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by

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One of the main outstanding problems for macroeconomic theory is that of accounting for the persistence of large scale unemployment. The rational expectations equilibrium theories of Lucas (1975), Sargent (1979, esp. Ch. 16) and others attempt to explain persistence by means of information lags, the effects of changes in capital stocks upon the marginal product of labor, and various adjustment costs. But even the most vigorous and effective proponents (e.g. Barro, 1981; Lucas, 1980) concede that these factors are unlikely to have a large enough effect to provide a satisfactory account of persistence in deep depressions.

Keynesian theories such as those of Patinkin (1965, Chs. 13, 14), Barro and Grossman (1976, Ch. 2), and Malinvaud (1977), which rely upon slowly adjusting wages and prices, and upon Clower's (1965) distinction between notional and effective demands[1] are often claimed to provide a more plausible account, on the grounds that prices are indeed slow to adjust (Gordon, 1981), and that the quantity-constraints of this approach make it consistent with the apparently involuntary nature of unemployment and with the observation that the typical business firm would usually be willing to increase its sales even at the current price (Hahn, 1983). There are, however, some well-known logical gaps in this approach.

The purpose of this paper is to explore an alternative approach based upon the common observation that the cost of transacting is generally larger the thinner the market. This idea has already been incorporated in two recent papers by Diamond, both of which deal with very explicit, but artificial models. The present paper shows how it can be incorporated less

explicitly into models of the same level of generality as standard macro models like that of Barro and Grossman (1971). The analysis retains many of the Keynesian features of the Barro-Grossman model, but eliminates some of its logical gaps. The formal argument is elementary, and less general than possible, for the purpose is to suggest the general fruitfulness of this approach rather than to attempt a definitive theory of unemployment.

Section 1 reviews the logical gaps of the non-clearing markets approach. Section 2 discusses the nature of transaction costs and shows how they can be introduced into the Barro-Grossman model. Section 3 shows how this introduction results in a concept of effective demand and supply even when prices are perfectly flexible. Section 4 takes a special case suggested by Diamond's analysis and shows two senses in which it could account for persistence. Section 5 discusses the meaning of involuntary unemployment in the light of this analysis. Section 6 discusses the connection between price flexibility and transaction costs, and section 7 offers some concluding observations.

1. Non-Clearing Markets

Our discussion in this section focusses upon Figure 1, reproduced in essence from Barro and Grossman (1971, p.86). Both the output and labor market are in excess supply at a real wage equal to its general equilibrium value, w^*. The supply of labor equals its notional demand, n^*. But the effective demand, as given by the schedule ABn', is only n', because collectively the firms find themselves unable to sell any more output than can be produced by this amount. Thus the amount $n^* - n'$ of unemployment exists.

Over time the money wage and the price level will fall in response to excess supplies. This will generally raise the aggregate demand for goods, shifting the vertical portion of the effective demand for labor schedule to the right, and thus increasing employment. Less than full employment will persist if the process of deflation is slow, or if the effect of deflation on aggregate demand is small. Involuntary unemployment will also persist, with w exceeding the supply price of labor, unless wages fall much more rapidly than prices, thus driving real wages down to the point where effective demand equals notional supply.

The main gap in this approach has been described succinctly by Barro (1979). It gives no clear account of why the mutually advantageous gains from trade implied by the fact that the supply price of labor at n' is less than its marginal product remain persistently unexploited. Defenders of the approach generally take this failure as a logical consequence of wage/price stickiness. But Barro points out that such stickiness can be explained by theories (e.g., Azariades, 1975) which predict that all potential gains from trade will be exhausted. He

supposes that people are aware of the exact nature of these potential trades and that their failure to execute them in the situation of Figure 1 can only reflect a perverse and unrealistic degree of irrationality.

A second gap has to do with the concept of effective demand. Each firm takes as given the real wage and a maximum amount of output that it can sell. But this begs the question of why a firm sees itself unable to move its sales constraint by undercutting its rivals. If firms made the conjecture most compatible with competitive analysis, namely that at slightly less than the going price it could supply the entire market, then the concept of effective demand would disappear,[2] for there would be no reason to take as given any quantity constraint that could be removed by a negligible reduction in price.

Third, no convincing explanation of the speed of wage and price adjustment has yet been provided. Of course this is also a logical gap in market-clearing theories, which fail to account for how the obvious co-ordination problems involved in finding a vector of market-clearing prices are solved. But the problem in this case is that when you ask why people remain without a job at the going wage in the situation depicted in Figure 1, the model gives you an answer that depends crucially upon the unexplained speeds of wage and price adjustment.

2. Transaction Costs

This paper argues that what's missing from the non-clearing markets approach that accounts for these gaps is a factor often stressed by supporters of the approach in their informal discussions of it; the costs of transacting. As Solow (1980) has observed, for example, the labor market does not function like a Walrasian auction house, for a variety of reasons, having to do with heterogeneity, indivisibility, inalienability, and moral hazard, all of which make it difficult to affect transactions. As Leijonhufvud (1968) has stressed,

it takes more than a willingness to work for the wage going for your services, or even slightly less, to become employed. You also have to find an employer willing to pay that wage, and to undertake all the costs of bargaining, screening, and relocating to conclude a deal. This can be especially difficult when potential employees are having sales problems of their own and have contractual commitments to other employees.

These problems arise in other markets as well. Laidler (1982) has argued cogently that the job of finding and executing the Walrasian equilibrium transactions in an economy is a non-trivial coordination problem that could plausibly take a lot of time.

The key to our approach is the common observation that transaction costs are highest in the thinnest of markets; that is, that the per-unit cost of transacting increases when the volume of trade in the market decreases. This observation refers not just to the well-known economies of scale resulting from lumpy set-up costs (e.g. Baumol, 1952; Tobin, 1956), but to a fundamental externality whereby one agent's trading costs in a market are increased by having less activity on the other side of the market. It usually refers to cross-market comparisons, as between over-the-counter shares versus regularly listed stock transactions, but it could apply equally to intertemporal comparisons within any given market, as when a decrease in the demand for housing results in an increase in a seller's expected time cost as well as a reduction in his expected sales price, or when a decrease in the demand for labor makes jobs harder to find as well as reducing wages.

Formally, this external economy of scale could be captured by showing how the per-unit costs of stock-out uncertainty faced by inventory-holding middlemen are reduced by an increase in the volume of trade. as argued by Howitt (1977) and by Clower and Howitt (1978). (This argument was implicit in Edgeworth's (1888) early economic application of the law of large numbers.) It could also be captured in a search model of any market where the expected time required to contact a potential trading partner depends upon the number of such partners. Indeed this externality is the essence of Jones's (1976) explanation of the emergence of money.

Diamond (1982a,b) has shown, using the latter approach, how this external economy of scale can explain "low-level" equilibria in very special models, which are suggestive of fixed price excess-supply equilibria but without the fixed prices. In these models, a widespread expectation of high costs of contacting trading partners can be self-fulfilling. It will discourage production, thereby resulting in a low volume of trade, thus bringing about the expected high cost by thinning out markets.

To apply this idea to more usual macro models in a more general (but less concrete) way than Diamond, we follow Hahn (1971), Hirshleifer (1973), Niehans (1971), and others, by supposing that traders are convened by an auctioneer (or a set of specialist auctioneers) able to find market-clearing wages and prices at no cost, but unable to arrange the trades costlessly. There are unspecified trading institutions in place that reduce but do not eliminate the trading costs faced by households and firms. In order to implement any trading plan each transactor must use a combination of resources restricted to lie in some feasible set. To incorporate the external economy of scale we suppose that this feasible set depends upon the volume

of trade in each market, which each agent takes as given.

Consider a simple monetary economy with only two markets--for labor and output, and two types of traders--identical firms and identical households. Each firm takes as given the real wage w, and the per-firm volume of trade in the labor market $\bar{x}_n$ and output market $\bar{x}_q$. It chooses a level of production q and of sales x_q^s, the difference between them being available as inputs into the transaction process. It also chooses to buy x_n^d units of labor, to employ n units in production and the rest in transacting. Its objective is to maximize profits subject to the constraints imposed by technology and the trading institutions. Thus its decision problem is to

$$
(1) \quad
\begin{cases}
\displaystyle\max_{\{q,n,x_q^s,x_n^d\}} \quad x_q^s - w\, x_n^d \\[2ex]
\text{Subject to} \quad q = f(n), \quad T^f(x_q^s, x_n^d; q - x_q^s, x_n^d - n; \bar{x}_q, \bar{x}_n) \geqq 0
\end{cases}
$$

where $f(\cdot)$ is a usual smooth production function with positive and decreasing marginal product and $f(0) = 0$, and $T^f(\cdot)$ is a smooth transformation function characterizing the boundary of the set of feasible trading plans.

Assume:

$$
(2) \quad T_i^f < 0 \quad i=1,2; \quad T_i^f > 0 \quad i=3,\dots,6.
$$

These conditions identify selling and buying as outputs of the trading process, and specify the external economy of scale. Assume furthermore that the

activity of buying labor using labor as the only input is always productive; i.e., that:

$$(3) \qquad T_2^f + T_4^f > 0.$$

Necessary conditions for an interior maximum on the boundary of the transaction technology are:

$$(4) \qquad f'(n) = \frac{T_4^f}{T_3^f}, \quad w = \frac{T_2^f + T_4^f}{T_3^f - T_1^f}$$

The Lagrangian multiplier on the transaction technology is

$$(5) \qquad \lambda^f = \frac{1}{T_3^f - T_1^f} > 0.$$

Each household takes as given w, $\bar{x}_q$, $\bar{x}_n$, its initial holding of real money balances, $\frac{M}{P}$, and its profit income π; and chooses its consumption y, expenditure of leisure ℓ, demand for real balances m, sale of labor x_n^s, and purchase of output x_q^d, so as to maximize utility subject to a budget constraint and a transactions technology constraint. Its decision problem is:

$$(6) \qquad \left\{ \begin{array}{l} \underset{\{y,\ell,m,x_q^d,x_n^s\}}{\text{Max}} \quad U(y,\ell,m) \\[2ex] \text{Subject to:} \quad x_q^d + m = w x_n^s + \pi + \frac{M}{P} \\[2ex] \text{and:} \quad T^h(x_q^d,x_n^s;x_q^d-y,\ell-x_n^s;\bar{x}_q,\bar{x}_n;m) = 0 \end{array} \right.$$

The smooth utility function U is increasing in y and m and decreasing in ℓ. The variable m enters to indicate the usefulness of money carried over to

next period. The transformation function T^h is perfectly analogous to T^f except for the additional term m, inserted to recognize the role of money in reducing transaction costs during the current period.[3] Assume:

(7) $\quad T_i^h < 0 \quad i=1,2; \quad T_i^h > 0 \quad i=3,\ldots,7; \quad T_1^h + T_3^h > 0.$

First-order conditions for an interior maximum are:

(8) $\quad -\dfrac{U_2}{U_1} = \dfrac{T_4^h}{T_3^h}, \quad w = \dfrac{T_4^h - T_2^h}{T_3^h + T_1^h}$

The Lagrangian multipliers on the transaction technology and the budget constraint are, respectively:

(9) $\quad \lambda^h = \dfrac{U_1}{T_3^h} > 0, \quad \mu^h = U_1 \left(1 + \dfrac{T_1^h}{T_3^h}\right) > 0.$

This set-up rules out several potentially interesting phenomena. First, no one faces a tradeoff between price and trading cost, as would be usual in most search markets. This tradeoff would probably be interesting to explore in the context of the issue of macroeconomic price flexibility, from which we abstract. Second, the marketing efforts on one side of the market have no direct effect on the trading costs of those on the other side, as when more advertising reduces search costs or when more active search by

workers reduces the need for recruiting effort. This could be introduced, for example, by including $(q - x_q^s)$ and $(x_n^d - n)$ as arguments of T^h as well as T^f. Third, no dynamic set-up costs are included such as exist when ongoing

relationships between specific trading partners allow a trader to continue trading as before without repeating the screening, bargaining, relocating, and other costs that were incurred at the time of the first contact with a trading partner. This might be included by introducing some dynamic elements into the problem, as in Sections 4 and 6 below. Fourth, guaranteeing the existence of unique continuous solutions to these problems will generally require convexity properties of T^f and T^h that rule out the lumpy set-up costs characteristic of trading technologies [see, for example, Heller, 1972]. Fifth, in the interest of simplicity, we abstract from all aspects of risk and uncertainty, as well as from any explicit treatment of the concepts of moral hazard, indivisibility, mobility, etc., usually invoked to explain the cost of transactions.

3. Effective Demand

Assume that given any values of the parametric variables there exist unique, continuous interior solutions to the decision problems (1) and (6). Then they can be written as the demand and supply functions:

$$(10) \quad x_q^s = \hat{x}_q^s(w; \bar{x}_q, \bar{x}_n), \quad x_n^d = \hat{x}_n^d(w; \bar{x}_q, \bar{x}_n)$$

and:

$$(11) \quad x_q^d = \hat{x}_q^d(w, \pi + \frac{M}{P}; \bar{x}_q, \bar{x}_n), \quad x_n^s = \hat{x}_n^s(w, \pi + \frac{M}{P}; \bar{x}_q, \bar{x}_n).$$

The realized quantities $\bar{x}_q$ and $\bar{x}_n$ affect demands and supplies, through the external cost of trading, independently of prices. As Clower (1965) observed, these independent effects are what distinguish Keynes's notion of effective

demand from the Walrasian notional demand. But (10) and (11) do not raise the problem discussed in Section 1, because each agent sees himself as free to trade more or less than $\bar{x}_q$ or $\bar{x}_n$.

Models of non-clearing markets with "manipulable" rationing schemes, like those of Heller and Starr (1979), and Gale (1979) bear a formal resemblance to the present set-up, for the T^f and T^h functions could perhaps be interpreted as rationing functions that indicate how much can potentially be traded (e.g. x_n^s) as a function of the actual amounts (e.g. $\bar{x}_n$) and of the demand or supply submitted to the market (e.g. ℓ). The crucial difference, however, is that according to this interpretation it would be x_n^s rather than ℓ which entered the utility function.

As with manipulable rationing schemes this approach generally permits the realized quantity in any market to affect the demands and supplies in that market, so as to accommodate such phenomena as discouraged workers. The Barro-Grossman approach, also followed by Benassy (1975) and others, does not permit such direct influences. Their approach also has the well-known disadvantage (e.g. Grandmont, 1977) that the demands and supplies of a trader cannot be derived from any single unified decision problem. The other commonly used approach of Drèze (1975), also permits direct effects, but prohibits traders from attempting to exceed their rationing constraint.

Another feature of this approach is that the realized quantity in one market will generally enter into the choice functions of agents on both sides of the market, in contrast to the usual non-clearing markets approach where only those on the long side are affected.

Finally, the dependency of choices on realized quantities does not depend in this approach upon the state of excess demands and supplies. Contrary to the usual approach it remains even if all markets are cleared.

The auctioneer in this approach finds values of w and P that clear both markets. To establish a full equilibrium he must also provide each agent with accurate information about the realized quantities $(\bar{x}_q, \bar{x}_n)$ and profits π, so that the expectations upon which choices are made are not subject to revision. Equilibrium is defined as a vector $(w, P, \bar{x}_q, \bar{x}_n, \pi)$ such that:

$$(12) \quad \hat{x}_q^d(w, \pi + \tfrac{M}{P}; \bar{x}_q, \bar{x}_n) - \hat{x}_q^s(w; \bar{x}_q, \bar{x}_n) = 0$$

$$(13) \quad \hat{x}_n^d(w; \bar{x}_q, \bar{x}_n) - \hat{x}_n^s(w, \pi + \tfrac{M}{P}; \bar{x}_q, \bar{x}_n) = 0$$

$$(14) \quad \bar{x}_q - \hat{x}_q^s(w; \bar{x}_q, \bar{x}_n) = 0$$

$$(15) \quad \hat{x}_n^d(w; \bar{x}_q, \bar{x}_n) - \bar{x}_n = 0$$

$$(16) \quad \hat{x}_q^s(w; \bar{x}_q, \bar{x}_n) - w\hat{x}_n^d(w; \bar{x}_q, \bar{x}_n) - \pi = 0.$$

If such an equilibrium does not exist, with a finite price level, then the degenerate situation: $(w, P, \bar{x}_q, \bar{x}_n; \pi) = (w, \infty, 0, 0, 0)$ of no activity provides an equilibrium, under the assumption that $T^f(a, b; 0, 0) < 0$ if $(a,b) \neq 0$, $T^h(c, d; 0, 0, 0) < 0$ if $(c,d) \neq 0$, and $T^f(0) = T^h(0) = 0$, i.e. that when no one is participating in any market and no money is held it is prohibitively expensive to trade in any market. Under what circumstances a non-degenerate equilibrium will exist is an open question.

"Solving" these equilibrium conditions requires both price and quantity adjustment. If (w,P) solve (12) and (13) then markets will clear in the usual sense, but equilibrium will not prevail unless $(\bar{x}_q, \bar{x}_n)$ solve (14) and (15). The quantity adjustments required can be thought of as a multiplier process, for (14) and (15) show how, for given prices, choices depend upon quantities and quantities upon choices. This suggests that, rather than being substitutes, as is sometimes thought, price and quantity adjustment are complementary and ought to be integrated within a single theory.

Unexploited gains from trade will persist in any equilibrium, despite full rationality and full market adjustment. The first-order conditions (4) and (8), together with conditions (2), (3), and (7), imply that $f'(n) > -\dfrac{U_2}{U_1}$. In terms of Figure 1 this means that S lies above D, implying unexploited gains from trade if transaction costs are ignored.

Even taking transaction costs into account there are unexploited gains. For consider an increase in the volume of trade on each market, with

$$d\bar{x}_q = dx_q^s = dx_q^d = 1$$

$$d\bar{x}_n = dx_n^d = dx_n^s = \text{Max}\left\{0, -\frac{T_1^f - T_3^f + T_5^f}{T_2^f + T_4^f + T_6^f}\right\} .$$

This change is feasible for the firm, since $dT^f \geqq 0$ if $dq = dn = 0$. By the usual envelope result, the change in profit will be

$$d\pi = \pi_2^*(w,\bar{x}_q,\bar{x}_n) + \pi_3^*(w,\bar{x}_q,\bar{x}_n)d\bar{x}_n$$

$$= \lambda^f(T_5^f + T_6^f d\bar{x}_n) > 0$$

where $\pi^*(\ )$ is the indirect profit function of the problem (1).

The change will also be feasible for the household, since it can choose any $(dy, d\ell, dm)$ satisfying $T_3^h dy - T_4^h d\ell = (T_1^h + T_3^h + T_5^h) + (T_2^h - T_4^h + T_6^h) d\bar{x}_n$,

and $dm = w dx_n^s - dx_q^d + d\pi + d(\frac{M}{P}) = w d\bar{x}_n - d\bar{x}_q + d(\bar{x}_q - w\bar{x}_n) + 0 = 0$. The change in utility will be

$$dU = u_2^* d\pi + u_3^* + u_4^* d\bar{x}_n = \mu^h d\pi + \lambda^h (T_5^h + T_6^h d\bar{x}_n) > 0$$

where $u^*(w, \pi + \frac{M}{P}, \bar{x}_q, \bar{x}_n)$ is the indirect utility function from (6).

4. Persistence of Unemployment

Unemployment in this model can be interpreted only as labor used by households in the activity of selling labor. To make clear the distinction between this and labor used in buying goods, and to bring out the possibilities of explaining persistence using this approach, consider the following special case, suggested by Diamond's model. Households have a utility function of the form $U(y, m) - c(\ell)$, with U homogeneous of degree one. (This eliminates income effects from the labor market.) Buying costs are absent, and selling costs consist of a given fraction of the good being sold, the size of which depends upon the volume of trade in that market. Thus the household will choose $x_q^d = y$, and will face a given $\bar{x}_n = n$ in equilibrium; its transaction constraint is thus:[4] $x_n^s = (1-u(n))\ell$, with $u'(n) < 0$. Notice that $u(n)$ is the rate of unemployment. The supply of labor schedule $\hat{\ell}(\ w\ , n)$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (+)\ (-)$
is defined by the first-order condition:

(17) $w(1-u(n))\lambda = c'(\ell)$

where $\lambda = U_y = U_m$ is the constant marginal utility of income. The demand for output function can be written as:

(18) $y = \theta(w(1-u(n))\hat{\ell}(w,n) + \pi + \frac{M}{P})$

where $\frac{1-\theta}{\theta}$ is the slope of the household expansion path in (y,m) space $(0 < \theta < 1)$ with $\frac{U_y}{U_m} = 1$.

Similarly, the firm's demand for labor function can be written as $\hat{n}(\underset{(-)}{w}, \underset{(+)}{y})$, characterized by:

(19) $w = (1-v(y))f'(n)$,

where $v(y)$ is the per-unit marketing cost of the firm.

Assume that:

(20) $u'(n) < 0$ for all $n \geqq 0$, $v'(y) < 0$ for all $y \geqq 0$

(21) $1-u(n) + nu'(n) > 0$ for all $n > 0$

(22) $1-v(y) + yv'(y) > 0$ for all $y > 0$

(23) $u(n) > 0$, $v(y) > 0$ for all n,y

(24) $u(0) = v(0) = 1$

Inequality (20) specifies the external economy; (21) and (22) rule out the case where a greater final demand reduces selling costs by so much that it can be satisfied with a smaller supply;(23) rules out costless sales. The crucial assumption for what follows is (24), which asserts that when there is no final demand it is impossible to find a buyer.[5]

Equilibrium requires that:

(25) $\ell = \tilde{\ell}(n) \equiv \dfrac{n}{1-u(n)}$, and

(26) $y = \tilde{y}(n) \equiv \{y \,|\, y - (1-v(y))f(n) = 0\}$.

For future reference note that:

(27) $\tilde{\ell}'(n) = \dfrac{1-u(n)+nu'(n)}{(1-u(n))^2} > 0$ for all $n > 0$

(28) $\tilde{y}'(n) = \dfrac{(1-v(y))^2 f'(n)}{1-v(y)+yv'(y)} > 0$ for all n such that $\tilde{y}(n)$ is well-defined.

From (17), (19), (25) and (26), the equilibrium quantity of employment is thus determined by the condition:

(29) $(1-v(\tilde{y}(n)))f'(n) = \dfrac{c'(\tilde{\ell}(n))}{\lambda(1-u(n))}$

which are Keynes's two classical postulates, in terms of "effective" marginal product and marginal disutility. Given this level of employment, the real wage will equal each side of (29), the level of output and of final demand will be f(n) and $\tilde{y}(n)$ respectively, and the price-level will be determined, from (18), according to the condition:

(30) $\dfrac{M}{P} = (\dfrac{1-\theta}{\theta})\tilde{y}(n)$

which is a "Cambridge quantity theory" equation.

Two more assumptions are required:

(31) $c'(0) \geqq 0$; $c'(\ell) \to \infty$ as $\ell \to \bar{\ell} > 0$; $c''(\ell) > 0$ for all $\ell \in (0,\bar{\ell})$.

(32) $f(0) = 0$; $(f'(n) > 0, f''(n) < 0)$ for all $n > 0$; $f(n) \to \infty$ as $n \to \infty$.

Figure 2 depicts (29). The curves with height $f'(n)$ and $c'(\tilde{\ell}(n))/\lambda$ are the usual notional demand and supply curves, as in Figure 1. Point A

would be the Walrasian equilibrium if $\tilde{\ell}$ were the identity function. The curves D and S are the "effective" curves given by the two sides of (29). By ℓ'Hôpital's rule, $\tilde{\ell}(n) \to \underline{\ell} \equiv \frac{-1}{u'(0)} > 0$ as $n \to 0$. From this, (24), and (31), the height of S becomes infinite as $n \to 0$ and as $n \to \tilde{\ell}^{-1}(\overline{\ell})$.

To characterize D we need to determine the range over which the implicit function $\tilde{y}(n)$ defined by (26) exists. Note that its inverse function, $\tilde{y}^{-1}(y) \equiv f^{-1}(\frac{y}{1-v(y)})$ is well defined and strictly increasing for all $y > 0$. Furthermore, from (22) and (32), $\tilde{y}^{-1}(y) \to \infty$ as $y \to \infty$. By (29) and ℓ'Hôpital's rule, $\tilde{y}^{-1}(y) \to \underline{n} = f^{-1}(\frac{-1}{v'(0)}) > 0$ as $y \to 0$. Thus the domain of $\tilde{y}$ is the range of $\tilde{y}^{-1}$; i.e. $(\underline{n}, \infty)$, with $\underline{n} > 0$.

From this and (24) it follows that $v(\tilde{y}(n)) \to 1$ as $n \to \underline{n}$, and thus D falls to the horizontal axis as $n \downarrow \underline{n}$, and cannot be extended below $\underline{n}$. Furthermore, since $f'(n)$ exceeds the height of D and is falling as n increases, therefore the height of D is bounded as $n \to \tilde{\ell}^{-1}(\overline{\ell})$.

It follows that an equilibrium must lie in the interval $(\underline{n}, \tilde{\ell}^{-1}(\overline{\ell}))$, and that S lies above D as $n \downarrow \underline{n}$ and as $n \uparrow \tilde{\ell}^{-1}(\overline{\ell})$. Therefore there must exist an even number of intersections of S with D, except in the razor's-edge case of tangency. As drawn in Figure 2 there are two equilibria. But since the unchanged curvature shown in Figure 2 cannot be guaranteed, there may be more than two. If D and S fail to intersect, the only equilibrium is the above-mentioned degenerate one of no activity.

Equilibria with lower n involve higher unemployment. Furthermore, as the appendix shows, they yield lower household utility. Thus one potential explanation of persistence provided by this approach is that in cases such as this if there exist any non-degenerate equilibria at all then there must exist at least one inferior equilibrium with low activity levels and a high rate of unemployment, which will persist unless disturbed.

Dynamic considerations must be added to see if these low activity levels will persist even when disturbed. A full treatment would include the price-quantity adjustments underlying the equilibrium. However, some insights may be gained by deferring this difficult task and asking what happens if there is simply a lag in the transaction costs; if the per unit costs of selling at t are $u(n_{t-1})$ and $v(y_{t-1})$. This might be rationalized by supposing that costs are reduced by "contacts" in a market, which are built up in the course of trading, take one period to develop, and last one period. Each period the equilibrium quantities (n_t, y_t) are determined according to the conditions:

$$(33) \qquad f'(n_t)(1-v(y_{t-1}))(1-u(n_{t-1}))\lambda - c'\left(\frac{n_t}{1-u(n_{t-1})}\right) = 0$$

$$(34) \qquad f(n_t)(1-v(y_{t-1})) - y_t = 0$$

These equations describe a moving equilibrium with perfect foresight. The appendix confirms what intuition suggests from Figure 2. The even-numbered equilibria such as B where the difference between the demand and supply prices is a decreasing function of employment are locally stable, and the odd ones locally unstable.

Two conclusions follow. First, low-level equilibria can be locally stable if these are more than the minimal number 2. Second, because of the non-uniqueness, the highest-level equilibrium can be stable locally but not globally; thus the model can exhibit Leijonhufvud's (1973) "corridor effects".

In the neighborhood of a stable equilibrium (n*,y*) the time path of n_t is approximately $n_t = n* + k_1\lambda_1^t + k_2\lambda_2^t$, where λ_1 and λ_2 are the roots of the linear approximation to the system. The appendix shows that both roots

are real, positive, and less than unity. Therefore $\Delta n_t = n_{t+1} - n_t$ never
reverts permanently to zero in finite time, and can change sign at most
once.

Thus a second sense in which the approach might be able to account
for persistence is that of Lucas (1975). Movements in employment, and
hence in unemployment, will generally show positive serial correlation. As
Barro (1981) has observed, such explanations generally require some "capital"
stock to carry forward the mistakes of the past. In this case the stock
consists of contacts between trading partners. A large increase in unemploy-
ment takes time to reverse itself because the lost contacts take time to be
re-established.

5. Involuntary Unemployment

Despite the fact that markets are clearing, the unemployment in this
model can be described as involuntary. As Patinkin (1965, pp. 313-315) has
observed, behavior is "involuntary" only if it takes place under some
"abnormal" constraint. The main problem that many critics seem to find with
the rational expectations equilibrium approach is that the only thing
"abnormal" with the constraints facing unemployed workers in those models
seems to be a lower than normal perceived real wage. This, they argue, does
not fit well with direct observations of labor markets during depression,
or with the absence of any marked pro-cyclical pattern in real wages.

The non-clearing market approach supposes that in addition to a real
wage the workers face a quantity constraint on the amount they can sell.
During a depression this quantity constraint is "abnormally" severe. This
account suffers from the problem referred to in Section 1 above that it

depends upon unemployed workers being unable or unwilling to undercut their rivals.

In the present approach unemployment is involuntary because of the abnormally large cost of selling labor during a depression. These costs can be interpreted as the costs of contacting potential employers who are actively hiring. This difficulty of finding a job is indeed what observers refer to when describing unemployment as involuntary. Our approach shows at least that this aspect can be incorporated in a very general way into formal equilibrium analysis.

6. Price Adjustment and Transaction Costs

One approach that has addressed the question of price formation and the possibility of a transactor moving his quantity constraints by offering to trade at different prices is that of Negishi (1976), Hahn (1978) and Woglom (1982). These authors all suppose that price-setting sellers face a kinked perceived demand curve which discourages them from changing price. As in oligopoly theory this kink yields an indeterminacy to equilibrium. Small enough changes in the overall level of demand will not persuade anyone to change price, and will therefore result in a changed level of output and employment.

While these authors have made interesting contributions to filling some of the gaps of the non-clearing markets approach, they have still not provided a very clear account of persistent unemployment. Negishi's analysis makes sense only if the wage rate in the economy is set by a single monopoly union, leaving the unemployed workers no opportunity to undercut the union-set wage. Woglom refrains from dealing explicitly with unemployment by assuming that the labor market clears in the usual sense. Hahn's analysis is done in

quite general abstract terms where labour is not distinguished from other
tradable objects. But if one is to interpret it as explaining persistent
unemployment, then it seems to say that workers remain unemployed because the
perceived rightward elasticity of demand for their services is so low as to
discourage them individually from offering to work for less.

The most obvious problem with this approach is how to account for the
kink. The traditional oligopoly explanation of conjectural rivals' asymmetric
reactions makes little sense in the case of the wage decisions of an individual
household. More recent explanations (Stiglitz, 1979) depend upon an
asymmetry between existing customers and potential searching customers.
An increase in price will drive away some existing customers, who
leave to search, but a reduction won't attract any more new customers,
who can learn of the price change only after having searched. The difficulty
with applying this to labor markets is that unemployed workers typically
have no existing customers.

These recent explanations clearly depend upon transaction costs similar
to those underlying the present approach. In order for the idea of existing
customers to make sense, there must be a positive marginal cost to con-
tacting additional trading partners. The rest of this section is intended
to confirm this dependency of price-inflexibility upon transaction costs by
showing that the same kind of indeterminacy follows from a suitably modified
version of the present approach, without the assumption that prices are set
by individual traders facing kinky supply or demand schedules.

Suppose that the main cost of selling is that of contacting new
trading partners when an increase in sales is attempted, and that no costs

are incurred when trading at the same level as the previous period. Specifically, suppose that the per-unit costs u(n) and v(y) are incurred only on sales in excess of those of the previous period.[6] Then the Kuhn-Tucker conditions for the firm and household in a dynamic equilibrium with $\ell = \ell_{-1}$ and $n = n_{-1}$ are

(35) $f'(n)(1-v(y)) \leqq w \leqq f'(n)$, and

(36) $\dfrac{c'(\ell)}{\lambda} \leqq w \leqq \dfrac{c'(\ell)}{\lambda(1-u(n))}$.

To interpret (35) notice that an increase in employment will cost the firm w but will yield the marginal product minus the additional marketing cost $f'(n)v(y)$. The first inequality says that to keep employment constant the firm must find such an increase unprofitable. The second inequality says that a decrease in employment, which saves the cost w but does not save on the marketing costs, which have already been incurred, must also be unprofitable. A similar interpretation can be given to (36).

Equilibrium requires $\{w,y,n,\ell\}$ to satisfy (35) and (36) as well as the market-clearing conditions $y = f(n)$ and $\ell = n$. The functions $\tilde{y}$ and $\tilde{\ell}$ no longer come into play because in equilibrium the volume of trade is constant, so no selling costs are incurred. Figure 3 illustrates the range of possible equilibria, which consist of all the points in the two shaded regions. Note that the existence of at least two separate regions depends upon S and D having the same shapes (U and inverted U) as in Figure 2. This will be the case if $c'(0)$ is strictly positive and $f'(0)$ is finite. Otherwise D and S may intersect only once. In any event there will be a set of equilibria of full measure in the w-n space. Indeed if D and S do not intersect, then any level of employment between 0 and the Walrasian equilibrium amount $\ell*$ will be consistent with equilibrium. As in Hahn's analysis, the Walrasian equilibrium will always be an equilibrium in this setup.

7. Conclusions

This paper has outlined a general approach to the theory of unemployment, which focuses upon transaction costs and their dependency upon the volume of trade, rather than upon sticky prices or misinformation. These aspects were abstracted from in order to show that they may not be crucial for understanding why large-scale unemployment persists.

But the persistence question should be distinguished from that of why large-scale unemployment arises. Existing macro theories suggest that the costs of gathering, and processing information, and the costs of coordinating price changes are crucial for understanding this question. Until some of these features are introduced, the present approach has no way of showing how unemployment will respond, for example, to an unanticipated change in the money supply, or of answering the question of whether fiscal or monetary policy might be used in pump-priming fashion to jog an economy out of a low-level unemployment equilibrium.

Although the paper dealt with transaction costs in a rather general way, further progress in making the approach operational, and in incorporating informational factors explicitly, will probably require detailed models of market interaction in which the nature of the transaction costs facing individuals are derived from more fundamental considerations.

Footnotes

[1]Although Clower's analysis postulates Patinkin's, which is essentially unchanged from the first edition of his book in 1956, nevertheless Patinkin uses the concept of the effective demand for labor.

[2]This problem was recognized by Patinkin (1965, p. 323, n. 9). It has also been dealt with by Hahn (1978) using the technique of conjectural equilibria. Hahn's approach will be discussed in Section 6 below.

[3]We are following the usual convention of assuming all money to be held by households. This is to keep the exposition on familiar grounds rather than because of empirical evidence which, if anything, suggests that the reverse convention would be more appropriate.

[4]To put this in the form of T^h, rewrite it as $-u(n)x_n^s+(1-u(\bar{x}_n))(\ell-x_n^s) = 0$. Note that strict equalities must be allowed in (7) to include this as a special case.

[5]An example satisfying (20)~(24) is $u(n) = e^{-\alpha n}$, $\alpha > 0$. In this case $1 - u(n) + nu'(n) = 1 - (1 + \alpha n)e^{-\alpha n} > 0$ for all $n > 0$.

[6]This kind of adjustment cost differs from that of Section 4 in being internal to the firm as well as in implying that total transaction costs are zero in equilibrium.

Appendix

Consider two equilibria, n_0 and n_1, each satisfying (29), with $0 < n_0 < n_1$. Define the welfare function $\phi(n) = U(\tilde{y}(n), \frac{1-\theta}{\theta} \tilde{y}(n)) - c(\tilde{\ell}(n))$. We want to show that $\phi(n_0) < \phi(n_1)$. Next, define

$$\Psi(\bar{n}) = \max_{\{y, \ell, n \geq 0\}} U(y, \frac{1-\theta}{\theta} \tilde{y}(\bar{n})) - c(\ell) \text{ subject to } f(n)(1 - v(\tilde{y}(\bar{n}))) = y$$

and $\ell(1 - u(\bar{n})) = n$. First-order conditions for an interior maximum to this problem are

$$\lambda f'(n)(1 - v(\tilde{y}(\bar{n}))) - \mu = 0$$

$$- c'(\ell) + \mu(1 - u(\bar{n})) = 0$$

$$U_1 - \lambda = 0$$

Inspection of these and (29) reveal that $\Psi(\bar{n}) = \phi(\bar{n})$ whenever $\bar{n}$ satisfies (29). Furthermore, $\Psi(\cdot)$ is a strictly increasing function since, by the usual envelope results, $\Psi'(\bar{n}) = \tilde{y}'(\bar{n})(U_2(\frac{1-\theta}{\theta} - \lambda f(n) v'(y)) - \mu \ell u'(\bar{n}) > 0$. Therefore, $\phi(n_0) = \Psi(n_0) < \Psi(n_1) = \phi(n_1)$.

The dynamic system (33), (34) can be written as

$$\begin{bmatrix} n_t \\ y_t \end{bmatrix} = F \begin{bmatrix} n_{t-1} \\ y_{t-1} \end{bmatrix}$$

since the Jacobian

$$\Phi_1 = \begin{bmatrix} f''(n)(1-v)(1-u)\lambda - c''(1-u)^{-1}, & 0 \\ f'(n)(1-v) & -1 \end{bmatrix}$$

is non-singular. Take any equilibrium (n^*, y^*). Linearizing F around (n^*, y^*) yields:

$$
\begin{bmatrix} \hat{n}_t \\ \hat{y}_t \end{bmatrix} = \begin{bmatrix} \alpha_1/\alpha_2 & , & -v'f'(1-u)\lambda/\alpha_2 \\ f'(1-v)\alpha_1/\alpha_2 , & -v'(f')^2(1-u)(1-v)\lambda/\alpha_2 - v'f \end{bmatrix} \begin{bmatrix} \hat{n}_{t-1} \\ \hat{y}_{t-1} \end{bmatrix} = A \begin{bmatrix} \hat{n}_{t-1} \\ \hat{y}_{t-1} \end{bmatrix}
$$

where $\alpha_1 = u'(-f'(1-v)\lambda - c''n(1-u)^{-2}) > 0$, and $\alpha_2 = -(f''(1-v)(1-u)\lambda - c''(1-u)^{-1}) > 0$.

Necessary and sufficient conditions for this linear system to have roots of less than one in absolute value are:

(i) $|\det A| < 1$

(ii) $|\operatorname{tr} A| < (1 + \det A)$

(This follows directly from Goldberg, p. 172.)

Note that $\det A = -\dfrac{\alpha_1}{\alpha_2} v'f > 0$. So, (i) is equivalent to $\alpha_3 \equiv \alpha_1 v'f + \alpha_2 > 0$. Expanding α_1 and α_2 produces:

$$
\alpha_3 = \underbrace{u'(-f'(1-v)\lambda - c''n(1-u)^{-2})}_{(+)}v'f - (f''(1-v)(1-u)\lambda - c''(1-u)^{-1})
$$

Since $\dot{y}v' = (1-v)fv' < 0$, and $1 - v + yv' > 0$, therefore

(37) $\qquad -1 < v'f < 0$

so that:

$$
\alpha_3 > u'(f'(1-v)\lambda + c''n(1-u)^{-2} - f''(1-v)(1-u)\lambda + c''(1-u)^{-1}
$$

$$
= -[J + v'\tilde{y}'f'(1-u)\lambda] > -J
$$

where $J = f''(1-v)(1-u)\lambda - v'\tilde{y}'f'(1-u)\lambda - u'f'(1-v)\lambda - c''(1-u+nu')(1-u)^{-2}$.

Thus a sufficient condition for (i) is $J < 0$.

Condition (ii) is equivalent to:

$$
\alpha_4 \equiv \alpha_1(1+v'f) - \alpha_2(1+v'f) - v'(f')^2(1-u)(1-v)\lambda < 0.
$$

Expanding α_1 and α_2, then gathering terms, produces:

$$\alpha_4 = (1+v'f)(J + v'\tilde{y}'f'(1-u)\lambda - \frac{v'(f')^2(1-u)(1-v)\lambda}{1+v'f})$$

But, from (28), $\tilde{y}' = \frac{f'(1-v)}{1+v'f}$. Therefore $\alpha_4 = (1+v'f)J$, and, by (37), (ii) is equivalent to: $J < 0$. Therefore $J < 0$ is necessary and sufficient for both roots of A to be less than one in absolute values. But it is easy to see that wherever (29) is solved $J < 0$ is also equivalent to having D cut S from above to the left. This shows that the odd numbered equilibria of (29) are locally unstable and the even ones locally stable, except in the razor's edge case of tangency, where $J = 0$.

The fact that the off-diagonal terms of A are both positive implies that both its roots are real. Since tr A > 0 and det A > 0, they are both positive.

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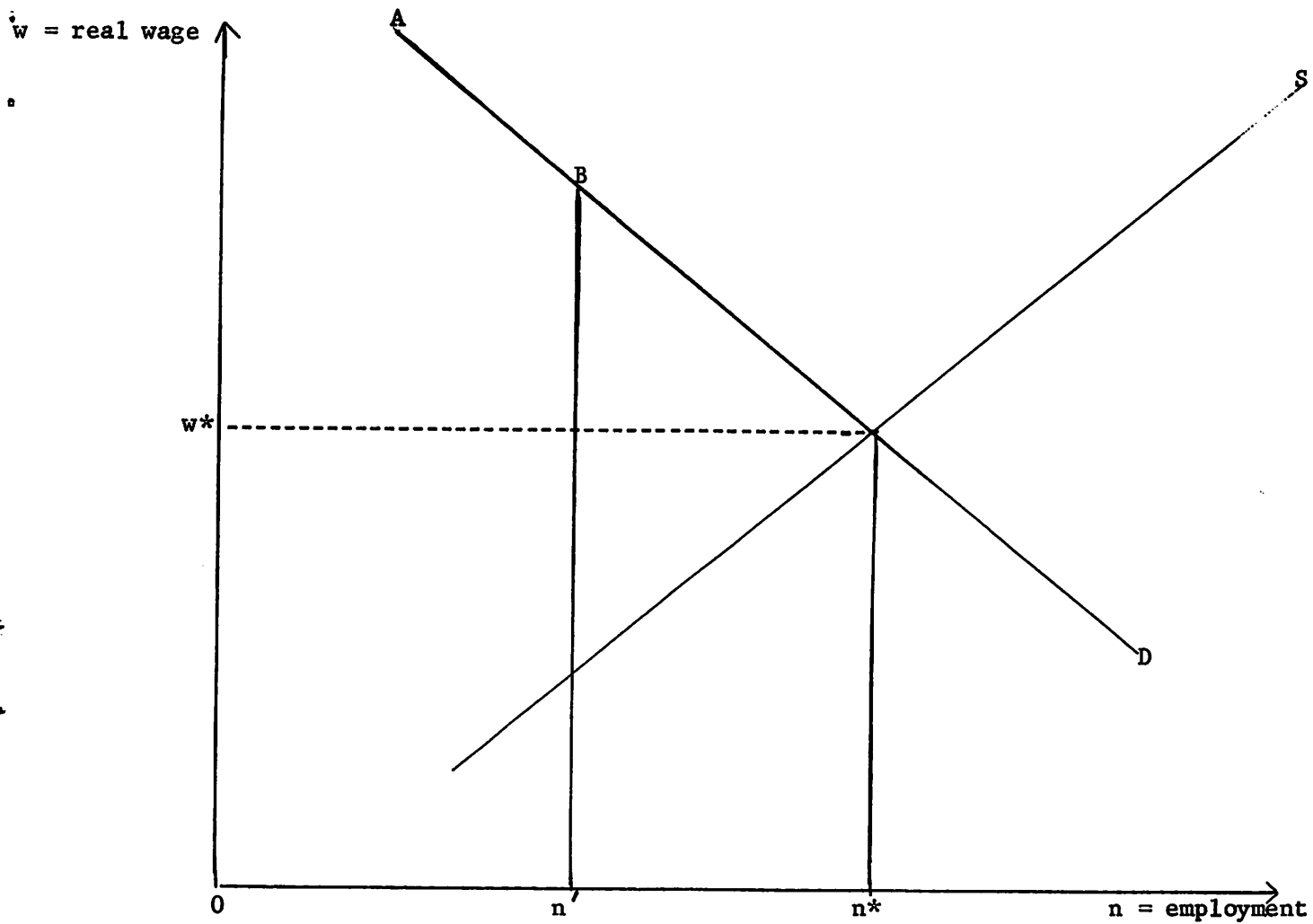   _Quarterly Journal of Economics_ 97 (February 1982): 89-107.
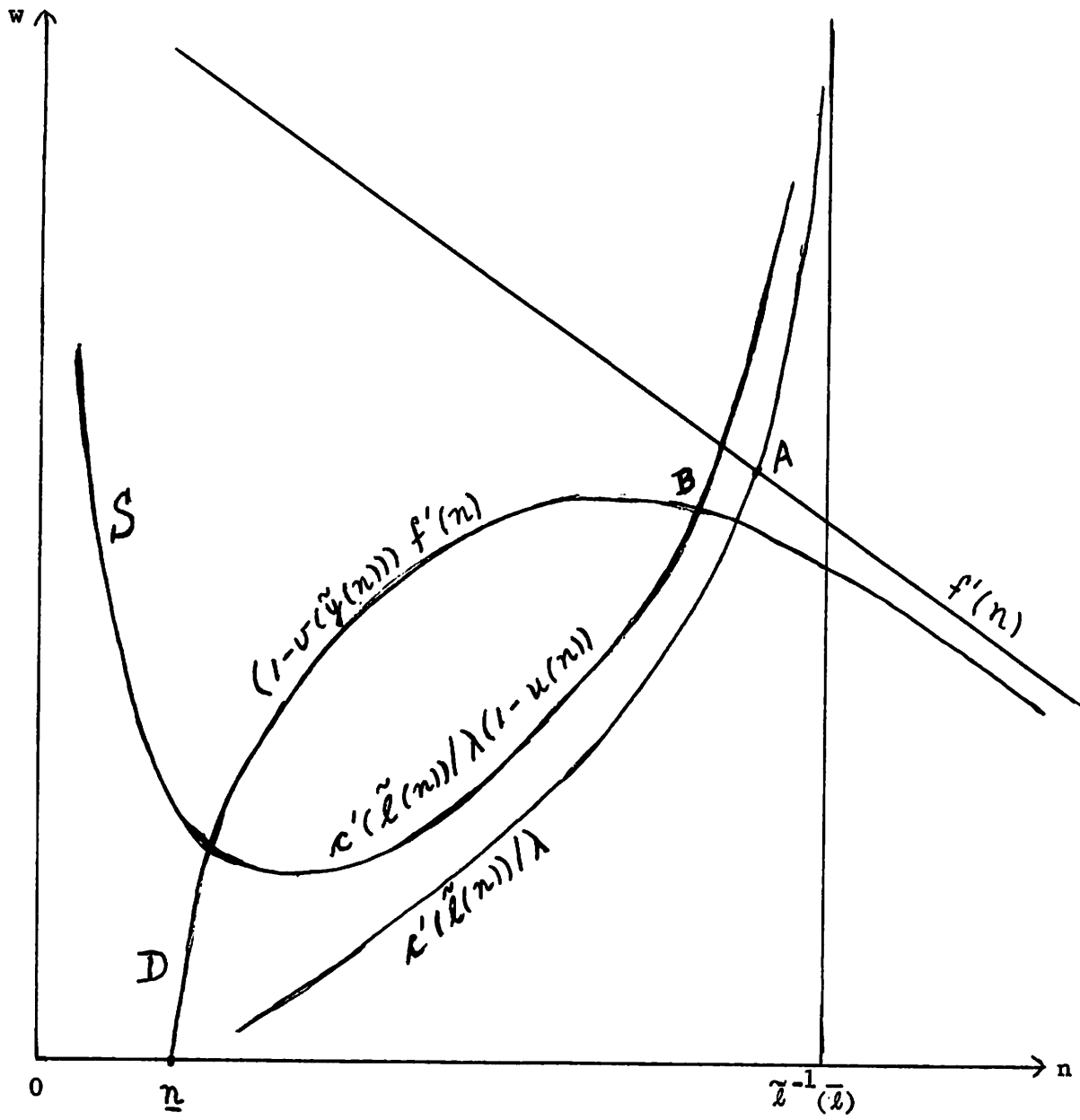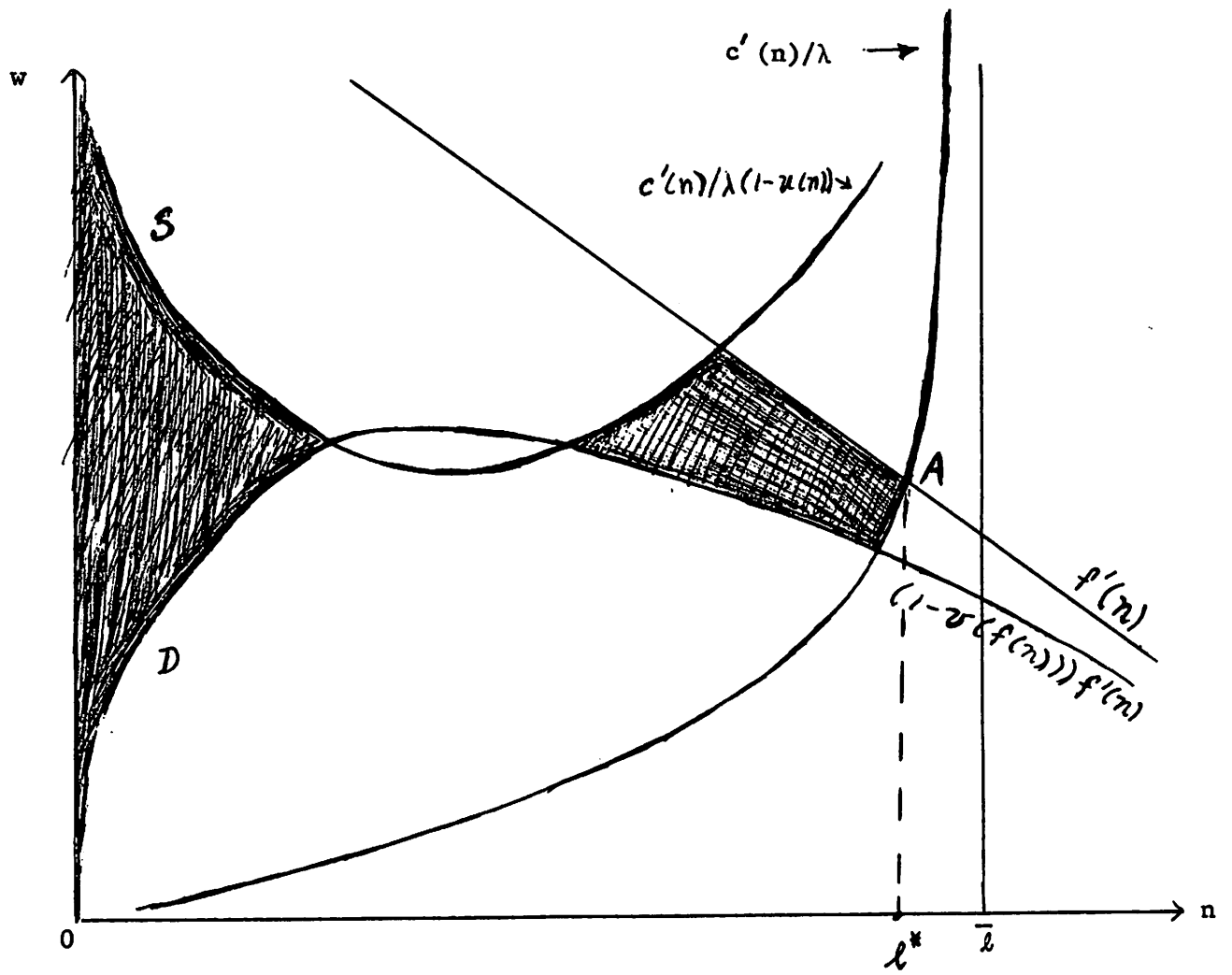
Figure 1

Figure 2

$c'(n)/\lambda$ $\longrightarrow$

$c'(n)/\lambda(1-u(n))$ $\rightarrow$

w

S

D

A

$f'(n)$

$(1-v(f(n)))f'(n)$

0

n

ℓ^*

$\overline{\ell}$

Figure 3