the single greatest impediment to competition' [5].

In this context, the problem of tracing electricity gains importance as its solution could enhance the transparency in the operation of the transmission system. Consider for example the Electricity Pool in the UK. The concept of the Pool inherently assumes that the electrical network is lossless. The transmission loss, which accounts for about 2% of generation and which in 1994/95 cost about £140m [6], is charged to the area suppliers through a uniform *pro-rata* charge. As this arrangement provides little incentive on suppliers, and none on generators, to take action to reduce losses, the industry regulator has urged the Pool 'to put forward proposals to deal with this issue as a matter of urgency' [6]. An electricity tracing method would make it possible to charge the suppliers and/or generators for the actual amount of losses caused and hence encourage efficiency. Obviously there is a question of whether or not a generator and/or supplier should be penalised for its geographical position but this question is beside the scope of this paper.

Recently a novel electricity tracing method has been proposed [7, 8] which, under the assumption that nodal inflows are shared proportionally between the nodal outflows, allows one to trace the flow of electricity in a meshed network. It is then possible to create a table, resembling a road distance table, which shows what amount of real and reactive power is supplied from a particular generator to a particular load and what is the share of each generator and load in each of the line flows. A similar approach has been independently proposed in [9].

In this paper the electricity tracing method is discussed in detail together with an algorithm of apportioning the transmission loss to individual loads or generators. This method is a generalisation of the loss

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allocation method originally applied to radial distribution networks [10]. This loss allocation methodology can be seen as an alternative to the more traditional marginal cost-based methodology which is preferred by orthodox economic theory. The marginal pricing, however, is known to result in volatile, and therefore unpredictable, charges [11] and may sometimes lead to negative marginal cost of losses rewarding the consumer for increasing his load [12]. The results of the loss-apportioning method presented in this paper do not depend on the choice of the marginal generator and always result in positive charges.

2 Assumptions

The proposed electricity tracing method is topological in nature, that is it deals with a general transportation problem of how the flows are distributed in a meshed network. The network is assumed to be connected and described by a set of n nodes, m directed links (transmission lines or transformers), $2m$ flows (at both ends of each link) and a number of sources (generators) and sinks (loads) connected to the nodes. Practically the only requirement for the input data is that Kirchhoff's Current Law must be satisfied for all the nodes in the network. In this respect the method is equally applicable to real and reactive power flows and direct currents. Neglecting the Kirchhoff's Voltage Law does not introduce any further errors as the law has been already used to obtain the flows.

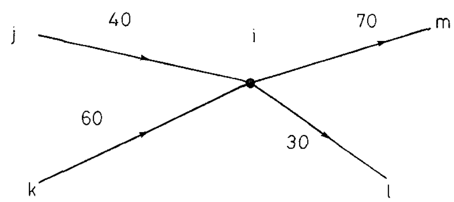

Fig. 1 *Proportional sharing principle*

The main principle used to trace the flow of electricity will be that of proportional sharing. This is illustrated in Fig. 1 where four lines are connected to node i, two with inflows and two with outflows. The total power flow through the node is P_i = 40 + 60 = 100MW of which 40% is supplied by line j–i and 60% by line k–i. As electricity is indistinguishable and each of the outflows down the line from node i is dependent only on the voltage gradient and impedance of the line, it may be assumed that each MW leaving the node contains the same proportion of the inflows as the total nodal flow P_i. Hence the 70MW outflowing in line i–m consists of 70 40/100 = 28MW supplied by line j–i and 70 60/100 = 42MW supplied by line k–i. Similarly the 30MW outflowing in line i–l consists of 30 40/100 = 12MW supplied by line j–i and 30 60/100 = 18MW supplied by line k–i.

The proportional sharing principle basically amounts to assuming that the network node is a perfect 'mixer' of incoming flows so that it is impossible to tell which particular inflowing electron goes into which particular outgoing line. This seems to agree with common sense and with the generally accepted view that electricity is indistinguishable.

As it is impossible to 'dye' the incoming flows and check the colour of the outflows, the proportional sharing principle can be neither proved nor disproved. This, however, is irrelevant as the principle will be applied here for nontechnical calculations. In this respect, the principle is fair as it treats all the incoming and outflowing flows in the same way. In other words, no particular generator or load is distinguished in any way.

3 Tracing electricity using average line flows

Tracing electricity can be seen as a transportation problem of determining how the power injected by generators is distributed between the lines and loads of the network. The algorithm proposed in this paper works only on lossless flows when the flows at the beginning and end of each line are the same. The simplest way of obtaining lossless flows from the lossy ones is by assuming that a line flow is an average over the sending- and receiving-end flows and by adding half of the line loss to the power injections at each terminal node of the line.

Consider for example a simple system shown in Fig. 2 with active and reactive power flows obtained from AC load flow program. A number on top or to the left of the line indicates a real power flow, while a number below or to the right of the line indicates a reactive power flow. A similar convention has been used for the generators and the loads. The total transmission loss in the network is equal to the sum of all the line losses and equals (225 – 218) + (83 – 82) + (173 – 171) + (60 – 59) + (115 – 112) = 14MW.

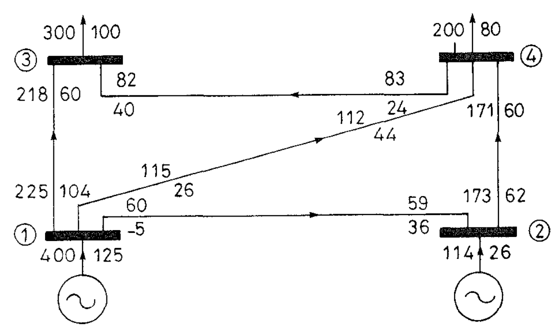

Fig. 2 *AC power flow in four-node network*

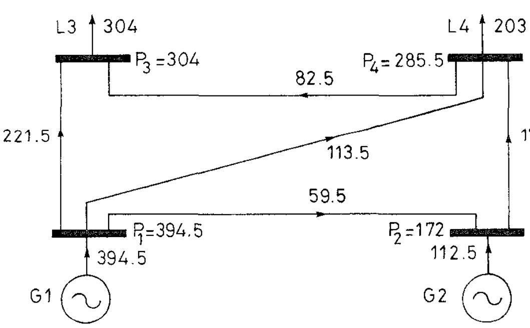

Fig. 3 *Lossless power flow*

Fig. 3 shows a lossless real power flow obtained from the lossy flow of Fig. 2. The algorithm for tracing the flow of electricity will be now derived in two versions. The downstream-looking algorithm will look at the nodal balance of outflows while the dual, upstream-looking algorithm, will look at the nodal balance of inflows.

3.1 Upstream looking algorithm

The total flow P_i through node i (i.e. the sum of inflows or outflows) may be be expressed, when looking at the inflows, as

$$P_i = \sum_{j \in \alpha_i^{(u)}} |P_{i-j}| + P_{Gi} \quad \text{for } i = 1, 2, \ldots, n \qquad (1)$$

where $\alpha_i^{(u)}$ is the set of nodes supplying directly node i (i.e. power must flow towards node i in the relevant lines), P_{i-j} is the line flow into node i in line j–i, and P_{Gi} is the generation at node i. As the losses have been eliminated, $|P_{j-i}| = |P_{i-j}|$.

The line flow $|P_{i-j}| = |P_{j-i}|$ can be related to the nodal flow at node j by substituting $|P_{i-j}| = c_{ji}P_j$, where $c_{ji} = |P_{j-i}|/P_j$, to give

$$P_i = \sum_{j \in \alpha_i^{(u)}} c_{ji}P_j + P_{Gi} \qquad (2)$$

which, on rearrangement, becomes

$$P_i - \sum_{j \in \alpha_i^{(u)}} c_{ji}P_j = P_{Gi} \quad \text{or} \quad \mathbf{A}_u\mathbf{P} = \mathbf{P}_G \qquad (3)$$

where $\mathbf{A}_u$ is the $(n \times n)$ upstream distribution matrix, $\mathbf{P}$ is the vector of nodal through-flows and $\mathbf{P}_G$ is the vector of nodal generations. The (i, j) element of $\mathbf{A}_u$ is equal to

$$[\mathbf{A}_u]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -c_{ji} = -|P_{j-i}|/P_j & \text{for } j \in \alpha_i^{(u)} \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Note that $\mathbf{A}_u$ is sparse and nonsymmetric. If $\mathbf{A}_u^{-1}$ exists [Note 1] then $\mathbf{P} = \mathbf{A}_u^{-1}\mathbf{P}_G$ and its ith element is

$$P_i = \sum_{k=1}^{n} [\mathbf{A}_u^{-1}]_{ik} P_{Gk} \quad \text{for } i = 1, 2, \ldots, n \qquad (5)$$

This equation shows that the contribution of the kth system generator to ith nodal power is equal to $[\mathbf{A}_u^{-1}]_{ik} P_{Gk}$. Note that the same P_i is equal to the sum of the load demand, P_{Li} and outflows in lines leaving node i. A line outflow in line i–l from node i can be therefore calculated, using the proportional sharing principle, as

$$|P_{i-l}| = \frac{|P_{i-l}|}{P_i} P_i = \frac{|P_{i-l}|}{P_i} \sum_{k=1}^{n} [\mathbf{A}_u^{-1}]_{ik} P_{Gk}$$
$$= \sum_{k=1}^{n} D_{il,k}^{G} P_{Gk} \quad \text{for all } l \in \alpha_i^{(d)} \qquad (6)$$

where $D_{i-l,k}^{G} = |P_{i-l}|[\mathbf{A}_u^{-1}]_{ik}/P_i$ and α_i^{d} is the set of nodes supplied directly from node i (that is power flows from those nodes to node i in the relevant lines). This equation defines $D_{i-l,k}$ as the topological generation distribution factor that is a portion of generation owing to the kth generator that flows in line i–l. This definition is similar to that used by Ng to define his generalised generation distribution factors [2]. His method, however, was based on the superposition theorem applied to the DC linearised system model so that his distribution factors represented the impact of a particular generation on the line flow which could well be negative. On the other hand, the topological distribution factors are based on topological analysis of network flows and represent the share of a particular generation in the total line flow. Consequently they are always positive.

Note 1: The problem of existence of $\mathbf{A}_u^{-1}$ and introduced later $\mathbf{A}_d^{-1}$ is beyond the scope of this paper.

Similarly, the load demand P_{Li} can be calculated from P_i as

$$P_{Li} = \frac{P_{Li}}{P_i} P_i = \frac{P_{Li}}{P_i} \sum_{k=1}^{n} [\mathbf{A}_u^{-1}]_{ik} P_{Gk} \quad \text{for } i = 1, 2, \ldots, n \qquad (7)$$

This equation shows that the contribution of the kth generator to the ith load demand is equal to $P_{Li}P_{Gk}[\mathbf{A}_u^{-1}]_{ik}/P_i$ and can be used to trace where the power of a particular load comes from.

Apply this algorithm to the example system of Fig. 3. Eqn. 3 takes the form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-59.5}{394.5} & 1 & 0 & 0 \\ \frac{-221.5}{394.5} & 0 & 1 & \frac{-82.5}{285.5} \\ \frac{-113.5}{394.5} & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 394.5 \\ 112.5 \\ 0 \\ 0 \end{bmatrix}$$

where $P_1 = 394.5$, $P_2 = 172$, $P_3 = 304$ and $P_4 = 285.5$. Inverting the matrix gives

$$\mathbf{A}_u^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1508 & 1 & 0 & 0 \\ 0.6882 & 0.289 & 1 & 0.289 \\ 0.4385 & 1 & 0 & 1 \end{bmatrix}$$

Eqn. 6 allows one to determine how the line flows are supplied from individual generators. A flow in line, say, 4–3 can be calculated as being supplied by (82.5/285.5) × 0.4385 × 394.5 = 49.99 MW from G1 and by (82.5/285.5) × 1 × 112.5 = 32.51 MW from G2.

Table 1 shows how the demand in each of the loads can be calculated, eqn. 7, as the sum of contributions from individual generators. This Table resembles a road distance table as an intersection between a row, corresponding to a load, and a column, corresponding to a generator, gives the amount of power supplied by a particular generator to a particular load.

Table 1: Distribution of power using upstream-looking algorithm

Load	Generation		Total
	G1	G2	
L3	0.6882 × 394.5 = 271.5	0.289 × 112.5 = 32.5	304
L4	(203/285.5) × 0.4385 × 394.5 = 123	(203/285.5) × 1 × 112.5 = 80	203
Total	394.5	112.5	507

Correctness of the results can be to some extent confirmed by inspection of flows in Fig. 3. For example, L4 gets its power directly from G2 and in a combined way from G1. The share obtained from G2 can be estimated, using the proportional sharing principle, as 112.5 – 112.5(82.5/285.5) = 80. The share obtained from G1 is equal to (113.5 – 113.5(82.5/285.5)) + (59.5 – 59.5(82.5/285.5)) = 123.

3.2 Downstream-looking algorithm

Now consider the dual, downstream-looking, problem when the nodal through-flow P_i is expressed as the sum of outflows

$$P_i = \sum_{l \in \alpha_i^{(d)}} |P_{i-l}| + P_{Li} = \sum_{l \in \alpha_i^{(d)}} c_{li}P_l + P_{Li} \quad \text{for } i = 1, 2, \ldots, n \qquad (8)$$

where $\alpha_i^{(d)}$ is, as before, the set of nodes supplied directly from node i and $c_{li} = |P_{l-i}|/P_l$. This equation can be rewritten as

$$P_i - \sum_{l \in \alpha_i^{(d)}} c_{li} P_l = P_{Li} \quad \text{or} \quad \mathbf{A}_d \mathbf{P} = \mathbf{P}_L \tag{9}$$

where $\mathbf{A}_d$ is the $(n \times n)$ downstream distribution matrix and $\mathbf{P}_L$ is the vector of nodal demands. The (i, l) element of $\mathbf{A}_d$ is equal to

$$[\mathbf{A}_d]_{il} = \begin{cases} 1 & \text{for } i = l \\ -c_{li} = -|P_{l-i}|/P_l & \text{for } l \in \alpha_i^{(d)} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Note that $\mathbf{A}_d$ is also sparse and nonsymmetric. Adding $\mathbf{A}_u$ and $\mathbf{A}_d$ gives a symmetric matrix which has the same structure as the nodal admittance matrix. If $\mathbf{A}_d^{-1}$ exists then $\mathbf{P} = \mathbf{A}_d^{-1}\mathbf{P}_L$ and its ith element is equal to

$$P_i = \sum_{k=1}^{n} [\mathbf{A}_d^{-1}]_{ik} P_{Lk} \quad i = 1, 2, \ldots, n \tag{11}$$

This equation shows how the nodal power P_i distributed between all the loads in the system. On the other hand, the same P_i is equal to the sum of the generation at node i and all the inflows in lines entering the node. Hence the inflow to node i from line i–j can be calculated using the proportional sharing principle as

$$|P_{i-j}| = \frac{|P_{i-j}|}{P_i} P_i = \frac{|P_{i-j}|}{P_i} \sum_{k=1}^{n} [\mathbf{A}_d^{-1}]_{ik} P_{Lk} = \sum_{k=1}^{n} D_{i-j,k}^{L} P_{LK} \quad \text{for all } j \in \alpha_i^{(u)} \tag{12}$$

where $D_{i-j,k}^{L} = |P_{i-j}|[\mathbf{A}_d^{-1}]_{ik}/P_i$ is the topological load distribution factor that is the portion of kth load demand that flows in line i–j. This definition is again similar to that of the generalised load distribution factor [3] based on DC load-flow sensitivity analysis. However, the topological factor represents the share (which is always positive) of the load in a line flow while the generalised factor determines the impact of the load on a line flow and may be negative.

The generation at a node is also an inflow and can be calculated using the proportional sharing principle as

$$P_{Gi} = \frac{P_{Gi}}{P_i} P_i = \frac{P_{Gi}}{P_i} \sum_{k=1}^{n} [\mathbf{A}_d^{-1}]_{ik} P_{Lk} \quad \text{for } i = 1, 2, \ldots, n \tag{13}$$

This equation shows that the share of the output of the ith generator used to supply the kth load demand is equal to $P_{Gi}P_{Lk}[\mathbf{A}_d^{-1}]_{ik}/P_i$ and can be used to trace where the power of a particular generator goes to. Comparing the shares in eqns. 7 and 13 gives

$$\frac{P_{Li}P_{Gk}[\mathbf{A}_u^{-1}]_{ik}}{P_i} = \frac{P_{Gk}P_{Li}[\mathbf{A}_d^{-1}]_{ki}}{P_k} \quad \text{or} \quad \frac{[\mathbf{A}_u^{-1}]_{ik}}{[\mathbf{A}_d^{-1}]_{ki}} = \frac{P_i}{P_k} \tag{14}$$

where i is any load node number and k is any generator node number. Hence. assuming that there is n_g generators and n_l loads in the system, it is necessary to determine $n_g n_l$ elements of matrix $\mathbf{A}_u^{-1}$ or $\mathbf{A}_d^{-1}$.

Now apply this algorithm to the example system of Fig. 3. Eqn. 9 takes the form

$$\begin{bmatrix} 1 & \frac{-59.5}{172} & \frac{-221.5}{304} & \frac{-113.5}{285.5} \\ 0 & 1 & 0 & \frac{-172}{285.5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-82.5}{304} & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 304 \\ 203 \end{bmatrix}$$

Inverting the matrix gives

$$\mathbf{A}_d^{-1} = \begin{bmatrix} 1 & 0.3459 & 0.8931 & 0.606 \\ 0 & 1 & 0.1635 & 0.6025 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.2714 & 1 \end{bmatrix}$$

Eqn. 12 allows to determine how the line flows supply individual loads. For example, the flow in line, say, 2–4 supplies load L3 with (172/285.5) × 0.2714 × 304 = 49.7 MW and load L4 with (172/285.5) × 1 × 203 = 122.3 MW.

Table 2: Distribution of power using downstream-looking algorithm

Generation	Load L3	Load L4	Total
G1	0.8931 × 304 = 271.5	0.606 × 203 = 123	394.5
G2	(112.5/172) × 0.1635 × 304 = 32.5	(112.5/172) × 0.6025 × 203 = 80	112.5
Total	304	203	507

Table 2 shows application of eqn. 13 to determine how the generation is distributed between each of the loads. This Table again resembles a road distance table. As the network considered is lossless, Table 2 is a transpose of Table 1. The following Section shows, however, that when the transmission losses are included, both algorithms give slightly different results.

4 Tracing electricity using gross flows

The lossless flow introduced in the previous Section was obtained by averaging the line flows and modifying power injections at both ends of the lines. An interesting version of the electricity tracing method can be obtained by assuming that the system is fed with the actual generation and no power is lost in the network. This will require then modifying the nodal demands but will leave the nodal generations unchanged.

Consider for example line 2–4 which carries 173 MW at the sending end and 171 MW at the receiving end. The line loss of 2 MW can be added to the receiving end flow (to give a gross line flow of 173 MW) so that the modified flows at both ends are made the same. This procedure can be repeated for all the lines of the system. The result, however, will not be satisfactory as the resultant power flows will not satisfy the Kirchhoff's Current Law. To understand this consider again line 2–4. The 173 MW at the sending end is not the 'true' gross flow as 1 MW out of power reaching node 2 has already been lost in line 1–2. Hence the true gross flow in line 2–4 is not 173, but 173 + 1 = 174.

In the simple system shown in Fig. 3 it is possible to find all the true gross flows by inspection. For more complicated networks, inspection will not suffice and a more formal method is necessary. Once the gross power flows (satisfying Kirchhoff's Current Law) are calculated, it is then straightforward to apply the electricity tracing method presented in Section 3.

Define an unknown gross nodal power $P_i^{(gross)}$ as a total power flow through node i which satisfies the Kirchhoff's Current Law and which would flow if the network was fed with the actual generation and no power was lost in the network. Similarly, let $P_{i-j}^{(gross)}$ be an unknown gross flow in line i–j which would flow if no power was lost. Obviously $|P_{i-j}^{(gross)}| = |P_{j-i}^{(gross)}|$. Taking as an example the real power flow shown in Fig. 3, one can find by inspection that $P_1^{(gross)} = 400$ as there is no line supplying node 1, $|P_{1-2}^{(gross)}| = |P_{2-1}^{(gross)}| = 60$, $P_2^{(gross)} = P_{1-2}^{(gross)} + P_{G2} = 60 + 114 = 174$, etc.

The gross nodal power, when looking at the inflows (upstream-looking algorithm), can be expressed as

$$P_i^{(gross)} = \sum_{j\in\alpha_i^{(u)}} |P_{i-j}^{(gross)}| + P_{Gi} \quad \text{for } i = 1, 2, \ldots, n \quad (15)$$

As $|P_{i-j}^{(gross)}| = |P_{j-i}^{(gross)}|$, the flow $P_{j-i}^{(gross)}$ can be replaced by $c_{ji}^{(gross)} P_j^{(gross)}$ where $c_{ji}^{(gross)} = |P_{j-i}^{(gross)}|/P_j^{(gross)}$. Normally the transmission losses are small so that it can be assumed that $|P_{j-i}^{(gross)}|/P_j^{(gross)} \simeq |P_{j-i}|/P_j$, where P_{j-i} is the actual flow from node j in line j–i and P_j is the actual total flow through node j. This corresponds to assuming that the distribution of gross flows at any node is the same as the distribution of actual flows. This is the only approximating assumption of the method. Under this assumption eqn. 15 can be rewritten as

$$P_i^{(gross)} - \sum_{j\in\alpha_i^{(u)}} \frac{|P_{j-i}|}{P_j} P_j^{(gross)} = P_{Gi} \quad (16)$$

$$\text{or} \quad \mathbf{A}_u \mathbf{P}_{gross} = \mathbf{P}_G$$

where $\mathbf{P}_{gross}$ is the unknown vector of gross nodal flows and $\mathbf{A}_u$ is the upstream distribution matrix calculated from the actual, not modified, flows. As $\mathbf{A}_u$ and $\mathbf{P}_G$ are known, the solution of eqn. 16 will give the unknown gross nodal flows.

Once the gross nodal flows have been determined, the gross line flows and gross demands can also be found using the proportional sharing principle. The gross flow in line i–l is

$$|P_{i-l}^{(gross)}| = \frac{|P_{i-l}^{(gross)}|}{P_i^{(gross)}} P_i^{(gross)} \cong \frac{|P_{i-l}|}{P_i} \sum_{k=1}^{n} [\mathbf{A}_u^{-1}]_{ik} P_{Gk} \quad \text{for all } l \in \alpha_i^{(d)} \quad (17)$$

while the gross demand at node i can be calculated as

$$P_{Li}^{(gross)} = \frac{P_{Li}^{(gross)}}{P_i^{(gross)}} P_i^{(gross)} \cong \frac{P_{Li}}{P_i} P_i^{(gross)} = \frac{P_{Li}}{P_i} \sum_{k=1}^{n} [\mathbf{A}_u^{-1}]_{ij} P_{Gk} \quad (18)$$

This equation is especially important as it shows what would be the load demand at a given node if a lossless network was fed with the actual generation. Hence the difference between the gross demand and the actual demand

$$\Delta P_{Li} = P_{Li}^{(gross)} - P_{Li} \quad (19)$$

gives the loss which is attracted by power flowing from all the generators to a particular load. In other words, the upstream-looking algorithm not only allows to determine participation of each generator in satisfying a particular load demand, but also allows to apportion the total transmission loss to individual loads in the network. This is a very important conclusion as it allows the charging of loads individually for the actual amount of power lost.

Now apply this algorithm to the real power flow shown in Fig. 2. Eqn. 16 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-60}{400} & 1 & 0 & 0 \\ \frac{-225}{400} & 0 & 1 & \frac{-83}{283} \\ \frac{-115}{400} & \frac{-173}{173} & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1^{(gross)} \\ P_2^{(gross)} \\ P_3^{(gross)} \\ P_4^{(gross)} \end{bmatrix} = \begin{bmatrix} P_{G1} = 400 \\ P_{G2} = 114 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

Solving this equation gives the following values of gross nodal powers: $\mathbf{P}_{gross} = [400\ 174\ 30976\ 289]^T$. This result confirms the earlier calculated values of the gross nodal powers. The gross load demands are $L_3^{(gross)} = (300/300) \times 309.76 = 309.76$ and $L_4^{(gross)} = 289 \times (200/283) = 204.24$. Hence the loss apportioned to L_1 is equal to 9.76 while the loss apportioned to L_4 is 4.24.

The share of the generation used to supply each of the loads can be calculated in a similar way to that described in the previous Section and is shown in Table 3.

Table 3: Results of upstream-looking algorithm of tracing gross flows

	G1	G2	Total	Loss
L3	276.32	33.44	309.76	9.76
L4	123.68	80.56	204.24	4.24
Total	400	114	514	14

The sum of the elements in each of the generator columns gives, as expected, the actual generation. The sum of file elements in each of the load rows gives the gross demand for each of the loads. The difference between the gross and actual demand gives the transmission loss associated with supplying a particular load and is shown in the last column of Table 3. Obviously the result is the same as that obtained previously.

5 Tracing electricity using net flows

This version of the method traces the flow of electricity in the network when transmission losses are completely removed from the line flows. This will require modifying the nodal generations while leaving the nodal demands unchanged. Consider again line 2–4. Removing the line loss of 2MW from the sending-end flow gives the net flow of 171MW. However, this 171MW is not the true net flow as it contains a component which will be lost in line 4–3 down the network. To deal with this problem introduce $P_i^{(net)}$ as an unknown net nodal power and $P_{i-j}^{(net)}$ as an unknown net flow in line i–j, both of which satisfy the Kirchhoff's Current Law and which would flow if transmission losses were completely removed from all the line flows. Obviously $|P_{i-j}^{(net)}| = |P_{j-i}^{(net)}|$. Taking as an example the real power flow shown in Fig. 2, by inspection $P_3^{(net)} = 300$ as there is no outflow from node 3, $|P_{4-3}^{(net)}| = |P_{3-4}^{(net)}| = 82$ and $P_4^{(net)} = P_{L4} + |P_{4-3}^{(net)}| = 200 + 82 = 282$. The net node power balance equation can now be defined, when looking at the outflows (downstream-looking algorithm), as

$$P_i^{(net)} = \sum_{l\in\alpha_i^{(d)}} |P_{i-l}^{(net)}| + P_{Li} = \sum_{l\in\alpha_i^{(d)}} c_{li}^{(net)} P_l^{(net)} + P_{Li}$$

$$\text{for } i = 1, 2, \ldots, n \quad (21)$$

where $c_{li}^{(net)} = |P_{l-i}^{(net)}|/P_l^{(net)}$. As the transmission losses are small, it can be assumed that $|P_{l-i}^{(net)}|/P_l^{(net)} \simeq |P_{l-i}|/P_l$ that eqn. 21 can be rewritten as

$$P_i^{(net)} - \sum_{i \in \alpha_i^{(d)}} \frac{|P_{l-i}|}{P_l} P_l^{(net)} = P_{Li} \quad \text{or} \quad \mathbf{A}_d \mathbf{P}_{net} = \mathbf{P}_L \tag{22}$$

where $\mathbf{P}_{net}$ is the unknown vector of net nodal flows and $\mathbf{A}_d$ is the downstream distribution matrix. As $\mathbf{A}_d$ and $\mathbf{P}_L$ are known, the solution of eqn. 22 will give the net nodal flows.

Now the net flow in line i–j can be calculated using the proportional sharing principle as

$$|P_{i-j}^{(net)}| = \frac{|P_{i-j}^{(net)}|}{P_i^{(net)}} P_i^{(net)} \cong \frac{|P_{i-j}|}{P_i} \sum_{k=1}^{n} [\mathbf{A}_d^{-1}]_{ik} P_{Lk} \tag{23}$$

for all $j \in \alpha_i^{(u)}$

while the net generation at node i can be calculated as

$$P_{Gi}^{(net)} = \frac{P_{Gi}^{(net)}}{P_i^{(net)}} P_i^{(net)} \cong \frac{P_{Gi}}{P_i} P_i^{(net)} = \frac{P_{Li}}{P_i} \sum_{k=1}^{n} [\mathbf{A}_d^{-1}]_{ij} P_{Lk} \tag{24}$$

This equation is especially important as it shows what would be the generation at a given node necessary to cover the system demand if the network was lossless. Hence the difference between the actual and net generation

$$\Delta P_{Gi} = P_{Gi} - P_{Gi}^{(net)} \tag{25}$$

gives the loss attracted by power flowing from a given generator to all the loads. In other words, the downstream-looking algorithm not only allows to determine how the output of a given generator is shared between all the loads, but also allows to apportion the total transmission loss to individual generators in the network. This is a very important conclusion as it allows to charge the generators individually for their share of the transmission loss.

Apply this algorithm to the real power flow shown in Fig. 2. Eqn. 22 gives

$$\begin{bmatrix} 1 & \frac{-59}{173} & \frac{-218}{300} & \frac{-122}{283} \\ 0 & 1 & 0 & \frac{-171}{283} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-82}{300} & 1 \end{bmatrix} \begin{bmatrix} P_1^{(net)} \\ P_2^{(net)} \\ P_3^{(net)} \\ P_4^{(net)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P_{L3} = 300 \\ P_{L4} = 200 \end{bmatrix} \tag{26}$$

Solving the equation gives the following vector of net nodal flows $\mathbf{P}_{net}$ = [387.72 170.4 300 282]^T. This solution confirms the results obtained earlier by inspection.

The net generations are $P_{G1}^{(net)}$ = (400/400) × 387.72 = 387.72 and $P_{G2}^{(net)}$ = (114/173) × 170.4 = 112.28. Hence the transmission losses apportioned to the generators are ΔP_{G1} = 400 − 387.72 = 12.28 and ΔP_{G2} = 114 − 112.28 = 1.72.

The share of the generation used to supply each of the loads obtained from eqn. 24 is shown in the Table 4.

Table 4: Results of downstream-looking algorithm for tracing net flows

	L3	L4	Total	Loss
G1	267.36	120.36	387.72	12.28
G2	32.64	79.64	112.28	1.72
Total	300	200	500	14

The sum of the elements in each of the load columns gives, as expected, the actual demand. However, the sum of the elements in the generation rows gives totals equal to the net generations. The difference between the actual and net generation is equal to the transmission loss associated with a particular generator and is shown in the last row of the Table.

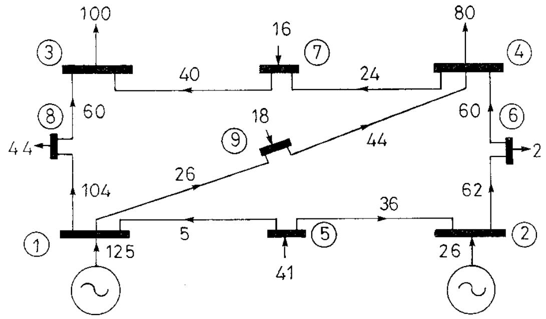

Fig. 4 *Reactive power flow with fictitious line nodes*

6 Tracing reactive power flow

As the only requirement for the proposed method is that Kirchhoff's Current Law must be obeyed, the method is equally well applicable to trace reactive power flows The main problem with reactive flows, however, is that the reactive power loss of a line may be quite considerable when compared with the flow itself. This makes it difficult to use average line flows, as in Section 3, and renders invalid assumptions used in eqns. 16 and 22 that the distribution of gross (or net) flows at any node is the same as the distribution of actual flows. To deal with this problem additional, fictitious, nodes need to be added in the middle of each line which will act as reactive power sources or sinks responsible for line generation/consumption. Fig. 4 shows the lossless reactive power flows obtained from the lossy one shown in Fig. 2. The nodes numbered from 5 to 9 are the fictitious line nodes. Nodes 5, 7, and 9 act as the reactive power sources while nodes 6 and 8 act as the reactive power sinks. Applying the downstream-looking algorithm expressed by eqn. 9 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\frac{26}{44} \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{24}{40} & 0 & 0 \\ -\frac{5}{130} & -\frac{36}{62} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{60}{104} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{40}{100} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{60}{100} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{44}{104} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Q_{L3}=100 \\ Q_{L4}=80 \\ 0 \\ Q_{L6}=2 \\ 0 \\ Q_{L8}=44 \\ 0 \end{bmatrix}$$

Inverting the matrix and applying an algorithm similar to that used to obtain Table 2 gives the distribution of generated reactive power shown in Table 5.

Table 5 allows one to trace how reactive power flows all over the network. This gives additional insight into the system operation and may have a number of other useful applications as for example in pricing the reactive power. Obviously, application of the algorithm to a large network would be cumbersome as a large matrix, of the order (n + m), needs to be inverted. However, as the matrix is highly sparse and not all elements of its inversion are needed, sparse inversion methods can be used. Moreover, reactive power tends to flow locally so that it could be possible to split the network into several parts and investigate them separately.

Table 5: Distribution of reactive power

Total	Sinks				Total
	3	4	6	8	
1	63.5	19.2	0	42.3	125
2	5.8	19.4	0.8	0	26
5	10.5	27.6	1.2	1.7	41
7	16	0	0	0	16
9	4.2	13.8	0	0	18
Total	100	80	2	44	226

7 Loss allocation algorithms

Another variation of the loss allocation method presented in Sections 4 and 5 can be developed which uses explicitly line losses in its formulation and introduces a concept of nodal losses. The resulting algorithm is a generalisation of the loss allocation method applicable only to radial distribution networks and proposed in [10]. The advantage of this approach is that it gives additional insight into the electricity tracing algorithm presented in Sections 4 and 5 and it allows to modify the loss-sharing formulas. Due to lack of space this is shown for the upstream-looking formulation only; the downstream-looking expressions can be derived in a similar way.

7.1 Proportional sharing of losses

Consider again eqn. 15. The gross nodal power $P_i^{(gross)}$ is greater then P_i, that is

$$P_i^{(gross)} = P_i + \Delta P_i^{(u)} \tag{27}$$

where $\Delta P_i^{(u)}$ is an unknown upstream nodal loss. Now consider the flow $P_{i-j}^{(gross)}$ (equal to $P_{j-i}^{(gross)}$) in line j–i supplying node i. The difference between $P_{i-j}^{(gross)}$ and P_{i-j} is greater than the line loss because some of the loss incurred in other lines supplying line j–i is passed over to that line. This can be expressed as

$$|P_{i-j}^{(gross)}| = |P_{i-j}| + \Delta P_{i-j} + \Delta P_{i-j}^{(u)} \tag{28}$$

where $\Delta P_{i-j} = ||P_{j-i}| - |P_{i-j}|| = \Delta P_{j-i}$ is the transmission loss in line j–i and $\Delta P_{i-j}^{(u)} = \Delta P_{j-i}^{(u)}$ is an unknown accumulated upstream line loss passed over from other lines in the system. Now it is necessary to assume a principle on which the nodal loss in the upstream node $\Delta P_j^{(u)}$ breaks down into components $\Delta P_{j-i}^{(u)}$, where $i \in \alpha_j^{(d)}$, to be passed down to all the lines leaving node j. The only requirement which must be met is that the sum of individual components must give the nodal loss, that is $\Delta P_j^{(u)} = \Sigma_{i \in \alpha_j(d)} \Delta P_{j-i}^{(u)} + \Delta P_{Lj}$ where ΔP_{Lj} is the component loss allocated to the jth load. The simplest distribution is obtained by assuming that the passed down components $\Delta P_{j-i}^{(u)}$ and ΔP_{Lj} are carried by flows leaving node j and therefore $\Delta P_j^{(u)}$ is shared proportionally to the values of the outflows. This can be expressed as

$$\Delta P_{j-i}^{(u)} = \frac{|P_{j-i}|}{P_j} \Delta P_j^{(u)} \quad \text{and} \quad \Delta P_{Lj} = \frac{P_{Lj}}{P_j} \Delta P_j^{(u)} \tag{29}$$

Substituting the first of the above equations and eqns. 27 and 28 into eqn. 15 gives

$$P_i + \Delta P_i^{(u)} = \sum_{j \in \alpha_i^{(u)}} \left(|P_{i-j}| + \Delta P_{j-i} + \frac{|P_{j-i}|}{P_j} \Delta P_j^{(u)} \right) + P_{Gi} \tag{30}$$

As the actual nodal flow can be expressed as $P_i = \Sigma_{j \in \alpha_i(u)} |P_{i-j}| + P_{Gi}$, eqn. 30 simplifies to

$$\Delta P_i^{(u)} - \sum_{j \in \alpha_i^{(u)}} \frac{|P_{j-i}|}{P_j} \Delta P_j^{(u)} = \sum_{j \in \alpha_i^{(u)}} \Delta P_{i-j} \tag{31}$$

or $\mathbf{A}_u \Delta \mathbf{P}_{node}^{(u)} = \Delta \mathbf{P}_{line}^{(u)}$

where $\mathbf{A}_u$ is the previously defined upstream distribution matrix, $\Delta \mathbf{P}_{node}^{(u)} = \mathbf{P}_{gross} - \mathbf{P}$ is the vector of unknown upstream nodal losses and $\Delta \mathbf{P}_{line}^{(u)}$ is a vector which ith element is equal to the sum of losses in all the lines supplying directly node i. Solving eqn. 31 gives the unknown vector $\Delta \mathbf{P}_{node}^{(u)}$ and the final allocation of the total transmission loss to the individual loads is obtained from the second of eqn. 28.

Now apply this algorithm to the test system shown in Fig. 2. The $\mathbf{A}_u$ matrix has already been calculated and is given by eqn. 20. The $\Delta \mathbf{P}_{line}^{(u)}$ vector is equal to

$$\Delta \mathbf{P}_{line}^{(u)} = \begin{bmatrix} 0 \\ 60 - 59 = 1 \\ (225 - 218) + (83 - 82) = 8 \\ (115 - 112) + (173 - 171) = 5 \end{bmatrix} \tag{32}$$

Solving eqn. 31 gives the vector of nodal losses, $\Delta \mathbf{P}_{node}^{(u)} = [0\ 1\ 9.76\ 6]^T$ which matches the $\mathbf{P}_{gross}$ vector obtained in Section 4. The two loads, $L3$ and $L4$ are responsible for a part of the nodal loss, proportional to the share of the demand in the total nodal power, eqn. 29. Hence the loss apportioned to load $L3$ is equal to (300/300) × 9.76 = 9.76 while the loss apportioned to load L_4 is (200/283) × 6 = 4.24. As expected, the allocation of the loss is the same as that shown in Table 3.

7.2 Nonproportional sharing of losses

Eqn. 29 shows that the loss allocation method presented so far is based on the implicit assumption that the loss is shared proportionally between any node outflows or, in other words, that it is averaged between the consumers (or generators) proportionally to their demand (generation). As the transmission loss is proportional to the current squared, this assumption of direct proportionality may seem to be unfair. A modification of the method may be therefore obtained by assuming that the nodal loss is shared between the outflows proportionally to some power of the outflow, that is by modifying eqn. 29 to

$$\Delta P_{j-i}^{(u)} = \frac{(P_{j-i})^\gamma}{P_j^{(\gamma)}} \Delta P_j^{(u)} \quad \text{and} \quad \Delta P_{Lj}^{(u)} = \frac{(P_{Lj})^\gamma}{P_j^{(\gamma)}} \Delta P_j^{(u)} \tag{33}$$

where $P_j^{(\gamma)} = (P_{Lj})^\gamma + \Sigma_{k \in \alpha_j(d)} (P_{j-k})^\gamma$ and γ is an exponent chosen for a given loss-sharing formula. Eqn. 31 is then modified to

$$\Delta P_i^{(u)} - \sum_{j \in \alpha_i^{(u)}} \frac{(P_{j-i})^\gamma}{P_j^{(\gamma)}} \Delta P_j^{(u)} = \sum_{j \in \alpha_i^{(u)}} \Delta P_{i-j} \tag{34}$$

or $\mathbf{A}_u^{(\gamma)} \Delta \mathbf{P}_{node}^{(u)} = \Delta \mathbf{P}_{line}^{(u)}$

where the (i, j) element of matrix $\mathbf{A}_u^{(\gamma)}$ is

$$[\mathbf{A}_u^{(\gamma)}]_{ij} = \begin{cases} 1 & \text{for } i = j \\ \frac{-(P_{j-i})^\gamma}{(P_{Lj})^\gamma + \sum_{k\in\alpha_j^{(d)}} (P_{j-k})^\gamma} & \text{for } j \in \alpha_i^{(u)} \\ 0 & \text{otherwise} \end{cases}$$

For $\gamma = 2$, the nodal loss is shared proportionally to the outflows squared while choosing $\gamma = 1$ gives the proportional sharing of the nodal loss. It is obviously possible to use a compromise value of the coefficient, e.g. $\gamma = 1.5$.

Now apply this algorithm to the test system shown in Fig. 2 when $\gamma = 2$. The equation to be solved is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-3600}{67450} & 1 & 0 & 0 \\ \frac{-50625}{67450} & 0 & 1 & \frac{-6889}{46889} \\ \frac{-13225}{67450} & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta P_1^{(u)} \\ \Delta P_2^{(u)} \\ \Delta P_3^{(u)} \\ \Delta P_4^{(u)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 8 \\ 5 \end{bmatrix}$$

and the solution is $\Delta\mathbf{P}_{node}^{(u)} = [0\ 1\ 8.8815\ 6]^T$. The final allocation of the transmission loss to individual loads is

$$\Delta P_{L3} = \frac{90000}{90000} 8.88 = 8.88 \quad \text{and} \quad \Delta P_{L4} = \frac{40000}{46889} 6 = 5.12$$

and the sum is obviously 14MW. In this particular case the loss associated with $L3$ has decreased (despite higher demand at this node) because most of the power supplied to this load comes directly from generator $G1$ using line 1–3 which is used exclusively by $L3$. Only a little part of the load demand comes via the rest of the system. As load $L4$ uses the network more heavily, its share of the total loss has increased.

8 Conclusions

Continuing trend towards deregulation and unbundling of transmission services has resulted in the need to assess what is the impact of a particular generator or the load on the power system. In this paper a new method of tracing the flow of electricity in meshed electrical networks has been proposed which may be applied to both real and reactive power flows. The method is of a topological nature and works on the results of a load flow program or a state estimation program. The method results in a table, resembling a road distance table, and allows one to assess how much of the real and reactive power output from a particular station goes to a particular load. It is also possible to assess contributions of individual generators (or loads) to individual line flows.

The electricity tracing method comes in two flavours. The upstream-looking algorithm analyses nodal inflows while its dual, downstream-looking algorithm, analyses the nodal outflows. The lossless real power flow required for the method can be obtained by one of three possible ways. The simplest way is to average the line flows over the sending and receiving-end values and adjust correspondingly the nodal injections. The second approach is to consider gross flows, that is flows which would exist if no power was lost in the network, while the third approach is to consider the net flows when all the losses are removed from the network. The upstream-looking algorithm applied to the gross flows determines how the power output from each of the generators would be distributed between the loads. The downstream-looking algorithm applied to the net flows determines how the demand of each of the loads would be distributed between individual generators if the transmission losses were removed from line flows.

Application of the method to the reactive power flow necessitates the use of additional, fictitious, nodes responsible for the reactive power generation and consumption in each of the lines. This allows one to assess how the reactive power generation from all the sources of reactive power, including lines, is distributed between all the sinks of reactive power in the system. This algorithm requires inverting a sparse matrix of the rank equal to sum of the number of nodes and the number of lines in the system.

One of the possible applications of the electricity tracing method lies in the apportioning of the transmission loss to individual generators or loads in the network. This can be done by accumulating the losses as the power flows to individual loads (or from individual generators). The nodal loss is assumed to be shared between nodal outflows proportionally to the square (or any other power) of the outflows. The loss allocation does not depend on the choice of the marginal generator and always results in positive charges. This algorithm requires solving a sparse linear equation of the rank equal to the number of network nodes.

It is envisaged that the proposed method could have wide applications in the deregulated electricity supply industry. Apart from giving additional insight into how power flows in the network, it can be used to set tariffs for transmission services based on the shared, as opposed to marginal, costs. This includes charging for the transmission loss and for the actual usage of the system by a particular generator or the load. The method can also be used to assess the contribution of individual sources of reactive power in satisfying individual reactive power demands and therefore be used as a tool for reactive power pricing.

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