# TRACKING CONTROL OF A MOBILE ROBOT BASED ON TAYLOR FORMULA 

Scaglia Gustavo<br>Mut Vicente<br>Rosales Andres

Universidad Nacional de San Juan(U.N.S.J.),Argentina


#### Abstract

In this work, a strategy to calculate the control actions for a mobile robot following a pre established trajectory is proposed. For this purpose, the Taylor series development of controlled variables is used and the control action is calculated to make the system follow the reference trajectory. As main result, an easy form to implement the strategy is obtained. Simulation and experimental results are presented, showing the advantages of the control strategy proposed.


Keywords - Control system design, discrete systems, models, tracking, trajectory control, numerical methods.

## I. INTRODUCCIÓN

Trajectory tracking is one of the main problems of control systems for which, in a search for a small tracking error, various controllers have been proposed in the literature, for example, Kim (2003) proposes a receding horizon tracking control for time-varying linear systems with constraints both on the control signal and on the tracking error, based on the minimization of a functional for finite-time costs. Besides, Linear Matrix Inequalities (LMI) are used in order to synthesize the controller. In Chem et al. (1995) a controller is proposed only for linear, non-varying systems which are exponentially stable and of nonminimum phase, which needs a set of input/output data.

In Fujimoto et al. (2001), a perfect-tracking control based on multirate feedforward control for linear systems is presented. Specifically, the design is made for a SISO system which, nevertheless, can be extended for a MIMO system. In Young-Hoon (2000), the case in which the dynamic equations of the tracking error are described by a set of non-linear, time-varying, periodic differential equations, is considered.

In Shuli (2005) a mobile robot controller that is based on the error model of Kanayama (1990) is proposed. As a result, instead of just one controller, two are hended, which are used depending on whether the angular
velocity is null or not. The work of Shuli (2005) shows only simulation results.

In Scaglia et al. (2005) a control based on numeric methods is shown, where the mobile robot linear speed depends on the current and desired orientation. In Scaglia et al. (2006a) the integral trapezoidal method is used to compute the control signal. The design is based on the model of Kanayama (1990) and here, conditions are established over the tracking error so that the equations system can always have solution. In (Scaglia G., et al., 2006b) the controller is computed solving the normal equations. In (Scaglia G., et al., 2007) the system evolution is approximated by means of a linear interpolation method, this allows a better system precision. The strategies previously mentioned can be applied to other systems, as can seen in Rosales et al. (2006), where the controller design for the RTAC system (also called TORA, a classic nonlinear problem, which is interesting due to the interaction between the traslational and rotational movements) is proposed in order that the system tends to the origin from any initial position. The proposed design is based on a system approximation using numerical methods and on the control signal calculus solving a linear equations system. Simulation results are presented; these results show tha advantages of the proposed control strategy which is obtained by using a simple design procedure.

In this work, a strategy based on Taylor's development of a variable to control is presented, in which the control signal is calculated modifying some terms of this mathematic development to obtain that the system output follows the reference signal. The main advantage of this methodology is that depending on the approximation by Taylor formula considered, different control signal expressions are obtained. Here, the proposed methodology is applied to a multivariable non-linear system (considering the problem of tracking trajectory of a mobile robot). Experimental and simulation results for this case show the advantages of the proposed methodology. The main contribution of this article is that the proposed methodology is based
upon easily understandable concepts and there is no need for complex calculations to get the control signal.

The paper is organized as follows: Section 2 states the problem and presents the methodology proposed to design the controller. Section 3 describes the application of the strategy to a typical robotic system described by a multi-variable nonlinear system as the mobile robot model is. Conclusions and suggested future work are detailed in Section 4.

## II. STATEMENT OF THE PROBLEM

A non-linear kinematic model for a mobile robot will be used (see Fig. 1), which is represented by Campion (1996),

$$
\left\{\begin{array}{l}
\dot{x}=V \cos \theta  \tag{1}\\
\dot{y}=V \sin \theta \\
\dot{\theta}=W
\end{array}\right.
$$

where, $V$ is the linear velocity of the mobile robot, $W$ is the angular velocity of the mobile robot, $(x, y)$ is the Cartesian position, $\theta$ is the orientation of the mobile robot, $\{R\}$ is the inertial frame and $\left\{R_{c}\right\}$ is the frame attached to the robot. The values of $x(t), y(t), \theta(t), V(t)$ and $W(t)$ at discrete time $t=$ $n T o$, where $T o$ is the sampling period, and $n \in\{0,1,2,3, \cdots\}$ will be denoted as $x_{n}, y_{n}, \theta_{n}, V_{n}$ y $W_{n}$. Then, the aim is to find the values of $V_{n}$ and $W_{n}$ so that the mobile robot may follow a pre-established trajectory $(x d(t), y d(t))$.


Fig. 1 Geometric description of the mobile robot.
The trajectory followed by the mobile robot is described by parametric equations as shown in Eq.(2),

$$
\left\{\begin{array}{l}
x=x(t)  \tag{2}\\
y=y(t) \\
\theta=\theta(t)
\end{array}\right.
$$

If the function $y=y(t)$ can be derived until $m+1$ order, with neighborhood of $t=n T o$ included, so the following Taylor's Formula is valid:

$$
\begin{align*}
y(t) & =y(n T o)+\dot{y}(n T o)(t-n T o)+\ddot{y}(n T o) \frac{(t-n T o)^{2}}{2!}+ \\
& +\dddot{y}(n T o) \frac{(t-n T o)^{3}}{3!}+\cdots+y^{(m)}(n T o) \frac{(t-n T o)^{m}}{m!}+R m(t) \tag{3}
\end{align*}
$$

Where the complementary term $\operatorname{Rm}(t)$ is calculated as Eq. (4),

$$
\begin{equation*}
R m(t)=y^{(m+1)}[n T o+(t-n T o) \xi] \frac{(t-a)^{m+1}}{(m+1)!}, \quad 0<\xi<1 \tag{4}
\end{equation*}
$$

If the time instant $t$ is close enough to $n T o$, so the Eq. (3) can be expressed as,

$$
\begin{align*}
y(t) \approx & y(n T o)+\dot{y}(n T o)(t-n T o)+\ddot{y}(n T o) \frac{(t-n T o)^{2}}{2!}+  \tag{5}\\
& \dddot{y}(n T o) \frac{(t-n T o)^{3}}{3!}+\cdots+y^{(m)}(n T o) \frac{(t-n T o)^{m}}{m!}
\end{align*}
$$

Similar expressions can be obtained for the state variables $x(t)$ and $\theta(t)$. Eq. (5) shows the different order derived influence over the state variable in a later instant of time $t$. In this paper, the value that this derivative must have in each sample time, is calculated and therefore, the control signals, too. So that the mobile robot follows the previous established trajectory, as shown in section III.

## III. CONTROLLER DESIGN

A first order Taylor approximation of Eq. (2) will be first considered, then,

$$
\left\{\begin{array}{l}
x_{n+1} \approx x_{n}+\dot{x}_{n} T o  \tag{6}\\
y_{n+1} \approx y_{n}+\dot{y}_{n} T o \\
\theta_{n+1} \approx \theta_{n}+\dot{\theta}_{n} T o
\end{array}\right.
$$

If we desire that the mobile robot goes from its Cartesian position $\left(x_{n}, y_{n}\right)$ to $\left(x d_{n+1}, y d_{n+1}\right)$, then from Eq. (6), $\dot{x}$ and $\dot{y}$ in time $n T o$ should have the following values,

$$
\left\{\begin{array}{l}
\dot{x}_{n}=\frac{x d_{n+1}-x_{n}}{T o}  \tag{7}\\
\dot{y}_{n}=\frac{y d_{n+1}-y_{n}}{T o}
\end{array}\right.
$$

Replacing Eq. (7) in Eq. (1),

$$
\left\{\begin{array}{l}
\dot{x}_{n}=V d \cos \theta d=\frac{x d_{n+1}-x_{n}}{T o}  \tag{8}\\
\dot{y}_{n}=V d \sin \theta d=\frac{y d_{n+1}-y_{n}}{T o}
\end{array}\right.
$$

where, $V d, \theta d$ are the linear velocity and orientation angle in instant $n T o$, respectively, necessaries to make the mobile robot go from $\left(x_{n}, y_{n}\right)$ to $\left(x d_{n+1}, y d_{n+1}\right)$.

Now, from Eq.(8),

$$
\begin{gather*}
\frac{\sin \theta d}{\cos \theta d}=\tan \theta d=\frac{y d_{n+1}-y_{n}}{x d_{n+1}-x n} \Rightarrow \theta d=\operatorname{atan} \frac{y d_{n+1}-y_{n}}{x d_{n+1}-x n}(9) \\
{\left[\begin{array}{c}
\cos \theta d \\
\sin \theta d
\end{array}\right] V d=\frac{1}{T o}\left[\begin{array}{l}
x d_{n+1}-x_{n} \\
y d_{n+1}-y_{n}
\end{array}\right]} \tag{10}
\end{gather*}
$$

Eq.(10) represents a two equations one unknown ( $V d$ ) system, which optimal solution by minimal square is (Strang G. ,1980),

$$
\begin{gather*}
{\left[\begin{array}{ll}
\cos \theta d & \sin \theta d
\end{array}\right]\left[\begin{array}{c}
\cos \theta d \\
\sin \theta d
\end{array}\right] V d=\frac{1}{T o}\left[\begin{array}{ll}
\cos \theta d & \sin \theta d
\end{array}\right]\left[\begin{array}{c}
x d_{n+1}-x_{n} \\
y d_{n+1}-y_{n}
\end{array}\right]} \\
V d=\frac{x d_{n+1}-x_{n}}{T o} \cos \theta d+\frac{y d_{n+1}-y_{n}}{T o} \sin \theta d \tag{12}
\end{gather*}
$$

In this work, we propose replace $\theta_{n+1}$ in Eq. (6) for $\theta d$ given by Eq. (9) and, in this way, calculate the $\dot{\theta}$ value, this means,

$$
\begin{equation*}
\dot{\theta}=\frac{\theta d-\theta_{n}}{T o} \tag{13}
\end{equation*}
$$

From Eqs. (12) and (13) the proposed controller for mobile robot is as follows,

$$
\left\{\begin{array}{lc}
V c_{n}= & k v^{2}\left(\frac{x d_{n+1}-x_{n}}{T o} \cos \theta d+\frac{y d_{n+1}-y_{n}}{T o} \sin \theta d\right)  \tag{14}\\
W c_{n}= & k w^{2} \frac{\theta d-\theta_{n}}{T o}
\end{array}\right.
$$

where the constants $k v^{2}, k w^{2}$ allow adjusting the system behavior and they satisfy $0<k v^{2} \leq 1$, and $0<k w^{2} \leq 1$.

Simulation and experimental studies were carried out with a mobile robot PIONEER 2DX available at the Instituto de Automática (INAUT) to test the proposed controller performance. The simulation software SAPHIRA of Active Media was also used (Konoologe K, 1998). Fig. 2 shows the Pioneer 2DX and the laboratory facilities where the experiences were carried out. In the PIONEER 2DX the value of the sample time To is 0.1 sec.


Fig. 2. Pioneer 2DX mobile robot and its environment
In order to test the performance of the proposed controller, a circumference of 600 mm . radius was used as a desired trajectory, with center on the origin of the coordinate system. The starting point for the robot was the center of the circumference, and an initial orientation $\theta=0^{\circ}$. From this starting point it evolves to the desired trajectory. The reference trajectory starts at
$(600,0) \mathrm{mm}$ and is generated at constant linear and angular velocities respectively known as Vref and Wref . Fig. 19 shows the system simulation (Koonolige $\mathrm{K}, 1998$ ) on the $x-y$ plane for $k v^{2}=k w^{2}=1$ in Eq. (14) and $\operatorname{Vref}=100 \mathrm{~mm} / \mathrm{sec}$.


Fig. 3: Simulation results: Simulated and Desired Trajectory, $V r e f=100 \mathrm{~mm} / \mathrm{sec}$.

It can also be noticed from this figure that the mobile robot follows the desired trajectory but in an oscillatory manner. In order to correct this problem, the control actions can be calculated by the minimization of a quadratic index, in which not only the tracking error but also the square of state variables derivatives have been considered. Thus, the state variables variation is minimizing as well as the error between the real and desired trajectory,

$$
\begin{align*}
& J=k_{1}^{2}\left[\left(x d_{n+1}-x_{n+1}\right)^{2}+\left(y d_{n+1}-y_{n+1}\right)^{2}\right]+k_{2}^{2}\left(\dot{x}_{n}^{2}+\dot{y}_{n}^{2}\right)+  \tag{15}\\
&+k_{3}^{2}\left(\theta d_{n+1}-\theta_{n+1}\right)^{2}+k_{4}^{2} \dot{\theta}_{n} \\
& \frac{\partial J}{\partial V_{n}}=k_{1}^{2}\left[-2 T o \cos \theta_{n}\left(x d_{n+1}-x_{n}-T o V_{n} \cos \theta_{n}\right)-\right.  \tag{16}\\
&\left.-2 T o \sin \theta_{n}\left(y d_{n+1}-y_{n}-\operatorname{ToV}_{n} \sin \theta_{n}\right)\right]+k_{2}^{2} 2 V_{n}=0 \\
& \frac{\partial J}{\partial W_{n}}=-2 \operatorname{Tok}_{3}^{2}\left(\theta d_{n+1}-\theta_{n}-\operatorname{ToW}_{n}\right)+2 k_{4}^{2} W_{n}=0 \tag{17}
\end{align*}
$$

From Eqs (16) y (17),

$$
\left\{\begin{align*}
V_{n} & =\underbrace{\frac{k_{1}^{2}}{k_{1}^{2}+\frac{k_{2}^{2}}{T o^{2}}}}_{k v^{2}}\left(\frac{\Delta x}{T o} \cos \theta_{n}+\frac{\Delta y}{T o} \sin \theta_{n}\right)  \tag{18}\\
W_{n} & =\underbrace{\underbrace{k_{3}^{2}+\frac{k_{3}^{2}}{T o^{2}}}_{3}}_{k w^{2}} \frac{\Delta \theta}{T o}
\end{align*}\right.
$$

where, $\Delta x=x d_{n+1}-x_{n}, \Delta y=y d_{n+1}-y_{n}$.
If Eqs. (14) and (18) are compared, then can be seen that, to minimize the state variables variations, the values of $k v^{2}$ y $k w^{2}$ should be chosen less than one. For this reason, we propose to reduce the values $k v^{2}$ and $k w^{2}$ to values $k v^{2}=0.2$ and $k w^{2}=0.2$.

In Fig. 4 the real trajectory of the mobile robot PIONEER 2DX in the $x-y$ plane is shown. On the other
hand, Figs. 5 and 6 show the time evolution of the $x, y$ coordinates.

Fig. 4 shows the mobile robot following the reference trajectory without making undesirable oscillations. Figs. 5 and 6 show the time evolution of coordinates $x$ and $y$ of the mobile robot while navigating, respectively. From Figs. 5 and 6, it can be noted that the mobile robot reaches the desired trajectory very quickly, and it follows this trajectory with an error smaller than 10 mm . In Figs. 4, 5 and 6, it can also be noted that the robot arrives at the end of the reference trajectory and it remains in that position without oscillations.


Fig. 4: Experimental result: Real and Desired Trajectory, $V r e f=100 \mathrm{~mm} / \mathrm{sec}$.


Fig. 5: Experimental results: Evolution of $\mathrm{x}(\mathrm{t}), \operatorname{Vref}=100$ $\mathrm{mm} / \mathrm{sec}$.


Fig. 6: Experimental results: Evolution of $\mathrm{y}(\mathrm{t}), \operatorname{Vref}=100$ $\mathrm{mm} / \mathrm{sec}$.

Fig. 7 shows experimental results obtained with the mobile robot PIONEER 2DX when the navigation velocity is increased to Vref $=200 \mathrm{~mm} / \mathrm{sec}$. and Wref $=$ $19.1 \mathrm{deg} / \mathrm{sec}$. It can be seen from Fig. 7 that the mobile robot follows the desired trajectory with a maximum error of 10 mm , which is very small when compared with the distance between wheel axes $(330 \mathrm{~mm})$. Figs. 8 and 9 show the time evolution of the real linear and
angular velocities, denoted as Vreal and Wreal, respectively, of the mobile robot and the control actions. As it can be observed, the linear and angular velocities of the mobile robot stay very close to the reference values generated by the desired trajectory.


Fig.7: Experimental results: Real and Desired Trajectory, Vref $=200 \mathrm{~mm} / \mathrm{sec} .$, Wref $=19.1 \mathrm{deg} / \mathrm{sec} ., k v^{2}=0.2, k w^{2}=$ 0.2.


Fig. 8: Experimental results: Real Linear velocity and Control Action $V c_{n}, V r e f=200 \mathrm{~mm} / \mathrm{sec}$.


Fig. 9: Experimental results: Real angular velocity and $W c_{n}$, Wref $=19.1 \mathrm{deg} / \mathrm{sec}$.

If a second order Taylor approximation is used, so

$$
\left\{\begin{array}{l}
x_{n+1} \approx x_{n}+\dot{x}_{n} T o+\ddot{x}_{n} \frac{T o^{2}}{2}  \tag{19}\\
y_{n+1} \approx y_{n}+\dot{y}_{n} T o+\ddot{y}_{n} \frac{T o^{2}}{2} \\
\theta_{n+1} \approx \theta_{n}+\dot{\theta}_{n} T o+\ddot{\theta}_{n} \frac{T o^{2}}{2}
\end{array}\right.
$$

By derivation of Eq. (19), we obtain,

$$
\left\{\begin{array}{l}
\dot{x}_{n+1} \approx \dot{x}_{n}+\ddot{x}_{n} T o  \tag{20}\\
\dot{y}_{n+1} \approx \dot{y}_{n}+\ddot{y}_{n} T o \\
\dot{\theta}_{n+1} \approx \dot{\theta}_{n}+\ddot{\theta}_{n} T o
\end{array}\right.
$$

Then, an equations system can be built with two equations and two unknown for each variable $x, y$,

$$
\begin{align*}
& {\left[\begin{array}{cc}
T o & T o^{2} / 2 \\
1 & T o
\end{array}\right]\left[\begin{array}{c}
\dot{x}_{n} \\
\ddot{x}_{n}
\end{array}\right]=\left[\begin{array}{c}
x d_{n+1}-x_{n} \\
\dot{x} d_{n+1}
\end{array}\right]}  \tag{21}\\
& {\left[\begin{array}{cc}
T o & T o^{2} / 2 \\
1 & T o
\end{array}\right]\left[\begin{array}{l}
\dot{y}_{n} \\
\ddot{y}_{n}
\end{array}\right]=\left[\begin{array}{c}
y d_{n+1}-y_{n} \\
\dot{y} d_{n+1}
\end{array}\right]} \tag{22}
\end{align*}
$$

obtaining,

$$
\begin{gather*}
\dot{x}_{n}=\frac{2}{T o}\left(x d_{n+1}-x_{n}\right)-\dot{x} d_{n+1}  \tag{23}\\
\dot{y}_{n}=\frac{2}{T o}\left(y d_{n+1}-y_{n}\right)-\dot{y} d_{n+1}  \tag{24}\\
\ddot{x}_{n}=\frac{2}{T o} \dot{x} d_{n+1}-\frac{2}{T o^{2}}\left(x d_{n+1}-x_{n}\right)  \tag{25}\\
\ddot{y}_{n}=\frac{2}{T o} \dot{y} d_{n+1}-\frac{2}{T o^{2}}\left(y d_{n+1}-y_{n}\right) \tag{26}
\end{gather*}
$$

where, Eqs. (23) to (26) represent the values that first and second derivatives of $x$ and $y$ variables should have, to make the mobile robot go from $\left(x_{n}, y_{n}\right)$ to $\left(x d_{n+1}, y d_{n+1}\right)$. From Eq. (1),

$$
\begin{align*}
& \dot{x}_{n}=V_{n} \cos \theta_{n}=\frac{2}{T o}\left(x d_{n+1}-x_{n}\right)-\dot{x} d_{n+1}  \tag{27}\\
& \dot{y}_{n}=V_{n} \sin \theta_{n}=\frac{2}{T o}\left(y d_{n+1}-y_{n}\right)-\dot{y} d_{n+1} \tag{28}
\end{align*}
$$

then,

$$
\begin{gather*}
V_{n}=\sqrt{\left(\frac{2}{T o}\left(x d_{n+1}-x_{n}\right)-\dot{x} d_{n+1}\right)^{2}+\left(\frac{2}{T o}\left(y d_{n+1}-y_{n}\right)-\dot{y} d_{n+1}\right)^{2}}  \tag{29}\\
\frac{\sin \theta_{n}}{\cos \theta_{n}}=\tan \theta_{n}=\frac{\frac{2}{T o}\left(y d_{n+1}-y_{n}\right)-\dot{y} d_{n+1}}{\frac{2}{T o}\left(x d_{n+1}-x_{n}\right)-\dot{x} d_{n+1}}  \tag{30}\\
\ddot{x}_{n}=\dot{V}_{n} \cos \theta_{n}-V_{n} \dot{\theta}_{n} \sin \theta_{n}=\frac{2}{T o} \dot{x} d_{n+1}-\frac{2}{T o^{2}}\left(x d_{n+1}-x_{n}\right)(31)  \tag{31}\\
\ddot{y}_{n}=\dot{V}_{n} \sin \theta_{n}+V_{n} \dot{\theta}_{n} \cos \theta_{n}=\frac{2}{T o} \dot{y} d_{n+1}-\frac{2}{T o^{2}}\left(y d_{n+1}-y_{n}\right)(32)  \tag{32}\\
{\left[\begin{array}{c}
\cos \theta_{n} \\
\sin \theta_{n}
\end{array}\right] \dot{V}_{n}=\left[\begin{array}{l}
\frac{2}{T o} \dot{x} d_{n+1}-\frac{2}{T o}\left(x d_{n+1}-x_{n}\right)+V_{n} \dot{\theta}_{n} \sin \theta_{n} \\
\frac{2}{T o} \dot{y} d_{n+1}-\frac{2}{T o^{2}}\left(y d_{n+1}-y_{n}\right)-V_{n} \dot{\theta}_{n} \cos \theta_{n}
\end{array}\right]}
\end{gather*}
$$

$$
\begin{equation*}
\dot{V}_{n}=\left[\frac{2}{T o} \dot{x} d_{n+1}-\frac{2}{T o^{2}}\left(x d_{n+1}-x_{n}\right)+V_{n} \dot{\theta}_{n} \sin \theta_{n}\right] \cos \theta_{n}+ \tag{33}
\end{equation*}
$$

$$
\begin{equation*}
+\left[\frac{2}{T o} \dot{y} d_{n+1}-\frac{2}{T o^{2}}\left(y d_{n+1}-y_{n}\right)-V_{n} \dot{\theta}_{n} \cos \theta_{n}\right] \sin \theta_{n} \tag{34}
\end{equation*}
$$

In this way, the linear velocity variation in the interval $\left[\begin{array}{cc}n T o & (n+1) T o\end{array}\right]$ will approximately have the shape given in Fig. 10.

As it is posible to use only one control signal so that it stays constant between each sampling period, we propose to define a new control signal whose value is defined by,

$$
\begin{equation*}
V_{\text {new }}=C 1 V_{n}+(1-C 1)\left(V_{n}+\dot{V}_{n} T o\right) ; \quad 0 \leq C 1 \leq 1 \tag{35}
\end{equation*}
$$

In a similar way, for the $W_{n}$ we have,

$$
\begin{gather*}
W_{n}=\frac{2}{T o}\left(\theta d_{n+1}-\theta_{n}\right)-\dot{\theta} d_{n+1}  \tag{36}\\
\dot{W}_{n}=\frac{2}{T o} \dot{\theta} d_{n+1}-\frac{2}{T o^{2}}\left(\theta d_{n+1}-\theta_{n}\right) \tag{37}
\end{gather*}
$$

And as Eq. (35), the expression obtained is

$$
W_{\text {new }}=C 2 W_{n}+(1-C 2)\left(W_{n}+\dot{W}_{n} T o\right) ; 0 \leq C 2 \leq 1(38)
$$



Fig. 10 Interpretación grafica de la variación de $V(t)$
In Fig. $11 x-y$ plane robot trajectory is shown when a reference trajectory is a 600 mm radius circumference,


Fig. 11: Experimental results: Real and Desired Trajectory, Vref $=200 \mathrm{~mm} / \mathrm{sec} .$, Wref $=19.1 \mathrm{deg} / \mathrm{sec} ., C 1=1$, $C 2=1$.


Fig. 12: Experimental results: Real and Desired Trajectory, $\operatorname{Vref}=200 \mathrm{~mm} / \mathrm{sec}$.

If Figs. 7 and 11 are compared, it can be seen that the performance of the controllers is very similar, and to remark the advantage of the use of Eqs. (35) and (38), a trajectory of a 1200 mm side square generated by a constant linear velocity reference ( $\operatorname{Vref}=200 \mathrm{~mm} / \mathrm{sec}$.), will be used. This reference represents a very hard and demanding trajectory. In Fig. 12 the trajectory followed by the mobile robot using Eq. (18) for the controller design, is shown. On the other hand, from Fig. 13, it can be seen the trajectory followed by the mobile robot when the control action is calculate using Eqs. (35) and (38), with $C_{1}=0.5, C_{2}=0.8$. In Fig. 14, the control signals and the evolution of the linear and angular velocities depending on the time instant are shown. It can be observed how the robot linear velocity decreases when it arrives to the square corner.


Fig. 13: Experimental results: Real and Desired Trajectory, $\operatorname{Vref}=200 \mathrm{~mm} / \mathrm{sec}$., $C 1=0.5, C 2=0.8$.


Fig. 14:Experimental results: a) Real Linear velocity and Control Action Vnew, b) Real angular velocity and Wnew.

It can be observed here, that when a higher order approximation is used, a better system response is obtained, when an abrupt change in the trajectory orientation exists.

## IV. CONCLUSIONS

In this work, a strategy to find the control actions to make the system follow a pre-established reference trajectory is proposed. For this purpose, the Taylor's development of the state of the system is made. This methodology is simple, and can be applied to different kinds of systems, and it is not necessary to make complex calculus to compute the control actions. As the system approximation order using Taylor's series is greater, different characteristics of the control signal can be obtained. In this way, it can be obtained, not only the amplitude but also the first derivative (slope), second derivative and so on, of the control action in a time instant $n T o$. This can be used to improve the system behaviour when the trajectories have abrupt variations.

Future work will entail the generalization of this methodology to cases where the states cannot be measured and, consequently, observers are needed.

## ACKNOWLEDGMENT

This work was partially funded by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET - National Council for Scientific Research), Argentina.

## REFERENCES

Campion G., Bastin G., d’Andrea-Novel B. (1996), "Structural Properties and Clasification of Kinematic and Dynamic Models of Wheeled Mobile Robots", IEEE Transact. Robotics and Automation, vol. 12, No 1, pages 47-62.
Chen M. and Chiou F. (1995), "Data-Based Tracking Control for Nonminimun-Phase Systems", Proc. American Control Conference, Seatle, Washington.
Fujimoto H., Hori Y. and Kawamura A. (2001), "Perfect Tracking Control Based on Multirate Feedforward with Generalizad Sampling Periods", IEEE Transact. Industrial Electronics, vol. 48, No 3, pp. 636-644.
Kanayama Y, Kimura Y., Miyazaki F., Noguchi T. (1990), "A Stable Tracking Control Method for an Autonomous Mobile Robot", Proc. IEEE Int. Conf. Robotics and Automation, 1990, 384-389.
Kim K. (2003), "Receding horizon tracking control for constrained linear continuous time-varying systems", IEE Proc. Control Theory and Applications, vol. 150, Issue: 5, ISSN: 1350-2379. Onpage(s): 534-538.
Konolige K. (1998), SAPHIRA MANUAL VERSION 6.1.
Morari M. and Zafiriou E. (1989), Robust Process Control, Prentice Hall Englewood Cliffs, New Jersey.
Rosales A., Scaglia G., Mut V., di Sciascio V. (2006), "Controller Designed by Means of Numeric Methods for a Benchmark Problem: RTAC (Rotational Translational Actuator)". CERMA2006, IEEE Computer Society, Cuernavaca, Morelos, Mexico.
Scaglia G., Postigo J.F., Mut V., Kuchen B. (2005), "A method for paths tracking in mobile robots", XI Reunión de Trabajo en Procesamiento de la Información y Control (RPIC), 21 al 23 de septiembre de 2005.
Scaglia G., Quintero O., Mut V., Di Sciascio F. (2006a), "Paths tracking in mobile robots using a trapezoidal integration method", XX Congreso Argentino de Control Automatico - AADECA 2006,Buenos Aires - Argentina.
Scaglia G., Mut V., Quintero O., Di Sciascio F. (2006b), "Diseño de un controlador basado en métodos algebraicos: Aplicación a robótica móvil". XII LatinAmerican Congress on Automatic Control, CLCA2006, Salvador, Bahia, Brazil.
Scaglia G., Mut V., Rosales A., Quintero O., "Tracking Control of a Mobile Robot using Linear Interpolation", Proc. 3rd Inter. Conf. on Integrated Modeling and Analysis in Applied Control and Automation, IMAACA 2007, vol. 1, pp. 11-15, ISBN: 978-2-9520712-7-7
Shuli S., (2005), "Designing approach on trajectory-tracking control of mobile robot", Robotics and ComputerIntegrated Manufacturing, vol. 21, Issue 1, pages 81-85.
Strang G. (1980), Linear Algebra and Its Applications, Academic Press, New York, 1980.
Young-Hoon Kim, In-Joongha Ha. (2000), "Asymptotic state tracking in a class of nonlinear systems via learningbased inversion", IEEE Transact. Automatic Control, On page(s): 2011 - 2027, vol. 45.

