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Train positioning via integration and fusion of GPS and inertial sensors

B. Cai & X. Wang
Dept. of Communications & Control Engineering,
Northern Jiaotong University, Beijing, P.R.China, 100044

Abstract

As new ATC and block system advanced, the accuracy of train position is required increasingly. Especially for the high-speed railway, train position is updating rapidly. One sensor source is not reliable. Instead we use multi-sensors to realize reliable and real-time positioning for the critical train control. In order to verify the feasibility of the proposed method, we selected GPS (for ablolute positioning) and inertial sensors (for relative positioning) as the different test sensors. For inertial sensors, we use gyroscopes and accelerometers to measure travel direction and distance respectively. GPS has good low-frequency behavior, but poor high-frequency behavior. Inertial sensors (and dead-reckoning) have the opposite characteristics. By using their complementary characteristics advantageously, we can implement accurate train positioning. To integrate the multi-sensors effectively, we developed the error compensation model for inertial sensors and used EKF (Extended Kalman Filter) to get the optimal train position estimation. Preliminary tests gave the prospective results and will be given.

1 Introduction

ITS (Intelligent Transportation Systems) is the hot topic for transportation recently. It covers the innovations of all transportation tools. Railway, the main transportation network in China, is facing challenges from the development of other transportation industry. To meet the increasing demands and better service motivates the researches and utilization of new technologies worldwide. There are many researches needed to be done for high-speed trains to be operated in China. Among those, ATC (Automatic Train Control) and ATP (Automatic Train

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Protection) play a vital role in achieving high-level reliability and safety. As the key element of ATC (ATP), the measurement of train position and speed attracts many researchers to investigate all kinds of different methods. Our research is based on the assumption that no single existing approach is suitable to all situations and environments(Cai [1], Zhao [2]). Instead, we try to implement accurate and reliable train positioning by adopting combination of different (complementary) methods.

Usually GPS/DGPS is adopted in vehicle positioning. It is associated with some inertial sensors as external/auxiliary or redundant information sources. Since there are many inertial sensors available for different purposes, it's really a hard work to make suitable selections considering performance/price ratio. On the other hand, different applications have their specific requirements. In a railway system under ATC, there are several different methods for the measurement of displacement and orientation. For train positioning, usually sensors include tachometers, transponders, track circuits, GPS, INS(Inertial Navigation System) and etc. Here we present the preliminary research results of the integration and fusion of GPS and inertial sensors.

2 GPS (Global Positioning System)

The NAVSTAR Global Positioning System (GPS) can provide three-dimensional position, velocity and accurate time information to a user anywhere in the world (Farrell [3]). It becomes one of the most popular navigation systems employed to locate the vehicle position. But it has problems with accuracy when employed for moving vehicles.

Main GPS error sources include common errors (receiver clock bias, satellite clock bias, atmospheric delay (propagation errors or tropospheric delay and ionospheric delay), SA(Selective Availability), ephemeris errors) and particular errors (multi-path, receiver noise).

The typical horizontal positioning accuracy for GPS with SA on is about 100m RMS in each axis. The differential GPS (DGPS) can achieve positioning accuracy of 2-8m RMS by removing common errors. But GPS signal cannot continuously provide the positioning information when GPS signal obstruction occurs in tunnels or under trees. As we talked above, auxiliary devices and techniques are needed to provide supplemental positioning information for vehicles under such cases. Normally DR or/and MM (Map Matching) can be used (Mirabadi [4], Hailes[5], Czommer [6]). GPS has good low-frequency behavior, but poor high-frequency behavior. It can be used as absolute positioning and speed measurement method for a train.

3 DR (Dead-Reckoning)

3.1 The principle of DR

Dead Reckoning (DR) positioning is the most popular way to solve the problem of GPS signal obstruction. Generally, the DR is composed of two or more

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sensors which measure heading and displacement of a vehicle (for us, it will be a train). It measures the vehicle's location relative to an initial position by integrating the position vectors on the basis of distance traveled and orientation. The displacement and heading direction can be obtained by integrating (once or twice) measurements from the accelerometer and heading sensors respectively. So DR can be used to provide continuous positioning information for the vehicle when GPS is unavailable. It has the opposite characteristics compared to GPS. It can be used for relative positioning. By using their complementary characteristics advantageously, we can implement accurate train positioning.

After sensor data being sampled and integrated and sensor fusion being performed, the vehicle position (x_n, y_n) and orientation (θ) at time t can be calculated from eqn (1) (Zhao[2]). The principle is shown in Fig. 1.

$$x_n = x_0 + \sum_{i=0}^{n-1} d_i \cos\theta_i \ , \qquad y_n = y_0 + \sum_{i=0}^{n-1} d_i \sin\theta_i \ ,$$

$$\theta = \theta_0 + \sum_{i=0}^{n-1} \omega_i \ . \qquad (1)$$

Where: (x0, y0) is the initial vehicle position at time t0, d_i is the distance traveled or the magnitude of the displacement between time t_{n-1} and time t_n, θ_i is the direction (heading) of the displacement vector, and ω_i is the angular velocity for the same time period.

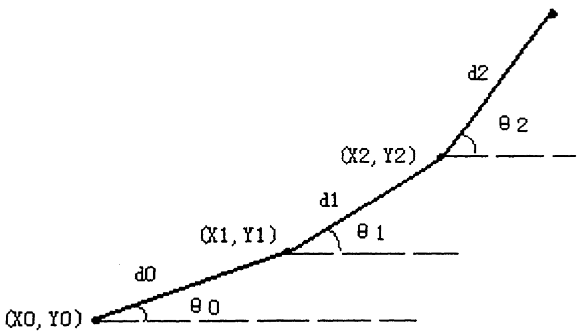

Figure 1: The principle of DR.

3.2 Error compensations of inertial sensors

3.2.1 The principle of inertial sensors

As above, it is possible to measure the acceleration, speed and position of a moving train using DR. A typical DR system basically consists of a gyroscope and accelerometers.

Linear accelerometers can be used to measure the linear acceleration of the

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vehicle in different dimensions. By integrating the acceleration signals both speed and position data can be derived. The measurement accuracy is decreased by many possible errors such as bias error, non-linearity of output, dead zone.

A gyroscope can be use to obtain accurate information of the trajectory of the train in a horizontal direction by measuring an angular rotation of the vehicle. The main error sources for a gyroscope include drift, non-linearity of output, and etc.

Because of the characteristics of gyroscope and accelerator, DR is determined to drift at a slow rate, so it has poor low-frequency behavior. A sample stationary test for a gyroscope and accelerator is shown in Fig. 2.

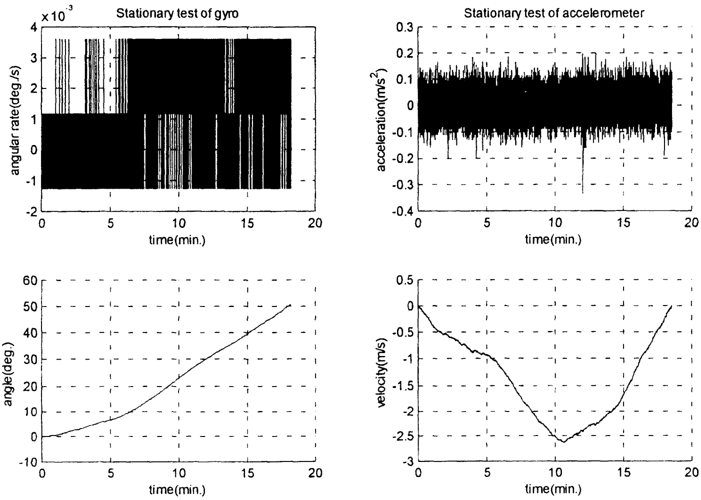

Figure 2: Stationary tests for a gyroscope and accelerator.

3.2.2 Error compensation model

In our DR system, a single-axis vibrating gyroscope is used as a heading sensor, and two accelerators are used as displacement sensors. Since a gyroscope measures the angular rate of the vehicle while the integrated heading is required for DR, a small error of the gyroscope may cause a large error accumulation. The same situation will be true with the accelerometers. A relatively high cost of a high accuracy gyroscope such as FOG and accelerometers limits their use in a car navigation system. Building error models for inertial sensors is motivated by an attempt to reduce the effect of unbounded position and orientation errors. Therefore an efficient gyroscope and accelerometer error compensation method will be critical to our system.

There are many different error compensation methods for inertial sensors. The more accurate, the more complicated and time-consumption.

Errors are divided into three parts: deterministic bias (or zero point offset), sensitivity error (or scale factor error) and zero point drift (warm-up bias and random bias). (Barshan [7], Kim [8])

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In the realm of inertial sensors, random error means a remaining error after accomplishing deterministic error compensation. In our system, a linear and nonlinear parametric model of the following form was fitted to the data from the gyroscopes and the accelerometers. It consists of n linear parameters and n nonlinear parameters (n=2).

$$\varepsilon_m(t) = C_1\lambda_1 + C_2\, e^{-\lambda_2 t} \quad \text{or} \quad \varepsilon_m(t) = CC_1 + C_2\, e^{-\lambda_2 t} \tag{2}$$

Actually, $C_1\lambda_1$ can be represented as one parameter CC_1, it includes deterministic bias and random bias and the other part in the eqn (2) denotes the warm-up bias. Three parameters CC_1, C_2, λ_2 can be tuned by fitting the error model to the sensor's output when zero input was applied. The best fitting parameters to the experimental data are tabulated in Table 1.

Table 1. Error Model Parameters for Gyro and Accelerometer

Inertial sensor	CC_1	C_2	λ_2
Gyro z	2545.4 A/D	-0.3816 A/D	0.0279
Accelerometer x	1920.4 A/D	-12.6535 A/D	0.0036
Accelerometer y	1971.1 A/D	-1.1765 A/D	0.2854

Adequacy of error models can be tested by checking that the residuals from the fitted model constitute a white, zero-mean process(Cai[1], Barshan [7]).

4 GPS/DR for train positioning

4.1 The integration and fusion of multi-sensors

It is clear that no single sensor can provide continuous, accurate vehicle position information. DR, GPS can be viewed as different sensors with different characteristics in our system. To obtain accurate and reliable train positioning, multi-sensor integration and fusion is required to provide the train with complementary, sometimes redundant information on its position and trajectory. Many researches have been done about it.(Mirabadi[4], Kobayashi[9] and etc.).
The key problem for the fusion of GPS and DR is the design of suitable algorithm. Among proposed methods, Kalman filter (or Extended Kalman filter) is regarded as the best approach.(Farrell[3], Brown[10])

4.2 GPS/DR integration

A Kalman filter is a recursive, linear, optimal, real time data processing algorithm used to estimate the so-called states of a dynamic system in a noisy environment. Kalman filter techniques were utilized in the integration of GPS and DR to obtain an optimal estimate of the current state of the system and a prediction of the future state of the system.

For non-linear systems, EKF (Extended Kalman filter) is used instead.

After discretization, eqn (2) becomes:

$$\varepsilon(k+1) = \frac{CC_1\lambda_2 T_s}{1+\lambda_2 T_s} + \frac{\varepsilon(k)}{1+\lambda_2 T_s} \text{ with } \varepsilon(0) = CC_1 + C_2 \qquad (3)$$

The state of system consists of 12 parameters, including 2 positions components, 2 velocities, 2 accelerations and 2 acceleration errors, orientation and angular rate, angle error and augular rate error. Then the state equation can be written as

$$X(k+1) = \Phi X(k) + u + w(k) \qquad (4)$$

with

$$X(k) = \begin{bmatrix} X_{ax}(k) \\ X_{ay}(k) \\ X_\theta(k) \end{bmatrix}, \quad \Phi = \begin{bmatrix} \Phi_{ax} & 0 & 0 \\ 0 & \Phi_{ay} & 0 \\ 0 & 0 & \Phi_\theta \end{bmatrix}, \quad u = \begin{bmatrix} u_{ax} \\ u_{ay} \\ u_\theta \end{bmatrix},$$

$$X_{ax}(k) \equiv \begin{bmatrix} x(k) \\ v_x(k) \\ a_x(k) \\ \varepsilon_{ax}(k) \end{bmatrix}, \quad X_{ay}(k) \equiv \begin{bmatrix} y(k) \\ v_y(k) \\ a_y(k) \\ \varepsilon_{ay}(k) \end{bmatrix}, \quad X_\theta(k) \equiv \begin{bmatrix} \theta(k) \\ \dot{\theta}(k) \\ \varepsilon_\theta(k) \\ \varepsilon_{\dot{\theta}}(k) \end{bmatrix}$$

$$\Phi_{ax} \equiv \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 & 0 \\ 0 & 1 & T_s & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{1+\lambda_{2x}T_s} \end{bmatrix},$$

$$\Phi_{ay} \equiv \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 & 0 \\ 0 & 1 & T_s & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{1+\lambda_{2y}T_s} \end{bmatrix}, \quad \Phi_\theta \equiv \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & \frac{1}{1+\lambda_{2\theta}T_s} \end{bmatrix},$$

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$$u_\theta \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{CC_{1\theta}\lambda_{2\theta}T_s}{1+\lambda_{2\theta}T_s} \end{bmatrix}, \quad u_{ax} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{CC_{1x}\lambda_{2x}T_s}{1+\lambda_{2x}T_s} \end{bmatrix}, \quad u_{ay} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{CC_{1y}\lambda_{2y}T_s}{1+\lambda_{2y}T_s} \end{bmatrix}$$

where: $w\ (k)$ is the process noise and its covariance matrix Q for Kalman filter can be derived:

$$Q_g = E\left\{w(k)w^T(k)\right\} = \begin{bmatrix} Q_a & 0 & 0 \\ 0 & Q_a & 0 \\ 0 & 0 & Q_\theta \end{bmatrix},$$

$$Q_a \equiv \begin{bmatrix} \dfrac{T_s^5}{20}\sigma_3^2 & \dfrac{T_s^4}{8}\sigma_3^2 & \dfrac{T_s^3}{6}\sigma_3^2 & 0 \\ \dfrac{T_s^4}{8}\sigma_3^2 & \dfrac{T_s^3}{3}\sigma_3^2 & \dfrac{T_s^2}{2}\sigma_3^2 & 0 \\ \dfrac{T_s^3}{6}\sigma_3^2 & \dfrac{T_s^2}{2}\sigma_3^2 & T_s\sigma_3^2 & 0 \\ 0 & 0 & 0 & T_s\sigma_4^2 \end{bmatrix},$$

$$Q_\theta \equiv \begin{bmatrix} \dfrac{T_s^3}{3}\sigma_1^2 & \dfrac{T_s^2}{2}\sigma_1^2 & 0 & 0 \\ \dfrac{T_s^2}{2}\sigma_1^2 & T_s\sigma_1^2 & 0 & 0 \\ 0 & 0 & \dfrac{T_s^3}{3}\sigma_2^2 & \dfrac{T_s^2}{2}\sigma_2^2 \\ 0 & 0 & \dfrac{T_s^2}{2}\sigma_2^2 & T_s\sigma_2^2 \end{bmatrix}.$$

If measurements are from inertial sensors and GPS, then the measurements will be north and east displacements and velocities from GPS, accelerations and angular rate from inertial sensors. The measurement equations are:

$$Z(k) = h[X(k)] + v(k) \tag{5}$$

with

$$Z(k) = \begin{bmatrix} dx_E & dy_N & V_E & V_N & a_x & a_y & \dot{\theta} \end{bmatrix}^T,$$

$$dx_E(k) = x(k) + v_{de}(k),$$
$$dx_N(k) = y(k) + v_{dn}(k),$$
$$V_E(k) = v_x(k) + v_{ve}(k),$$
$$V_N(k) = v_y(k) + v_{vn}(k),$$
$$Z_{ax}(k) = -\sin\theta(k)a_y(k) + \cos\theta(k)a_x(k) + \varepsilon_{ax}(k) + v_{ax}(k),$$
$$Z_{ay}(k) = \cos\theta(k)a_y(k) + \sin\theta(k)a_x(k) + \varepsilon_{ay}(k) + v_{ay}(k),$$
$$Z_{\dot\theta}(k) = \dot\theta(k) + \varepsilon_{\dot\theta}(k) + v_{\dot\theta}(k),$$

$$H_k = \left[\frac{\partial h}{\partial X}\right]_{\hat{X}_k^-}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -a_y\cos\theta - a_x\sin\theta & -a_y\sin\theta + a_x\cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T_{\hat{X}}$$

Measurement covariance matrix R:

$$v(k) = \begin{bmatrix} v_{de} & v_{dn} & v_{ve} & v_{vn} & v_{ax} & v_{ay} & v_{\dot\theta} \end{bmatrix}^T ,$$

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$$R_k = E\{v(k)v^T(k)\} = \begin{bmatrix} \sigma_{de}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{ne}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{ve}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{vn}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{ax}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{ay}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\theta}}^2 \end{bmatrix}$$

4.3 Field test results

To verify the methods proposed, we took a land test around a square course. As shown in Fig. 3, the result of DR alone without the aid from GPS indicates that the orientation of vehicle is correct, but the displacements have very big errors caused by the error accumulation of accelerometers since we obtain the displacements by double integral of acceleration. The result of GPS/DR fusion is very prospective. Since GPS (absolute positioning) can eliminate the error accumulation of DR (relative positioning), more accurate train positioning can be achieved by the integration of GPS/DR.

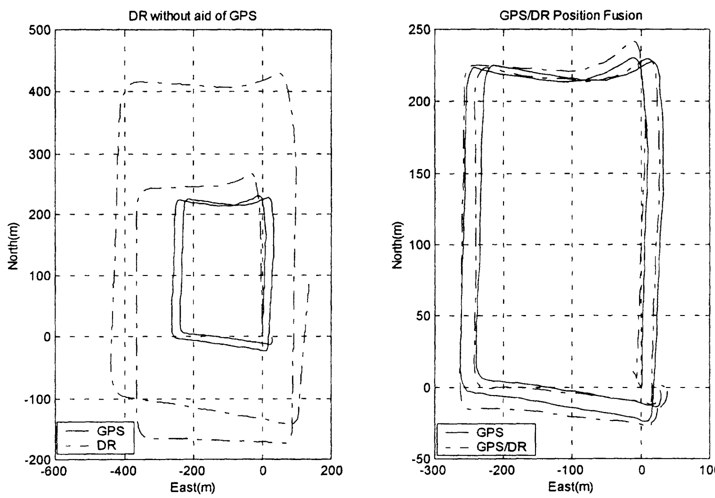

Figure 3: Field tests of DR without and with GPS aid.

5 Conclusions

Inertial sensors have good high frequency response and poor low frequency

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response. Its accumulated errors have significant effect on the positioning performance. In order to evaluate the performance of inertial sensors (gyroscope and accelerometers), error compensation model is developed and the parameters are tuned. By adopting multi-sensor integration and fusion, more accurate and reliable measurement for train speed and position can be obtained. It'll play an important role in the next generation of ATP and ATC systems.

Our preliminary field test indicated that the gyroscope is very suitable for measuring the orientation of the vehicle, but accelerometer we used behaved inaccurately due to the effect of gravity. By integrating DR and GPS, the performance of the system is greatly improved. The orientation obtained were reliable and useful over quite long time, while the position estimates obtained were reliable only over shorter periods. In both cases, the error models developed for these sensors substantially increased estimate accuracy.

Now we're doing more research work about the fusion of other possible sensors such as tachometers, transponders, track circuits, and etc. Our next goal is to make the system to be re-configurable and more flexible by using some AI techniques such as neural network, fuzzy logic.

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