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INFORMATION TECHNOLOGY MODELLING

Complexus 2006;3:97–107 DOI: 10.1159/000094192

Published online: August 25, 2006

# Traffic Dynamics in Scale-Free Networks

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**Key Words**

Scale-free network · Transmission Control Protocol · Internet · Simulations

**Abstract**

We study traffic dynamics in growing scale-free networks. Both the scale-free structure of the network and the adaptive nature of the dynamics which controls traffic in the network are considered in the model. The model is investigated with computer simulations and analytically for the case of a scale-free tree. For the scale-free tree, an exact formula and its power law approximation of the complementary cumulative distribution function of link load (edge betweenness) is presented. We examine whether the scaling properties of the network affect the performance of the transport mechanism and estimate the average number of competing transport mechanisms at bottlenecks. We find that bottlenecks tend to appear on the periphery of the network as the performance increases. Various bandwidth allocation strategies are compared. We show that the best performance is achieved when capacity is distributed proportionally to the expected load of links. We demonstrate that it is necessary to study both the topology and the dynamics of the transport mechanism to understand the whole system.

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## Simplexus

We live in a networked world. The world's ecosystems consist of species linked together within intricate food webs, just as our communities are complex webs of social ties. The Internet is a vast network of computers linked by transmission lines, and the living cell depends for its function on a staggeringly complex web of interactions among a great number of genes, proteins and other small molecules. Within the past decade, researchers have learned that the architecture of these networks has a surprisingly universal character, in that many are 'scale free' – the distribution of network nodes according to their 'degree' (number of links) follows a power-law $P(k) \sim k^{-\gamma}$, where $k$ is the number of links and $\gamma$ is a constant, typically between 2 and 3. The term 'scale free' refers to the fact that a power-law function has no inherent scale to it, so there is no 'typical' number of links for a node, but great heterogeneity within the network. The scale-free character of such networks also tends to be reflected in other properties such as the 'betweenness' – a measure, for each network link, of how often it lies on the shortest path between other nodes in the network.

Given the scale-free character of these features, it is natural to wonder how network topology might influence the dynamics of processes that occur in such networks. In the Internet, for example, the flow of information across the network can be smooth and efficient, or it can be afflicted with frequent traffic jams and bottlenecks. Roughly speaking, the betweenness of a link reflects how crucial it is as a connecting artery within a network. Hence, it seems plausible that variations in the precise nature of the scale-free structure of the network might strongly influence its performance. Such understanding would clearly be of value to engineers who might try to alter network structure in search of better performance.

Following this line of inquiry, Fekete et al. here examine the dynamics of informa-

## 1 Introduction

The statistical properties of complex networks have been investigated extensively in the physics community in recent years [1–4]. With the increasing computing power of modern computers, analysis of large-scale networks and databases has become possible. It has been shown that the degree statistics of many natural and artificial networks follows power law. Examples for such networks vary from social interconnections and scientific collaborations [5, 6] to the World Wide Web [7] and the Internet [8, 9]. These networks are usually referred to as *scale-free* networks.

The first mathematical model of complex networks, the random graph theory was developed by Erdős and Rényi (ER) [10]. In this model, the number of nodes is fixed and connections are established randomly. Although the ER model leads to rich theory, it fails to predict the power law distributions observed in scale-free networks. Barabási and Albert (BA) proposed a more suitable model of these networks [11, 12]. The BA model is also based on the random graph theory, but, in addition, it involves two key principles: (1) *growth*, that is the size of the network increases during development, and (2) *preferential attachment*, that is new network elements are connected to higher degree nodes with higher probability.

The concepts of graph theory are used throughout this paper. A graph consists of vertices (nodes) and edges (links). Edges are ordered or un-ordered pairs of vertices, depending on whether an ordered or un-ordered graph is considered, respectively. The order of a graph is the number of vertices it holds, while the degree of a vertex counts the number of edges adjacent to it. A path is also defined by the most natural way: it is a vertex sequence, where any two consecutive elements form an edge. The graph is called connected, if for any vertex pair there exists a path which starts from one vertex and ends at the other.

The study of complex networks usually deals with the structural properties of networks, like degree distribution, shortest path distribution, degree-degree correlations, or clustering. Furthermore, some complex networks also involve a dynamical system which governs traffic in the network. The matter of importance in such systems is the performance of the dynamics. Therefore, exploring the influence of network structure upon traffic dynamics is essential. Moreover, one should be interested in distributing the available resources to obtain the best performance for a given network structure.

From this point of view 'betweenness' is the most important attribute. Betweenness measures the number of shortest paths passing through a certain network element. *Node betweenness* has been studied recently by Goh et al. [13], who argued that it follows power law in scale-free networks, and the exponent $\delta \approx 2.2$ is independent from, in a certain range, the degree distribution. Szabó et al. [14] used rooted deterministic trees to model scale-free BA trees, and found scaling exponent $\delta_t = 2$.

The study of complex networks usually deals with the structural properties of networks, like degree distribution, shortest path distribution, degree-degree correlations, or clustering. Furthermore, a complex network may also involve a dynamical system which governs traffic in the network. In this paper we study scale-free networks with embedded flow dynamics. The dominant algorithm which controls the data traffic in the Internet is the Transmission Control Protocol (TCP) [15]. For the detailed analysis of TCP mechanism we refer to Jacobson [16]. Since TCP performance affects overall network performance, TCP modelling is an important issue that has attracted research interests during the last years. Traditional approaches to performance evaluation packet networks have normally relied on attempts to describe as closely as possible the dynamics of network elements over a discrete

tion flow on a set of simple Internet-like networks, with the aim of deriving some basic insights regarding the influence of topology on dynamical performance. Unfortunately, the detailed structure of the real Internet is still far too intricate to be modelled with complete accuracy. Consequently, they study a well-known class of much simpler scale-free networks for which they can work out important characteristics analytically. Using these analytical results, and exploiting a judicious simplification in modelling the flow of information over these networks – using a 'fluid' model based on a coarse-grained approximation of the fine-grained 'TCP' dynamics of real Internet traffic – they find several interesting results that might, with further development, be useful in suggesting strategies for improving the real Internet.

The authors introduce their simplified networks in their section 2; see their figure 1 for a diagram. As they describe, a well-known prescription for 'growing' scale-free networks works through a mechanism of so-called 'preferential attachment'. The prescription works as follows. Start with a small 'seed' network, having only a handful of nodes and links among them. Next, add new nodes to the network, one by one. At each step, attach each new node to several of the already existing nodes, choosing these at random, but with one bias: links to existing nodes should be established with probability proportional to their degree. As originally shown by Albert and Barabasi in 1999, a network grown in this way will, when it becomes large, have a scale-free character.

The models used by Fekete et al. follow the Barabási-Albert prescription very closely, but differ very slightly. In particular, the authors take the likelihood of linking to an already existing node to be proportional to $a + q$, where $q$ is a node's degree and $a$ a variable parameter whose value influences the scaling properties of the network. (Note that $a$ can also be given conveniently in terms of another paramestate space. A new class of semi-analytical models has recently been introduced in the networking arena, and today appears to be the most promising approach for scalable and accurate performance analysis of large IP networks. This new approach, that is often called 'fluid models', adopts an abstract deterministic description of the average network dynamics through a set of ordinary differential equations, thus neglecting the short-term, packet-by-packet description of the stochastic network dynamics. We will present a simple model, which considers both the scale-free structure of the Internet and the adaptive nature of the underlying dynamics using fluid models. We stress that the main goal of this paper is to study the TCP-like (adaptive) dynamics on growing scale-free networks, not to model the Internet.

## 2 The Network Model

It has been shown that the structure of the autonomous systems in the Internet is a scale-free tree [17]. An autonomous system is a large segment of the Internet, which usually belongs to one organization, for example to a university, a large company, or a national office. In order to keep our model analytically tractable, we model the whole Internet with a simple scale-free rooted BA tree, extended with initial attractiveness [18]. Shortcuts, correlations with the geographical distribution of the population [19], and other details are neglected in our model.

### 2.A The BA Model

The construction of the network proceeds in discrete time steps according to the BA model. Let us denote time with $\tau \in \mathbb{N}$. Initially, at $\tau = 0$, the graph consists only of a single vertex without any edges. Then, in every time step, a new vertex is connected to the network with a single, *directed* edge. Note that the initial vertex is distinguished from all the other ones, since it has only incoming connections; we refer to it as *root vertex*. The target of the new edge is selected randomly from the present vertices of the graph. The probability that a new vertex connects to an old one is proportional to the attractiveness of the old vertex $v$. Attractiveness is defined as

$$A(v) = a + q,$$

where parameter $a > 0$ denotes the initial attractiveness and $q$ is the in-degree of vertex $v$. The scaling properties of the network can be smoothly controlled by parameter $a$: it has been shown that the probability distribution of in-degrees is $P(q) \sim (q + a)^{-(2+a)}$ [18]. Note that the special case $a = 1$ practically repeats the original BA model. Indeed, except for the root node, the attractiveness of every vertex becomes equal to its degree if $a = 1$; this is exactly the definition of the attractiveness in the BA model [11]. On the other hand, the model tends to Poisson-type ER graph if $a \to \infty$, since preferential attachment disappears in the limit, and $P(q) \sim e^{-q}$.

We refer to a connected subgraph as a *cluster* in this paper (fig. 1). To calculate the number of shortest paths passing through a given edge, it is sufficient to know the size of the cluster attached to the given edge $n$. If the size of the network is $N$, then from elementary combinatorics it follows that the number of shortest paths, that is the betweenness, or shortly the load of the particular edge is

$$L = (n + 1)(N - n - 1). \quad (1)$$

The probability distribution of cluster size for any finite $N$ can be given exactly:

$$\mathbb{P}_N(n) = \frac{N-\alpha}{N-1}\frac{1-\alpha}{(n+1-\alpha)(n+2-\alpha)}, \quad (2)$$

where $0 \leq n < N - 1$, and $\alpha = 1/(1 + a)$. The details of the calculations are published in [20].

For $1 \ll N$ and $1 \ll n \ll N$ equation 2 can be approximated with

$$\mathbb{P}_N(n) \approx (1-\alpha)\frac{1}{n^2},$$

ter, $\alpha = 1/(1 + a)$.) For small values of $a$, this rule is essentially equivalent to the BA algorithm. For $a$ very large, however, it instead gives a random graph with a well-defined number of links per node; that is, a graph lacking scale-free structure. This is a useful variation, as later in the paper, by varying the value of the parameter $a$, the authors can conveniently compare the performance of both scale-free and non-scale-free networks. After introducing their basic model, the authors go on in section 2 to derive some further mathematical results (useful in later calculations) for the networks generated by these models. In particular, they derive an analytical result for the probability distribution (and expected value) of the betweenness of the network links.

Having defined their basic model for network topology, the authors next introduce their 'fluid approximation' for the dynamics of information flow in the network. Real information traffic in the Internet is, of course, discrete and carried by individual packets according to the well-known 'TCP' protocol – a scheme that manages the flow of information back and forth between two Internet hosts. The basic TCP traffic mechanics are fairly simple: when a packet goes from one computer to another, the receiver sends an 'acknowledgement' packet back to the sender. The TCP algorithm uses this acknowledgement to control traffic by increasing the total 'throughput' for the sender, if the acknowledgement is received, and decreasing the throughput if not, as this indicates congestion. One further possibility complicates the dynamics: as flows originating in different parts of the network compete for bandwidth, it will occasionally happen that the buffer of router gets full and has to drop some packets.

To produce a convenient approximation of these dynamics, Fekete et al. introduce what they term the AIMD model. The idea is to seek a rough description of the dynamics that ignores the granularity of the packets. As the authors note in their equawhere the scaling exponent $v = 2$ is independent of $\alpha$; therefore it is universal in the class of evolving scale-free trees.

### 2.B Betweenness

The probability distribution of betweenness $L$ can be given by the following transformation formula of random variables:

$$\mathbb{P}_N(L) = \sum_{n=0}^{N-2} \delta_{L,(n+1)(N-n-1)} \mathbb{P}_N(n).$$

However, this expression is difficult to handle. An alternative description of a random variable is the complementary cumulative distribution function (CDF), defined as $F^c(x) = \mathbb{P}(L \geq x)$, that is the probability that the value of the random variable exceeds $x$.

From equation 1 it is obvious that the load of an edge exceeds $L$ if and only if $n_L \leq n < N - (n_L + 1)$, where

$$n_L = \left\lceil \frac{N-2}{2} - \frac{N}{2}\sqrt{1 - \frac{4L}{N^2}} \right\rceil,$$

and $\lceil \cdot \rceil$ denotes the ceil function. It immediately follows that the complementary CDF of the load is

$$F_N^c(L) = \sum_{n=n_L}^{N-n_L+2} \mathbb{P}_N(n) =$$

$$\frac{N-\alpha}{N-1} \frac{(1-\alpha)(N-2n_L-1)}{(n_L+1-\alpha)(N-n_l-\alpha)}. \quad (3)$$

If $N \ll L \ll N^2/4$ then the complementary CDF can be approximated by the following power law

$$F_N^c(L) \approx (1-\alpha) N \frac{1}{L}.$$

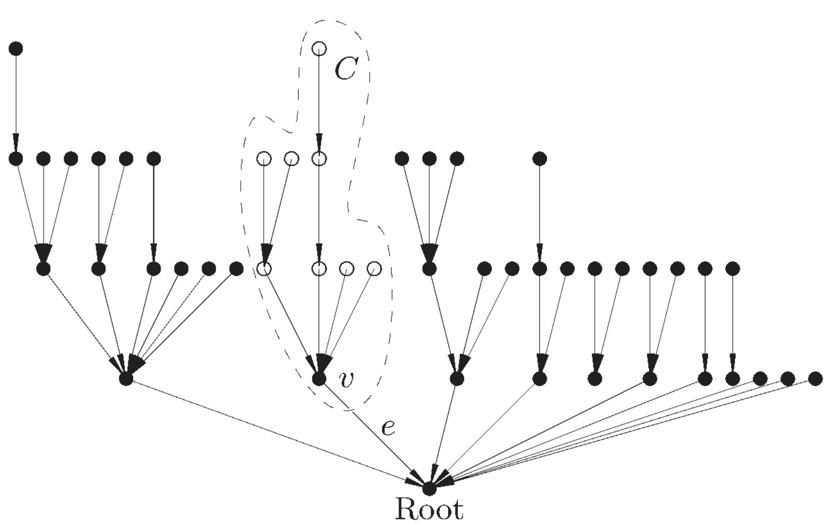


**Fig. 1.** Schematic illustration of the evolving network at time $\tau$. Vertex $v$, connected to the network at $\tau_e$, denotes the root of cluster $C$. Variable $n = |C| - 1$ denotes the number of nodes in $C$, except $v$ (marked by ○).

tion 4, the throughput increases or decreases according to the measured round-trip time between two nodes. In the absence of any congestion, these times should be effectively constant. Hence, the TCP algorithm should tend to increase the throughput linearly with time, as in their equation 5. But packets will be lost, at some point, whenever the capacity of some link in the network is surpassed, as defined in equation 6. At this point, a different dynamics comes into play. The authors stipulate, by definition, that when a link becomes congested, one of the TCP connections running through it (there will normally be many) will lose a packet. The loser is chosen at random, and this TCP then reduces its throughput by a factor of two.

These equations provide a rough description of network flow, following throughput rates, rather than the detailed movement of specific packets. In this sense, these equations define a fluid approximation, even though they remain faithful to the underlying principles of network routing. In addition to this definition of the traffic flow dynamics, the authors also have to make a plausible model for the number and placement of such TCP connections within the network. For this they follow a very simple prescription: every pair of nodes in the network has a chance $p = 1/(N-1)$ of having an established TCP connection. This means that, on average, there will be roughly the same number of connections as there are nodes in the network.

The authors are leading up to a series of simulations of flow over these TCP connections that represent the real 'payoff' of the paper. In preparation for such simulations, they now introduce some further definitions. In particular, they define the 'performance' $Q^{(i)}$ of the $i^{th}$ TCP connection as the long-term average of its throughout. Referring to earlier studies, they then derive an expression for the expected value of this quantity, and note that the performance of

Finally, the expectation value of the edge betweenness is calculated:

$$\mathbb{E}_N[L] = \sum_{L=0}^{\infty} L\mathbb{P}_N(L) = \sum_{n=0}^{N-2}(n+1)(N-n-1)\,\mathbb{P}_N(n)$$
$$= (1-\alpha)\frac{(N-\alpha)(N+1-2\alpha)}{N+1}\left[\Psi(N-\alpha)-\Psi(1-\alpha)\right]-(1-\alpha)(2N+1-2\alpha),$$

where $\Psi(z) = d[\ln\Gamma(z)]/dz$ is the digamma function [21]. Since $\Psi(z) \sim \ln z$ as $z \to \infty$, therefore

$$\mathbb{E}_N[L] = (1-\alpha)N\ln N + O(N),$$

if $N \to \infty$.

## 3 The Model of Network Dynamics in the Internet

Data is transferred between source and target computers through intermediate *routers*. Before transmission data is cut into smaller units, called *packets*. This way, if some part of the file is lost or gets corrupted, then only the damaged or lost parts should be retransmitted, not the whole file. The TCP algorithm administers the departure, arrival and retransmission of packets.

Interactions of different TCP flows inevitably cause congestion in the network. Packets are temporarily queued in buffers, but when a buffer is full, incoming packets are dropped by routers. When a packet successfully reaches its destination, the receiver sends an acknowledgement (ACK) packet back to the source. The elapsed time between packet departure and ACK arrival is called *round-trip time*, $T_{RTT}$.

An important feature of the TCP algorithm is that it can adapt its throughput to the changing network conditions. The throughput, that is the amount of bits transferred per unit time, is increased when arriving ACK indicates successful transmission, and decreased when missing ACK implies congestion.

In this section the fluid approximation of the TCP algorithm, the additive increase-multiplicative decrease (AIMD) model is discussed, supposing that the topology of the network does not change. Modelling dynamics on a fixed topology is legitimate when the time scales describing the development of the network topology and the dynamical process superposed to the network differ widely. A good example is Internet traffic, whose modelling requires time resolutions from milliseconds up to a day [22–24], compared with the months required for significant topological changes [25].

Detailed description of the AIMD model can be found in Baccelli and Hong [26]. The TCP standard is given in Postel [15].

### 3.A The AIMD Model

Let us suppose that $N_{TCP}$ number of TCPs are operating in the network and their throughput is denoted by $X^{(1)}(t)$, $X^{(2)}(t),\ldots,X^{(N_{TCP}(t))}$. A heuristic, but reasonable assumption of the AIMD model is that between consecutive packet loss events the development of throughput $X^{(i)}, 0 < i \leq N$ can be approximated by the following differential equation [27]:

$$\frac{dX^{(i)}(t)}{dt} = \frac{P}{T_{RTT}^{(i)}(t)^2}, \quad (4)$$

where

$$T_{RTT}^{(i)}(t)$$

denotes the round-trip time of the $i$-th TCP connection at time $t$, and $P$ is the packet size. In fixed topology, round-trip times may vary due to queuing delays. If queuing delays are negligible, however, then round-

the network as a whole should depend on the distribution of the edge 'capacity', $C_e$, which gives the total information-transfer capacity of each link in the network.

At this point, however, the authors have not included any mention of the capacity $C_e$ of an edge in their definition of their scale-free networks. So far, all network links have equal weight and equal capacity. As capacity figures so importantly in their analysis, however, the authors now have to give this some attention. In the real world, of course, capacity tends to be selected by local forces as they respond to shifting demands, by buying better fibre-optic cables or faster satellite links, for example. Following this natural idea, the authors assume that the capacity of any edge will simply be proportional to its betweenness. This makes intuitive sense, as links of higher betweenness should tend to carry more traffic (because more TCP connections should pass through this link). In their figure 3, the authors show the results of numerical simulations (backing up their earlier analytical results) indicating how the betweenness (and hence capacity) is distributed through the network. Using this distribution leads the authors, finally, to their equation 9 for the expected 'performance' $Q^{(i)}$ and its dependence on the scaling parameter $a$ and the average capacity $\bar{C}$.

Finally, the authors turn to their simulations. The results, in their figure 4, show the distribution of TCP performance in the network. Each curve shows – for a particular network architecture – the probability that a TCP link in the network has throughput $Q^{(i)}$ greater than some value $Q$. The figure shows results for four specific kinds of networks: for a series of scale-free networks, with values of $\alpha = 1/3, 1/2, 2/3$, and also for $\alpha = 0$ (not a scale-free network). The general conclusion is, as illustrated in table 1, that performance is improved for higher values of the scaling parameter $\alpha$.

The authors finally explore how the performance of the traffic dynamics might be trip times are constants $T^{(i)}_{\rm RTT}(t) \equiv T^{(i)}_{\rm RTT}$, and (4) can be solved:

$$X^{(i)}(t) = X^{(i)}(0) + \frac{P}{T^{(i)\,2}_{\rm RTT}} t. \quad (5)$$

Note that equation 4 is applicable only if the packet loss ratio is modest (<1–2%) [28].

Equations 4 and 5 are valid only between packet loss events $t_n$, which occur when the total throughput on edge $e$ first reaches capacity $C_e$ of that particular edge:

$$\sum_{i\in I_e}\left(X^{(i)}_n + \frac{P}{T^{(i)\,2}_{\rm RTT}}\Delta t_{n+1}\right) = C_e, \quad (6)$$

where $I_e$ denotes the set of TCPs which share edge $e$, $X^{(i)}_n = X^{(i)}(t_n)$ is the throughput of the $i$-th TCP at $t_n$, and $\Delta t_n = t_n - t_{n-1}$ is the elapsed time between the $n$-th and the previous congestion events. The first moment when (6) holds is

$$\Delta t_{n+1} = \min_e \left(\frac{C_e - \sum_{i\in I_e} X^{(i)}_n}{\sum_{i\in I_e} P/T^{(i)\,2}_{\rm RTT}}\right). \quad (7)$$

At congestion events, some TCPs that share the congested link lose packets. The AIMD model makes sure that the packet losses can be modelled by a stationary stochastic process; the owners of the lost packets are selected randomly and independently. The probability $p_s$ that a TCP flow experiences packet loss is called *synchronization parameter*.

According to the TCP congestion control mechanism, those TCPs which lose packets halve their throughput. The schematic time evolution of the total throughput and the throughput of a chosen TCP is shown in figure 2.

### 3.B The Model of TCP Connections

The hosts of the source and the destination of the TCP connections are located altered with different bandwidth allocation strategies. Specifically, they explore four alternatives to the assumption that capacity is allocated in proportion to the betweenness of a link. As their table 2 indicates, however, none of the most obvious alternatives delivers superior performance. This offers satisfying confirmation of what would seem intuitively likely – that the network as a whole should perform more efficiently if those links that play a most central role in the network (those of high betweenness) are also given the highest capacities.

These studies may not be applicable directly to the Internet, but they demonstrate a promising route for the simulation of dynamics on networks of plausible topology that can be used to help inform Internet managements. Further work will be required, of course, to judge the implications for the real Internet, especially as the 'AB' networks used here are known not to replicate all of the key topological properties of the real Internet. Future work will, of course, have to explore how the results found here generalize to other scale-free networks, especially those that resemble the real Internet more closely.

*Mark Buchanan*

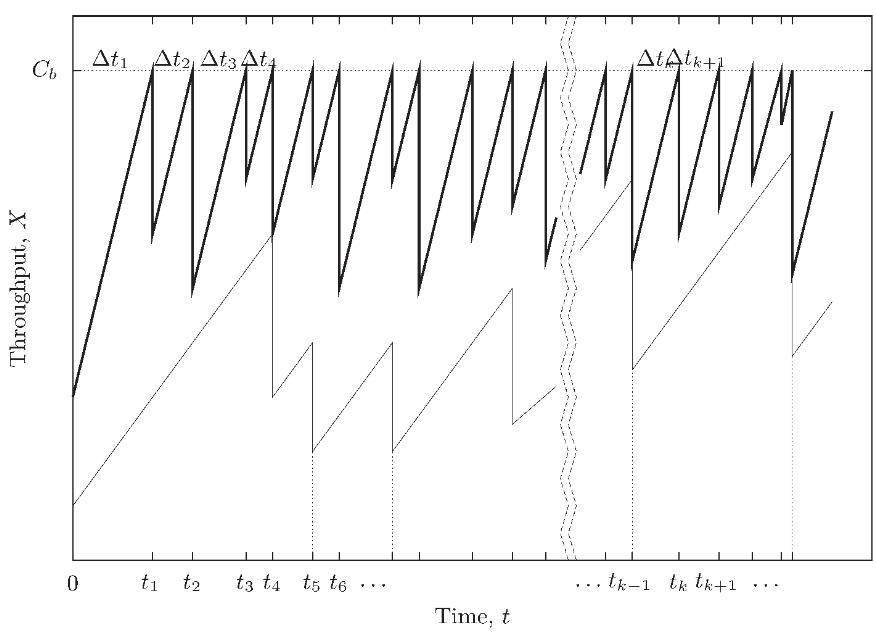


**Fig. 2.** Schematic time evolution of the total throughput (thick line) and the throughput of one TCP (thin line) in the AIMD model for constant round-trip time. Packet losses occur at $t_1$, $t_2$,... moments, when the total throughput reaches the capacity of the bottleneck link $C_b$. Time intervals between consecutive packet loss events are $\Delta t_1$, $\Delta t_2$, ...

randomly in the Internet. The actual location of the connections might be influenced by many factors including the importance and the availability of the computers, the user's language, behaviour and preference.

We assume in our model that TCP connections are established randomly in the network, and the distribution of both the source and the destination of the TCPs are homogeneous. That is, every pair of nodes may establish a directed TCP connection with the same, uniform probability, $p = \frac{1}{N-1}$. Therefore, the average number of TCP connections is $E[N_{TCP}] = N$ in the network. Moreover, data transfers are considered to be persistent in our model.

For a more realistic model, one should take finite file sizes and the heterogeneous TCP connections into consideration.

## 4 Discussions

The model we outlined in the previous sections was studied with extensive numerical simulations. First, we validate the analytic results that we obtained in section 2. Then, the influence of the network topology on the performance is discussed.

### 4.A Validation of Edge Betweenness Distribution

The exact formula 3 presented in section 2.B shows that possible values of the edge betweenness are $L = (N-1), 2(N-2), 3(N-3), \ldots, [N/2](N-[N/2])$, that is the CDF is constant between the above integer values.

Simulations confirm the validity of equation 2. With computer simulations $N = 10^5$ node random networks were generated with $\alpha = 0$ (ER), $\alpha = 1/3$, $\alpha = 1/2$ and $\alpha = 2/3$ (BA) parameter values. Empirical distributions, obtained from 100 independent simulations, formula 3 and power law approximations are compared in figure 3.

The expected staircase structure of the distribution can be clearly seen. The power law approximation fits the complementary CDF in the range $N \ll L \ll N^2/4$ accurately.

### 4.B Influence of the Network Structure on TCP Performance

We study in this section how the structure of the topology ($\alpha$) affects the 'performance' of the TCPs in the network. Let us define performance first: if $N_{TCP}$ number of TCPs are operating in the network where bandwidths $\{C_e\}$ have been allocated to the links, then the performance of the $i$-th TCP, $Q^{(i)}$, is defined as the time average of its throughput $X^{(i)}(t)$:

$$Q^{(i)} = \bar{X}^{(i)} \equiv \lim_{t\to\infty} \frac{1}{t}\int_0^t X^{(i)}(u)\,du.$$

Let us consider $m_e$ number of TCPs, which utilize a bottleneck link and let us suppose that the competing TCPs share the available bandwidth equally. It has been shown [25] that the expected performance of such TCPs is

$$\mathbb{E}\left[Q^{(i)}\right] = \left(1 - \frac{p_s}{2}\right)\frac{C_e}{m_e},$$

where $C_e$ denotes the capacity of the bottleneck link, and $p_s$ is the synchronization parameter, introduced in the AIMD model above. For the sake of simplicity, let the synchronization parameter be so small that only one packet is dropped at every congestion epoch: $p_s m_e \approx 1$. With this assumption

$$\mathbb{E}\left[Q^{(i)}\right] = \left(1 - \frac{1}{2m_e}\right)\frac{C_e}{m_e}. \tag{8}$$

The performance of the network is obviously influenced by the bandwidth distribution $\{C_e\}$. For the performance of different network structures to become comparable, the average bandwidth $\bar{C} = 1/(N-1)\sum_e C_e$ is fixed. Furthermore, the limited amount of capacity is distributed with the same strategy in networks with a different scaling parameter α.

In case of homogeneous TCP distribution, the expected number of TCPs which

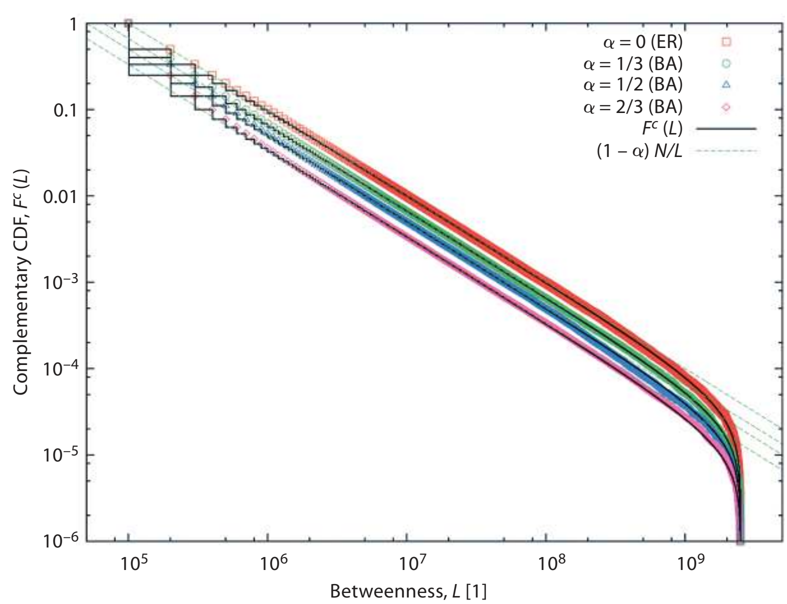


**Fig. 3.** Complementary CDF of edge betweenness, obtained from 100 realizations of $N = 10^5$ node networks, is shown at $\alpha = 0$, $\alpha = 1/3$, $\alpha = 1/2$, and $\alpha = 2/3$ parameter values. Equation 3 (solid line) and power law approximation $(1-\alpha)$ $N/L$ (dotted line) are also plotted.

share a given link is proportional to the betweenness $L_e$, that is the number of shortest paths passing through the particular link:

$$\mathbb{E}[m_e] = \frac{\mathbb{E}[N_{\text{TCP}}] L_e}{N(N-1)} = \frac{L_e}{N-1},$$

where the expected number of TCPs of our model, $\mathbb{E}[N_{\text{TCP}}] = N$, is substituted. In this section, mean field approximation is applied for distributing capacity, that is capacity is allocated proportionally to the edge betweenness: $C_e = C_0 L_e$. The normalization coefficient $C_0$ can be given by:

$$C_0 = \frac{\bar{C}}{\mathbb{E}_N[L]} \approx \frac{\bar{C}}{(1-\alpha) N \ln N} + O(1/N)$$

Using the above equations, the following formula can be obtained from equation 8 for the expected performance of the $i$-th TCP:

$$\mathbb{E}\left[Q^{(i)}\right] \approx \left(1 - \frac{1}{2m}\right) \frac{\bar{C}}{(1-\alpha)\ln N},$$

where $m$ is the number of TCPs, including the $i$-th TCP, which share the bottleneck link.

Network performance can be characterized by the complementary CDF of TCP performance: $F^c(Q) = \mathbb{P}(Q^{(i)} > Q)$. On figure 4 CDF of TCP performance is shown on normal-log plot for $N = 10^4$ node networks with $\alpha = 0, 1/3, 1/2, 2/3$ parameter values. Empirical distributions were obtained from simulations running for $100N$ congestion epochs on three realizations of random networks of each $\alpha$ value. The mean of the link capacity was set to $\bar{C} = 10^5$ [$b/s$]. The inset shows complementary CDF of TCP performance on log-log plot.

A point of inflection can be observed in figure 4 at every $\alpha$ parameter. The behaviour of the CDF is markedly different below and above the point of inflection. The sharp difference in the CDF implies that TCPs can be divided into two categories according to whether their performance is over or below the point of inflection.

The tail of the complementary CDF of TCP performance, above the point of inflection (see inset of fig. 4), consists of TCPs whose throughput is much higher than the expected performance (equation 9). These TCP operate in the core of the network, where every link along the path of their connection has large bandwidth. Moreover, they either hardly need to compete with other TCPs for the available bandwidth, or they win the competition at congestion epochs. The relative number of such TCPs is approximately 15% if $\alpha = 0$, and it is decreasing with the growth of parameter $\alpha$.

Below the point of inflection performances of TCPs are limited by low-bandwidth links, located on the periphery of the network, and by congested bottlenecks inside the network. If we suppose that the point of inflection approximately equals the expected throughput of TCPs at bottleneck links $\mathbb{E}[Q^{(i)}]$, then the number of TCPs competing at bottlenecks can be estimated from equation 9. In table 1 the location of the point of inflection $Q_I$ and the estimated number of TCPs at bottleneck link is shown for networks with different scaling parameter $\alpha$. Estimates show that as scaling parameter $\alpha$ increases bottlenecks tend to form on the outer links where only 1–2 TCP share the links.

Overall performance of the networks, measured by the average TCP performance

$$Q = \frac{1}{N_{\text{TCP}}} \sum_{i=0}^{N_{\text{TCP}}} Q^{(i)},$$

is also shown in table 1. We found that the overall performance $Q$ also increases with parameter $\alpha$. It follows from above that the scaling properties of the topology influence the TCP performance. It is reasonable to suppose that the interaction between the topology and the dynamical system is mu-

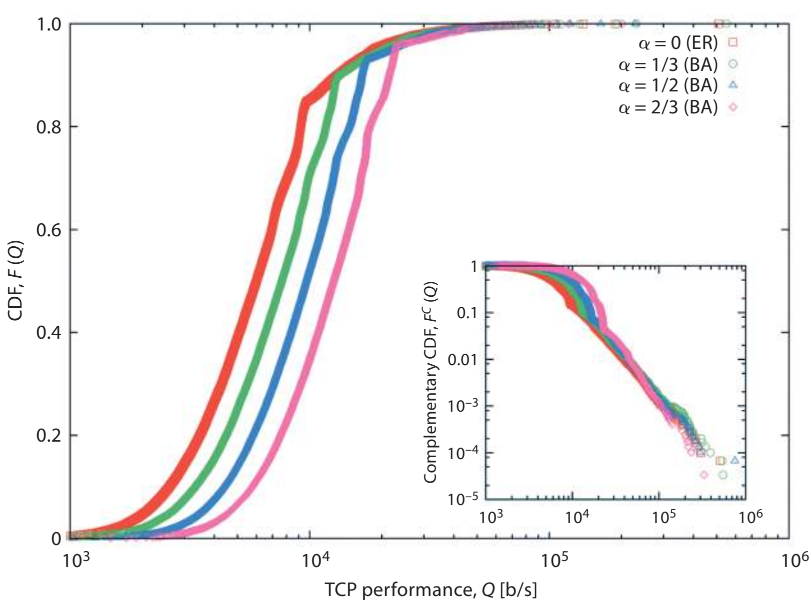


**Fig. 4.** CDF of TCP performance, obtained from three realizations of $N = 10^4$ node networks, is shown on normal-log plot at $\alpha = 0$, $\alpha = 1/3$, $\alpha = 1/2$ and $\alpha = 2/3$ parameter values. Average capacity is set to $\bar{C} = 10^5$ [$b/s$] for every network. Simulation lasted for $100N$ congestion epochs. Inset: Complementary CDF of TCP performance on log-log plot.

**Table 1.** Average TCP performance $Q$, the point of inflection of the CDF of TCP performance $Q_I$ and the estimated number of TCPs at bottlenecks for networks with different scaling parameter $\alpha$ are shown

| $\alpha$ | $Q[b/s]$ | $Q_I\ [b/s]$ | m |
|---|---|---|---|
| 0 | 7,785 | 9,655 | 4.52 |
| 0.3333 | 9,259 | 13,057 | 2.52 |
| 0.3891 | 9,346 | 14,456 | 2.68 |
| 0.5 | 11,184 | 17,200 | 2.40 |
| 0.6666 | 13,790 | 23,118 | 1.72 |

tual, that is the evolution of the network can by influenced by the dynamical system to reach optimum TCP performance as well.

### 4.C Performance of Other Bandwidth Distribution Strategies

In this section different bandwidth distribution scenarios are compared. The topology of the network and the average capacity are kept fixed, and only link capacities are changed in simulations. The scaling parameter of the topology is chosen to be $\alpha = 1/2$ for numerical simulations. Besides the mean field bandwidth distribution strategy discussed in the previous section, the following scenarios are considered:

Uniform: capacity is the same for every link: $C_e = \bar{C}$,

Minimum: capacity is proportional to the following minimum: $\min(q_A, q_B)$,

Maximum: capacity is proportional to the following maximum: $\max(q_A, q_B)$,

Product: capacity is proportional to the following product: $q_A \cdot q_B$,

where $q_A$ and $q_B$ denote the in-degrees of the nodes which compose a particular link.

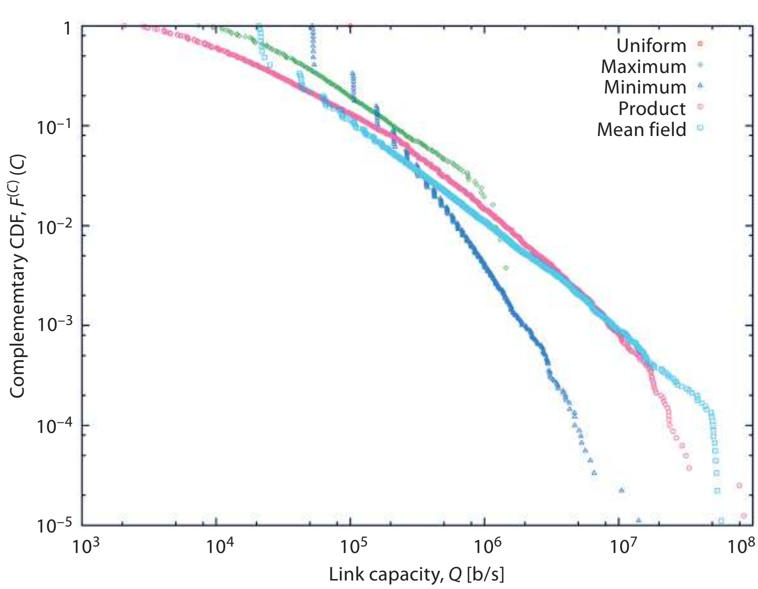


**Fig. 5.** Comparison of the complementary CDF of link capacity is shown for different bandwidth distribution strategies on log-log plot. Data is obtained from 10 realizations of $N = 10^4$ node networks with scaling parameter $\alpha = 1/2$. Average capacity is set to $\bar{C} = 10^5$ $[b/s]$ for every network. The following scenarios are considered: uniform, maximum, minimum, product, and mean field.

The uniform scenario is presented as a reference. It can be considered as the worst case scenario, when no information is available on the details of the network. Minimum, maximum and product strategies take the local structure of the network into account, and the more connection the link possesses, the more capacity they allocate for the particular link. The difference between the three strategies is whether they prefer loosely, moderately or highly connected links.

The complementary CDF of link capacities, obtained as the result of the above bandwidth distribution strategies, are compared in figure 5. CDF of the uniform strategy is degenerated, and the structure of the network is not taken into consideration in this case. The maximum strategy prefers the lower bandwidths at the cost of a cutoff at about $10^6$ $b/s$ capacity. The minimum strategy also prefers lower bandwidths at the cost of high bandwidths, but no cutoff exists. The complementary CDF of minimum strategy also resembles the mean field distribution with a different scaling exponent. The product strategy prefers the midrange, and it underestimates both the low and the high capacity range, compared to the mean field strategy.

Simulation results of the CDF of TCP performance is shown in figure 6 for the above-mentioned bandwidth distributions. The performance of mean field strategy is clearly the best. The next two best performing strategies, the minimum and the product, perform almost the same, although they prefer completely different bandwidth ranges. It follows that the whole bandwidth range must be taken into consideration in any bandwidth distribution strategy to reach the optimum network performance. The performance of the maximum strategy is considerably worse than that of the previous two. Finally, the uniform bandwidth distribution is the worst of all: its performance is just a few percent of the mean field scenario's perfor-

**Table 2.** Network performance for different bandwidth distribution strategies is shown

| Strategy | Q[b/s] |
|---|---|
| Uniform | 740.79 |
| Maximum | 2,391.94 |
| Minimum | 6,574.69 |
| Product | 5,279.5 |
| Mean field | 11,284.6 |

mance. The network where this strategy is applied is heavily congested, since the bottlenecks form in the core of the network.

Network performances, that is average TCP performances, are shown in table 2 for the different bandwidth distribution strategies. The measured network performances confirm the qualitative analysis of figure 6. Table 2 shows that mean field bandwidth allocation strategy is almost twice more effective than the second, minimum strategy, and it is more than twice as good as the product strategy. The performance of a network with maximum bandwidth distribution strategy is just about one fifth of the performance of the same network when mean field strategy is used. Moreover, the performance of the uniform scenario is even less than the third of the second worst, maximum strategy.

## 5 Conclusions

A complex model of the network embedded with dynamics has been studied in this paper. Both the scale-free structure of the network and the TCP dynamics, which controls traffic, are considered in the model. The topology of the network has been modelled by a growing scale-free random graph model, where the scaling properties of the network can be changed with parameter $\alpha$. The TCP dynamics has been approximated by the fluid AIMD model. We have assumed in the model that TCP connections are distributed homogeneously in the network, that is TCP connections are established between every pair of nodes with the same probability. It follows that the expected number of TCPs which utilize a link is proportional to the link load (edge betweenness), that is the number of shortest paths passing through the particular link.

We can summarize the main conclusions of this paper as follows:

- For the case of the scale-free tree we analytically computed conditional cluster size distribution, total cluster size distribution, complementary CDF of link load (edge betweenness), and the expectation value of the link load (edge betweenness; see section 2.B). The exact formula and the approximation for the complementary CDF of link load have been validated by numerical simulations in section 4.A.
- The purpose of TCP connections is to transfer data in the network. The performance of a TCP connection is measured by its throughput, that is its average transfer rate. We have investigated in section 4.B whether TCP performance is influenced by the scaling properties of the network. For the comparison of different networks the bandwidth allocation strategy has been fixed to the mean field strategy. It has been shown that the TCP performance increases as the scaling parameter $\alpha$ increases. It follows that the network topology influences the performance of the TCP.
- From the analysis of the CDF of TCP performance we have estimated the number of TCPs at bottleneck links. We have found that the number of TCPs at bottlenecks decreases as parameter $\alpha$ increases. It follows that bottlenecks move to the periphery of the network, when network performance is higher. This is understandable, since a bottleneck in the core of the network can reduce the performance of more TCP than a bottleneck on the periphery of the network.

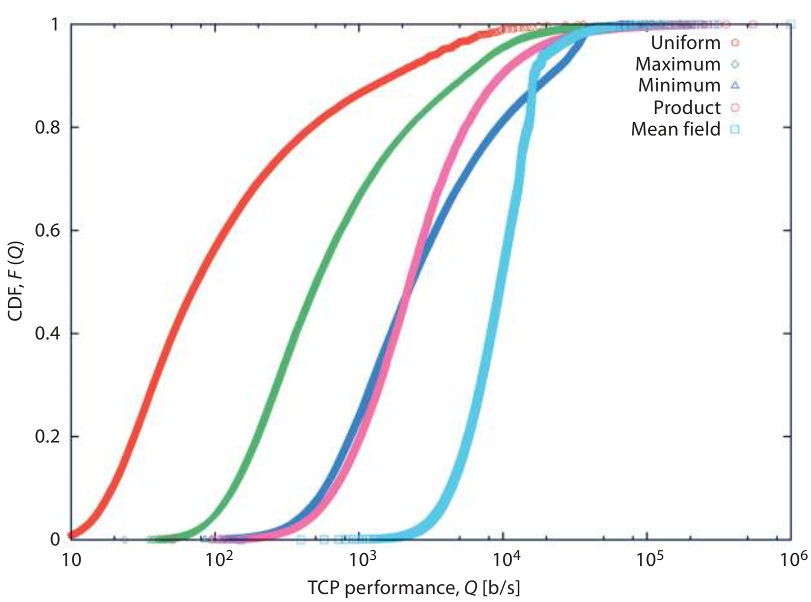


**Fig. 6.** Comparison of the CDF of TCP performance is shown for different bandwidth distribution strategies on normal-log plot. Data is obtained from 10 realizations of $N = 10^4$ node networks with scaling parameter $\alpha = 1/2$. Average capacity is set to $\bar{C} = 10^5$ [$b/s$] for every network. Simulation lasted for $100N$ congestion epochs. The following scenarios are considered: uniform, maximum, minimum, product, and mean field.

• We have investigated TCP performance in networks which were built on various bandwidth allocation strategies in section 4.C. We have found that mean field strategy performs about twice as well as the minimum and the product strategies, five times as well as the maximum, and it is more than fifteen times as good as the uniform strategy. These results indicate that the mean field bandwidth distribution strategy provides the optimum TCP performance.


## Acknowledgement

The authors thank for the partial support of the National Science Fund Hungary (OTKA T37903), the National Office for Research and Technology (NKFP 02.032.04) and the EU IST FET Complexity EVERGROW Integrated Project.


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