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TRADE AND PROTECTION WITH MULTI-STAGE PRODUCTION

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Abstract

This paper analyzes trade in manufactured goods that are produced via a vertical production structure with many stages, where some value is added at each to an intermediate product to yield a good-in-process ready for the next stage. We consider the stage at which a good is traded to be an economically endogenous variable, with comparative advantage determining the pattern of production specialization by stages across countries. We study how endowment changes and policy shifts move the margin of comparative advantage, which thus provides a channel for resource allocation adjustment that is additional to the usual ones of factor substitution and changes in the quantity of output.

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I. INTRODUCTION

The importance of intermediate goods in international trade is widely recognized, and many models have been constructed to analyze positive and normative issues raised by such trade. The early models of effective protection, e.g. Balassa (1965), Corden (1966), Jones (1971), Bhagwati and Srinivasan (1973), assumed a vertical two-stage production structure. Pure intermediate goods were produced using primary factors at an upstream stage, and combined with further primary factors to yield the final goods at the downstream stage. With both kinds of goods tradeable, the effect of a tariff structure on resource allocation could be studied. Some recent models use this structure for different questions. For example, Sanyal and Jones (1980) consider the pattern and effects of trade in intermediate goods alone, by assuming that all goods must undergo further processing in their ultimate destination, before being consumed. Grossman (1981) examines content protection which requires that a certain proportion of value added be domestic. A parallel strand of literature, e.g. Vanek (1963), Melvin (1969) and Warne (1971), investigates the implications of interindustry flows when all industries produce final goods that may also serve as inputs to production in other industries. The usual generalization of all of these two-stage production structures is to a complete input-output model where goods may be intermediate, final or both, as in Woodland (1977).

An alternative extension would better describe the reality of many manufacturing industries. This is a vertical structure with many stages, where some value is added at each stage to an intermediate product to yield a "good-in-process" ready for the next stage. Different stages may have different technologies or factor intensities. Considerations of comparative advantage determine the pattern of production specialization by stages across countries.

The most important new feature of this view is that the stage at which a good is traded is itself an economically endogenous variable. Endowment changes or policy shifts can move this margin of comparative advantage, thus providing a channel for resource allocation adjustment that is additional to the usual ones of factor substitution (movement along an isoquant) and changes in the quantity of output (movement between isoquants).[1/]

The practical importance of this kind of production specialization and trade has risen in recent years, as transnational firms have increasingly integrated their production on a worldwide basis. In the automobile industry, the manufacture of component parts and the assembly of the product is arranged in this way in the European market, and sometimes even more widely. In this context, the adjustment mechanism is easily illustrated. Increased content protection requirements will lead to more parts of each car being produced domestically, and not to a shift in the proportions of wholly domestic versus wholly imported cars. Other examples can be found in assembly of electronic equipment, and even of shirts being shipped to the far east to have buttons sewn on. In some cases the product, in the course of these stages, may cross international boundaries more than once.

In this paper we study trade of this kind by modelling the production process as a continuum of stages. Stages are distinguished by factor intensities, and comparative advantage determines the pattern of production and trade. Our model bears some formal similarity to the models of trade with a continuum of goods of Dornbusch, Fischer and Samuelson (1977, 1980) and

Falvey (1981), but our concern is with the effects of factor accumulation and protection on the allocation of resources to the entire sector represented by the continuum.[2/] Also, since most of the production in our continuum yields intermediate goods, demand for this output is derived demand.

The industry with a continuum of intermediate stages, on which we concentrate most of our attention, is called "manufacturing" for sake of brevity. The rest of output is aggregated into one final good which is produced directly from primary factors and called "agriculture". We consider a semi-small country, i.e. one which can affect its commodity terms of trade but not the factor prices or incomes in the rest of the world.

We find that production specialization with respect to the stages in manufacturing is complete, i.e. the country with the highest wage-rental ratio concentrates production at the most capital-intensive end of the spectrum of stages. However, as in the simple sector-specific capital model, comparative advantage is not governed by factor endowment considerations alone. Turning to effects of policies, we find that increased protection by either tariff or content requirement is successful in the very limited sense of expanding the range of processes undertaken domestically. But it runs a very real risk of failing by decreasing the quantity of output of the manufactured good by so much that resources are shifted away from this sector. This extends the earlier results for content protection of Grossman (1981) to adjustment at the margin of production stages, and is consistent with the intuition developed in the effective protection literature, that tariff schedules which incorporate duties on intermediate goods may be anti-protective for a sector as a whole. Finally, as in more familiar models, a small tariff, by improving the final good terms of trade, must raise social welfare provided non-distorting policies are available to redistribute income.

II. THE MANUFACTURING SECTOR

The stages of manufacturing are indexed by i, ranging over the interval [0, 1] . The upstream end of the range is $i = 0$, and the downstream end is $i = 1$. We assume that all goods with index $i < 1$ are pure intermediates, i.e. consumers demand only output that has completed the entire continuum of stages. The intermediate good at stage $i + di$ is produced from one unit of stage i output, and a capital-labor mix of cost $f(w, r, i)di$, where w is the wage rate and r the rental rate on capital. Thus the inputs of the good in process from an earlier stage are naturally in fixed proportions, but capital and labor can be substituted for each other in adding value. The function f has the usual properties of a unit cost function with respect to w and r; in particular the optimum labor-capital ratio is f_w/f_r .

We do not consider any time to be required in this production process. For the final good to emerge, all that is then required is that it should pass once through each stage, and the ordering of the stages is immaterial. We will find it convenient to choose the order by increasing labor intensity, so that

$$\partial(f_w/f_r)/\partial i > 0 \qquad (1)$$

Now consider two countries with factor prices (w_1, r_1) and (w_2, r_2) which have equal costs in operating the stage of production from i^* to $i^* + di$, i.e.

$$f(w_1, r_1, i^*) = f(w_2, r_2, i^*) \qquad (2)$$

Choose the numbering so that $w_1 > w_2$ and $r_1 < r_2$. As shown in Figure 1, the two countries have a common unit cost contour at i* . By (1), the cost contour map twists clockwise as i increases.

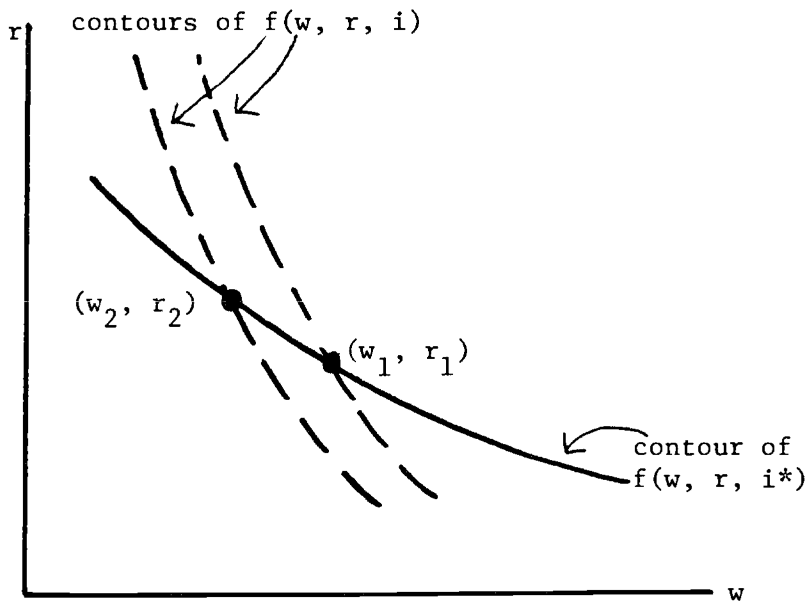

Figure 1

Then, for i > i*, country 1 must be on a higher cost contour than 2, i.e.

$$f(w_1, r_1, i) > f(w_2, r_2, i) \text{ for } i > i^* \quad (3)$$

Therefore comparative advantage for stages downstream from i* lies with country 2, and upstream from i* with 1. In other words, the high-wage country will specialize in carrying out the stages at the capital-intensive end of the range, and the low-wage country at the labor-intensive end. The marginal point of indifference i* given by (2) is of course endogenous to the full model, and it moves as endowments, tariffs etc. change factor prices in one or both countries.

The upshot is that country 1 produces all the stages up to i*, and exports the good at this stage to country 2, which completes the rest of the stages to the downstream end. Our choice of ordering precludes any reversals or multiple switches of comparative advantage. Since we are neglecting transport costs, this is a harmless simplification.

The unit cost of production, and therefore the price, of the final good can be found by summing the incremental unit costs over all stages, and the cost at each stage depends on the country undertaking that stage. Thus

$$p = \int_{o}^{i*} f(w_1, r_1, i)di + \int_{i*}^{1} f(w_2, r_2, i)di \tag{4}$$

III. EQUILIBRIUM

We now embed this model of manufacturing into a complete model of trade. The manufacturing sector and its output is denoted by x . The agricultural sector and its output is y; this good is chosen as the numeraire, and the relative price of the x-good is p . Agriculture uses inputs of labor L_y and land T; manufacturing uses labor L_x and capital K . Total labor L is mobile between the sectors, but land and capital are specific to their sectors.

We concentrate on a semi-small country with wage w and capital rental r. We assume that the factor-endowment ratios in the home country are sufficiently different from those in the rest of the world to prevent factor-price equalization. This is the only interesting case for comparative statics, since the specialization of stages obtained in Section 2 collapses to total indifference when factor prices are equal.[3/] For sake of brevity we confine ourselves to the case where w/r is less than that in the rest of the world. So this country produces all stages downstream from the marginal index i*. The value added per unit of the x-good is

$$F(w, r, i^*) \equiv \int_{i^*}^{1} f(w, r, i)\,di \tag{5}$$

We treat the rest of the world as one country, and denote variables pertaining to it by means of a bar overhead. The factor prices $\bar{w}$, $\bar{r}$ are independent of the home country's behavior, and can be suppressed where not required. The value added per unit of x in the rest of the world is

$$\bar{F}(i^*) \equiv \int_{o}^{i^*} f(\bar{w}, \bar{r}, i)\,di \tag{6}$$

We abbreviate $f(\bar{w}, \bar{r}, i)$ as $\bar{f}(i)$.

Let z denote the rental of land, and g(w, z) the unit cost function in agriculture. We assume that the home country produces something in both sectors.[4/] Then the price-cost equations are

$$g(w, z) = 1 \tag{7}$$

$$\bar{F}(i^*) + F(w, r, i^*) = p \tag{8}$$

The price partial derivatives of the unit cost functions give the optimum factor inputs per unit output. Therefore the factor-market clearing conditions are

$$y\, g_z(w, z) = T \tag{9}$$

$$x\,F_r(w,\ r,\ i^*) = K \tag{10}$$

$$y\,g_w(w,\ z) + x\,F_w(w,\ r,\ i^*) = L \tag{11}$$

Next we have the condition governing the marginal stage of manufacturing production

$$\bar{f}(i^*) = f(w,\ r,\ i^*) \tag{12}$$

Finally, we need an output-market clearing condition. The home country's national income is

$$Q = xT + rK + wL \tag{13}$$

Let its demand for the final output of the manufactured good be $D(p,\ Q)$. Let the rest of the world's demand by $\bar{D}(p)$; any other variables affecting $\bar{D}$ are outside the control of the semi-small home country and can be omitted. Then we have

$$D(p,\ Q) + \bar{D}(p) = x \tag{14}$$

In (7) - (14) we have eight equations to determine the factor prices w, z and r, the output relative price p, the output quantities x and y, national income Q, and most importantly, the marginal stage i^* .

IV. COMPARATIVE STATICS: FACTOR ACCUMULATION

The above equilibrium will be shifted by exogenous changes in K, T and L . In this section we study the effects of factor accumulation on the patterns of production and trade. Here we assume that free trade prevails; analysis of commercial policy will be taken up in the next section.

The comparative static effects of changes in factor endowments are derived by total differentiation of (7) - (14). After some substitution, we derive a matrix equation in terms of changes in three crucial variables: w, p and i* . Let a carat over a variable indicate a proportional change, e.g. $\hat{w} \equiv dw/w$. Then we find

$$\begin{bmatrix} \lambda_{Lx}\beta_x + \lambda_{Ly}\beta_y & -\lambda_{Lx}\beta_x/v & -\lambda_{Lx}(\alpha_K - \alpha_L)f/F \\ \alpha_K - 1 & -\alpha_K/v & \pi \\ \theta_{Lx}\beta_x & -(\eta + \theta_{Lx}\beta_x/v) & -\alpha_K f/F \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{p} \\ di^* \end{bmatrix} =$$

$$\begin{bmatrix} \lambda_{Lx} & \lambda_{Ly} & -1 \\ 0 & 0 & 0 \\ 1 - \gamma\eta_Q\theta_K & -\gamma\eta_Q\theta_T & -\gamma\eta_Q\theta_L \end{bmatrix} \begin{bmatrix} \hat{K} \\ \hat{T} \\ \hat{L} \end{bmatrix} \qquad (15)$$

The symbols are defined as follows. The fraction of the labor force allocated to sector n, for n = x, y, is denoted by λ_{Ln} . The distributive share of factor m, for m = K, T, L, in a relevant sector n is θ_{mn}, while that in national income is θ_m . For m = K, L, we set $\alpha_m \equiv \theta^*_{mx}/\theta_{mx}$, where θ^*_{mx} is the distributive share of factor m in the marginal manufacturing stage i*, e.g. $\theta^*_{Lx} \equiv w\, f_w(w, r, i^*)/f(w, r, i^*)$. The elasticity of factor substitution in agriculture is σ_y . That in manufacturing, in the aggregate sense holding i* constant, is σ_x, i.e. $\sigma_x \equiv F\, F_{wr}/F_w F_r$.[5] The elasticity of demand for labor in sector n is β_n, and equals $\sigma_n/(1 - \theta_{Ln})$.[6] The rate at which relative costs of manufacturing stages change between the countries as the index i rises from the marginal value i* is denoted by π, i.e. $\pi \equiv [\bar{f}_i(i^*) - f_i(w, r, i^*)]/f(w, r, i^*)$. The share of domestic value added in a unit of manufacturing output is v, i.e. $v \equiv F/p$. We define $\eta_Q \equiv QD_Q/D$, the income elasticity of home demand for manufactures, and $\gamma \equiv D/x$, the share of home consumption in their world output. Finally, the total price elasticity of world demand for manufactures multiplied by minus one is written η, which we assume to be positive.[7]

Note that $\alpha_K > 1 > \alpha_L$, since the marginal stage i* is the most capital intensive of the stages undertaken domestically, and each α is the ratio of the marginal to average factor share. Also, $\pi > 0$ because comparative cost advantage shifts to the home country as i increases.

Writing Σ for the 3-by-3 matrix on the left hand side of (15), it is now straightforward to show that

$$\det \Sigma = \lambda_{Lx}\beta_x\left(\pi\eta + \frac{f}{vF}\right) + \frac{\lambda_{Ly}\beta_y}{v}\left(\pi\eta + \pi\theta_{Lx}\beta_x + \alpha_K^2\frac{f}{F}\right)$$

$$+ \lambda_{Lx}\,\eta\,\frac{f}{F}(\alpha_K - \alpha_K)(\alpha_K - 1) \qquad (16)$$

$$> 0$$

The key economic variable of interest in this model is the marginal stage of processing, i^*. Therefore we focus our comparative static analysis on its response to factor endowment changes. Consider first the effect of an increase in the capital stock. By application of Cramer's rule to (15), we have

$$K\frac{\partial i^*}{\partial K} = \frac{-1}{\det \Sigma}\left\{\eta\lambda_{Lx}(\alpha_K - 1) + \frac{\lambda_{Lx}\sigma_x}{v}(1 - \mu_x v) + \frac{\alpha_K\lambda_{Ly}\beta_y}{v}(1 - \mu_x\theta_{Kx}v)\right\} \tag{17}$$

where μ_x is the marginal propensity to consume manufactures at home. The expression in (17) is unambiguously negative if the agricultural good is not inferior, i.e. $\mu_x < 1$. At this level of aggregation, it makes sense to assume that both goods are normal, and we do so henceforth.

This result accords with intuition. A rise in the capital stock, by increasing the wage-rental ratio, allows the home country to compete in more capital-intensive activities, and thereby expands the range of processes in which it specializes. In other words, our labor-rich country will import intermediate goods at an earlier, more capital-intensive, stage as it accumulates capital, regardless of whether any of the production technologies admit factor substitution.

It is not necessarily true, though, that capital accumulation will cause mobile resources to reallocate to the manufacturing sector, even as the range of processes performed at home expands. Differentiating $L_y/T = g_w/g_z$ and using the definition of β_y, we have $\hat{L}_y = -\beta_y\hat{w} + \hat{T}$. Therefore the direction of flow of labor is revealed by the response of w, which is given by

$$\frac{K}{w}\frac{\partial w}{\partial K} = \frac{\lambda_{Lx}}{\det \Sigma}\left\{\frac{\alpha_K f}{vF}\left[\alpha_L(1 - \mu_x v) + \mu_x v\right] + \pi\left[\eta - \frac{\sigma_x}{v}(1 - \mu_x v)\right]\right\} \tag{18}$$

If η, the total price elasticity of world demand for manufactures, is small, the second term in the brackets will be negative and the agricultural sector may expand. This effect is more likely the smaller is μ_x, the home marginal propensity to consume manufactures, and the larger is σ_x, the elasticity of factor substitution in manufacturing. The most likely outcome, however, is for capital accumulation to cause an increase in the net imports of agricultural goods,[8/] and an increase in the domestic value added per unit of manufactured output.

We next consider the effects of an increase in the endowment of land. The response of the marginal manufacturing stage to such growth is ambiguous. The comparative static derivative is

$$T\frac{\partial i^*}{\partial T} = \frac{1}{\det \Sigma}\left\{\mu_x\frac{\theta_T}{\theta_x}(\lambda_{Lx}\beta_x + \lambda_{Ly}\beta_y\alpha_K) + \lambda_{Ly}\left[\theta_{Lx}\beta_x/v - \eta(\alpha_K - 1)\right]\right\} \tag{19}$$

where θ_x is the share of sector x in national product. An important case to consider, based on empirical evidence for several less developed countries, is one in which factor substitution possibilities at any given stage are limited, but processes vary greatly in their relative intensities of factor usage.[9/] With σ_x and σ_y small, and $(\alpha_K - 1)$ large, (18) is negative. For these parameter values, accumulation of the specific factor in agriculture has the same effect as that in manufacturing, namely an expansion of the range of stages produced by the low-wage home country. In fact, with fixed-coefficient technologies in both sectors, we have

$$T\frac{\partial i^*}{\partial T} = - \frac{\lambda_{Ly}F}{\lambda_{Lx}f(\alpha_K - \alpha_L)}$$

which is independent of demand conditions and depends only on the relative sizes of the sectors and the factor intensity differences between the marginal and average processes. Alternatively, when η is small, an increase in the endowment of land is seen to cause the home country to specialize in a smaller number of stages.

The intuition for these results is as follows. An increase in the endowment of land, ceteris paribus, increases the demand for labor in agriculture. This pushes up the wage rate, and thus the wage-rental ratio in manufacturing. This alone would tend to expand the range of manufacturing processes undertaken domestically. But the additional income that accrues to the new land is partially spent on manufactures, raising their price and therefore the demand for capital. This effect is large when η is small, i.e. when world demand for manufactures is price inelastic. It in turn pushes up the rental rate on capital. Then the equilibrium wage-rental ratio can actually fall, and the marginal index i* can rise. As a further point of interest, we note that the reallocation of labor in response to an increase in the endowment of land is also ambiguous. Substituting once again into $\hat{L}_y = \hat{T} - \beta_y \hat{w}$ gives

$$\frac{T}{L_y}\frac{\partial L_y}{\partial T} = \lambda_{Lx}\left\{\beta_x(\pi\eta + \frac{f}{vF}) + \frac{f}{F}\eta(\alpha_K - \alpha_L)(\alpha_K - 1) - \mu_x\beta_y\frac{\theta_T}{\theta_x}[\pi\beta_x + \frac{f}{F}\alpha_K(\alpha_K - \alpha_L)]\right\} \qquad (21)$$

which is more likely to be positive the larger the price elasticity of world demand, the smaller the home marginal propensity to consume manufactures, and the smaller the elasticity of substitution between capital and labor in agriculture.

An increase in the labor supply also has an ambiguous effect on the margin i*. Applying Cramer's rule once again, we have

$$L\frac{\partial i^*}{\partial L} = \frac{1}{\det \Sigma}\{\alpha_K \mu_x \beta_y \lambda_{Ly}\theta_L/\theta_x$$

$$+ \eta(\alpha_K - 1) - \beta_x \theta_{Lx}(1/v - \mu_x)\} \qquad (22)$$

A small elasticity of substitution between land and labor in agriculture contributes to an expansion in the range of home processes, while a small elasticity of substitution between capital and labor in manufacturing has the opposite effect. If both are small the marginal domestic stage is necessarily more labor-intensive. Note that the range of home processing is more likely to increase in response to an increase in labor endowment the smaller the price elasticity of world demand for manufactures, the smaller the home marginal propensity to spend on manufactures, and the smaller the factor intensity differences across manufacturing stages. The immediate effect of an increase in labor supply is a fall in the wage rate. However, if the demand for capital falls greatly, as will be the case if the increase in national income is spent on agricultural goods and if the price elasticity of demand for manufactures is small, any fall in the wage-rental ratio in manufacturing will be small. Since $\pi di^* = -\theta_{Kx}(\alpha_K - 1)(\hat{w} - \hat{r}) + \hat{p}/v$, a small decrease in w/r is consistent with a decrease in i* so long as $(\alpha_K - 1)$ is small, and p falls.

To sum up, we have shown that factor endowment changes affect the patterns of production and trade with multi-stage processing in a complex way. There is some presumption, especially if elasticities of factor substitution are small and the price elasticity of world demand for manufactures is large, that an exogenous increase in the supply of either specific factor will cause the range of domestic manufacturing processes to expand, whereas an increase in the supply of the mobile factor will have the opposite effect. But since resources may be allocated to, or withdrawn from, the manufacturing sector on either of two margins, one intensive and one extensive, the complexities and ambiguities of general equilibrium comparative statics are unavoidable. Effects from the demand side can easily overturn the intuitive supply-side impacts, unlike the simple two-sector models.

V. COMPARATIVE STATICS: TARIFFS AND CONTENT PROTECTION

In this section we study the effects of two forms of protection. First, we consider a uniform tariff at an ad valorem rate t on imports of all manufactured goods, starting from an initial level of t = 0.[10] Secondly, we investigate a content protection scheme which requires that v, the fraction of value added domestically, be raised by a small amount starting from that which obtains in a free-trade equilibrium. Such an increase is mandated by the government, and is enforced by a threat of some economic sanction for noncompliance.

When a uniform tariff schedule is in place, the equilibrium conditions must be modified slightly. We have to replace (8) by

$$\bar{F}(i^*)(1 + t) + F(w, r, i^*) = p \tag{8'}$$

and (12) by

$$\bar{f}(i^*)(1 + t) = f(w, r, i^*) \tag{12'}$$

to include the tariff costs faced by private producers. Further, we must include tariff revenues in national income, so (13) is replaced by

$$Q = xT + rK + wL + t\bar{F}(i^*)x \tag{13'}$$

Taking total differentials of the new system and evaluating at t = 0 we have the comparative statics of the small tariff in the implicit form

$$\Sigma \begin{bmatrix} \hat{w} \\ \hat{p} \\ di^* \end{bmatrix} = \begin{bmatrix} -\lambda_{Lx}\beta_x(1 - v)/v \\ -[1 + \alpha_K(1 - v)/v] \\ -\theta_{Lx}\beta_x(1 - v)/v \end{bmatrix} dt \tag{23}$$

Does a uniform tariff protect the manufacturing sector? The answer depends on the desired sense of 'protection'. Consider first the effect on the range of processes undertaken at home, given by

$$\frac{di^*}{dt} = \frac{-1}{\det \Sigma}\left\{\eta\lambda_{Lx}\beta_x/v + \eta\lambda_{Ly}\beta_y[1 + \alpha_K(1 - v)/v] + \lambda_{Ly}\beta_y\theta_{Lx}\beta_x/v\right\} \tag{24}$$

$$< 0$$

Therefore a tariff is unambiguously protective in the limited sense of expanding the range of home activity to include more capitale-intensive stages of processing. However, perhaps a better measure of protection for the manufacturing sector as a whole is in terms of the reallocation of labor. By this measure a tariff runs the real risk of being anti-protective, because it raises the cost of intermediates and thereby lowers the effective protection for the downstream stages. Thus a uniform tariff fails to protect whenever the fall in manufacturing output it causes results in the release of more labor than is required to perform the new activities at the extensive margin. We use the solution to (23) in $\hat{L}_y = -\beta_y \hat{w}$ to write

$$\frac{1}{L_y}\frac{dL_y}{dt} = \frac{\beta_y}{\det \Sigma}\left\{\lambda_{Lx}\eta\frac{f}{F}(\alpha_K - \alpha_L)(1 + \alpha_K\frac{1-v}{v}) + \frac{\lambda_{Lx}\beta_x}{v}\left[(1-v)\pi\eta - \frac{f}{F}\right]\right\} \quad (25)$$

For small σ_x this is positive, and a tariff is necessarily anti-protective in the resource allocation sense. It is interesting to note that whenever (25) is positive, dw/dt is negative. Since

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{\det \Sigma}\left\{\pi\lambda_{Ly}\beta_y\theta_{Lx}\beta_x\frac{1-v}{v} + \frac{f}{F}\alpha_K\lambda_{Ly}\beta_y\left(1 + \alpha_K\frac{1-v}{v}\right)\right.$$

$$\left. + \frac{\lambda_{Lx}\beta_x}{v}\frac{f}{F}\right\} > 0 \quad (26)$$

a tariff may well reduce the real reward for the mobile factor in terms of both final goods, a possibility that does not exist in the standard two-good sector-specific-factor model.

A final question concerns the aggregate welfare effects of a small tariff. If the home demand functions result from maximization of a Bergson-Samuelson social welfare function, we can write the change in social welfare as

$$dU/\lambda = dQ - D\,dp \quad (27)$$

where U is social welfare, and λ is its pure income derivative. Now from (13')

$$dQ = Tdz + Ldw + Kdr + \bar{F}(i^*)xdt$$

on evaluation at $t = 0$. Note that $L = L_x + L_y$, and (7) gives $Tdz + L_y dw = 0$, while from (8) we have $L_y dw + Kdr = x(dp - \bar{F}(i^*)dt)$. With the aid of these, (27) becomes

$$dU/\lambda = (x - D)dp \tag{28}$$

which is just the terms-of-trade effect on the final manufactures exports. It is perhaps surprising that the expression in (28) is so simple. It is understood by recognizing that the extra cost of inputs to domestic producers is exactly offset by the tariff revenue that is raised, and since at free trade the costs of undertaking the marginal stage are equalized, the distortion created by a small tariff is a second-order effect.

We have seen in (26) that dp/dt is positive. Thus our semi-small country benefits by using a small tariff to alter the terms of trade in its favor. The anti-protective effect in resource allocation is only a distributive loss, not an aggregate one. Provided the government can redistribute income optimally at home, a small tariff provides protection in the economically relevant sense after all.

Finally, we consider an alternative form of protection that is being increasingly used by governments. This is a content requirement for multi-stage industries of the sort modelled in this paper. Under such a policy regime domestic firms are required to achieve a specific percentage of domestic value added in their final product. Economic penalties are imposed for failure to comply. An important outgrowth of many content requirement schemes has been an expansion in the range of intermediates processes in which the domestic firms specialize, rather than an increase in the domestic share of traded intermediates. This aspect of content protection is not captured in previous analyses, e.g. Corden (1971) or Grossman (1981), which are couched in terms of standard two-stage production with all goods traded and complete non-specialization. The prominence given to specialization of production by stages of processing in the model developed here makes it particularly appropriate for the study of content protection.

A single modification of the equilibrium conditions in Section III is required under a content protection scheme. The costs at stage i* need no longer be equal in the two countries; instead, the margin must adjust to fulfill the content requirement. Thus we replace (12) by

$$F(w, r, i^*) = vp \tag{12''}$$

where v is now an exogenous policy variable. In other words, the government dictates that the share of domestic in total value added be greater than occurs in free trade, and enforces the policy by unspecified penalties that are assumed sufficient to induce compliance. The relative domestic and foreign unit costs at the marginal stage are now determined endogenously. We differentiate the equilibrium conditions totally as v changes, and evaluate the derivatives at an initial free-trade equilibrium, where of course the two unit costs happen to be equal. After the usual substitutions, we are left with the matrix equation

$$\begin{bmatrix} \lambda_{Lx}\beta_x + \lambda_{Ly}\beta_y & -\lambda_{Lx}\beta_x/v & -\lambda_{Lx}(\alpha_K - \alpha_L)f/F \\ 0 & -(1 - v)/v & f/F \\ -\theta_{Lx}\beta_x & -(\eta + \theta_{Lx}\beta_x/v) & -\alpha_K f/F \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{p} \\ di^* \end{bmatrix} = \begin{bmatrix} 0 \\ -\hat{v} \\ 0 \end{bmatrix} \quad (29)$$

Let Σ'' be the 3-by-3 matrix on the left hand side. Then

$$\det \Sigma'' = \frac{f}{F}\left\{\lambda_{Lx}\beta_x\left(\eta + \frac{1 - v}{v}\right) + \lambda_{Ly}\beta_y\eta + \frac{\lambda_{Lx}\beta_x + (1 - v)\alpha_K}{v}\right\} > 0 .$$

Content protection, like a tariff, unambiguously expands the range of domestic manufacturing activities. The comparative static derivative of the marginal stage is

$$v\frac{di^*}{dv} = \frac{-1}{\det \Sigma''}\left\{\lambda_{Lx}\beta_x\eta + \lambda_{Ly}\beta_y\left(\eta + \theta_{Lx}\beta_x/v\right)\right\} \quad (30)$$

But, again like a uniform tariff, and for much the same reasons, content protection can be anti-protective for the sector as a whole.[11] The direction of reallocation of labor is found from $\hat{L}_y = -\beta_y\hat{w}$ and

$$\frac{v}{w}\frac{dw}{dv} = -\frac{\lambda_{Lx}}{\det \Sigma''}\frac{f}{F}\left\{\eta\,(\alpha_K - \alpha_L) - \beta_x/v\right\} \quad (31)$$

Therefore content protection will cause a move of resources away from manufacturing in exactly those cases which may be of empirical relevance, namely when the elasticity of substitution between capital and labor in manufacutring is small and the differences in factor intensities across stages are large. Also,

$$\frac{v}{p}\frac{dp}{dv} = \frac{1}{\det\ \Sigma''}\frac{f}{F}(\lambda_{Lx}\beta_x + \lambda_{Ly}\beta_y\alpha_k) > 0 \tag{32}$$

so the real return to labor is again reduced in terms of both goods in these cases. However, the aggregate welfare again increases since the commodity terms of trade improve.

VI. CONCLUDING COMMENTS

In this paper we have constructed a model of trade in intermediate goods that is very different from, and in some ways complementary to, the usual approaches. We have avoided the circularity of input-output models by stipulating a unidirectional sequence of production stages. At each stage the good in process combines in naturally fixed proportions with a labor-capital composite, thus removing the substitution between intermediate and primary inputs which was a prominent feature of neoclassical models of effective protection. In return, we are able to capture some complex realities of manufacturing processes. We are able to determine production specialization endogenously, and determine which intermediate goods will be traded. Commercial policy works by shifting the stage at which a country will import a good in process: a channel which corresponds to factual observations in many industries. Well-known 'paradoxes' of effective protection have their counterparts in this setting; in addition there are more subtle ways in which attempts at protection may fail under empirically likely circumstances.

The model is readily adapted to deal with policies like establishment of duty-free industrial zones. Conversion of the semi-small country model into a full two-country model presents only algebraic difficulties, and allows discussion of issues like direct foreign investment. These are topics for future research.

FOOTNOTES

1. Empirical studies of the choice of techniques in less developed countries by Stewart (1972) and Pack (1976) have found that although factor substitution possibilities may be quite limited for a given production process, differences in factor intensities across processes can greatly magnify the total opportunity for substitution between capital and labor in a given sector. We find this potential magnification to be a prominent feature in the comparative statics of our model.

2. In particular, Falvey (1981) presents a model where production is most similar in structure to ours. Falvey assumes, as we do, that production in the continuum sector requires a sector-specific factor, but differs from us in assuming fixed proportions between this factor and the mobile factor. He uses this model to study factor-endowment motivated intra-industry trade, and thus assumes that all output is for final demand.

3. An alternative route would be to assume that our ordering of stages by labor intensities coincides literally with the sequencing of production, and that a small amount of transport costs ensures that factor prices are generically different across countries.

4. This will be the case as long as the endowments of all factors are positive, and the production functions in each sector satisfy the Inada conditions.

5. The aggregate σ_x is the logarithmic derivative of the cost-minimizing capital-labor ratio in the manufacturing sector as a whole with respect to the wage-rental ratio, for a given range of stages performed domestically. It is related to the elasticities of substitution between capital and labor at the individual stages by a complicated expression that is exactly analogous to the relationship between the elasticities of substitution in the individual sectors of a two-sector model and the aggregate elasticity of substitution in supply, as derived in Jones (1965, p. 563).

6. See Mussa (1974).

7. We have $\eta \equiv -[\gamma\eta_p + (1-\gamma)\bar{\eta}_p - \mu_x] = -[\gamma\eta_p^c + (1+\gamma)\bar{\eta}_p^c - (1+\gamma)\mu_x]$ where $\eta_p \bar{\eta}_p$ and η_p^c, $\bar{\eta}_p^c$ are respectively the uncompensated and compensated home and foreign price elasticities, and μ_x the home marginal propensity to consume manufactures. Therefore, if manufactures are a normal good at home, η is necessarily positive.

8. These net imports may be positive or negative.

9. See footnote 1 above.

10. In principle there could be a tariff schedule t(i) varying with the index i . This may result in the goods in process crossing international boundaries several times, and may also break the pattern of complete specialization by stages. Further, since our ordering of the index i does not

necessarily correspond to the physical sequencing of stages, even with a uniform tariff there may be several boundary crossings of intermediate goods. In these cases the tariff is understood to be on a value-added basis to avoid cascading, e.g. as in the application of offshore assembly provisions.

11. Grossman (1981) found a similar result in a partial equilibrium model of content protection in which all adjustment occurs through an increase in the ratio of domestically produced goods to imported intermediate goods of a given kind.

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