

## Traffic Behavior in a Two-lane System consisting of a Mixture of Taxies and Cars

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### Abstract

In this paper, we study a two-lane traffic system consisting of a mixture of cars and taxies. The taxies occasionally stop to pick up or drop off passengers. The empty taxi rate, getting on rate (defined as average number of passengers that get on taxi in unit time), fundamental diagram, passenger carry capability, and lane-changing frequency are investigated under different sets of parameters. Some insights are obtained and explained. For example, (i) the flow rate still decreases even if the taxies stop less frequently; (ii) getting on rate does not monotonically increase with the pick-up rate. The diagram of the maximum value of getting on rate is also presented, so that an optimum taxi number could be suggested. We expect our works could be useful for taxi administration and traffic management.

**Key Words:** empty taxi rate; getting on rate; fundamental diagram; passenger carry capability; lane-changing frequency

### Introduction

Recently, traffic congestion has become a serious problem all over the world and has resulted in increases in economic expenses and environment pollutions. Therefore, it has been an urgent problem to understand complex traffic behavior and then to guide the traffic planning, designing, controlling and management.

To simulate various complex behavior of traffic flow, several kinds of models have been proposed, such as hydrodynamic models, car-following models, cellular automaton (CA) models and gas-kinetic models (Chowdhury *et al.*, 2000; Helbing, 2001; Hoogendoorn *et al.*, 2005; Kerner, 2004; Maerivoet *et al.*, 2005; Nagatani, 2002; Nagel *et al.*, 2003). Recently, CA models have been shown to be able to emulate much of the behavior of observed traffic flow, therefore, using CA has

become a well-established method to model, analyze, understand, and even forecast the behavior of real road traffic. One of the most well-known CA model is NaSch model proposed in 1992 (Nagel *et al.*, 1992). Although it is very simple, it can reproduce some real traffic phenomena, such as spontaneous appearance of jams and start-stop waves.

Because the roadway usually has two or more lanes in real traffic, the extension of the single lane traffic flow model to two-lane or multi-lane situations is very important and has been widely studied (Chowdhury *et al.*, 1997; Knospe *et al.*, 1999, 2002; Nagatani, 1996; Nagel *et al.*, 1998; Rickert *et al.*, 1996; Wagner *et al.*, 1997). On the other hand, taxis play an important role in city traffic. Nevertheless, to our knowledge, the two-lane traffic flow considering a mixture with taxis has not been studied.

In this paper, we study a two-lane traffic system consisting of a mixture of cars and taxis. The taxis occasionally stop to pick up or drop off passengers. Extensive simulations have been carried out. Traffic behaviors including empty taxi rate, getting on rate (defined as average number of passengers that get on taxi in unit time), fundamental diagram, passenger carry capability, and lane-changing frequency are investigated under different sets of parameters.

This paper is organized as follow. In the next section, the NaSch model and lane-changing rules are introduced. The simulation results are presented and discussed in Section III. The conclusion is given in section IV.

## Model

This section briefly introduces the model of the heterogeneous two-lane system. We assume that the system consists of only two kinds of vehicles: cars and taxis. Both cars and taxis move according to NaSch model. For simplicity, we suppose taxis only drive on the right lane while cars can change lane. Although we believe traffic behaviors will not change qualitatively, a further study in which taxis also could change lane needs to be carried out in future work.

Next we briefly recall the definition of NaSch model. In NaSch model, the position, speed, acceleration as well as time are treated as discrete variables. The speed  $v$  of each vehicle can take one of the  $V_{\max}+1$  allowed integer values  $v=0, 1, 2, \dots, V_{\max}$ . A lane is represented by a one-dimensional lattice, and each of the lattice sites represents a "cell" which can be either empty or occupied by at most one "vehicle" at a given instant of time. At each time step  $t \rightarrow t+1$ , the parallel updating rules are as follows:

(1) Acceleration:

$$v_n(t+1) = \min(v_n(t)+1, V_{\max})$$

(2) Braking:

$$v_n(t+1) = \min(v_n(t+1), d_n(t))$$

(3) Random deceleration with probability  $p$ :

$$v_n(t+1) = \max(v_n(t+1)-1, 0)$$

(4) Vehicle motion:

$$x_n(t+1) = x_n(t) + v_n(t+1)$$

Here,  $x_n(t)$ ,  $v_n(t)$  denotes the position and velocity of vehicle  $n$  at time  $t$ .  $d_n(t) = x_{n+1}(t) - x_n(t) - l$  is the gap between vehicle  $n$  and its preceding vehicle  $n+1$ , where  $l$  is the length of the vehicle.  $V_{max}$  is the maximum velocity and  $p$  is the randomization parameter.

Step 1 reflects the general tendency of the drivers to drive as fast as possible, if allowed to do so, without crossing the maximum speed limit. Step 2 is intended to avoid collision between the vehicles. The randomization in step 3 takes into account the different behavioral patterns of the individual drivers, especially, nondeterministic acceleration as well as overreaction while slowing down, as well as stochastic disturbances.

The taxis pick up and drop off passengers as follows. If a taxi is empty, it will pick up passengers with probability  $P_{on}$ . It will take  $T_{st}$  for a taxi to stop to pick up passengers. For simplicity, the traveling distance of the passengers is supposed to be randomly distributed between  $L_1$  and  $L_2$ . When passengers arrive at the destination, the taxi will stop for another  $T_{st}$  to drop them off.

Now we introduce lane-changing rules for the cars. Here we consider symmetric lane-changing rules. The details of lane-changing rules are defined as follows. When

- Incentive criterion:

$$d_n < \min(v_n + 1, V_{max})$$

$$d_{n,other} > d_n$$

- Safety condition:

$$d_{n,other,back} > d_{safety}$$

are satisfied, a car will change lane with probability  $P_{changelane}$ . Here, the subscript 'other' denotes the term defined on the other lane and 'back' denotes the term behind the car. In the rules, incentive criterion (1) means the car on the current lane is hindered, incentive criterion (2) means the car can move faster on the other lane. Safety condition means it is safe to change lane.

In simulations, the cars and taxis are assumed to drive on a two-lane circular road. The length of each lane is set to 2000 cells. Each cell corresponds to 7.5 m on a real road, and a vehicle occupies 1 cell. So the road corresponds to 15 km in real traffic. One time step in the model corresponds to 1 second in reality. The maximum velocities of both cars and taxis are set to 3 cells/s, which correspond to 81 km/h in reality. The parameters for the taxis are  $N_{taxi}=100$ ,  $P_{on}=0.01$ ,  $T_{st}=40s$ ,  $L_1=300$  cells (2.25 km) and  $L_2=1500$  cells (11.25 km) unless otherwise mentioned. The lane-changing probability  $P_{changelane}$  is set to 1 for simplicity.

In this section, we present and analyze the simulation results. Firstly, empty rate of taxis is investigated. Figure 1(a) shows the rate under different values of  $P_{on}$ . It can be seen that the rate decreases with the increase of  $P_{on}$ . When  $P_{on} \geq 0.1$ , there are only few empty taxis in the system. Moreover, the rate essentially remains a constant at small density and it decreases with the increase of density at large density. This is due to at large density, average velocity of traffic flow decreases with increase of density. Thus, average occupation time of taxis increases.

Simulation Results

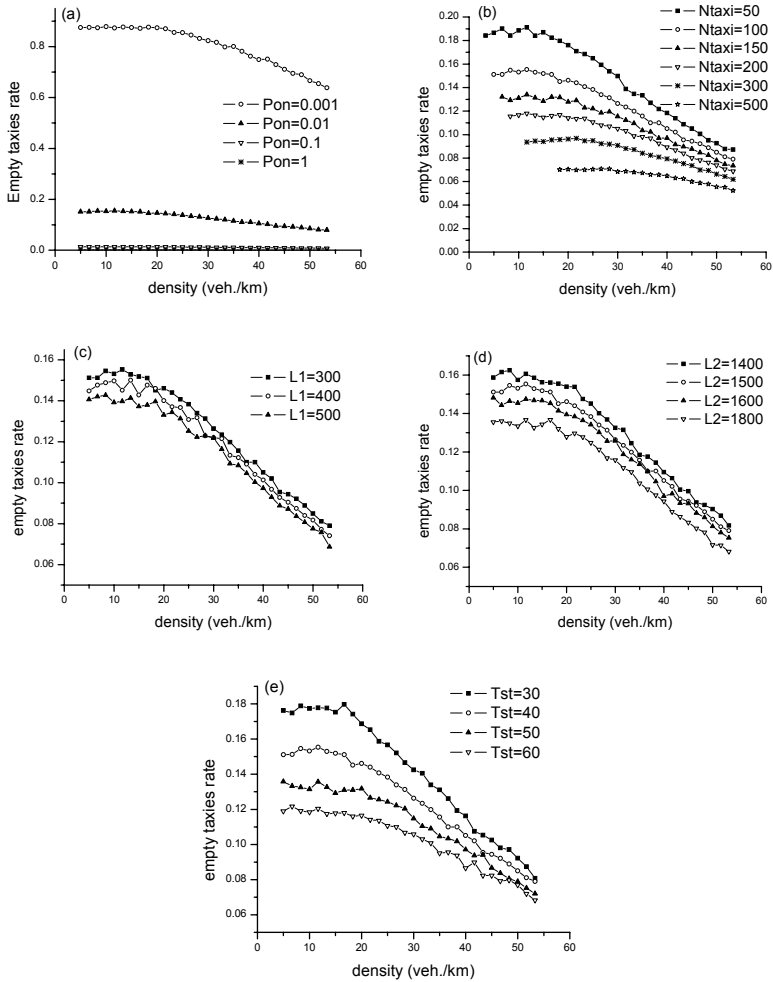


Figure 1. Empty taxi rate under different parameters (a)  $P_{on}$  (b)  $N_{taxi}$  (c)  $L_1$  (d)  $L_2$  (e)  $T_{st}$

Figure 1(b) shows empty taxi rate under different values of  $N_{taxi}$ . One can see that at a given density, the more taxis in the system, the smaller the rate is, provided  $P_{on}$  does not change. This is because average velocity of traffic flow decreases with the increase of taxi number.

Empty taxi rate decreases with the increase of  $L_1$  and  $L_2$ , as shown in Figure 1(c)(d). It is because average occupation time of taxis increases with the increase of average travel distance of passengers. The rate also decreases with the increase of  $T_{st}$ , as shown in Figure 1(e). It is because on the one hand, average occupation time of taxis increases (it is assumed a taxi is occupied in the pick up and drop off process), on the other hand, average velocity of traffic flow will decrease with the increase of  $T_{st}$ .

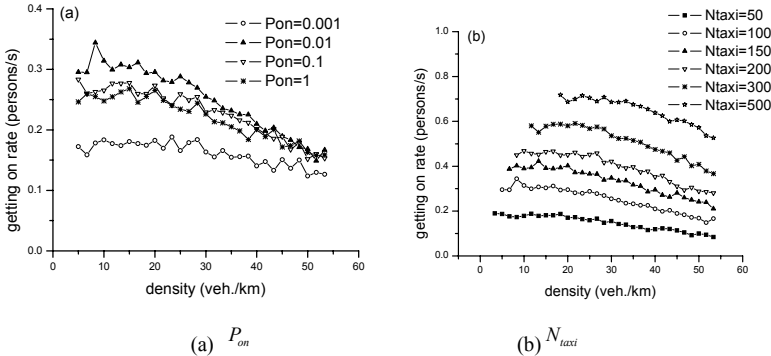


Figure 2. Getting on rate  $G$  under different parameters

Secondly, we study getting on rate  $G$ , which is defined as average number of passengers that get on taxi in unit time. Clearly the rate is a product of taxi number, empty taxi rate,  $P_{on}$ , and average number of passengers to take a taxi, which is assumed to be two in this paper. Figure 2(a) shows  $G$  under different values of  $P_{on}$ . One can see that it first increases and then decreases with the increase of  $P_{on}$ . This is because empty taxi rate decreases with the increase of  $P_{on}$ . As a product, maximum  $G$  could be achieved at an intermediate value of  $P_{on}$ . This is unexpected because large  $P_{on}$  means more passengers want to take taxi and large  $G$  is equivalent to more income of taxi driver. The unexpected result might be related to pick-up and drop-off process of taxis. The time interval between drop-off and pick-up becomes smaller with the increase of  $P_{on}$ . In the limit case  $P_{on}=1$ , the interval is zero, which means a taxi will stop  $2T_{st}$  one time. As a result, the traffic jam induced by stopped taxis is much more difficult to dissolve, which lowers down average velocity of traffic flow. The decrease of average velocity decreases empty taxi rate and accordingly decreases  $G$ .

Figure 2(b) shows  $G$  under different values of  $N_{taxi}$ . It can be seen that as a product,  $G$  increases with the increase of  $N_{taxi}$ , despite empty taxi rate decreases with the increase of  $N_{taxi}$ . This means the decrease rate of empty taxi rate is smaller than increase rate of  $N_{taxi}$ .

The dependence of  $G$  on  $L_1$ ,  $L_2$  and  $T_{st}$  is similar to that of empty taxi rate shown in Figure 1(c)-(e), because taxi number and  $P_{on}$  are constants in these results.

We study the dependence of maximum  $G$  on taxi number and car number, which is

shown in Figure 3. One can see with a fixed  $N_{car}$ , the maximum value first increases and then decreases with the increase of  $N_{taxi}$ . Thus, an optimum  $N_{taxi}$  could be chosen. If we fix  $N_{taxi}$ , the maximum value decreases with the increase of  $N_{car}$ .

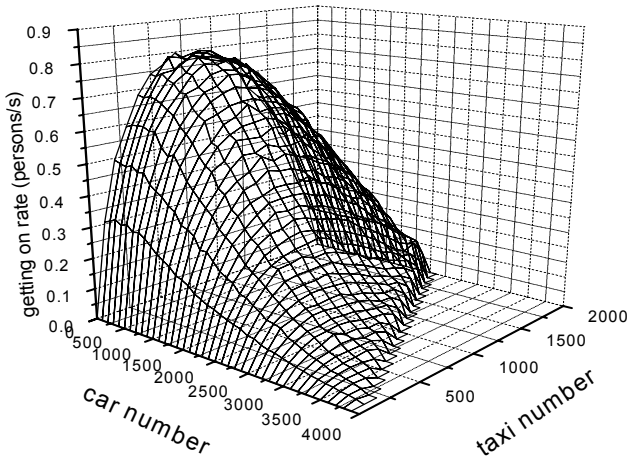


Figure 3. The maximum value of getting on rate  $G$  under different sets of taxi number and car number

Next we study the fundamental diagram. Figure 4(a) shows the fundamental diagrams under different value of  $P_{on}$ . One can see that when  $P_{on}$  is small, the flow rate decreases fast with the increase of  $P_{on}$ . However, it still slightly decreases with the increase of  $P_{on}$  when  $P_{on}$  is large, especially at intermediate densities, even if the taxies stop (to pick up and drop off passengers) less frequently ( $G$  decreases with  $P_{on}$  when  $P_{on}$  is large). This might be, as explained above, due to time interval between drop-off and pick-up process of taxies decreases with the increase of  $P_{on}$ .

Figure 4(b) shows the fundamental diagrams under different value of  $N_{taxi}$ . It can be clearly seen the flow rate remarkably decreases with the increase of  $N_{taxi}$ . Figure 4 (c) and (d) shows both  $L_1$  and  $L_2$  have trivial impact on the flow rate of the system. The effect of  $T_{st}$  on the fundamental diagrams is shown in Figure 4(e). One can see that the flow rate in the intermediate density range notably decreases with the increase of  $T_{st}$ , but the flow rate in the low density range is essentially independent of  $T_{st}$ .

Figure 5 shows carry capability of the system. Here we suppose one car carries only one passenger (i.e., the driver himself). As mentioned before, an occupied taxi carries two passengers averagely, and an unoccupied taxi carries zero passenger. One can see a qualitative change happens from the fundamental diagram. The carry capability is essentially independent of  $N_{taxi}$  at large densities. Our results indicate in order to optimize traffic flow, one should not solely concentrate on flow rate.

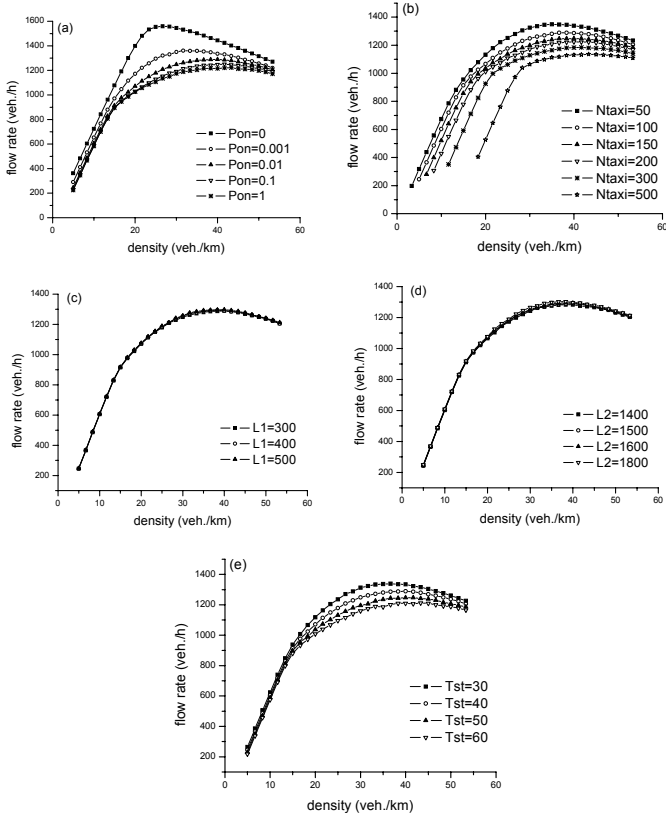


Figure 4. Fundamental diagrams under different parameters  
 (a)  $P_{on}$  (b)  $N_{taxi}$  (c)  $L_1$  (d)  $L_2$  (e)  $T_{st}$

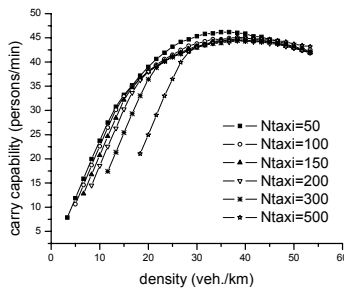


Figure 5. Carry capacity of the system under different  $N_{taxi}$

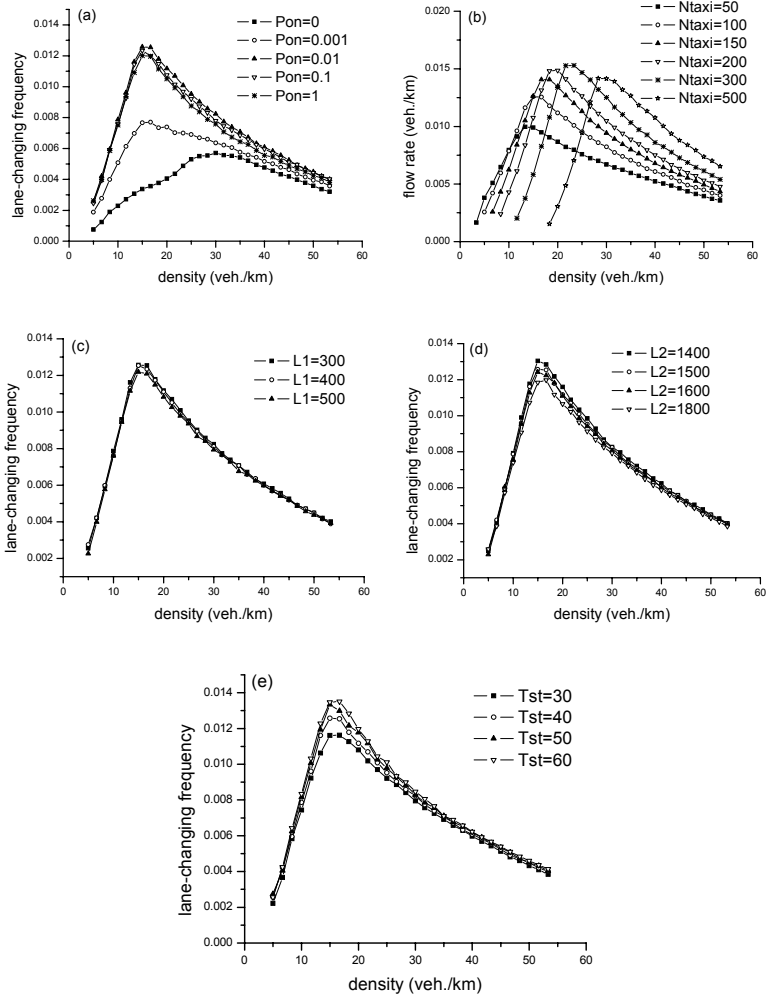


Figure 6. Lane-changing frequencies under different parameters

(a)  $P_{on}$  (b)  $N_{taxi}$  (c)  $L_1$  (d)  $L_2$  (e)  $T_{st}$

Now we investigate lane-changing frequency, which might be relevant to traffic safety because many car accidents are induced in lane changing process. Figure 6(a) shows the lane-changing frequencies under different values of  $P_{on}$ . It can be seen that



the lane-changing frequency first increases fast and then decreases slightly with the increase of  $P_{on}$ . This is in consistent with the stop frequency of taxis: the more frequently taxis stop (i.e., the larger  $G$  is), the larger lane changing frequency is.

Figure 6(b) illustrates the lane-changing frequencies under different values of  $N_{taxi}$ . It is found that the maximum value of lane changing frequency firstly increases and then decreases with the increase of  $N_{taxi}$ . In addition, the density corresponding to the maximum value moves right with the increase of  $N_{taxi}$ . As a result, it can be clearly seen that in intermediate density range, the lane-changing frequency first increases and then decreases with the increase of  $N_{taxi}$ .

Figure 6(c) and (d) shows the lane-changing frequency under different values of  $L_1$  and  $L_2$ . One can see that the lane-changing frequency slightly decreases with the increase of  $L_1$  or  $L_2$ . This is also in consistent with the stop frequency of taxis. Figure 6(e) shows lane-changing frequency increases with the increase of  $T_{st}$  in the intermediate density range, despite taxis stop less frequently.

## Conclusion

In this paper, we have investigated a traffic system consisting of a mixture of cars and taxis. The taxis stop to pick up and drop off passengers on the right lane. The cars can change lane when hindered by preceding vehicles.

We have studied empty taxi rate, getting on rate, fundamental diagram, carry capability, and lane changing frequency in the mixed traffic system. The dependence on pick up rate  $P_{on}$ , the system density, the taxi number  $N_{taxi}$ , taxi stop time  $T_{st}$  and two travel distance parameters  $L_1$  and  $L_2$  is investigated. The diagram of the maximum value of getting on rate is also presented, so that an optimum taxi number could be suggested. Some unexpected results are reported. For example, (i) the flow rate still decreases even if the taxis stop less frequently; (ii) getting on rate does not monotonically increase with the pick-up rate. These results might be due to time interval between drop-off and pick-up process decreases with the increase of pick up rate.

In our future work, some more realistic factors need to be considered. For example, (i) the taxis could also change lane; (ii) the inhomogeneous distribution of passengers. Furthermore, the study also needs to be extended to consider traffic network instead of a single road.

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