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Trajectory planning for a quadrotor helicopter

Y. Bouktir, M. Haddad, T. Chettibi

Abstract—**A simple direct method able to generate time-optimal trajectories for a micro quadrotor helicopter is presented. It is based on modeling the quadrotor trajectory as a composition of a parametric function $\mathbf{P}(\lambda)$ defining the quadrotor path, and a monotonically increasing function $\lambda(t)$, specifying the motion on this path. The optimal evolutions of $\mathbf{P}(\lambda)$ and $\lambda(t)$, which are approximated using B-spline functions, are found using a nonlinear optimization technique. The proposed method accounts for the most important constraints inherent to the system behavior, such as underactuation, obstacles avoidance and limits on actuator torques and speeds.**

Keywords: quadrotor, dynamics, trajectory, nonlinear optimization.

I. Introduction

Unmanned air vehicles (UAVs) are self-propelled aerial robots. They can be equipped with various instruments and payloads, making them capable of performing various civilian or military tasks. Among existing small UAVs, we find quadrotors which are Vertical Take-Off and Landing (VTOL) four rotor helicopters (Fig.1). They are controlled simply by changing the rotation speed of the four rotors. The front and rear rotors (2, 4) rotate in a clockwise direction while the left and right rotors (1, 3) rotate in a counter-clockwise direction to balance the torque created by the spinning rotors. The up/down motion is achieved by increasing/decreasing the rotors speed while maintaining an equal individual speed. The forward/backward, left/right motions are achieved through a differential control strategy of rotors speed. Thanks to this configuration, quadrotors are able to hover, takeoff, and land in small areas and enable them to perform tasks that fixed-wing craft are unable to do. Although the mechanical design of the quadrotor is simple, its particular dynamics makes the vehicle control relatively difficult. This is due principally to the fact that the system is underactuated: there are only four rotors which generate four inputs thrusts (T_i) to control the six degrees of freedom of the crossing body during fly.

In recent years, there have been a number of papers dealing with various problems inherent to the exploitation of quadrotors. For example, dynamic modeling issues were addressed in references [1, 2, 3, 4, 5, 6, 7] whereas, nonlinear control laws, such as feedback linearization control, visual control, back-stepping control and sliding-mode control were studied in many papers such as references [3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. However, trajectory planning problems of quatrotors were not widely studied. Yang *et al.* [12] treated the problem of time-optimal control of a quadrotor by using a nonlinear programming method coupled with a genetic algorithm. They succeeded to generate in simulation minimum time point-to-point trajectories under various technological constraints. Cowling *et al.* [18] presented an optimal trajectory planner with a linear control scheme to follow a reference trajectory. Authors exploited the differential flatness of the quadrotor to address the optimization problem within the output space. In reference [19], a model predictive control based trajectory tracking system for small unmanned helicopters is presented. It is based on a linear model predictive controller and showed, in simulation, a good robustness to parameter uncertainty. In reference [20], authors addressed the stabilization with motion planning problem of a standard quadrotor. They showed that the system presents a flat output and exploited this fact in the treatment of the motion generation problem. They proposed an efficient tracking feedback controller based on receding horizon point to point steering. This work was later extended to the case of a bidirectional X4 flyer [21]

In this paper, we propose a simple numerical method to treat the problem of generating minimum time trajectories for a quadrotor aerial robot under various constraints. It is based on previous works developed at our laboratory [22, 23] dealing with the problem of trajectory generation for serial manipulators and mobile robots. The trajectory generation problem is cast as a nonlinear optimization problem by parameterizing both of the robot path and the associated motion profile using a set of control points which are fitted by Bspline functions. The resulting problem is solved using the sequential quadratic programming method (SQP). Finally, simulation results corresponding to point-to-point tasks with free and imposed path in free and encumbered environments are presented to illustrate the efficiency of the proposed approach.

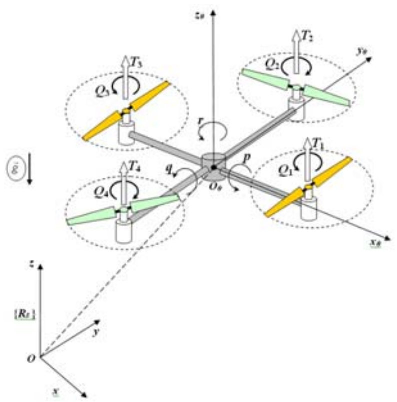

Fig. 1. Description of the quadrotor motion

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II. QUADROTOR MODELING

The quadrotor helicopter is actuated by four DC motors driving four rotors connected to the extremities of a crossing body (fig. 1). Varying the speed of all rotors, thereby changing the lift forces, generates the motion of the quadrotor. Two frames will be used to study the system motion: an inertial earth frame $\{R_E\}$ $(O, \mathbf{x}, \mathbf{y}, \mathbf{z})$, and a body-fixed frame $\{R_0\}$ $(O_0, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$, where O_0 is supposed to be at the mass center of the quadrotor. $\{R_0\}$ is related to $\{R_E\}$ by a vector (x, y, z) describing the position of the center of gravity in $\{R_E\}$, and three independent angles (α, β, γ). The relationship between the time variation of Euler angles and the components of ${}^0\mathbf{\Omega}_0$ is as follows [6, 7]:

$$ {}^0\mathbf{\Omega}_0 = \mathbf{H}\dot{\mathbf{\Theta}} \qquad (1a) $$

where $\mathbf{\Theta} = [\gamma\ \beta\ \alpha]^T$ and $\mathbf{H}$ is given by:

$$ \mathbf{H} = \begin{bmatrix} 1 & 0 & -s\beta \\ 0 & c\gamma & c\beta\, s\gamma \\ 0 & -s\gamma & c\beta\, c\gamma \end{bmatrix} \qquad (1b) $$

The rotation matrix ${}^E\mathbf{R}_0$ that defines the orientation of $\{R_0\}$ relative to the earth frame $\{R_E\}$ can be written in a general form as follows [24]:

$$ {}^E\mathbf{R}_0 = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} \qquad (2) $$

The linear velocity of the mass center of the crossing body O_0, expressed in $\{R_E\}$, is:

$$ {}^E\mathbf{V}_0 = [\dot{x}\ \dot{y}\ \dot{z}]^T \qquad (3) $$

In reference [6], a complete dynamic model including rotors dynamics and external perturbations is developed using a multibody approach. Hereafter, we consider a simplified dynamic model which is obtained under the following assumptions:

- The linear and angular momentums of rotors are neglected.
- Joints relating rotors to the crossing body are supposed perfect (no friction).
- The crossing body is supposed symmetrical and the corresponding inertia matrix $\mathbf{I}_0$ is given in $\{\boldsymbol{R_0}\}$ by: $\mathbf{I}_0 = diag\,(I_{xx}, I_{yy}, I_{zz})$.
- The translation velocity of the quadrotor is small, therefore the aerodynamic forces and the corresponding moment are neglected.

According to the momentum theory of rotors [25, 26], the aerodynamic efforts applied to the quadrotor are four trust efforts T_i and four resistive moments Q_i, (i = 1, ..., 4), given by:

$$ \begin{cases} T_i \mathbf{z}_0 = b\dot{\theta}_i^2 \mathbf{z}_0 \\ Q_i \mathbf{z}_0 = d\dot{\theta}_i^2 \mathbf{z}_0 \end{cases} \quad i = 1...4 \qquad (4) $$

The direct dynamic model of the quadrotor is [6] :

$$ \begin{cases} \ddot{x} = (c\alpha\, s\beta\, c\gamma + s\alpha\, s\gamma)\, u_1 \\ \ddot{y} = (s\alpha\, s\beta\, c\gamma - c\alpha\, s\gamma)\, u_1 \\ \ddot{z} = -g + c\beta\, c\gamma\, u_1 \\ \ddot{\gamma} = u_2 + \dot{\alpha}\, \dot{\beta}\, I_1 \\ \ddot{\beta} = u_3 + \dot{\alpha}\, \dot{\gamma}\, I_2 \\ \ddot{\alpha} = u_4 + \dot{\gamma}\, \dot{\beta}\, I_3 \end{cases} \qquad (5a) $$

where
$$ \begin{cases} u_1 = \sum_{i=1}^{4} T_i / m_0 \\ u_2 = l(T_2 - T_4)/I_{xx} \\ u_3 = l(T_3 - T_1)/I_{yy} \\ u_4 = (Q_4 + Q_2 - Q_1 - Q_3)/I_{zz} \end{cases} \qquad (5b) $$

and
$$ \begin{cases} l = \|\overrightarrow{O_0 O_i}\| \quad i = 1,...,4 \\ I_1 = (I_{yy} - I_{zz})/I_{xx},\ I_2 = (I_{zz} - I_{xx})/I_{yy},\ I_3 = (I_{xx} - I_{yy})/I_{zz} \\ b > 0 \text{ and } d > 0 \end{cases} $$

Note that:

- b and d are coefficients depending upon the blade's Reynolds Number, Mach number and angle of attack as well as other factors [27]. They are generally identified experimentally as done in ref. [7, 28].
- The quantities u_i can be seen as equivalent control inputs of our system since they are a linear combination of effective input torques τ_i given by:

$$ \tau_i = (-1)^{i+1}\, d\, \dot{\theta}_i^2 \qquad (\mathrm{i} = 1,..., 4) \qquad (6) $$

The inverse form of (5a) is possible:

$$ \begin{cases} u_1 = \sqrt{\ddot{x}^2 + \ddot{y}^2 + (g + \ddot{z})^2} & u_2 = \ddot{\gamma} - \dot{\alpha}\, \dot{\beta}\, I_1 \\ u_3 = \ddot{\beta} - \dot{\alpha}\, \dot{\gamma}\, I_2 & u_4 = \ddot{\alpha} - \dot{\gamma}\, \dot{\beta}\, I_3 \end{cases} \qquad (7) $$

The inverse dynamic model (7) allows us to compute the equivalent input controls as a function of the system kinematics.

III. STATEMENT OF THE OPTIMAL TRAJECTORY PLANNING PROBLEM

The quadrotor is required to move from an initial configuration to a final one; both are characterized by null velocities. Solving the optimal trajectory planning problem involves the determination of the transfer time T, the trajectory $\Im(t) = (x(t), y(t), z(t), \alpha(t), \beta(t), \gamma(t))$ and the corresponding input controls $\Gamma(t) = (\tau_1(t), \ldots, \tau_4(t))$ such as the initial and final states are matched, constraints are respected and a cost function is minimized. Hereafter, we adopt exclusively the minimum time cost function:

$$ F^{obj} = \int_0^T dt \qquad (8) $$

Boundary conditions inherent to the achievement of the desired task are those imposed on the quadrotor:

– Configuration $\Im(0) = \Im^{ini}$ and $\Im(T) = \Im^{fin}$ (9a)

– Velocity $\dot{\Im}(0)=\vec{0}$ and $\dot{\Im}(T)=\vec{0}$ (9*b*)

The other constraints that may have to be satisfied during the quadrotor fly are:

- *bounds on the quadrotor configurations*
$$\Im^{min} \le \Im(t) \le \Im^{max} \quad (10a)$$
- *bounds on the actuator velocities*:
$$\dot{\theta}_i^{min} \le \dot{\theta}_i(t) \le \dot{\theta}_i^{max} \quad i = 1, \ldots, 4 \quad (10b)$$
- *bounds on the actuator torques*:
$$\tau_i^{min} \le \tau_i(t) \le \tau_i^{max} \quad i = 1, \ldots, 4 \quad (10c)$$
- *obstacles avoidance :*
$$Col(\Im(t)) = false \quad (10d)$$

The constraint (10*a*) traduces the fact that the quadrotor will move in a limited space and its orientation should be compatible with the simplification hypotheses (small angles) proposed in [6]. The constraint (10*b*) arises from the fact that each rotor should turn in a specific direction and the corresponding motor has a limited speed. The limited power of actuators implies also bounds on input torques as indicated in (10*c*). These limits are systematically projected on the amplitude of the equivalent control inputs u_i, (i =1, …, 4). When obstacles are present in the workspace, the constraint (10*d*) will hold during the quadrotor fly. The function *Col* indicates whether the robot at a given configuration is in collision with an obstacle or not.

Moreover, the quadrotor has only four motors actuating the four rotors whereas the system has six *dof.* This means that the system is underactuated. The exploitation of such a system involves the identification of the dependency existing between the six *dof*, i.e. between the elements of $\Im(t)$.

From the three first equations of (5*a*), it is straightforward to write:

$$\begin{cases} \ddot{x}\, c\alpha + \ddot{y}\, s\alpha - (\ddot{z} + g) tg\beta = 0 \\ \ddot{x}\, s\alpha - \ddot{y}\, c\alpha - \sqrt{\ddot{x}^2 + \ddot{y}^2 + (g + \ddot{z})^2}\, s\gamma = 0 \end{cases} \quad (11a)$$

Relation (11*a*) represents two nonholonomic constraints of second order traducing the dependency existing between the quadrotor kinematic parameters. Using this relation, it is possible to deduce two parameters from a non linear combination of the four other parameters (and their time derivatives), e.g.:

$$\beta = arctan\left(\frac{\ddot{x}\, c\alpha + \ddot{y}\, s\alpha}{\ddot{z} + g}\right), \gamma = arcsin\left(\frac{\ddot{x}\, s\alpha - \ddot{y}\, c\alpha}{\sqrt{\ddot{x}^2 + \ddot{y}^2 + (g + \ddot{z})^2}}\right) \quad (11b)$$

In addition to the previous constraints, there is another constraint arising from the relationship existing between the quantities u_i and the effective input torques τ_i, (i = 1, …, 4), which are proportional to $\dot{\theta}_i^2$. In fact, from relations (4), (5*b*) and (6), it is easy to establish that:

$$(u_1\, u_2\, u_3\, u_4)^T = \mathbf{\Lambda}\left(\dot{\theta}_1^2\ \dot{\theta}_2^2\ \dot{\theta}_3^2\ \dot{\theta}_4^2\right)^T \quad (12a)$$

or
$$(u_1\, u_2\, u_3\, u_4)^T = \frac{1}{d} diag(1,-1,1,-1)\, \mathbf{\Lambda}\, (\tau_1\ \tau_2\ \tau_3\ \tau_4)^T \quad (12b)$$

where:

$$\mathbf{\Lambda} = \begin{pmatrix} \frac{b}{m_0} & \frac{b}{m_0} & \frac{b}{m_0} & \frac{b}{m_0} \\ 0 & \frac{lb}{I_{xx}} & 0 & -\frac{lb}{I_{xx}} \\ -\frac{lb}{I_{yy}} & 0 & \frac{lb}{I_{yy}} & 0 \\ -\frac{d}{I_{zz}} & \frac{d}{I_{zz}} & -\frac{d}{I_{zz}} & \frac{d}{I_{zz}} \end{pmatrix}$$

Hence, the feasibility of any set of inputs u_i, i = 1,…, 4, is conditioned by the following constraint:

$$\mathbf{\Lambda}^{-1}(u_1\quad u_2\quad u_3\quad u_4)^T > \vec{0} \quad (13)$$

The satisfaction of inequality (13) ensures the existence of the vector $\left(\dot{\theta}_1^2\, \dot{\theta}_2^2\, \dot{\theta}_3^2\, \dot{\theta}_4^2\right)^T$. Hence, the corresponding input torques may be achieved by the quadrotor actuators. In contrast, violating this constraint means that the considered trajectory involves unfeasible control inputs, thus it should be modified.

The problem defined by relations (8, 9*a*-*b*, 10*a*-*d*, 11*a*, 13) is a generic optimal control problem and may be solved using either direct or indirect methods [29]. In the following section, we present a simple direct numerical method able to treat this problem.

IV. The Proposed Approach

Using (11*b*), it is possible to express all elements of the optimization problem (cost function and constraints) as a function of the time evolution of only four configuration parameters, namely {$x(t)$, $y(t)$, $z(t)$, $\alpha(t)$} and their time derivatives. In fact, the definition of a trajectory candidate may be done by defining the evolution of {$x(t)$, $y(t)$, $z(t)$, $\alpha(t)$}, for $t \in [0\quad T]$, while accounting for boundary conditions (9) and bounds (10a) on the quadrotor configurations. Whereas, from relation (11*b*) the time evolution of the two other configuration parameters can be deduced. By application of the inverse dynamic model (7), the input controls are calculated. After that, the remaining constraints (10*b*, 10*c*, 10*d*, 13) may be checked.

The remaining question is how to generate the trajectory candidates? For this purpose, we propose to consider any trajectory candidate as a composition of : (*i*) a parametric form $\mathbf{P}(\lambda) = (x(\lambda), y(\lambda), z(\lambda), \alpha(\lambda)), \lambda \in [0\quad 1]$, defining the quadrotor path, and (*ii*) a monotonically increasing function $\lambda(t)$, $t \in [0\quad T]$, specifying the motion on this path. i.e.:

$$\Im(t) = \mathbf{P}(\lambda) \circ \lambda(t) \quad (14)$$

The optimal evolutions of $\mathbf{P}(\lambda)$ and $\lambda(t)$, which are approximated using B-spline functions fitting a set of control points, are found using a nonlinear optimization technique.

The construction of $\mathbf{P}(\lambda)$ and $\lambda(t)$ may be done as follows:

- Generate $\mathbf{P}(\lambda), \lambda \in [0\quad 1]$ (Fig.2*a*), by means of a quintic B-spline model. The 5th degree ensures the continuity of the 2nd order derivatives of angles β and γ. This B-spline is built by using a set of N_p control points (5 points at least) [30], and generated within the admissible workspace defined by (10*a*) and accounting for constraints (9*a*, 10*d*).

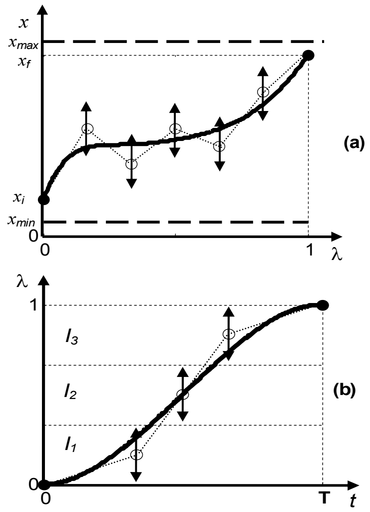

Fig. 2. Path and motion profiles
a) A path component profile b) The motion profile

- Build the motion profile $\lambda(t)$ on the interval [0 T] using a quintic B-spline model (Fig.2b), generated by N_m control points uniformly distributed along the time scale, and accounting for the following boundary conditions:

$$\lambda(0) = 0 \quad \dot{\lambda}(0) = 0 \quad \ddot{\lambda}(0) = 0 \quad \dddot{\lambda}(0) = 0$$

and $$\lambda(T) = 1 \quad \dot{\lambda}(T) = 0 \quad \ddot{\lambda}(T) = 0 \quad \dddot{\lambda}(T) = 0$$

These conditions ensure the compatibility of the resulting trajectory $\Im(t)$ with constraints (9*b*). The increasing monotony of the motion profile is ensured by generating the Nm control points in I_{Nm} intervals such as (Fig.2 b):

$$I_i = \left[\frac{i-1}{N_m}, \frac{i}{N_m}\right] \quad i = 1,\ldots, N_m$$

The design parameters of $\mathbf{P}(\lambda)$ and $\lambda(t)$, which are the coordinates of the B-spline function control points and the value of T, become the unique unknowns of the trajectory generation problem. Their optimal values may be found easily using, for example, the sequential quadratic programming technique [31].

V. Simulation Results

Hereafter, three examples are discussed. The two first examples show the possibility of applying the proposed approach for a point to point motion with a free or imposed path between two limit configurations. These examples are executed in a free-obstacle space. The third example shows the ability of the approach to handle the trajectory planning problem in an encumbered space. Also, this last example highlights that the necessary minimum time to move between two configurations is not given obligatory by a motion on the shortest path. These examples are simulated using parameters of Table 1.

TABLE I
Simulation Parameters

$m = 0.5\ kg$	$I_{xx} = 0.0622\ kg.m^2$	$z^{min} = 0;\ z^{max} = 14m$
$L = 0.2\ m$	$I_{yy} = 0.0733\ kg.m^2$	$\alpha^{max} = -\alpha^{min} = 10°$
$b = 4.74E\text{-}5$	$I_{zz} = 0.0964\ kg.m^2$	$\beta^{max} = -\beta^{min} = 10°$
$d = 2.35E\text{-}7$	$I_r = 1E\text{-}4\ kg.m^2$	$\gamma^{max} = -\gamma^{min} = 10°$
$g = 9.81 m.s^{-2}$	$x^{min} = 0;\ x^{max} = 14m$	$\dot{q}_i^{max} = 180 rd.s^{-1}$
$\tau^{max} = -\ \tau^{min} = 0.05$Nm	$y^{min} = 0;\ y^{max} = 14m$	

A. *Example* 1

The quadrotor is assumed to move from an initial configuration $\Im^{ini} = (1,1,0,0)$ to a final one $\Im^{fin} = (12,6,9,0)$ in a 3D non-encumbered space. A quintic B-spline, generated using nine free control points is used to represent the parameterized path $\mathbf{P}(\lambda)$. The motion on this path is built also by using a quintic Bspline but with only two free control points. The minimum transfer time found in simulation is $T = 6.25$ seconds. The path and the evolution of the position and the orientation of the quadrotor are depicted on Figure 3.

B. *Example* 2

In this example, the quadrotor is supposed to achieve the same displacement as in example 1, but along a predefined linear path. In this case, the path function $\mathbf{P}(\lambda)$ is already defined and only $\lambda(t)$ should be optimized. The motion on this path is always represented by a quintic B-spline generated with two free control points. The minimum transfer time found in simulation is $T = 7.22$ seconds and the evolution of the six dof of the quadrotor is given in Figure 4.

C. *Example* 3

In this example, the environment is cluttered with obstacles as shown in figures 5a and 5b and the motion is confined to a fixed altitude z = 5m. We are interested in finding the minimum time transfer linking the configurations $\Im^{ini} = (1.5, 0.5, 5, 0)$ and $\Im^{fin} = (8.5, 8.5, 5, 0)$. Two alternatives are studied: moving from $\Im^{ini}$ to $\Im^{fin}$ (i) freely or (ii) along the shortest safe path. In the first case, $\mathbf{P}(\lambda)$ is generated using a quintic B-spline fitting twelve free control points. The motion profile $\lambda(t)$ is constructed, in both cases, by using a quintic B-spline and six free control points. The minimum transfer time found for case (i) is $T = 10.57$ seconds, while for case (ii) is $T = 19.41$ seconds. The optimal evolutions of the four actuator torques, for both cases, are shown in Figures 5c and 5d successively.

VI. Conclusion

In this paper, the problem of minimum time trajectory planning has been addressed accounting for the main constraints inherent to the system and its environment. A simple direct numerical method, based on an adequate parametrization of the quadrotor trajectory and using a nonlinear optimization technique, has been

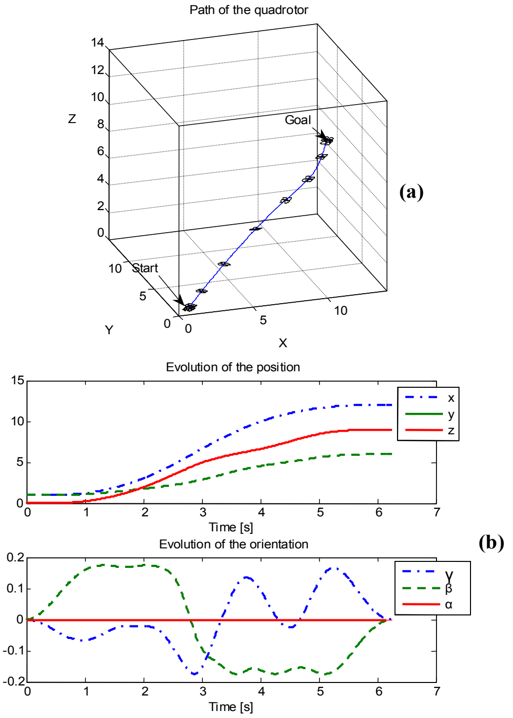

Fig. 3. Point to point motion with free path
a) Path of the quadrotor b) Evolution of the six dof of the quadrotor

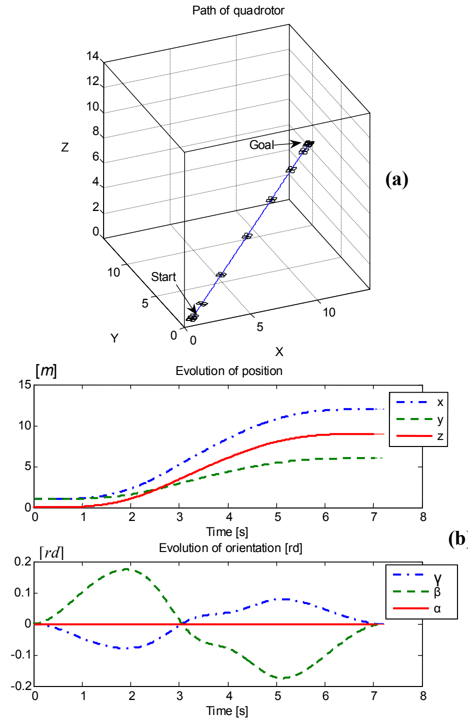

Fig. 4. Point to point motion with an imposed linear path
a) Path of the quadrotor b) Evolution of the six dof of the quadrotor

proposed. Any trajectory candidate was modeled as a composition of a path function and a monotonically increasing motion function. Both of these functions were generated using b-spline functions fitting a set of control points. In addition to the transfer time, the locations of these control points were considered as the principal unknowns of the trajectory optimization problem and can be found by using classical NL optimization techniques. The proposed approach is able to treat various problem formulations, involving for example, obstacles avoidance or path following. Although, the method was exposed for the minimum time transfer problems, it can easily be extended to handle more general cost function formulations.

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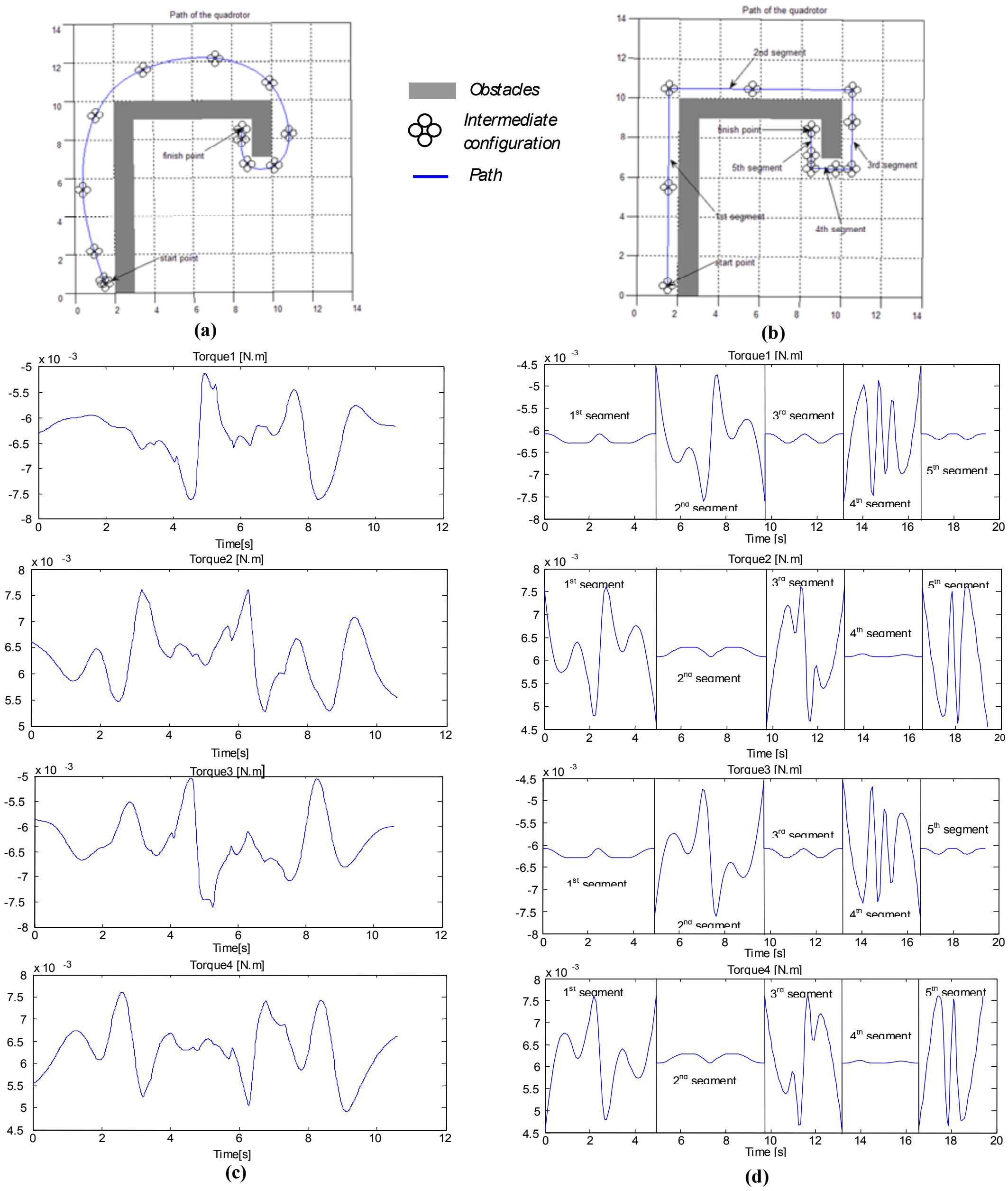

Fig. 5. Point to point motion of the quadrotor: a) The free Path of the quadrotor b) The imposed path of the quadrotor c) Evolution of actuators torques for the free displacements d) Evolution of actuators torques for the imposed displacements